# Analytic false-vacuum decay rate in the thin-wall approximation

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WORKSHOP ON STANDARD MODEL AND BEYOND

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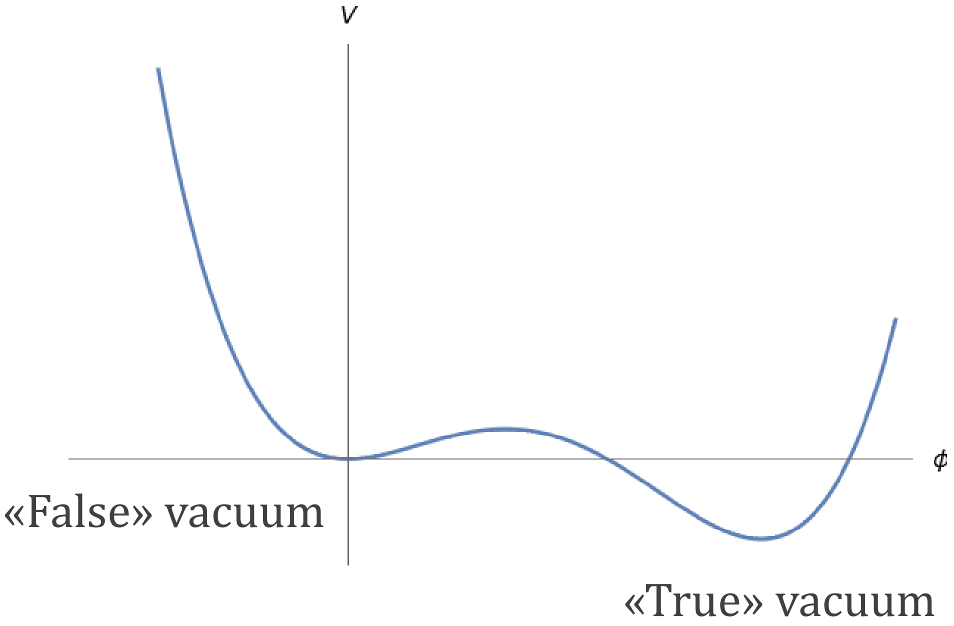
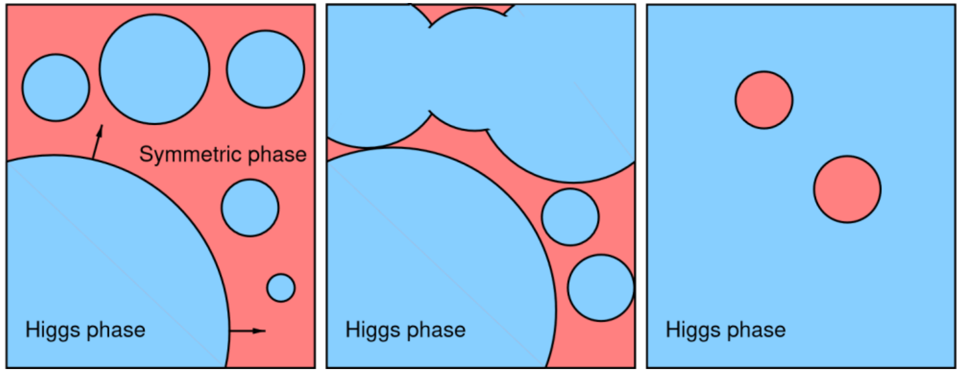
# VACUUM DECAY

- Easiest example in field theory: single scalar $\phi$



- Metastability of the false vacuum
- Decay to the true vacuum (tunnel under the barrier)
- Bubble nucleation: 1° order phase transition
- Bubble expansion: conversion of false vacuum to true vacuum



[*Hindmarsh, Lüben, Lumma, Pauly,* 2008.09136]

