Torsional String Newton-Cartan Geometry for Non-Relativistic Strings

Workshop on New Developments in Quantum Gravity and String Theory

Corfu, Sept. 15, 2021

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based on work:

2107.006542 (Bidussi,Harmark,Hartong,,NO,Oling)

2011.02539 (JHEP) (Harmark,Hartong,,NO,Oling)

1907.01663 (JHEP) (Harmark,Hartong,Menculini,NO,Oling)

& 1810.05560 (JHEP) (Harmark,Hartong,Menculini,NO,Yan)

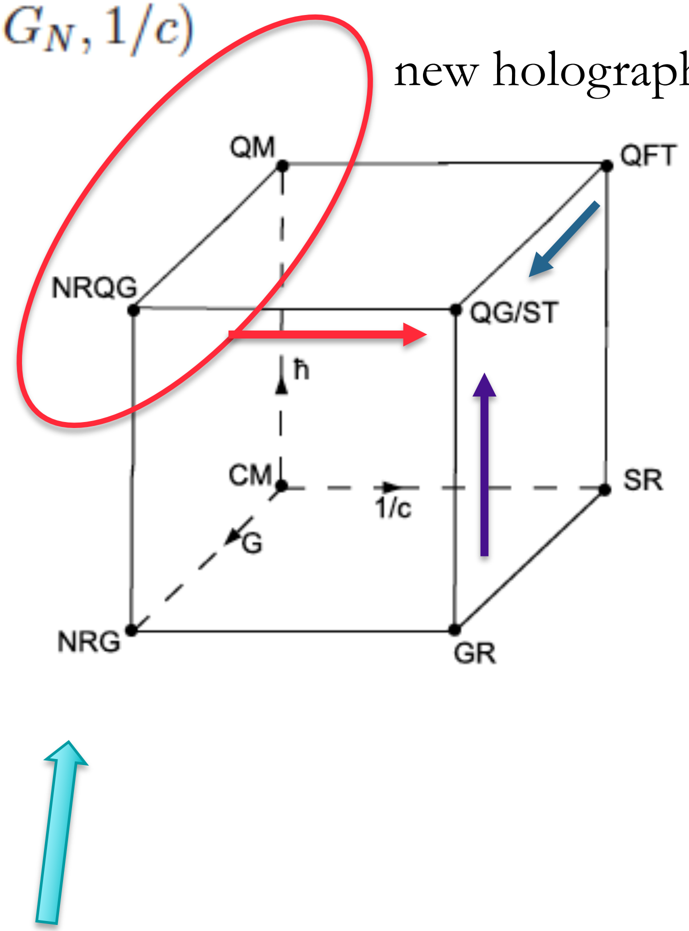
1705.03535 (PRD) (Harmark,Hartong,NO),

& work to appear with Bidussi,Harmark,Hartong,Oling

Non-relativistic physics and cube of physical theories

$(\hbar, G_N, 1/c)$

new holographic dualities ?



a third route towards (relativistic) quantum gravity

how does this fit with string theory/holography ?

already (classical) non-relativistic gravity (NRG) is more than just Newtonian gravity

Non-Lorentzian geometries

recent progress in understanding non-relativistic corners of:
gravity, quantum field theory and string theory:

→ builds on improved understanding of non-Lorentzian geometries
= spacetimes with local symmetries other than Lorentz

NL geometries appear in:

- bdry geometries in non-AdS holography (e.g. Lifshitz flat space)
- covariant formulations of PN approximation in GR
- covariant formulations of non-Lorentzian fluids and CMT systems
- Horava–Lifshitz gravity, non-relativistic versions of CS, JT
- double field theory
- near-BPS limits of string theory on AdS5 $\times$ S5
- **non-relativistic limits of String Theory**

Why non-relativistic (NR) string theory ?

- perhaps simpler (UV complete) theory
- non-relativistic gravity via beta functions
- limit of AdS/CFT and novel sigma models
- certain NR strings contained in double field theory
- rich limit of string theory

→ what is the landscape of NR string theories ?

