Reduction of Couplings: Finite Unified Theories, reduced models and their predictions

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Dedicated to the memory of Graham G. Ross,
with gratitude and affection

- What happens as we approach the Planck scale? or just as we go up in energy...
- What happened in the early Universe?
- How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?
- How do we go from a fundamental theory to eW field theory as we know it?
- How do particles get their very different masses?
- What about flavour?
- **Where is the new physics??**

Search for understanding relations between parameters

addition of symmetries.

$N = 1$ SUSY GUTs.

Complementary approach: look for RGI relations among couplings at GUT scale $\longrightarrow$ Planck scale

$\Rightarrow$ **reduction of couplings**

resulting theory: less free parameters $\therefore$ more predictive

Zimmermann 1985

Remarkable: reduction of couplings provides a way to relate two previously unrelated sectors

gauge and Yukawa couplings

Kapetanakis, M.M., Zoupanos (1993), Kubo, M.M., Olechowski, Tracas, Zoupanos (1995,1996,1997); Oehme (1995); Kobayashi, Kubo, Raby, Zhang (2005); Gogoladze, Mimura, Nandi (2003,2004); Gogoladze, Li, Senoguz, Shafi, Khalid, Raza (2006,2011); M.M., Tracas, Zoupanos (2014)

Reduction of Couplings

A RGI relation among couplings $\Phi(g_1, \ldots, g_N) = 0$ satisfies

$$\mu\, d\Phi/d\mu \;=\; \sum_{i=1}^{N} \beta_i\, \partial\Phi/\partial g_i \;=\; 0.$$

g_i = **coupling,** β_i **its** β **function**

Finding the $(N-1)$ independent Φ's is equivalent to solve the reduction equations (RE)

$$\beta_g\,(dg_i/dg) = \beta_i\ ,$$

$i = 1, \cdots, N$

- Reduced theory: only one independent coupling and its β function
- complete reduction: power series solution of RE

$$g_a = \sum_{n=0} \rho_a^{(n)}\, g^{2n+1}$$

- uniqueness of the solution can be investigated at one-loop valid at all loops Zimmermann, Oehme, Sibold (1984,1985)
- The complete reduction might be too restrictive, one may use fewer Φ's as RGI constraints
- SUSY is essential for finiteness

finiteness: absence of ∞ renormalizations

$$\Rightarrow \quad \beta^N = 0$$

may be achieved through RE

- SUSY no-renormalization theorems
  - $\Rightarrow$ only study one and two-loops
  - RE guarantee that is gauge and reparameterization invariant to all loops

Reduction of couplings: the Standard Model

It is possible to make a reduced system in the Standard Model in the matter sector:
solve the REs, reduce the Yukawa and Higgs in favour of α_S gives

$$\alpha_t/\alpha_s = \frac{2}{9}\,; \quad \alpha_\lambda/\alpha_s = \frac{\sqrt{689}-25}{18} \simeq 0.0694$$

border line in RG surface, Pendleton-Ross infrared fixed line
But including the corrections due to non-vanishing gauge couplings up to two-loops, changes these relations and gives

$$M_t = 98.6 \pm 9.2 GeV$$

and

$$M_h = 64.5 \pm 1.5 GeV$$

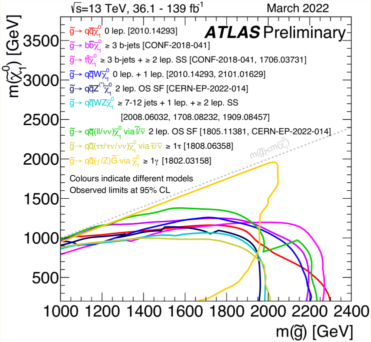
Both out of the experimental range, but pretty impressive

Kubo, Sibold and Zimmermann, 1984, 1985

SUSY in RE

Many of the reduced systems imply SUSY, even if it was not assumed a priori

Moreover: adding SUSY improves predictions $\Rightarrow$ SUSY + reduction of couplings natural



- Solution to the hierarchy problem
- Light SUSY in varios SUSY models incompatible with LHC data
- e.g.: Different assumptions on parameters of MSSM or NMSSM lead to different predictions

Figure from https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PUBNOTES/ATL-PHYS-PUB-2022-013/

