FIMP Dark Matter in Heterotic M-Theory

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Graham Ross:

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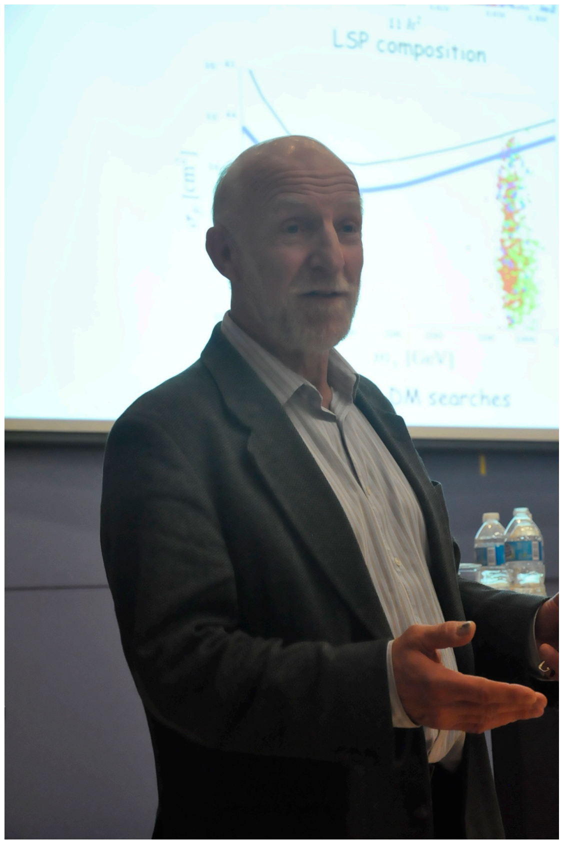
"Status of the SUSY MSSM"

Part A—



A World Renowned Scientist

Part B—

A Warm and Gentle Human Being

R. Deen, B. A. Ovrut and A. Purves, "Supersymmetric Sneutrino-Higgs Inflation,"

Y. Cai, R. Deen, B. A. Ovrut and A. Purves, "Perturbative reheating in Sneutrino-Higgs cosmology,"

Observable Sector Higgs-Sneutrino Inflation:

The observable sector of the B − L MSSM heterotic theory has the exact N = 1 supersymmetric particle content as the MSSM with the addition of three right-handed neutrino chiral multiplets–one for each of the three families. The gauge group is that of the standard model, with an extra gauged $U(1)_{B-L}$ factor. Although the B − L MSSM arises from compactification of heterotic M-theory on a Calabi-Yau threefold with $h^{1,1} = 3$ and, hence, has three Kähler moduli, Higgs-Sneutrino inflation is developed by considering only the "universal" modulus. That is, the moduli of the cosmological theory are the dilaton S and a single Kähler modulus T . The Kähler potential of the observable sector, which arises by restricting the full $h^{1,1} = 3$ theory to the universal modulus, is given by

$$K = -\kappa_4^{-2} ln(S+\bar{S}) - 3\kappa_4^{-2} ln(T+\bar{T} - \mathcal{G}_{\mathcal{I}\bar{\mathcal{J}}} C^{\mathcal{I}}_{(o)} \bar{C}^{\bar{\mathcal{J}}}_{(o)})$$

$C^{\mathcal{I}}_{(o)}$ are the dimensionless scalars of the B − L MSSM observable sector. $\mathcal{G}_{\mathcal{I}\bar{\mathcal{J}}}$ are generically complex structure dependent hermitian matrices. That is, no-scale supergravity.