NR strings

NR strings on flat spacetime = Gomis-Ooguri string

Gomis,Ooguri(2000); Danielsson et al.(2000);

→ Newton-Cartan geometries when spacetime is curved

- NR strings on torsional Newton-Cartan (TNC) from null-reduction & further limits thereof (related to AdS/CFT)

  Harmark,Hartong,NO(2017); Harmark,Hartong,Menculini,NO,Yan(2018); Gallegos,Gursoy,Zinnato(2019), Harmark,Hartong,Menculini,NO,Oling(2019); Kluson(2018/19), …. Bidussi, Harmark,Hartong,NO,Oling(2021);

- string Newton-Cartan generalization (SNC)

  Andriga et al (2012), Bergshoeff,Gomis,Yan(2018); Gomis,Oh,Yan(2019),Bergshoeff,Gomis,Rosseel,Simsek,Yan(2019); Kluson (2018/19), …

also

- tensionless strings e.g. Bagchi,Gopakumar(2009) Bagchi,Banerjee,Parekh(2019)
- Galilean strings Battle,Gomis,Not(2016))
- relation to double field theory Morand,Park(2017);Berman,Blair,Otsuki(2019);Blair(2019)

issues with previous formulations:

- string action exhibits Stueckelberg symmetry (overparametrization of fields)

- Z_A symmetry: puts constraints on the torsion of the spacetime: not seen when doing null reduction

- closure of algebra requires an extra symmetry generator with no corrsponding field in target space geometry field

- This talk: revisit formulation of non-relativistic (NR) string theory & target space geometry

Main result

→ find formulation in which geometry contains 2-form field that couples to tension curent and transforming under string Galilei boosts

i.e. 2-form is intrinsic part of the geometry
(in parallel with NR particle and its coupling to Newton-Cartan geometry)

- follows from both null reduction & $c \rightarrow$ infinity limit

- geometry arises from gauging novel algebra:
  F-string Galilei algebra
  = Inonu-Wigner contraction of
  Poincare algebra + syms of Kalb-Ramond field

Outline

- intro to Newton-Cartan geometry for particles:
  c → infinity limit and null reduction

- Torsional string Newton-Cartan geometry (TSNC)
  from c → infinity limit
  - preliminaries: KR B-field from string Poincare
  - NR string action with TSNC targe space
  - symmetries of TSNC geometry

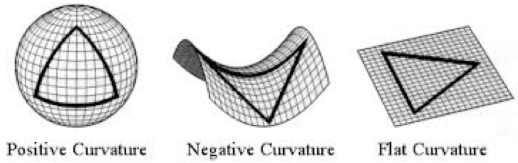
- bigger picture: c → infinity vs 1/c-expansions

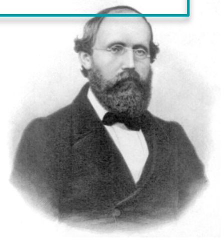
- outlook

Space-Time symmetries and Geometry

local symmetries of space and time ←→ geometry of space and time

Einstein: Lorentz ←→ (pseudo-)Riemannian geometry





Cartan: Galilean ←→ Newton-Cartan geometry

[Eisenhart,Trautman,Dautcourt,Kuenzle,Duval,Burdet,Perrin,Gibbons,Horvathy,Nicolai,Julia…] ..

- geometrize Poisson equation of Newtonian gravity
  falling observers see Galilean laws of physics

TNC geometry from null reduction

Lorentzian metric with null isometry

$$ds^2 = g_{MN} dX^M dX^N = 2\tau_\mu dx^\mu (du - m_\nu dx^\nu) + h_{\mu\nu} dx^\mu dx^\nu \,,$$

signature $(0, 1, \ldots, 1)$

torsional Newton–Cartan (TNC) geometry: $\tau_\mu \;,\quad h_{\mu\nu} \;,\quad m_\mu \,,$