Predictions in Finite Grand Unified Theories

Dimensionless sector of all-loop finite $SU(5)$ model

$M_{top} \sim 178$ GeV (1993)

large $\tan\beta$, heavy SUSY spectrum

Kapetanakis, M.M., Zoupanos, Z.f.Physik (1993)

$M_{top}^{exp} 176 \pm 18$ GeV found in 1995

$M_{top}^{th} \sim 172.5$ 2007

M_{top}^{exp} $173.1 \pm .09$ GeV 2013

$M_{Higgs}^{th} \sim 122 - 126$ GeV 2007

M_H^{exp} 126 ± 1 GeV 2013

Very promising, a more detailed analysis was clearly needed

Heinemeyer M.M., Zoupanos, JHEP (2007); Phys.Lett.B (2013), Symmetry (2018)

Finiteness

Finiteness = absence of divergent contributions to renormalization parameters $\Rightarrow \beta = 0$
Possible in SUSY due to improved renormalization properties

A chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g has a superpotential

$$W = \frac{1}{2}\, m^{ij}\, \Phi_i\, \Phi_j + \frac{1}{6}\, C^{ijk}\, \Phi_i\, \Phi_j\, \Phi_k \ ,$$

Requiring one-loop finiteness $\beta_g^{(1)} = 0 = \gamma_i^{j(1)}$ gives the following conditions:

$$\sum_i T(R_i) = 3 C_2(G)\,, \qquad \frac{1}{2} C_{ipq} C^{jpq} = 2\delta_i^j g^2 C_2(R_i)\,.$$

$C_2(G)$ quadratic Casimir invariant, $T(R_i)$ Dynkin index of R_i, C_{ijk} Yukawa coup., g gauge coup.

- **restricts the particle content of the models**
- **relates the gauge and Yukawa sectors**

- One-loop finiteness $\Rightarrow$ two-loop finiteness

Jones, Mezincescu and Yao (1984,1985)

- One-loop finiteness restricts the choice of irreps R_i, as well as the Yukawa couplings
- Cannot be applied to the susy Standard Model (SSM): $C_2[U(1)] = 0$
- The finiteness conditions allow only SSB terms

It is possible to achieve all-loop finiteness $\beta^n = 0$:

Lucchesi, Piguet, Sibold

1. One-loop finiteness conditions must be satisfied
2. The Yukawa couplings must be a formal power series in g, which is solution (isolated and non-degenerate) to the reduction equations

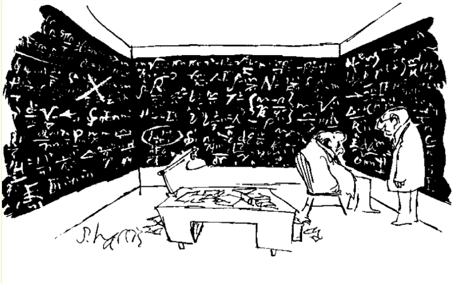
SUSY breaking soft terms

Supersymmetry is essential. It has to be broken, though...

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\, h^{ijk}\, \phi_i\phi_j\phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\phi_j + \frac{1}{2}\,(m^2)^j_i\, \phi^{*\,i}\phi_j + \frac{1}{2}\, M\,\lambda\lambda + \text{H.c.}$$

h trilinear couplings (A), b^{ij} bilinear couplings, m^2 squared scalar masses, M unified gaugino mass

Introduce over 100 new free parameters ☹



"Whatever happened to *elegant* solutions?"

RGI in the Soft Supersymmetry Breaking Sector

The RGI method has been extended to the SSB of these theories.