The Inflaton: The inflaton is a linear combination of the up, neutral Higgs scalar H_u^0 and the left-handed and right-handed sneutrinos $\nu_{L,3}, \nu_{R,3}$. Specifically,

$$\phi_1 = \frac{1}{\sqrt{3}} H_u^0 + \nu_{L,3} + \nu_{R,3}$$

To canonically normalize the kinetic energy, one defines a real scalar field ψ by

$$\phi_1 = \sqrt{3}\ tanh(\frac{\psi}{\sqrt{6}})$$

Potential Energy: The associated potential energy is given by

$$V = V_{\rm soft} + V_F$$

The relevant part of the ψ potential energy is given by the soft supersymmetry breaking potential

$$V_{\rm soft} = 3m^2 tanh^2(\frac{\psi}{\sqrt{6}})$$

where

$$m^2 = \frac{1}{3}(m^2_{H_u^0} + m^2_{\nu_{L,3}} + m^2_{\nu_{R,3}})$$

In order to satisfy the Planck 2015 cosmological data, the soft mass parameter m must take the value

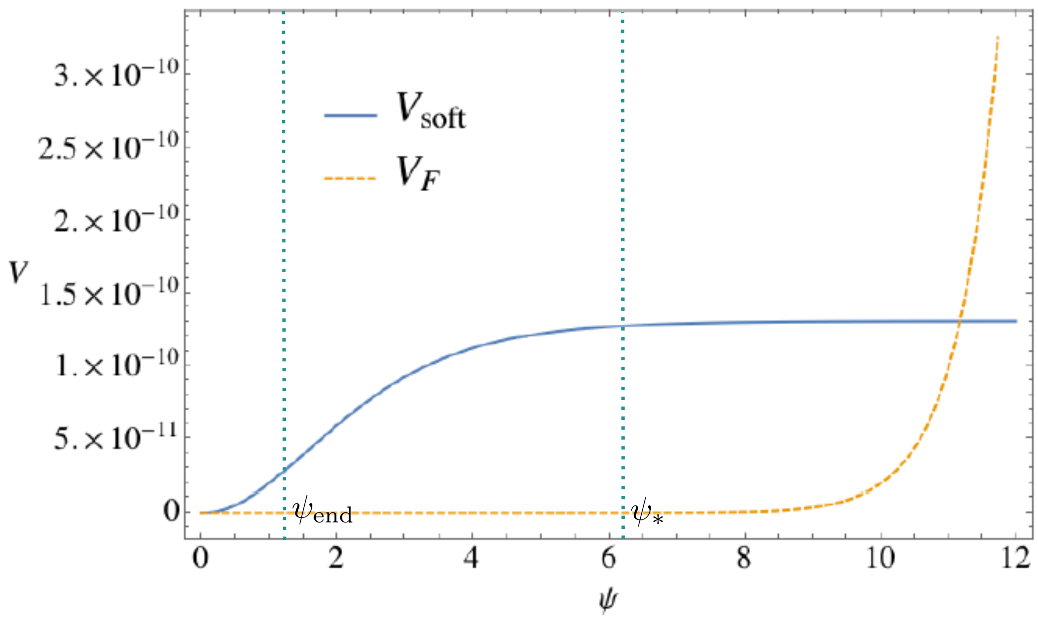
$$m = 1.58 \times 10^{13} {\rm GeV}$$

in dimensionful units.

Since V_{soft} is a soft supersymmetry breaking term in the effective Lagrangian, it follows that supersymmetry in this inflationary B – L MSSM must be broken at a high scale

$$m_{SUSY} \sim \mathcal{O}(10^{13}\text{GeV})$$

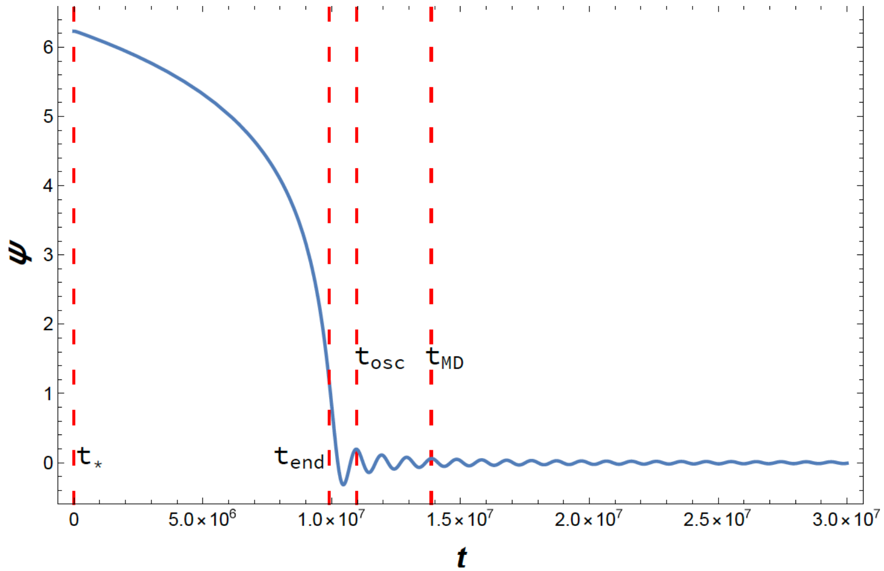
Setting the reduced Planck mass $\kappa_4^{-1} = \frac{M_P}{\sqrt{8\pi}} = 2.435 \times 10^{18}\text{GeV}$ to unity, we find