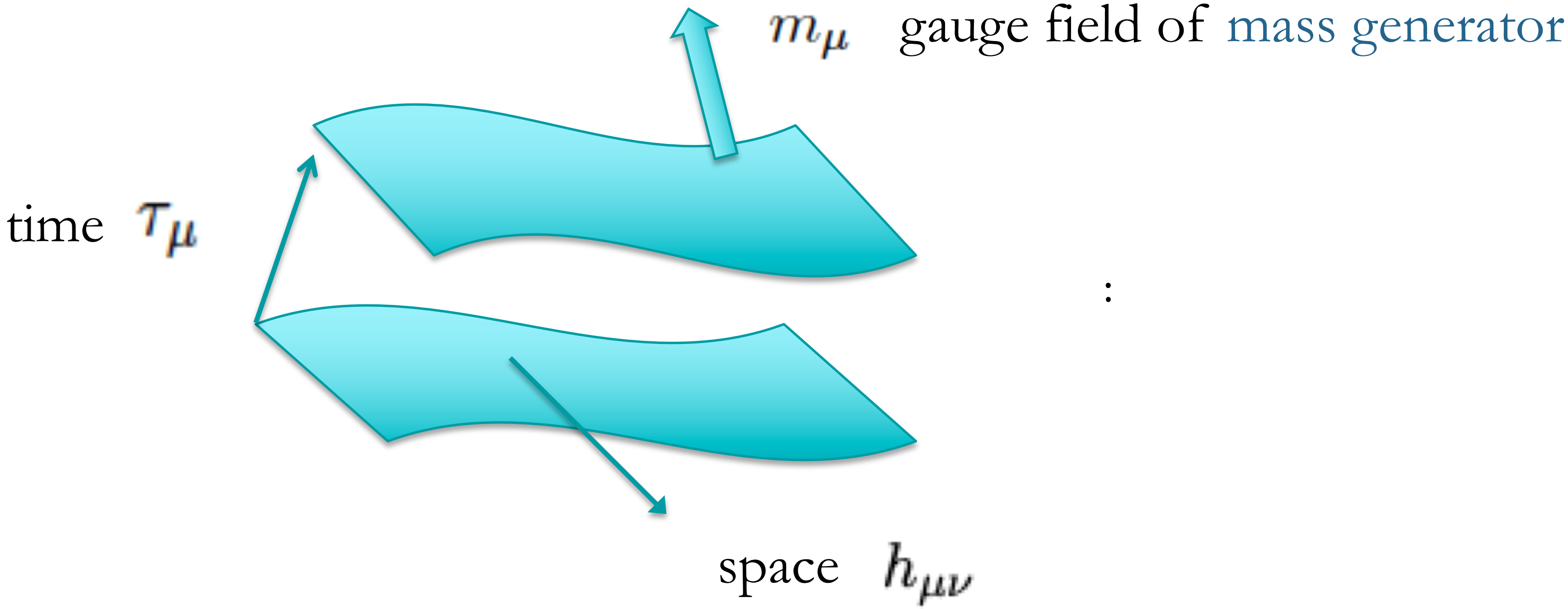
local syms:

$$\delta\tau_\mu = \mathcal{L}_\xi \tau_\mu \,, \quad \delta h_{\mu\nu} = \mathcal{L}_\xi h_{\mu\nu} + \lambda_\mu \tau_\nu + \lambda_\nu \tau_\mu \,,$$
$$\delta m_\mu = \mathcal{L}_\xi m_\mu + \lambda_\mu + \partial_\mu \sigma \,,$$

$v^\mu \lambda_\mu = 0.$ Galilean (Milne) boosts

σ U(1) (mass) parameter

torsional Newton-Cartan geometry



NC = no torsion → $\tau_\mu = \partial_\mu t$ absolute time

TTNC = twistless torsion → $\tau_\mu = \mathrm{HSO}$ preferred foliation equal time slices

TNC no condition on τ_μ

Warmup: Non-Relativistic particle from null reduction

null-reduction of relativistic particle $S = \int \frac{1}{2e} g_{MN} \dot{X}^M \dot{X}^N d\lambda =$

→ reduce on target space with null Killing vector :

conserved momentum in null direction: $p_u = m$

$$S = \frac{m}{2} \int \frac{h_{\mu\nu} \dot{X}^\mu \dot{X}^\nu}{\tau_\rho \dot{X}^\rho} d\lambda - m \int m_\mu \dot{X}^\mu d\lambda$$

[Kuchar], [Bergshoeff et al]

kinetic term

potential term: coupling to m_μ

$m_0 \sim$ Newtonian potential

$$T^\mu = m \int d\tau \, \partial_\tau X^\mu \delta(x - X(\tau))$$ mass current

- action has TNC local target space symmetries

Other properties

- geodesic equation on flat NC space with:

$$m_t = \Phi_{\text{Newt}}$$ → Newton's law

- TNC geometry can also be obtained by gauging Bargmann algebra

Andringa,Bergshoeff,Gomis,de Roo (2021)

$$[G_a, P_b] = -\delta_{ab} N \quad , \quad [G_a, H] = -P_a$$ & rotations

↗ mass generator

(just as pseudo-Riemmanian geometry follows from gauging Poincare)