- One- and two-loop finiteness conditions for SSB have been known for some time
  Jack, Jones, et al.
- It is also possible to have all-loop RGI relations in the finite and non-finite cases
  Kazakov; Jack, Jones, Pickering
- SSB terms depend only on g and the unified gaugino mass M
  universality conditions

$$h = -MC, \qquad m^2 \propto M^2, \qquad b \propto M\mu$$

  but charge and colour breaking vacua
- Possible to extend the universality condition to a sum-rule for the soft scalar masses
  $\Rightarrow$ better phenomenology
  Kawamura, Kobayashi, Kubo; Kobayashi, Kubo, M.M., Zoupanos

Soft scalar sum-rule for the finite case

Finiteness implies

$$C^{ijk} = g \sum_{n=0} \rho^{ijk}_{(n)} g^{2n} \Rightarrow h^{ijk} = -MC^{ijk} + \cdots = -M\rho^{ijk}_{(0)} g + O(g^5)$$

If lowest order coefficients $\rho^{ijk}_{(0)}$ and $(m^2)^i_j$ satisfy diagonality relations

$$\rho_{ipq(0)}\rho^{jpq}_{(0)} \propto \delta^j_i \,, \qquad (m^2)^i_j = m^2_j \delta^i_j$$

for all p and q.

The following soft scalar-mass sum rule is satisfied, also to all-loops

$$(\ m^2_i + m^2_j + m^2_k\ )/MM^\dagger = 1 + \frac{g^2}{16\pi^2}\Delta^{(2)} + O(g^4)$$

for $i, \ j, \ k$ with $\rho^{ijk}_{(0)} \neq 0$, where $\Delta^{(2)}$ is the two-loop correction =0 for universal choice

Kobayashi, Kubo, Zoupanos

based on developments by Kazakov et al; Jack, Jones et al; Hisano, Shifman; etc

Also satisfied in certain class of orbifold models, where massive states are organized into $N = 4$ supermultiples

Several aspects of Finite Models have been studied

- $SU(5)$ Finite Models studied extensively

  Rabi et al; Kazakov et al; López-Mercader, Quirós et al; M.M, Kapetanakis, Zoupanos; etc
- One of the above coincides with a non-standard Calabi-Yau $SU(5) \times E_8$ Greene et al; Kapetanakis, M.M., Zoupanos
- Finite theory from compactified string model also exists (albeit not good phenomenology) Ibáñez
- Criteria for getting finite theories from branes Hanany, Strassler, Uranga
- $N = 2$ finiteness Frere, Mezincescu and Yao
- Models involving three generations Babu, Enkhbat, Gogoladze
- Some models with $SU(N)^k$ **finite** $\Longleftrightarrow$ **3 generations,** good phenomenology with $SU(3)^3$ Ma, M.M, Zoupanos
- Relation between commutative field theories and finiteness studied Jack and Jones
- Proof of conformal invariance in finite theories Kazakov
- Inflation from effects of curvature that break finiteness

  Elizalde, Odintsov, Pozdeeva, Vernov

$SU(5)$ Finite Models

Example: two models with $SU(5)$ gauge group. The matter content is

$$3\,\overline{\mathbf{5}} + 3\,\mathbf{10} + 4\,\{\mathbf{5} + \overline{\mathbf{5}}\} + \mathbf{24}$$

The models are finite to all-loops in the dimensionful and dimensionless sector. In addition:

- The soft scalar masses obey a sum rule
- At the M_{GUT} scale the gauge symmetry is broken and we are left with the MSSM
- At the same time finiteness is broken
- The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs $\{\mathbf{5} + \overline{\mathbf{5}}\}$ which couple to the third generation

The difference between the two models is the way the Higgses couple to the **24**

Kapetanakis, Mondragón, Zoupanos; Kazakov et al.

The superpotential which describes the two models takes the form

$$W = \sum_{i=1}^{3} \left[\frac{1}{2} g_i^u \, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \, \mathbf{10}_i \overline{\mathbf{5}}_i \, \overline{H}_i \right] + g_{23}^u \, \mathbf{10}_2 \mathbf{10}_3 H_4$$

$$+ g_{23}^d \, \mathbf{10}_2 \overline{\mathbf{5}}_3 \, \overline{H}_4 + g_{32}^d \, \mathbf{10}_3 \overline{\mathbf{5}}_2 \, \overline{H}_4 + \sum_{a=1}^{4} g_a^f \, H_a \, \mathbf{24} \, \overline{H}_a + \frac{g^\lambda}{3} (\mathbf{24})^3$$

find isolated and non-degenerate solution to the finiteness conditions

The unique solution implies discrete symmetries, $Z_n \times Z_m \times \ldots$
We will do a partial reduction, only third generation