Inflation begin at ψ_* and ends at ψ_{end}. The value of the potential at the beginning of inflation is

$$V_{\text{soft}}^{1/4} = 7.97 \times 10^{15}\text{GeV}$$

As shown in "Perturbative reheating in Sneutrino-Higgs cosmology,"



at the end of inflation, the inflaton enters an oscillatory phase with

$$V(\psi) \simeq \frac{1}{2} m^2 \psi^2$$

At the beginning of this phase at t_{osc} we find

$$V_{\text{osc}}^{1/4} \simeq 2.332 \times 10^{15} \text{GeV}$$

Reheating Epoch:

The Universe then undergoes a period of reheating (or inflaton domination) which ends when the bath of produced supersymmetric standard model particles comes into thermal equilibrium–the beginning of the matter dominated era. The reheating temperature at the end of the inflaton dominated era was found to be

$$T_{RH} \simeq 1.13 \times 10^{13}\mathrm{GeV}$$

An important quantity during the reheating period is the so-called maximal temperature defined to be

$$T_{max} = \sqrt{V_{osc}^{1/4} T_{RH}}$$

Using the above, we find that

$$T_{max} \simeq 1.623 \times 10^{14}\mathrm{GeV}$$

The inflaton dominated period can be characterized by the temperature interval

$$T_{max} > T > T_{RH}$$

Masses: When N = 1 supersymmetry is spontaneously broken in the hidden sector, chiral matter fermions–both in the observable and hidden sectors–do not acquire soft supersymmetry breaking mass terms. However, in the oscillatory regime, the inflaton field ψ develops a time-dependent VEV given by the square root of

$$\langle\psi^2(t)\rangle \simeq \frac{1}{2\delta}\int_{t-\delta}^{t+\delta}\psi^2(\bar{t})d\bar{t}$$

where $\delta \simeq 2\pi/m$. It follows from the above that in this reheating phase $\psi \propto H_u^0$.

Hence, observable sector chiral fermions $\psi_o^{\mathcal{I}}$ develop a time-dependent non-zero mass given by

$$M_{\psi_o^{\mathcal{I}}} = y_{\psi\psi_{(o)}^{\mathcal{I}}\psi_{(o)}^{\mathcal{I}}}\sqrt{\langle\psi^2\rangle}$$

where $y_{\psi\psi_{(o)}^{\mathcal{I}}\psi_{(o)}^{\mathcal{I}}}$ is the Yukawa coupling parameter in the inflaton-two Weyl fermion interaction. In contrast, the hidden sector chiral fermions cannot receive such contributions and remain massless. Similarly, neither the observable nor hidden sector gauge bosons acquire soft supersymmetry breaking masses. However, as with the observable chiral fermions, the observable sector gauge bosons do have time-dependent masses generated by the inflaton VEV during the reheating period. Just as in the case of fermions, the gauge bosons in the hidden sector do not receive such contributions and, therefore, remain massless.

Finally, the case for the observable and hidden sector gauginos is more complicated. Bottom line: They both get soft supersymmetry breaking masses and the observable sector gauginos also get contributions to their masses generated by the VEV of the inflaton during the reheating period.