# VACUUM DECAY RATE

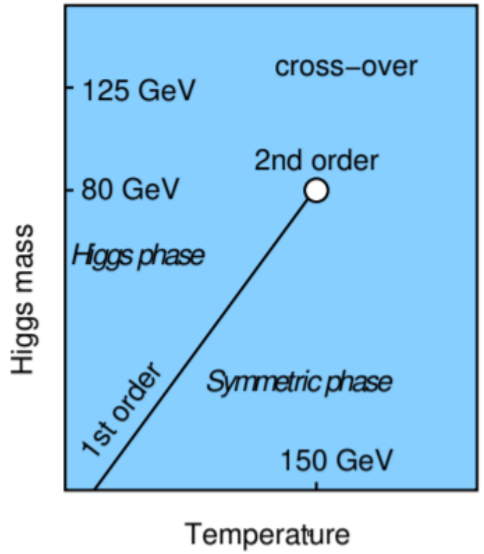
- 1-loop decay rate (per unit volume) for generic Euclidean dimension:

$$\frac{\Gamma}{\mathcal{V}} = A\ e^{-B/\hbar}(1+\mathcal{O}(\hbar)) \quad \Longrightarrow \quad \boxed{\frac{\Gamma}{\mathcal{V}} = \underbrace{\left(\frac{S_R}{2\pi\hbar}\right)^{\frac{D}{2}} \left|\frac{\det'\mathcal{O}}{\det\mathcal{O}_{\mathrm{FV}}}\right|^{-\frac{1}{2}}}_{\text{fluctuations}} \underbrace{e^{-\frac{S_R}{\hbar}-S_{\mathrm{ct}}}}_{\text{classical solution («bounce»)}}(1+\mathcal{O}(\hbar))}$$

- $D = 3$ : Thermal field theory, $D = 4$ : Quantum field theory

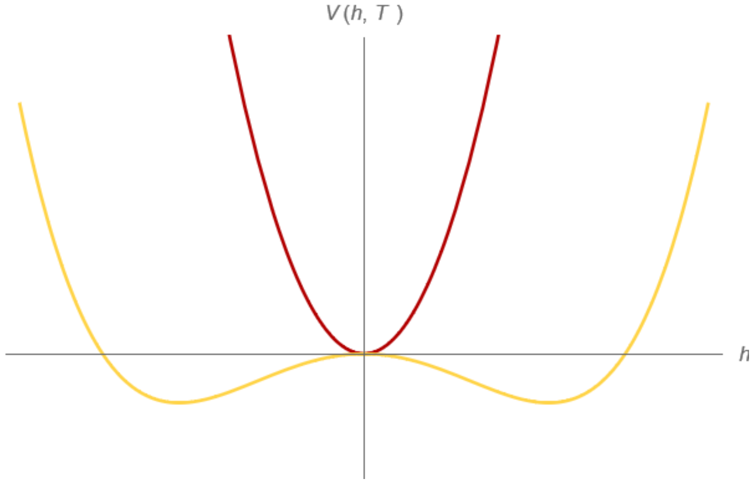
# WHY VACUUM DECAY?

- Early universe: 1° order cosmological phase transition is a sign of BSM physics!
- Example: Electroweak phase transition (EWPT)
  - $\sim 10^{-11} s$ , $\sim 100$ GeV
  - Crossover in SM, can be 1° order even in simplest extensions



[*Hindmarsh, Lüben, Lumma, Pauly,* 2008.09136]

SM:



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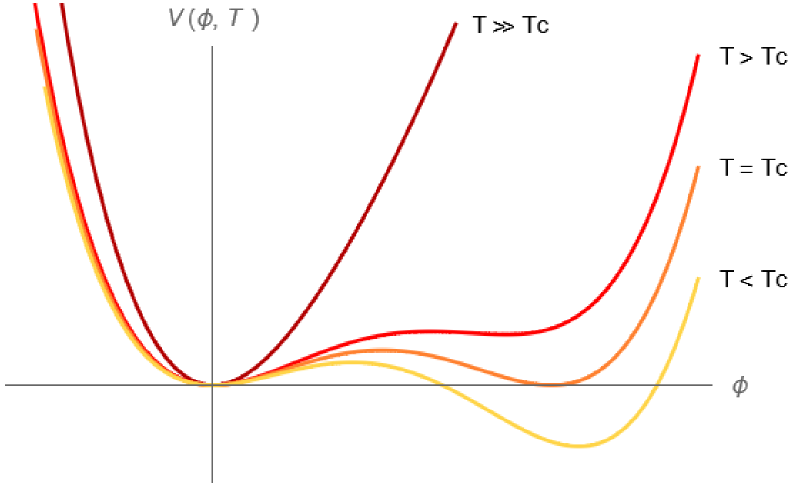