The finiteness relations give at the M_{GUT} scale

Model A

- $g_t^2 = \frac{8}{5}\, g^2$
- $g_{b,\tau}^2 = \frac{6}{5}\, g^2$
- $m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$
- $m_{H_d}^2 + m_{\mathbf{5}}^2 + m_{\mathbf{10}}^2 = M^2$

- **3 free parameters:**
  M, $m_{\mathbf{5}}^2$ and $m_{\mathbf{10}}^2$

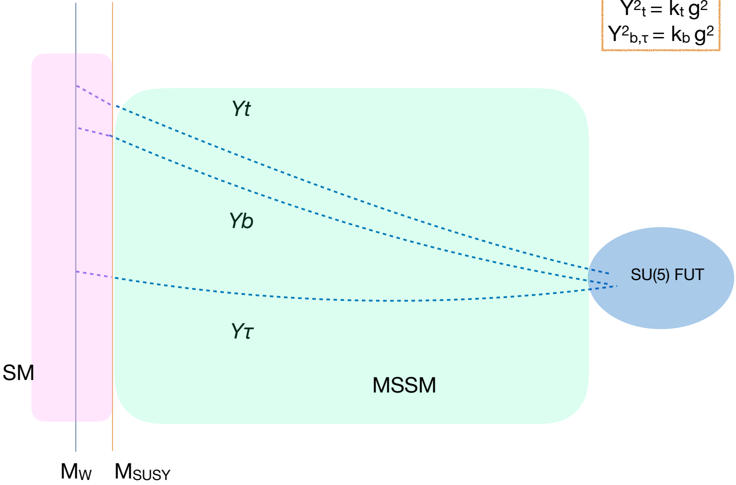
Model B

- $g_t^2 = \frac{4}{5}\, g^2$
- $g_{b,\tau}^2 = \frac{3}{5}\, g^2$
- $m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$
- $m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}$
- $m_{\mathbf{5}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3}$

- **2 free parameters:**
  M, $m_{\mathbf{5}}^2$

FUT

$Y^2_t = k_t\, g^2$
$Y^2_{b,\tau} = k_b\, g^2$



$m_t = Y_t\, v_u$ $\quad v_u / v_d = \tan\beta$
$m_{b,\tau} = Y_{b,\tau}\, v_d$ $\quad v_d = m_\tau^{exp} / Y_\tau$

Phenomenology

The gauge symmetry is broken below M_{GUT}, and what remains are boundary conditions of the form $C_i = \kappa_i g$, $h = -MC$ and the sum rule at M_{GUT}, below that is the MSSM.

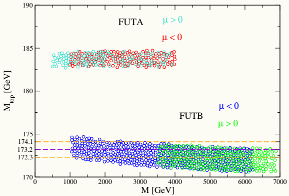
- Fix the value of $m_\tau \Rightarrow \tan\beta \Rightarrow M_{top}$ and m_{bot}
- We assume a unique susy breaking scale
- The LSP is neutral
- The solutions should be compatible with radiative electroweak breaking
- No fast proton decay

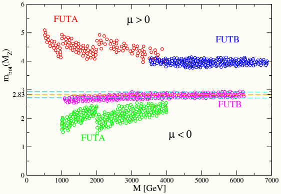
We also

- Allow 5% variation of the Yukawa couplings at GUT scale due to threshold corrections
- Include radiative corrections to bottom and tau, plus resummation **(very important!)**
- Estimate theoretical uncertainties

TOP AND BOTTOM MASS

We can discriminate among solutions $\Rightarrow$ region for M points to heavy s-spectrum





Predictions:

- FUTA: $M_{top} \sim 182 \sim 185$ *GeV*
  FUTB: $M_{top} \sim 172 \sim 174$ *GeV*
  Theoretical uncertainties $\sim 4\%$
- large $\tan\beta$
- Δb and $\Delta\tau$ included
  resummation done.
  Depend mainly
  on $\tan\beta$ and unified gaugino mass M.
- **FUTB $\mu < 0$ favoured**

Now include the rest...