Anomalous U(1) Hidden Sector:

The Kähler potential is now extended to

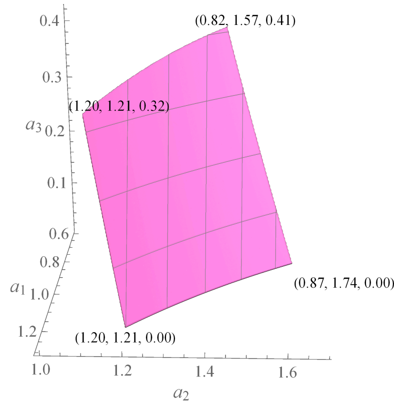
$$K = -\kappa_4^{-2} ln(S + \bar{S}) - 3\kappa_4^{-2} ln(T + \bar{T} - \mathcal{G}_{\mathcal{I}\bar{\mathcal{J}}} C^{\mathcal{I}}_{(o)} \bar{C}^{\bar{\mathcal{J}}}_{(o)} - \mathcal{G}_{L\bar{M}} C^{L}_{(h)} \bar{C}^{\bar{M}}_{(h)})$$

where $C^{L}_{(h)}$ are the dimensionless scalars of the B – L MSSM hidden sector and $\mathcal{G}_{L\bar{M}}$ are generically complex structure dependent hermitian matrices. Here, we consider a specific example where the hidden sector gauge group is the line bundle

$$L = \mathcal{O}_X(2, 1, 3)$$

with an anomalous U(1) structure group. This vacuum will satisfy all phenomenological and mathematical constraints if the three real components a^i, i = 1,2,3 of the Kähler moduli lie within the so-called "magenta" region.

The Magenta Region:



For specificity, we will assume this is the case.

Hidden Sector Spectrum:

For this particular line bundle, with $L \oplus L^{-1}$ embedded in the E_8 so that

$$E_8 \to E_7 \times U(1)$$

the low energy hidden sector spectrum is the one reproduced in the table

	$U(1) \times E_7$	Cohomology	Index χ	
	~~$(0, \underline{\mathbf{133}})$~~	~~$H^*(X, \mathcal{O}_X)$~~	~~0~~	Strong coupling Gluon-Gauginos condensation
Heavy anomalous mass	~~$(0, \underline{\mathbf{1}})$~~	~~$H^*(X, \mathcal{O}_X)$~~	~~0~~	
	~~$(-1, \underline{\mathbf{56}})$~~	~~$H^*(X, L)$~~	~~8~~	Strong coupling nucleation
	~~$(1, \underline{\mathbf{56}})$~~	~~$H^*(X, L^{-1})$~~	~~-8~~	
	~~$(-2, \underline{\mathbf{1}})$~~	~~$H^*(X, L^2)$~~	~~58~~	
Left chiral supermultiplets	$(2, \underline{\mathbf{1}})$	$H^*(X, L^{-2})$	-58	

Masses: Due to the fact that all 58 chiral multiplets have identical -1 charge, the associated superpotential vanishes. Therefore, the supersymmetric masses of all such scalars and fermions are zero. Furthermore, the fermions cannot receive soft supersymmetry breaking masses. $\Longrightarrow$ the fermions cannot be dark matter. This leaves the 58 complex scalar component fields $C^{L}_{(h)}$ as dark matter candidates. These scalars can indeed get soft supersymmetry breaking masses. To lowest order,

$$m^2_{L\bar{M}} = m^2_{SUSY} e^{K_T/3} \mathcal{G}_{L\bar{M}}$$

Since $m_{SUSY} \sim \mathcal{O}(10^{13}\text{GeV})$ then, if $e^{K_T/3}\mathcal{G}_{L\bar{M}} \sim 1$ for some diagonal components, it follows that these particles have soft scalar masses of order 10^{13}GeV. However, if some diagonal components have $e^{K_T/3}\mathcal{G}_{L\bar{M}} \ll 1$, then their masses are $\ll m_{SUSY}$. These light scalars do not contribute strongly to dark matter. Similarly, diagonal components with $e^{K_T/3}\mathcal{G}_{L\bar{M}} \gg 1$ have their contribution to dark matter Boltzmann suppressed. Denote the number of scalars with mass $m_{SUSY} \sim \mathcal{O}(10^{13}\text{GeV})$ by $\hat{N}_{(h)}$. Then

$$0 \leq \hat{N}_{(h)} \leq 58$$

Moduli Portals:

Assume that the S and T moduli have been stabilized at real VEVs ⟨s⟩ and ⟨t⟩ respectively, and that these VEVs satisfy the condition that the anomalous Fayet-Iliopoulos (FI) term vanishes. Then the fluctuations δS and δT are related to the moduli mass eigenstates ξ^1 and ξ^2 by the linear relation

$$\begin{pmatrix} \delta S \\ \delta T \end{pmatrix} = \mathbf{U}^{-1} \begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix}$$

for a known matrix U. Diagonalizing, we find that the modulus eigenstate ξ^1 gets a very heavy anomalous mass of order the unification scale and, therefore, can be integrated out. However, after supersymmetry breaking ξ^2 gets a lighter mass of $\mathcal{O}(m_{SUSY})$. Writing

$$\xi^2 = \eta^2 + i\phi^2$$

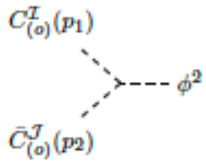
we find that both components are mass eigenstates with

$$m_{\phi^2} \sim m_{\eta^2} \simeq \mathcal{O}(m_{SUSY})$$

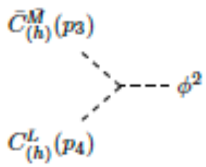
Thus, ϕ^2 and η^2 are the moduli portals.

Moduli Interactions: The interaction vertices for both observable sector and hidden sector fields with the ϕ^2 and η^2 portal moduli are the following. a) Couplings to ϕ^2

- matter scalars to moduli:

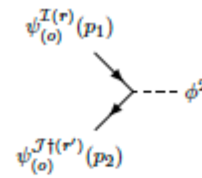


$$= \frac{2}{\sqrt{3}} \frac{1}{(1+\frac{\beta^2}{3})^{1/2}} \delta_{\mathcal{I}\bar{\mathcal{J}}}\, p_{1\mu} p_2^{\mu}\, \kappa_4$$

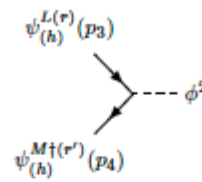


$$= \frac{2}{\sqrt{3}} \frac{1}{(1+\frac{\beta^2}{3})^{1/2}} \delta_{\mathcal{L}\bar{\mathcal{M}}}\, p_{3\mu} p_4^{\mu}\, \kappa_4$$

- matter fermions to moduli:

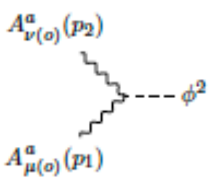


$$= \frac{2}{\sqrt{3}} \frac{1}{(1+\frac{\beta^2}{3})^{1/2}} \delta_{\mathcal{I}\bar{\mathcal{J}}}\, \kappa_4 M_{\Psi^{\mathcal{I}}_{(o)}} \bar{u}^r(p_1) v^{r'}(p_2)$$

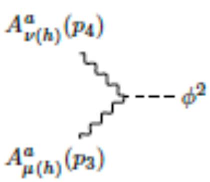


$$= \frac{2}{\sqrt{3}} \frac{1}{(1+\frac{\beta^2}{3})^{1/2}} \delta_{\mathcal{L}\bar{\mathcal{M}}}\, \kappa_4 M_{\Psi^{\mathcal{L}}_{(h)}} \bar{u}^r(p_3) v^{r'}(p_4)$$

- gauge fields to moduli:

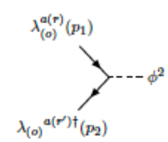


$$= -\eta_{\mu\nu} p_1^{\mu} p_2^{\nu} \langle s \rangle \frac{\kappa_4}{\pi \hat{\alpha}_{\rm GUT}}$$