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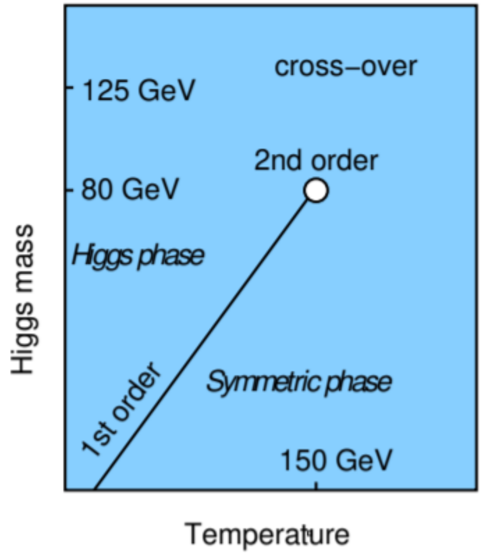
Potential to be probed via detection of GWs background



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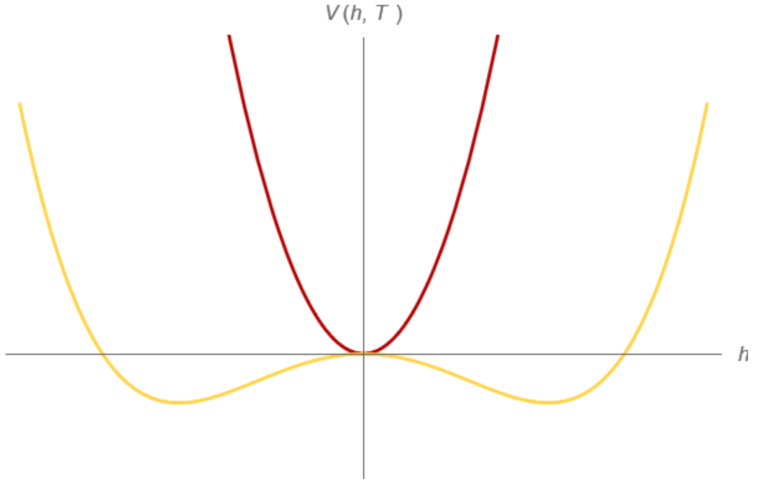
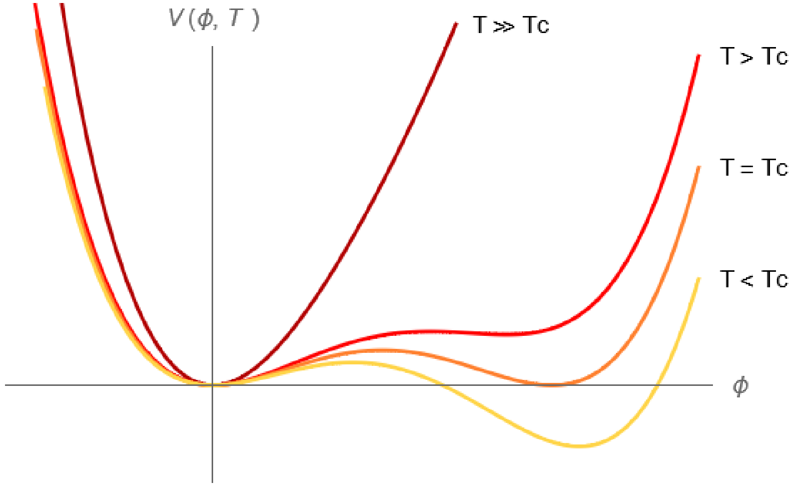
SM:

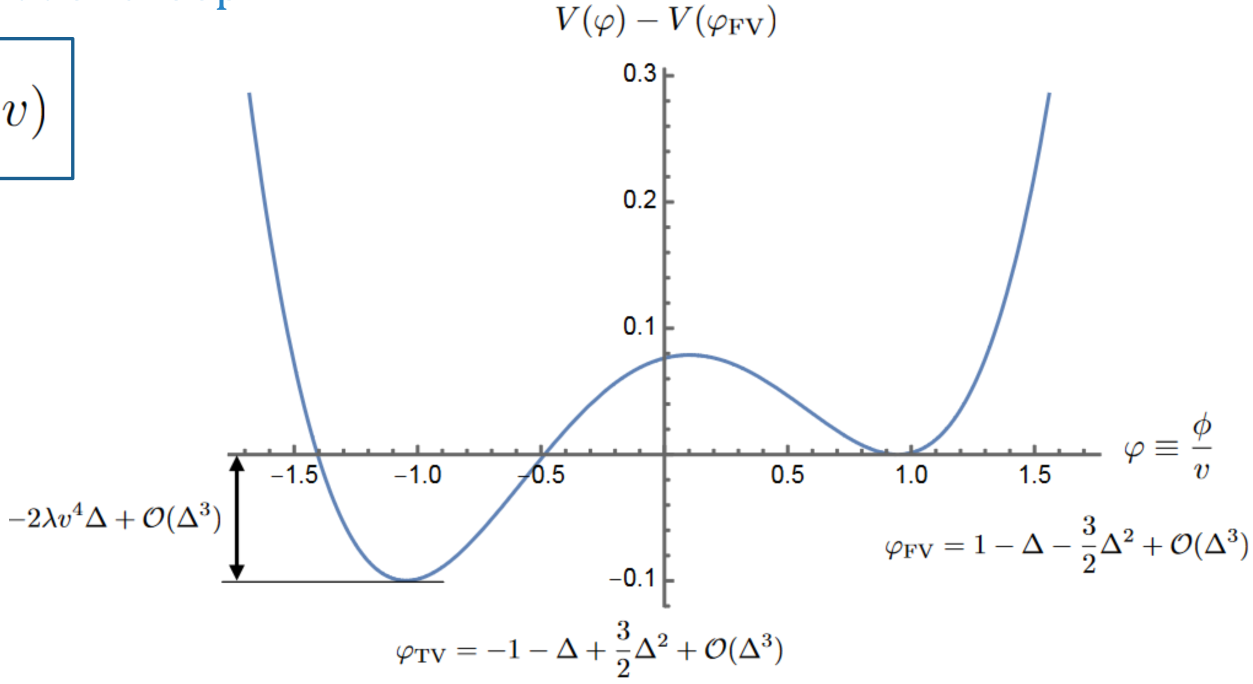


BSM:



# VACUUM DECAY – THIN WALL

- Long-standing problem [*Coleman*, 10.1103/PhysRevD.15.2929 ; *Coleman, Callan*, 10.1103/PhysRevD.16.1762 ]
- We derived a closed-form false vacuum decay rate at one loop
- Potential: $$\boxed{V = \frac{\lambda}{8}\left(\phi^2 - v^2\right)^2 + \lambda\,\Delta\,v^3\,(\phi - v)}$$
- Thin wall: $0 < \Delta \ll 1$ (expansion parameter)