Once top was found, we look for the solutions that satisfy the following constraints:

Facts of life:

- Right masses for top and bottom
- B physics observables

$\mathrm{BR}(b \to s\gamma) SM/MSSM$:
$|BRbsg - 1.089| < 0.27$

$\mathrm{BR}(B_u \to \tau\nu) SM/MSSM$:
$|BRbtn - 1.39| < 0.69$

$\Delta M_{B_s} SM/MSSM$: 0.97 ± 20

$\mathrm{BR}(B_s \to \mu^+\mu^-) =$
$(2.9 \pm 1.4) \times 10^{-9}$

Results:
$M_H =\sim 121 - 126$ GeV

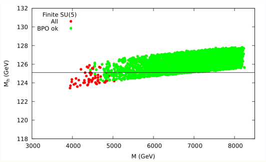
Heavy s-spectrum

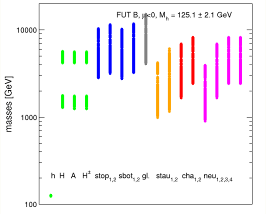
Heinemeyer, MM, Zoupanos, JHEP 2008

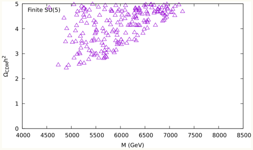
Once the Higgs was found, we can use the experimental value as constraint $\Rightarrow$ restrict more M and s-spectrum

Masses, s-spectrum

With latest FeynHiggs and experimental constraints:







- Top and bottom quark masses within 2σ
- Heavy SUSY spectrum $\Rightarrow$ consistent with non-observation
- Only third generation included
- Lightest neutralino 100% of DM $\Rightarrow$ Over abundance of DM
- R parity breaking $\Rightarrow$ neutrino masses and gravitino as DM
- Possible to extend to 3 generations

- Finiteness provides us with an UV completion of our QFT
- Boundary conditions for RGE of the MSSM
- RGI takes the flow in the right direction for the third generation and Higgs masses
  also for susy spectrum (high)

- Are there other finite models?
- Can it give us insight into the flavour structure?
- Can we have successful reduction of couplings in a SM-like theory?

$SU(N)^k$

3 generations $\leftrightarrow$ finite

Consider the gauge group

$$\mathrm{SU}(N)_1 \times \mathrm{SU}(N)_2 \times \cdots \times \mathrm{SU}(N)_k$$

with n_f copies of
$(N, \bar{N}, 1, \ldots, 1) + (1, N, \bar{N}, \ldots, 1) + \cdots + (\bar{N}, 1, 1, \ldots, N)$.

The one-loop β-function

$$\beta = \left(-\frac{11}{3} + \frac{2}{3}\right) N + n_f \left(\frac{2}{3} + \frac{1}{3}\right) \left(\frac{1}{2}\right) 2N = -3N + n_f N \,. \quad (1)$$

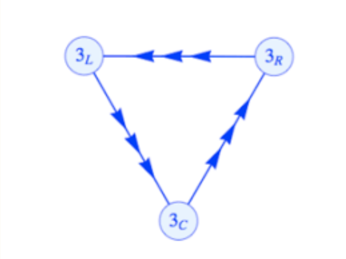
$\Rightarrow n_f = 3$ is a solution of $\beta = 0$, independently of the values of N and k.

2-loop $SU(3)^3$ out of several possibilities

$SU(3)^3$ 2-loop finite trinification model, parametric solution of reduction equations

$$f^2 = r\left(\frac{16}{9}\right)g^2\,,\quad f'^2 = (1-r)\left(\frac{8}{3}\right)g^2\,,$$

r parameterizes different solutions to boundary conditions, f, f' Yukawa for quarks and leptons respectively



- Finiteness implies 3 generations
- Good top and bottom masses, depend on a parameter
- Large tan β
- Heavy SUSY spectrum
- Possibility of having neutrino masses
- Consistent with seesaw mechanism
- DM neutralino, consistent with DM relic density

Reduced MSSM not finite, but reduced

Can we have successful reduction of couplings in a SM-like theory? YES, with SUSY
We assume a covering GUT, reduced top-bottom system