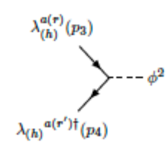
$$= -\eta_{\mu\nu} p_3^{\mu} p_4^{\nu} \langle s \rangle \frac{\kappa_4}{\pi \hat{\alpha}_{\rm GUT}}$$

- gauginos to moduli:



$$= \frac{\kappa_4}{2\pi \hat{\alpha}_{\rm GUT}} \bar{u}^r(p_1)(\not{p}_1 - \not{p}_2) v^{r'}(p_2)$$

$$= \langle s \rangle \frac{\kappa_4 M_{\lambda^a_{(o)}}}{2\pi \hat{\alpha}_{\rm GUT}} \bar{u}(p_1) v(p_2) \ ,$$

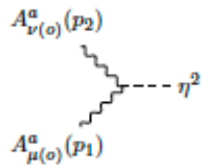


$$= \frac{\kappa_4}{2\pi \hat{\alpha}_{\rm GUT}} \bar{u}^r(p_3)(\not{p}_3 - \not{p}_4) v^{r'}(p_4)$$

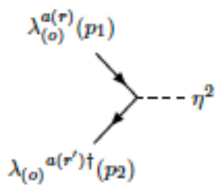
$$= \langle s \rangle \frac{\kappa_4 M_{\lambda^a_{(h)}}}{2\pi \hat{\alpha}_{\rm GUT}} \bar{u}(p_3) v(p_4) \ .$$

b) Couplings to η^2

- observable gauge fields/gauginos to axion:



$$= -\eta_{\mu\nu} p_1^\mu p_2^\nu \langle s \rangle \frac{\kappa_4}{\pi \hat{\alpha}_{\mathrm{GUT}}} ,$$



$$= \frac{\kappa_4}{2\pi \hat{\alpha}_{\mathrm{GUT}}} \bar{u}^r(p_1)(\not{p}_1 - \not{p}_2) v^{r'}(p_2)$$

$$= \langle s \rangle \frac{\kappa_4 M_{\lambda^a_{(o)}}}{2\pi \hat{\alpha}_{\mathrm{GUT}}} \bar{u}(p_1) v(p_2) \ .$$

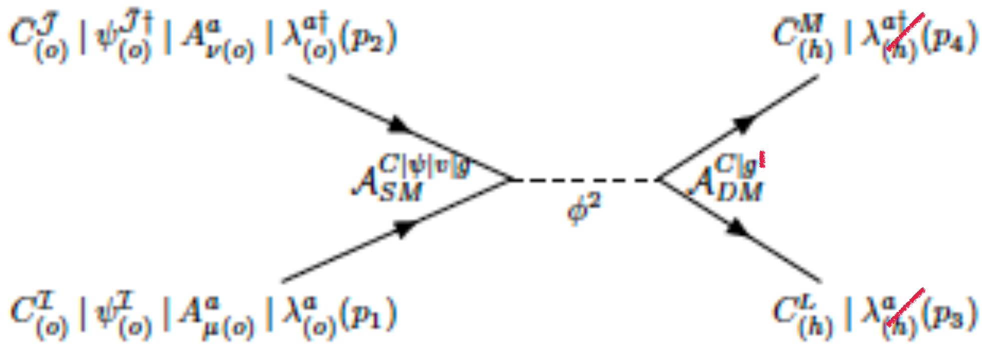
For the specific anomalous hidden sector discussed above, all gauge and gaugino fields are integrated out. $\Longrightarrow$ No hidden sector gauge or gaugino couplings to η^2.

Dark Matter Production Mechanism—Anomalous U(1) Hidden Sector:

As discussed previously, the only possible dark matter candidates are the 58 left-chiral supermultiplets

$$(C^L_{(h)}, \psi^L_{(h)}), \ L = 1, \dots, 58$$

However, all $\psi^L_{(h)}$ fermions remain massless and, therefore, the amplitude to produce them vanishes. On the other hand, all scalars $C^L_{(h)}$ get a non-zero soft-supersymmetry breaking mass of which $0 \leq \hat{N}_{(h)} \leq 58$ have a mass $m_{SUSY} \sim \mathcal{O}(10^{13}\mathrm{GeV})$. These are our dark matter candidates.
Using the above vertices, we find that the dark matter production mechanisms are



where

$$A^C_{SM} = \frac{2}{\sqrt{3}} \frac{1}{(1+\frac{\beta^2}{3})^{1/2}} \delta_{\mathcal{I}\bar{\mathcal{J}}}\, p_{1\mu} p_2^\mu\, \kappa_4 \equiv \delta_{\mathcal{I}\bar{\mathcal{J}}}\, p_{1\mu} p_2^\mu\, \kappa_4 \alpha^C_{\mathrm{SM}}\,,$$

$$A^\psi_{SM} = \frac{2}{\sqrt{3}} \frac{1}{(1+\frac{\beta^2}{3})^{1/2}} \delta_{\mathcal{I}\mathcal{J}}\, \kappa_4 M_{\Psi^{\mathcal{I}}_{(o)}} \bar{u}^r(p_1) v^{r'}(p_2) = \delta_{\mathcal{I}\mathcal{J}}\, \kappa_4 M_{\Psi^{\mathcal{I}}_{(o)}} \bar{u}(p_1) v(p_2) \alpha^\psi_{\mathrm{SM}}\,,$$

$$A^v_{SM} = p_{1\mu} p_2^\mu \langle s \rangle \frac{\kappa_4}{\pi \hat{\alpha}_{\mathrm{GUT}}} \equiv p_{1\mu} p_2^\mu\, \kappa_4 \alpha^v_{\mathrm{SM}}\,,$$

$$A^g_{SM} = \langle s \rangle \frac{\kappa_4 M_{\lambda_{(o)}}}{2\pi \hat{\alpha}_{\mathrm{GUT}}} \bar{u}(p_1) v(p_2) = \kappa_4 M_{\lambda_{(o)}} \bar{u}(p_1) v(p_2) \alpha^g_{\mathrm{SM}}\,.$$

and

$$A^C_{DM} = \frac{2}{\sqrt{3}} \frac{1}{(1+\frac{\beta^2}{3})^{1/2}} \delta_{\mathcal{I}\mathcal{J}}\, p_{3\mu} p_4^\mu\, \kappa_4 \equiv \delta_{\mathcal{I}\mathcal{J}}\, p_{3\mu} p_4^\mu\, \kappa_4 \alpha^C_{\mathrm{DM}}$$

In the above, the "alpha" parameters are

$$\alpha^C_{\rm SM} = \alpha^\Psi_{\rm SM} = \alpha^C_{\rm DM} = \frac{2}{\sqrt{3}}\frac{1}{(1+\frac{\beta^2}{3})^{1/2}} \quad \text{and} \quad \frac{1}{2}\alpha^v_{\rm SM} = \alpha^g_{\rm SM} = \frac{\langle s\rangle}{2\pi\hat{\alpha}_{\rm GUT}}$$

where, for FI=0

$$\beta = \frac{\langle s\rangle}{\langle t\rangle}\frac{3}{\pi\epsilon_s}$$

Results for the B-L MSSM Vacuum:

Assuming the VEVs of S and T lie on the "magenta" region shown above, we find that

$$\beta \in [4.0, 6.8] \Rightarrow \alpha^C_{\rm SM} = \alpha^\Psi_{\rm SM} = \alpha^C_{\rm DM} = \frac{2}{\sqrt{3}}\frac{1}{(1+\frac{\beta^2}{3})^{1/2}} \in [0.28, 0.46]\ .$$

and

$$\langle\alpha_u\rangle = \frac{1}{26.64} \Rightarrow \frac{1}{2}\alpha^v_{\rm SM} = \alpha^g_{\rm SM} = \frac{1}{2\pi\alpha_u} = 4.2$$

Using these results, restricting temperature T to be in the reheating regime

$$T_{max} > T > T_{RH}$$

and using the results for all observable sector masses in that regime, we find that the dominant production mode is

$$AA \longrightarrow CC$$

with

$$|\mathcal{M}_{AA\to CC}|^2 \simeq \alpha_{\rm SM}^{v}\alpha_{\rm DM}^{C}\,\kappa_4^4 s^2$$

and $\sqrt{s} \sim T$.