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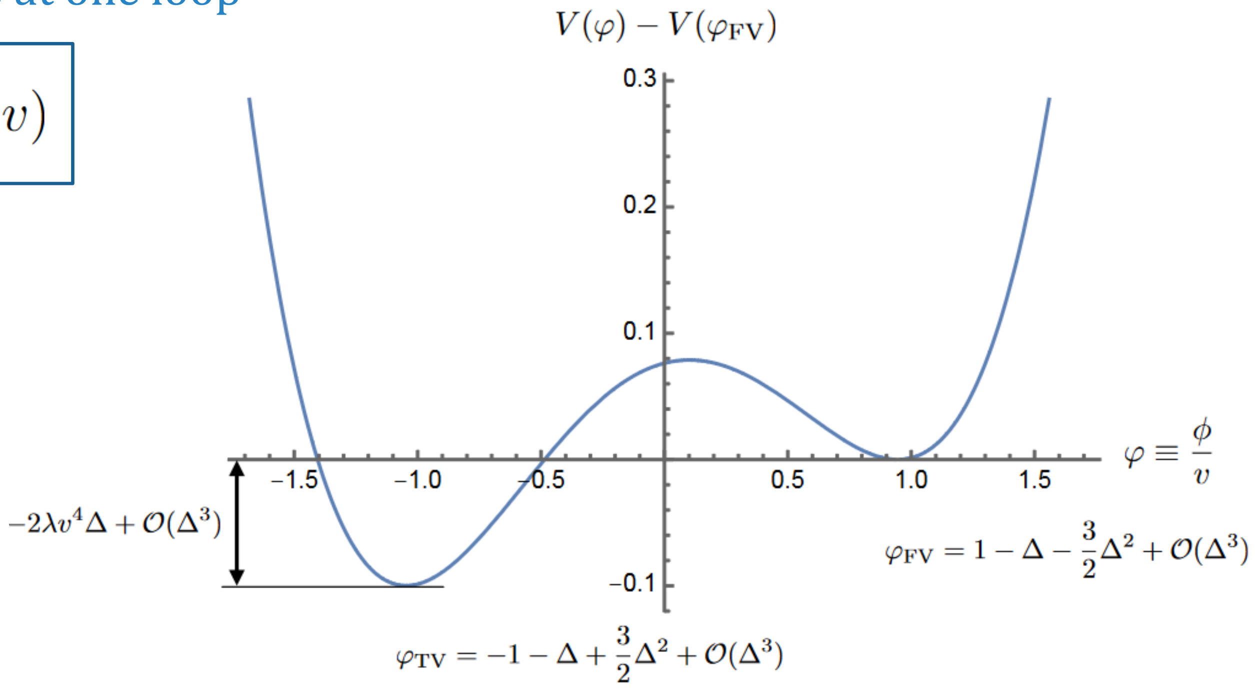
$$S = \int_D \left(\frac{1}{2}\dot{\phi}^2 + V - V_{\rm FV}\right)$$

$$\Downarrow$$

$$\frac{\Gamma}{\mathcal{V}} = \left(\frac{S_R}{2\pi\hbar}\right)^{\frac{D}{2}} \left|\frac{\det'\mathcal{O}}{\det\mathcal{O}_{\rm FV}}\right|^{-\frac{1}{2}} e^{-\frac{S_R}{\hbar} - S_{\rm ct}}\,(1 + \mathcal{O}(\hbar))$$

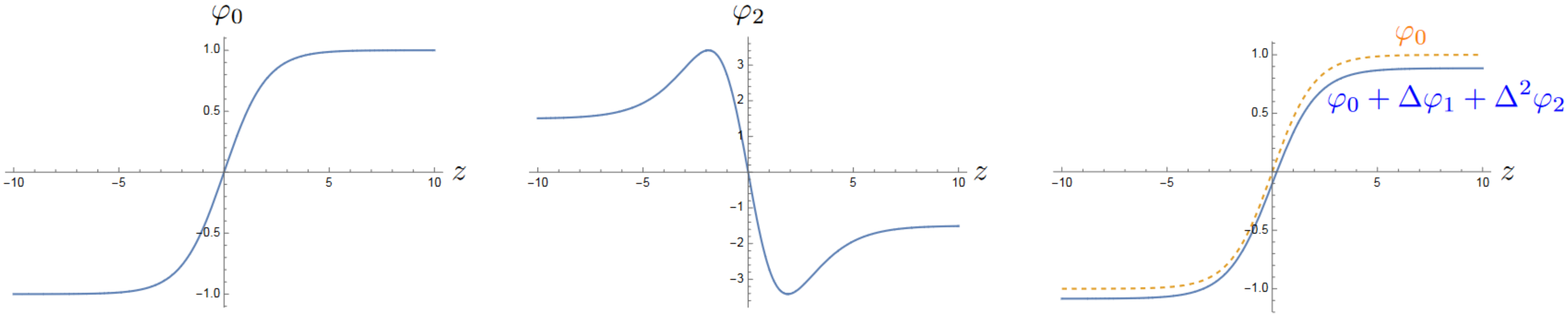
$$\mathcal{O} = -\partial_\mu\partial^\mu + V^{(2)}$$