Y_τ not reduced, its reduction gives imaginary values

$$\frac{Y_t^2}{4\pi} = G_t^2 \frac{g_3^2}{4\pi} + c_2 \left(\frac{g_3^2}{4\pi}\right)^2 ; \quad \frac{Y_b^2}{4\pi} = G_b^2 \frac{g_3^2}{4\pi} + p_2 \left(\frac{g_3^2}{4\pi}\right)^2$$

where

$$G_t^2 = \frac{1}{3} + \frac{71}{525}\rho_1 + \frac{3}{7}\rho_2 + \frac{1}{35}\rho_\tau , \qquad G_b^2 = \frac{1}{3} + \frac{29}{525}\rho_1 + \frac{3}{7}\rho_2 - \frac{6}{35}\rho_\tau$$

$$\rho_{1,2} = \frac{g_{1,2}^2}{g_3^2} = \frac{\alpha_{1,2}}{\alpha_3}, \qquad \rho_\tau = \frac{g_\tau^2}{g_3^2} = \frac{\frac{Y_\tau^2}{4\pi}}{\alpha_3}$$

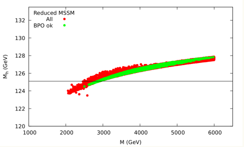
$\rho_{1,2}, \rho_\tau$ corrections from the non-reduced part, assumed smaller as energy increases

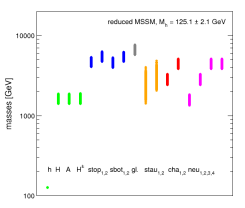
c_2 and p_2 can also be found (long expressions not shown)

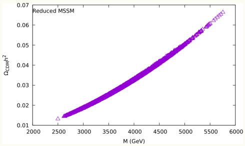
Higgs mass and s-spectrum

RMSSM has lightest s-spectrum!

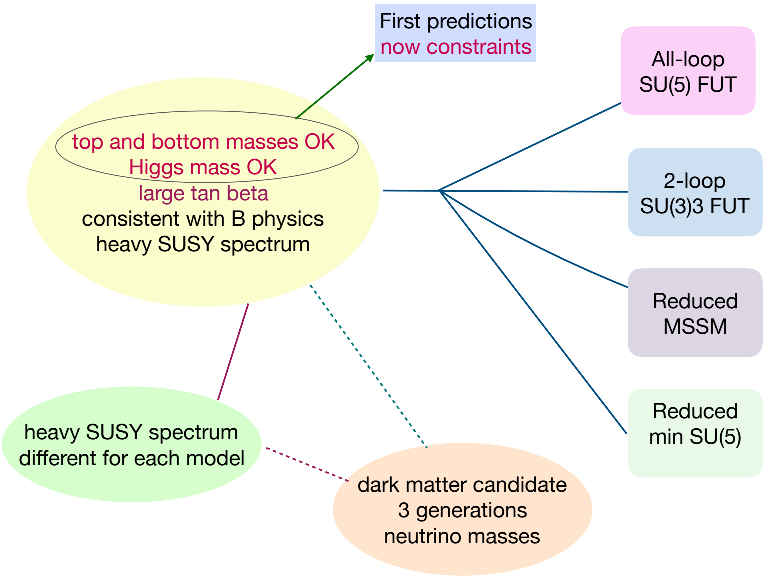
- Possible
  to have reduction of couplings
  in MSSM, third family of quarks
- Up to now
  only attempted in SM or in GUTs
- Reduced system further
  constrained by phenomenology:
- Large tan β
- SUSY spectrum $M_{LSP} \geq 1\ TeV$
- DM abundance OK (below
  limit), possible to add a SUSY axion







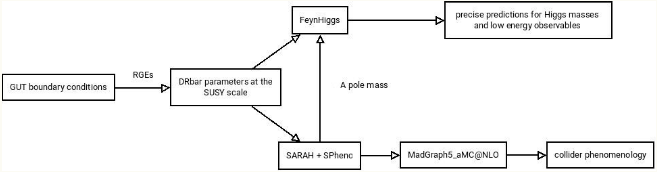
GYU from reduction of couplings at work



Experimental challenge

- Can they be tested at HL-LHC or FCC?
- Constraints: Top, bottom, and Higgs masses, B physics
- tan β always large, heavy s-spectrum common to all, but details differ
- Test models, calculate expected cross sections at 14 Tev (HL-LHC) and 100 TeV (FCC)

Heinemeyer, Kalinowski, Klotarski, MM, Patellis, Tracas, Zoupanos, Eur. Phys. J. C (2021) 81:185