Dark Matter Relic Density:

D. Chowdhury, E. Dudas, M. Dutra and Y. Mambrini, "Moduli Portal Dark Matter,"
M. Dutra, "The moduli portal to dark matter particles,"

After inflation ends, there are two cosmological regimes

$$\text{ID}: \quad T_{max} > T > T_{RH}\ ,$$
$$\text{RD}: \quad T_{RH} > T > T_0\ ,$$

where

$$T_{max} = 1.623 \times 10^{14}\text{GeV} \quad \text{and} \quad T_{RH} = 1.13 \times 10^{13}\text{GeV}$$

and T_0 is the present temperature of the Universe. Note that

$$m_{DM} \simeq m_{\phi^2} \simeq m_{\eta^2} \sim \mathcal{O}(10^{13}\ \text{GeV})$$

The formula for the dark matter relic density is given by $\Omega h^2 = m_{DM} n_{DM}/\rho$

$$\Omega h^2 \simeq \Omega h^2_{\rm ID} + \Omega h^2_{\rm RD}$$
$$\approx 4 \times 10^{24}\, m_{\rm DM} \left(1.07 \times T_{\rm RH}^7 \int_{T_{\rm RH}}^{T_{\rm max}} dT \frac{R(T)}{T^{13}} + \int_{T_0}^{T_{\rm RH}} dT \frac{R(T)}{T^6}\right)$$

R_T is the dark matter production rate at temperature T. When the temperature of the Universe drops below the dark matter particle mass, the production rate becomes exponentially suppressed by the Botzmann factor

$$R(T) \sim e^{-m_{\rm DM}/T}$$

Since $m_{DM} \sim \mathcal{O}(10^{13}{\rm GeV}) \sim T_{RH}$, it follows that for T in the RD regime $R(T) \to 0$. Therefore the expression for the relic density simplifies to

$$\Omega h^2 \simeq 4 \times 10^{24}\, m_{\rm DM} \left(1.07 \times T_{\rm RH}^7 \int_{T_{\rm RH}}^{T_{\rm max}} dT \frac{R(T)}{T^{13}} \right)$$

and depends only on the reheating regime $T_{max} > T > T_{RH}$. Using the result $|\mathcal{M}_{AA\to CC}|^2 \simeq \alpha_{\rm SM}^v \alpha_{\rm DM}^C\, \kappa_4^4 s^2$ one can derive $R(T)$. The result is

$$R(T) = N_{(h)} \frac{\pi^3}{21,600}\, [\alpha_{\rm SM}^v]^2 [\alpha_{\rm DM}^C]^2\, \kappa_4^4 T^8$$

Inserting this into the above, we find that

$$\Omega h^2 \approx (8.7 \times 10^{21}) N_{(h)}\, \frac{\pi^3}{21,600}\, [\alpha_{\rm SM}^v]^2 [\alpha_{\rm DM}^C]^2\, m_{DM}\, \kappa_4^4 T_{RH}^3$$

This will produce the measured relic density $\Omega h^2 = 0.12$ for

$$N_{(h)}[\alpha_{\rm SM}^{v}]^2[\alpha_{\rm DM}^{C}]^2\, m_{DM} \approx 3.4 \times 10^{14}{\rm GeV}\ .$$

This can indeed be achieved for the α_{SM}^{v} and α_{DM}^{C} in the “magenta” region ranges given above and for

$$\hat{N}_{(h)} \sim \mathcal{O}(1-14)$$

Small changes in the supersymmetry breaking scale, the magenta region and so on allows one to satisfy the above constraint for any number of hidden sector scalars—including

$$\hat{N}_{(h)} = 58$$

These calculations have been extended to hidden sectors with non-anomalous structure groups. Result: One can still obtain the observed dark matter relic density over a wide range of input parameters.

Conclusion:

Producing hidden sector scalar dark matter using a “freeze-in” mechanism in heterotic M-theory appears to be a natural explanation for the observed dark mater relic abundance!