NR particle from limit of extremal particle

action of charged relativistic particle:

$$S = -mc\int\sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}d\lambda + q\int A_\mu\dot{x}^\mu d\lambda$$

- time-space split in metric: $g_{\mu\nu} = -c^2T_\mu T_\nu + h_{\mu\nu}$

expand for large c:

$$S = -mc^2\int\left[T_\mu - \frac{q}{mc^2}A_\mu\right]\dot{x}^\mu d\lambda + \frac{m}{2}\int\frac{h_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}{T_\rho\dot{x}^\rho}d\lambda + \mathcal{O}(c^{-2})$$

extremal particle

$$q = mc^2.$$

divergent term. cancels with:

$$T_\mu = \tau_\mu + \frac{1}{2c^2}m_\mu\,,$$

$$A_\mu = \tau_\mu - \frac{1}{2c^2}m_\mu\,,$$

algebra level:
IW contraction

$$H = cP_0 + Q\quad,\qquad N = \frac{1}{2c^2}(cP_0 - Q)$$

energy mass

Mimic the procedure for strings

fundamental strings are extremally charged under B-field

what is the analogue of Poincare x U(1) (extremal charged particle) for strings ?

→ understand the symmetry of the Kalb-Ramond B-field

Kalb-Ramond B-field from string Poincare

metric and B-field symmetries

$$\bar{\delta} g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu} \quad , \qquad \bar{\delta} B_{\mu\nu} = \mathcal{L}_\xi B_{\mu\nu} + 2\partial_{[\mu}\lambda_{\nu]}.$$

can be obtained from string extension of Poincare:
(with extra set of translational generators (cf. doubled field theory)

$$[M_{\underline{ab}}, M_{\underline{cd}}] = \eta_{\underline{ac}} M_{\underline{bd}} - \eta_{\underline{bc}} M_{\underline{ad}} + \eta_{\underline{bd}} M_{\underline{ac}} - \eta_{\underline{ad}} M_{\underline{bc}},$$
$$[M_{\underline{ab}}, P_{\underline{c}}] = \eta_{\underline{ac}} P_{\underline{b}} - \eta_{\underline{bc}} P_{\underline{a}},$$
$$[M_{\underline{ab}}, Q_{\underline{c}}] = \eta_{\underline{ac}} Q_{\underline{b}} - \eta_{\underline{bc}} Q_{\underline{a}},$$

Lie algebra valued connection:

$$\mathcal{A}_\mu = e_\mu^{\underline{a}} P_{\underline{a}} + \frac{1}{2}\omega_\mu{}^{\underline{ab}} M_{\underline{ab}} + \pi_\mu^{\underline{a}} Q_{\underline{a}};$$

$$g_{\mu\nu} = \eta_{\underline{ab}}\, e_\mu^{\underline{a}} e_\nu^{\underline{b}}. \qquad B_{\mu\nu} = \eta_{\underline{ab}}\, e_{[\mu}^{\underline{a}} \pi_{\nu]}^{\underline{b}}.$$

(see e.g. Ne'eman,Regge/D'Auria,Fre)

NR string action from c → infinity limit

$$S = S_{\rm NG} + S_{\rm WZ},$$

$$S_{\rm NG} = -cT_{\rm F}\int d^2\sigma\sqrt{-\det g_{\alpha\beta}}, \qquad S_{\rm WZ} = -c\frac{T_{\rm F}}{2}\int d^2\sigma B_{\alpha\beta}\epsilon^{\alpha\beta}.$$

use vielbein decompostion of the NSNS target space:

$$g_{\mu\nu} = \eta_{\underline{ab}}e_{\mu}^{\underline{a}}e_{\nu}^{\underline{b}} \quad , \qquad B_{\mu\nu} = \eta_{\underline{ab}}e_{[\mu}^{\underline{a}}\pi_{\nu]}^{\underline{b}},$$

split tangent space into
A=0,1: longitudinal
a=2,..d-1: transverse

$$e_{\mu}^{\underline{a}} = (cE_{\mu}^{A}, e_{\mu}^{a}) \quad , \qquad \pi_{\mu}^{\underline{a}} = (c\Pi_{\mu}^{A}, \pi_{\mu}^{a}),$$