Results for FUT SU(5): CDM, Higgs and s-spectra

	M_H	M_A	$M_{H^\pm}$	$M_{\tilde g}$	$M_{\tilde\chi_1^0}$	$M_{\tilde\chi_2^0}$	$M_{\tilde\chi_3^0}$	$M_{\tilde\chi_4^0}$	$M_{\tilde\chi_1^\pm}$	$M_{\tilde\chi_2^\pm}$
FUTSU5-1	5.688	5.688	5.688	8.966	2.103	3.917	4.829	4.832	3.917	4.833
FUTSU5-2	7.039	7.039	7.086	10.380	2.476	4.592	5.515	5.518	4.592	5.519
FUTSU5-3	16.382	16.382	16.401	12.210	2.972	5.484	6.688	6.691	5.484	6.691
	$M_{\tilde e_{1,2}}$	$M_{\tilde\nu_{1,2}}$	$M_{\tilde\tau}$	$M_{\tilde\nu_\tau}$	$M_{\tilde d_{1,2}}$	$M_{\tilde u_{1,2}}$	$M_{\tilde b_1}$	$M_{\tilde b_2}$	$M_{\tilde t_1}$	$M_{\tilde t_2}$
FUTSU5-1	3.102	3.907	2.205	3.137	7.839	7.888	6.102	6.817	6.099	6.821
FUTSU5-2	3.623	4.566	2.517	3.768	9.059	9.119	7.113	7.877	7.032	7.881
FUTSU5-3	4.334	5.418	3.426	3.834	10.635	10.699	8.000	9.387	8.401	9.390

Table 5: Masses for each benchmark of the Finite $N = 1$ $SU(5)$ (in TeV).

scenarios $\sqrt{s}$	FUTSU5-1 100 TeV	FUTSU5-2 100 TeV	FUTSU5-3 100 TeV	scenarios $\sqrt{s}$	FUTSU5-1 100 TeV	FUTSU5-2 100 TeV	FUTSU5-3 100 TeV
$\tilde\chi_2^0\tilde\chi_3^0$	0.01	0.01		$\tilde\nu_i\tilde\nu_j^*$	0.02	0.01	0.01
$\tilde\chi_3^0\tilde\chi_4^0$	0.03	0.01		$\tilde u_i\tilde\chi_1^-, \tilde d_i\tilde\chi_1^+ + h.c.$	0.15	0.06	0.02
$\tilde\chi_2^0\tilde\chi_1^+$	0.17	0.08	0.03	$\tilde q_i\tilde\chi_1^0, \tilde q_i^*\tilde\chi_1^0$	0.08	0.03	0.01
$\tilde\chi_3^0\tilde\chi_2^+$	0.05	0.03	0.01	$\tilde q_i\tilde\chi_2^0, \tilde q_i^*\tilde\chi_2^0$	0.08	0.03	0.01
$\tilde\chi_4^0\tilde\chi_2^+$	0.05	0.03	0.01	$\tilde\nu_i\tilde e_j^*, \tilde\nu_i^*\tilde e_j$	0.09	0.04	0.01
$\tilde g\tilde g$	0.20	0.05	0.01	$Hb\bar b$	2.76	0.85	
$\tilde g\tilde\chi_1^0$	0.03	0.01		$Ab\bar b$	2.73	0.84	
$\tilde g\tilde\chi_2^0$	0.03	0.01		$H^+b\bar t + h.c.$	1.32	0.42	
$\tilde g\tilde\chi_1^+$	0.07	0.03	0.01	H^+W^-	0.38	0.12	
$\tilde q_i\tilde q_j, \tilde q_i\tilde q_j^*$	3.70	1.51	0.53	HZ	0.09	0.03	
$\tilde\chi_1^+\tilde\chi_1^-$	0.10	0.05	0.02	AZ	0.09	0.03	
$\tilde\chi_2^+\tilde\chi_2^-$	0.03	0.02	0.01				
$\tilde e_i\tilde e_j^*$	0.23	0.13	0.05				
$\tilde q_i\tilde g, \tilde q_i^*\tilde g$	2.26	0.75	0.20				

Table 6: Expected production cross sections (in fb) for SUSY particles in the FUTSU5 scenarios.