- reparametrize longitudinal vielbeins:

$$E_{\mu}^{A} = \tau_{\mu}^{A} + \frac{1}{2c^2}\pi_{\mu}^{B}\epsilon_{B}{}^{A},$$

$$\Pi_{\mu}^{A} = \epsilon^{A}{}_{B}\tau_{\mu}^{B} + \frac{1}{2c^2}\pi_{\mu}^{A},$$

divergent term in action cancels since F-string is extremal

NRST action on TSNC target space

after limit:

kinetic potential

$$S_{\rm NR} = -\frac{T}{2}\int d^2\sigma \left[\sqrt{-\tau}\,\eta^{AB}\tau_A^\alpha\tau_B^\beta h_{\alpha\beta} + \epsilon^{\alpha\beta}m_{\alpha\beta}\right],$$

$$h_{\mu\nu} = e^a_\alpha e^b_\beta \delta_{ab} \qquad m_{\mu\nu} = \eta_{AB}\tau^A_{[\mu}\pi^B_{\nu]} + \delta_{ab}e^a_{[\mu}\pi^b_{\nu]}.$$

torsional string Newton–Cartan geometry : $\tau_\mu^A, \quad h_{\mu\nu}, \quad m_{\mu\nu}.$

$m_{\mu\nu}.$ couples to worldsheet tension current $J_{\rm T}^{\mu\nu} = T\int d^2\sigma\,\epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \delta(x - X(\sigma^\alpha)),$

conserved charges: $M^\mu = \int d\sigma J^{0\mu}.$

gravitational mass: $M^v = 2\pi T R.$ (for compact longitudinal direction v)

m_{0v} = gravitational potential

- weak equivalence principle for NR strsing

Symmetries of the action

- gauge: $$\bar{\delta} m_{\mu\nu} = 2\partial_{[\mu}\lambda_{\nu]}.$$

- string Galilean boosts:

$$\bar{\delta} h_{\mu\nu} = -\lambda_{Ab}\left(\tau_\mu{}^A e_\nu^b + \tau_\nu{}^A e_\mu^b\right) \quad , \quad \bar{\delta} m_{\mu\nu} = -2\epsilon_{AB}\lambda^B{}_c\tau_{[\mu}^A e_{\nu]}^c.$$

→ string analogue of the symmetries of
NR particle coupling to Newton-Cartan

Question: what is underlying symmetry algebra ?

F-string Galilean (FSG) algebra

- decompose string $\underline{a} = (A, a)$

Poincare algebra: longitudinal, transverse

$$P_A, \quad Q_A, \quad P_a, \quad Q_a, \quad J_{AB} = \epsilon_{AB} J = M_{AB}, \quad J_{ab} = M_{ab}, \quad c\,G_{Ab} = M_{Ab}.$$

$$H_A = c(P_A + Q_B \epsilon^B{}_A) \quad , \quad N_A = \frac{1}{2c}(\epsilon_A{}^B P_B + Q_A)$$ (basis transformation)

after IW contraction (c → infinity):

$$[G_{Ab}, H_C] = \eta_{AC} P_b + \epsilon_{AC} Q_b,$$
$$[G_{Ab}, P_c] = -\delta_{bc} \epsilon_A{}^B N_B,$$
$$[G_{Ab}, Q_c] = -\delta_{bc} N_A,$$

& futher commutators involving SO(1,1) x SO(d-2) rotations

- symmetry trafos follow from FSG-valued connection:

$$\mathcal{A}_\mu = \tau_\mu^A H_A + e_\mu^a P_a + \omega_\mu J + \frac{1}{2}\omega_\mu{}^{ab} J_{ab} + \omega_\mu{}^{Ab} G_{Ab} + \pi_\mu^A Z_A + \pi_\mu^a Q_a,$$

Non-relativistic strings from null reduction

- start from Polyakov action (including NSNS) and reduce along null isometry

- implement conservation of string momentum along null isometry using Lagrange multipliers

- go to dual formulation that exchanges the (fixed) momentum along null direction for fixed winding of string along compact dual direction

→ action of non-relativistic strings moving in torsional string Newton-Cartan target space

& FSG symmetries can also be derived

Limit vs. 1/c expansion

- limit geometry: type I (cancellation of divergent term)

- geometry from expansion: type II
(each term in the action generates more gauge fields)

type II studied for:

- NR particle (and coupling to non-relativistic gravity from expanding GR) van den Bleeken (2018), Hansen,Hartong,NO (2019, 2020)
- NR string Hartong,Have (2021)

spectrum:
(compact long. direction)
center of mass velocity << c

$$E = \frac{c^2 w R_{\text{eff}}}{\alpha'_{\text{eff}}} + \frac{N_{(0)} + \tilde{N}_{(0)}}{w R_{\text{eff}}} + \frac{\alpha'_{\text{eff}}}{2 w R_{\text{eff}}} p^2_{(0)} + \mathcal{O}(c^{-2})$$

	origin	target space geometry of probe action	torsion/foliation constraints	important special cases
NR particle (type I)	$c \to \infty$ limit of extremal particle/null reduction of massless particle	(type I) TNC geometry: τ_μ, $h_{\mu\nu}$, m_μ	none	NC geometry: $d\tau = 0$
NR string (type I)	$c \to \infty$ limit of string with critical electric field/null reduction of relativistic string	(type I) TSNC geometry: τ_μ^A, $h_{\mu\nu}$, $m_{\mu\nu}$	none	SNC geometry: $d\tau^A = \omega \varepsilon^A{}_B \wedge \tau^B$
NR particle (type II) plus relativistic corrections	$1/c^2$ expansion of uncharged massive relativistic particle	(type II) TNC geometry: LO: τ_μ NLO: τ_μ, $h_{\mu\nu}$, m_μ NNLO: τ_μ, $h_{\mu\nu}$, m_μ, $\Phi_{\mu\nu}$, B_μ	dynamically determined by both the EOM of the target space fields and the embedding scalars	NC geometry: $d\tau = 0$
NR string (type II) plus relativistic corrections	$1/c^2$ expansion of uncharged relativistic string	(type II) TSNC geometry: LO: τ_μ^A NLO: τ_μ^A, $h_{\mu\nu}$, m_μ^A	dynamically determined by both the EOM of the target space fields and the embedding scalars	$d\tau^A = \alpha^A{}_B \wedge \tau^B$ with $\alpha^A{}_A = 0$

Table 1. Overview of the different approaches to NR particles and strings.

Outlook

obtain beta functions and effective spacetime actions in new TSNC formulation

recent results on beta-functions:

- SNC string

Gomis,Oh,Yan(2019)
Bergshoeff,Gomis,Rosseel,Simsek,Yan(2019)
Yan,Yu(2019)
Bergshoeff et al (2021), Yan (2021)

$$D_{[M}\tau_{N]}{}^{A} = 0\,.$$

- TNC string

Gallegos,Gursoy,Zinnato(2019)
Gallegos,Gursoy,Verma,Zinnato(2020

→ describe the dynamics of (versions of) non-relativistic gravity

examine torsion conditions using new formulation

Outlook

- include dilaton field in analysis

- Hamiltonian analysis (including for non-relativistic world-sheet models)

  Kluson (2021), Bidussi,Harmark,Hartong,NO,Oling (to appear)

- open strings and branes

  - non-relativistic open string sector and DBI actions

  Gomis,Yan,Yu (2020)

  - connection to NR D/M-branes

  Kluson/Blair,Gallegos,Zinnato (2021)

  - generalize procedure to non-relativistic limit of extremal p-branes TSNC analogue for p-branes (incl. D/M)

  Bidussi,Harmark,Hartong,NO,Oling (in progress)

  -

- supersymmetric generalization of stringy Poincare (include RR fields) NR limit & relations to exceptional geometry

  - non-perturbative dualities in NR string theory

The end

Comparison to SNC algebra

string Galilei algebra

$$
\begin{aligned}
[J_{ab}, J_{cd}] &= \delta_{ac} J_{bd} - \delta_{bc} J_{ad} + \delta_{bd} J_{ac} - \delta_{ad} J_{bc}\,, \\
[J, G_{Ab}] &= \epsilon^{C}{}_{A} G_{Cb}\,, \\
[J_{ab}, G_{Cd}] &= \delta_{ad} G_{Cb} - \delta_{bd} G_{Ca}\,, \\
[J, H_A] &= \epsilon^{B}{}_{A} H_B\,, \\
[G_{Ab}, H_C] &= \eta_{AC} P_b\,, \\
[J_{ab}, P_c] &= \delta_{ac} P_b - \delta_{bc} P_a\,.
\end{aligned}
$$

SNC by extending with:

$$
\begin{aligned}
[G_{Ab}, G_{Cd}] &= \delta_{bd} Z_{AC}\,, \\
[J, Z_A] &= \epsilon^{B}{}_{A} Z_B\,, \\
[G_{Ab}, P_c] &= -\delta_{bc} Z_A\,, \\
[Z_{AB}, H_C] &= \eta_{AC} Z_B - \eta_{BC} Z_A\,.
\end{aligned}
$$

- string Galilei boosts do not commute
- extra generator Z_AB to close algebra

Comparison to NRST action on SNC

form of action:

$$S = -\frac{T}{2}\int d^2\sigma\left[\sqrt{-\tau}\,\tau^{\alpha\beta}\bar{h}_{\mu\nu} + \epsilon^{\alpha\beta}B_{\mu\nu}\right]\partial_\alpha X^\mu \partial_\beta X^\nu\,,$$

Bergshoeff et al

$$\bar{h}_{\mu\nu} = h_{\mu\nu} + \eta_{AB}\left(\tau_\mu^A m_\nu^B + \tau_\nu^A m_\mu^B\right),$$

Stueckelberg symmetry:

$$\bar{h}_{\mu\nu} \to \bar{h}_{\mu\nu} + 2C^A_{(\mu}\tau^B_{\nu)}\eta_{AB} \quad , \quad B_{\mu\nu} \to B_{\mu\nu} - 2C^A_{[\mu}\tau^B_{\nu]}\epsilon_{AB},$$

allows to go to a gauge in which $m_\mu^A = 0.$

→ actions agree

Non-relativistic world-sheet theories

can take a further world-sheet NR limit

-→ new class of sigma models that are
also non-relativistic on worldsheet:

exhibits 2D GCA

$$[L_n\,, L_m] = (n-m)L_{n+m}\,, \qquad [L_n\,, M_m] = (n-m)M_{n+m}\,.$$

- NR WS theories directly related to near-BPS limits of AdS/CFT (spin-matrix theory (SMT))

simplest example: LL model appearing from continuum limit of Heisenberg spin chains

Stringy side of SMT gives NR sigma models

- using AdS/CFT dictionary SMT limit can be formulated as limit of type IIB string theory on AdS5xS5

- turns out to correspond to sigma-model that precisely realizes our scaling limit: i.e. contained in the class of non-relativistic world-sheet string theory !

→ the LL model (and generalizations for other near-BPS sectors) is an example of our novel class of non-relativistic ws. string theories & exhibits the GCA infinite-dim symmetry

- the extra target space dimension = position along the spin chain (zero momentum because of cyclicity of trace)

- strongly suggests: bulk description of SMT is a type of non-relativistic gravity

new class of flat-fluxed backgrounds obtained recently: analogue of flat Minkowski space using Penrose type limits

The end

SU(2) case and LL model as non-rel 2D CFT

start with AdS5xS5 in appropriate coordinates

consider BPS bound $E \geq Q = J = J_1 + J_2.$ $\quad g_s \to 0 \quad , \quad \frac{E-Q}{g_s} = \text{fixed}$

$E = i\partial_t$ and $J = -i\partial_\gamma.$ $\quad t = x^0 - \frac{1}{2}u \quad , \quad \gamma = x^0 + \frac{1}{2}u$

after limit $c = \frac{1}{\sqrt{4\pi g_s N}}$ to infinity

$$\tau = d\tilde{x}^0 \quad , \quad m = -\frac{\cos\theta}{2}d\phi \quad , \quad h_{\mu\nu}dx^\mu dx^\nu = \frac{1}{4}(d\theta^2 + \sin^2\theta d\phi^2)$$

U(1)xGal NR background R x S2 and non-zero "magnetic" flux

gives LL model

$$S = \frac{Q}{4\pi}\int d^2\sigma \left[\dot{\varphi}\cos\theta - \frac{1}{4}\left[(\theta')^2 + \sin^2\theta\left(\varphi'\right)^2\right]\right]$$