Results for RMSSM: CDM, Higgs and s-spectra

	M_H	M_A	$M_{H^\pm}$	$M_{\tilde{g}}$	$M_{\tilde{\chi}_1^0}$	$M_{\tilde{\chi}_2^0}$	$M_{\tilde{\chi}_3^0}$	$M_{\tilde{\chi}_4^0}$	$M_{\tilde{\chi}_1^\pm}$	$M_{\tilde{\chi}_2^\pm}$
RMSSM-1	1.393	1.393	1.387	7.253	1.075	3.662	4.889	4.891	1.075	4.890
RMSSM-2	1.417	1.417	1.414	7.394	1.098	3.741	4.975	4.976	1.098	4.976
RMSSM-3	1.491	1.491	1.492	7.459	1.109	3.776	5.003	5.004	1.108	5.004
	$M_{\tilde{e}_{1,2}}$	$M_{\tilde{\nu}_{1,2}}$	$M_{\tilde{\tau}}$	$M_{\tilde{\nu}_\tau}$	$M_{\tilde{d}_{1,2}}$	$M_{\tilde{u}_{1,2}}$	$M_{\tilde{b}_1}$	$M_{\tilde{b}_2}$	$M_{\tilde{t}_1}$	$M_{\tilde{t}_2}$
RMSSM-1	2.124	2.123	2.078	2.079	6.189	6.202	5.307	5.715	5.509	5.731
RMSSM-2	2.297	2.139	2.140	2.139	6.314	6.324	5.414	5.828	5.602	5.842
RMSSM-3	2.280	2.123	2.125	2.123	6.376	6.382	5.465	5.881	5.635	5.894

Table 11: Masses for each benchmark of the Reduced MSSM (in TeV).

Since $M_A \lesssim 1.5$ TeV and large $\tan\beta$, RMSSM is excluded by searches $H/A \to \tau\tau$ at ATLAS.

Prospects for FCC

Model	top/bottom masses	Higgs mass	SUSY spectra	heavy Higgs spectra	CDM
$\sim$ FUT $SU(5)$	OK/OK	OK	$\gtrsim$ 2.0 TeV	$\gtrsim$ 5.5 TeV	too much
✓ FUT $SU(3)^3$	OK/OK	OK	$\gtrsim$ 1.5 TeV	$\gtrsim$ 6.4 TeV	feasible
$\sim$ RMin $SU(5)$	OK/bot 4σ	OK	$\gtrsim$ 1.2 TeV	$\sim$ 2.5 TeV	too much
✗ RMSSM	OK/OK	OK	$\sim$ 1.0 TeV	$\sim$ 1.3 TeV	OK

- RMSSM already excluded by LHC searches
- The rest testable only at FCC-hh at 2 σ, only part at 5 σ
- Exception: $SU(3)^3$ heavy Higgs sector testable at FCC-hh
- In $SU(5)$ models you can have neutrino masses and gravitino as DM $\Rightarrow$ $\not{R}$

Conclusions

- Reduction of couplings: powerful principle implies Gauge Yukawa Unification $\Rightarrow$ predictive models
- Possible SSB terms $\Rightarrow$ satisfy a sum rule among soft scalars
- Finiteness $\Rightarrow$ reduces greatly the number of free parameters
  - completely finite theories $SU(5)$
  - 2-loop finite theories $SU(3)^3$

- Reduced non-finite models:
  - min $SU(5)$
  - RMSSM
- Successful prediction for top quark and Higgs boson mass
- Large $\tan\beta$
- Satisfy BPO constraints (not trivial)
- Heavy SUSY spectrum
- Most of the spectra too heavy to be tested at FCC:
  - RMSSM excluded
  - $SU(3)^3$ heavy Higgs sector could be tested

Outlook

Some open questions and future work in reduction of couplings

- Are there more finite and reduced models? Yes...
- Do all fermions acquire masses the same way? ??
- Is it possible to include the three generations in a reduced or finite model? Yes...
- How to incorporate flavour? possible, aided by symmetries
- How to include neutrino masses? Yes... $\not{R}$ for $(SU5)$, natural for $SU(3)^3$
- Is it indispensible to have SUSY for successful reduced theories? So far it looks like that
- How to make better use symmetries ⇔ reduction of couplings? ?