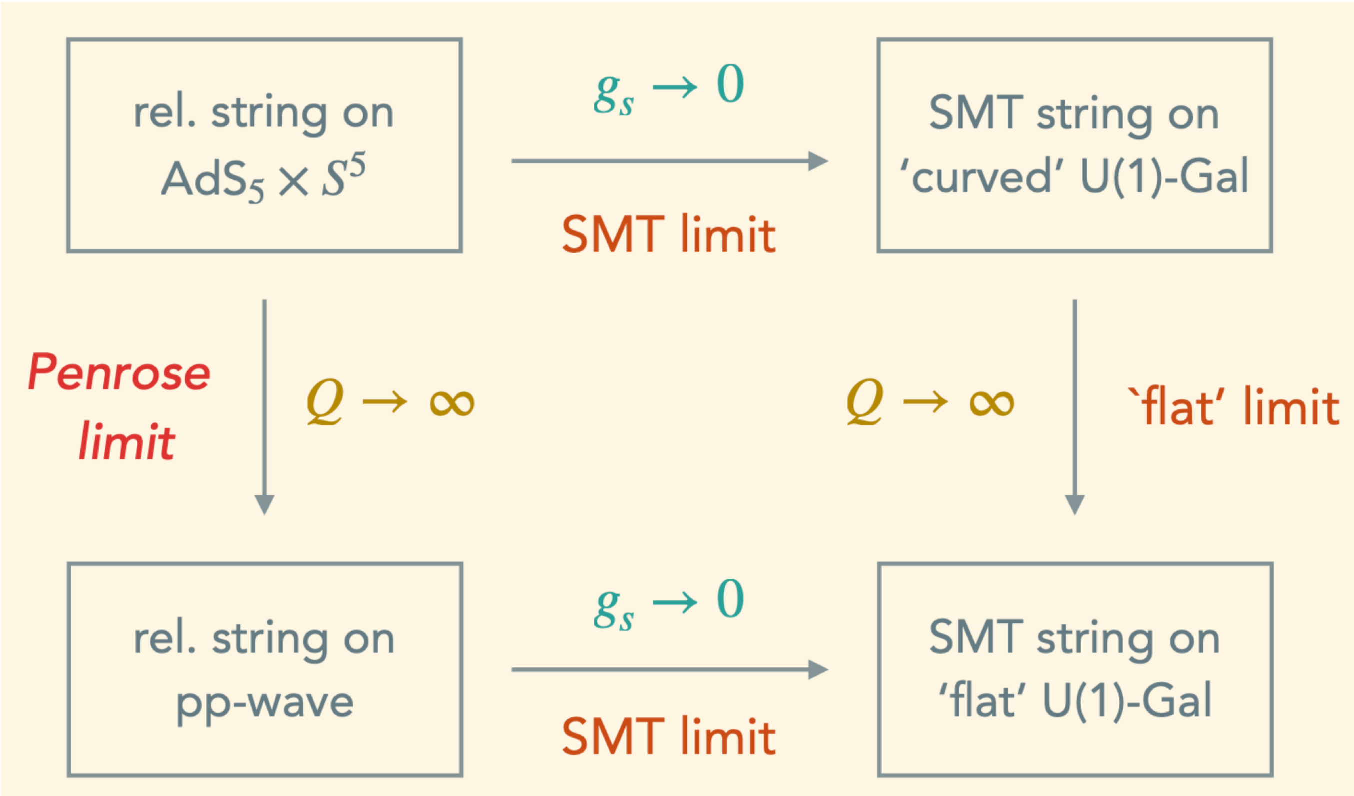
free magnon limit: S2 -> R2 → U(1)-Gal geometry
(corresponds to pp-wave limit)

$$\tau = d\tilde{x}^0, \quad m = \frac{1}{2}xdy, \quad h = \frac{1}{4}\left(dx^2 + dy^2\right)$$

action

$$S = \frac{1}{4\pi}\int d^2\sigma\left(x\dot{y} - \frac{1}{4}\left[(x')^2 + \left(y'\right)^2\right]\right)$$

Penrose limit and SMT limit commute



FF (flat-fluxed) backgrounds
→ natural starting point to quantize the theory

flat WS gauge
& "light-cone" gauge:

$$S = -\frac{Q}{2\pi}\int d^2\sigma \left[m_\mu \dot{X}^\mu + \frac{1}{2}h_{\mu\nu}X'^\mu X'^\nu\right]$$

Outlook

- comparison of TNC & SNC beta functions

- inclusion of WS fermions, ws/target space SUSY
  recent results using connection to double field theory Blair(2019)

- non-relativistic open strings and D-branes Gomis,Yan,Yu(2020)
- role of RR backgrounds
- T-duality Bergshoeff et al S/U-duality ?

- further study (quantization) of NR world-sheet theories
  - role of GCA

- strings in type II TNC backgrounds ? Hansen,Hartong,NO(2028/2019)
  (i.e. related to NR gravity from 1/c expansion of GR)

The end