«Bounce» action



# THIN WALL BOUNCE AND RADIUS

- Thin wall expansion: $\frac{\phi}{v} \equiv \varphi = \sum \varphi_n \, \Delta^n \quad , \qquad z = \sqrt{\lambda} v \; \rho - r \qquad \Longrightarrow \qquad r = \frac{1}{\Delta} \sum r_n \, \Delta^n$
- Solve e.o.m. order by order $\ddot{\phi} + \frac{D-1}{\rho} \dot{\phi} = \frac{\mathrm{d}V}{\mathrm{d}\phi} \, , \qquad \dot{\phi}(0) = \dot{\phi}(\infty) = 0 \, ,$
- Bounce: $\phi(0) = \phi_{\text{in}} \, , \qquad \phi(\infty) = \phi_{\text{FV}} \, .$



- Then fix radius by extremizing the action at that order :

$$\frac{\mathrm{d}S_0}{\mathrm{d}r_0} = 0 \quad \Rightarrow \quad r_0 = \frac{D-1}{3} \qquad\qquad r_1 = 0 \, , \qquad\qquad r_2 = \frac{6\pi^2 - 40 + D(26 - 4D - 3\pi^2)}{3(D-1)}$$

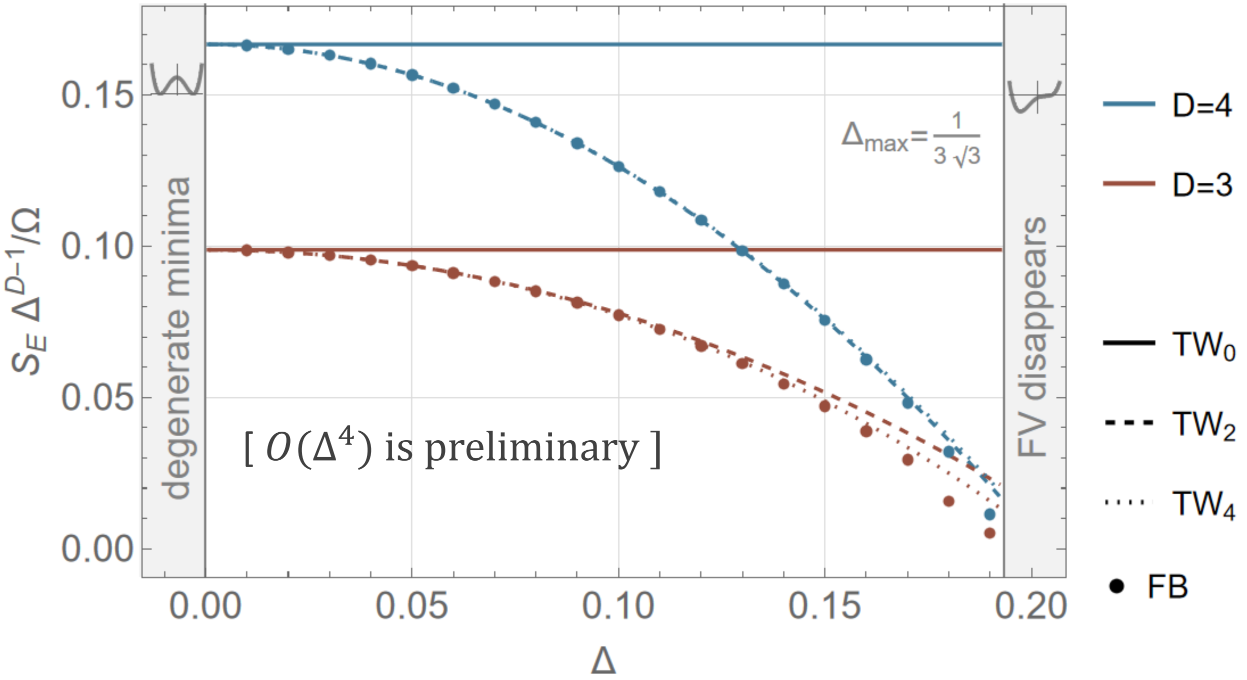
# THIN WALL BOUNCE ACTION

- Action in general $D$ :

$$S_0 = \frac{\Omega\, v^{4-D}}{\lambda^{D/2-1}\boxed{\Delta^{D-1}}}\left(\frac{D-1}{3}\right)^{D-1}\frac{2}{3D}$$

$$\boxed{S = S_0\left(1+\boxed{\Delta^2}\left(\frac{1+D\,(25-8D-3\pi^2)}{2(D-1)}\right)\right)}$$

New!



- Counterterms & running in *4D*: dim.reg. and MSbar

$$S_R + S_{\text{ct}} = S\left(1-\frac{9\lambda_0}{(4\pi)^2}\left(\frac{1}{\varepsilon}+\ln\frac{\mu}{\mu_0}\right)\right)$$

$\mu_0^2 \sim V_{\text{FV}}^{(2)} \sim \lambda_0 v^2$

*FindBounce (FB) Mathematica package*:
[*Guada, Nemevšek, Pintar*, 2002.00881]

# THIN WALL FLUCTUATIONS

- Most important achievement: inclusion of fluctuations
- *O(D)* symmetry: separate variables, expand in hyperspherical multipoles

$$\left|\frac{\det' \mathcal{O}}{\det \mathcal{O}_{\rm FV}}\right|^{-\frac{1}{2}} = \left|\prod_{l=0}^{\infty} \frac{\det' \mathcal{O}_l}{\det \mathcal{O}_{l\rm FV}}\right|^{-\frac{1}{2}}$$

$$\mathcal{O}_l = -\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} - \frac{D-1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho} + \frac{l\,(l+D-2)}{\rho^2} + V^{(2)}$$

- Gel’fand-Yaglom theorem:

$$\boxed{\frac{\det \mathcal{O}_l}{\det \mathcal{O}_{l\rm FV}} = R_l(\infty)^{d_l}}$$

satisfies

$$\boxed{\ddot{R}_l + 2\left(\frac{\dot{\psi}_{l\rm FV}}{\psi_{l\rm FV}}\right)\dot{R}_l = \left(V^{(2)} - V^{(2)}_{\rm FV}\right)R_l}$$

$$d_l = \frac{(2l+D-2)(l+D-3)!}{l!(D-2)!}$$

$$R_l(\rho=0) = 1$$

$$\dot{R}_l(\rho=0) = 0$$

$$\mathcal{O}_{l\rm FV}\,\psi_{l\rm FV} = 0\,,\ \psi_{l\rm FV}(\rho\sim 0) \sim \rho^l$$

- Expected behavior:

$$R_l(\infty) = \begin{cases} <0, & l=0\,, \\ 0, & l=1\,, \\ 1, & l\gg 1\,. \end{cases}$$

# THIN WALL FLUCTUATIONS

- Introduce $\nu = l + \frac{D}{2} - 1$
- Low multipoles $\Delta^2\nu^2 \sim O(\Delta^2)$

$$\frac{d\psi^2_{l\mathrm{FV}}}{dz^2} = \left(1 - 3\Delta - 3\Delta^2 + \Delta^2\left(\frac{\nu^2 - \frac{1}{4}}{r_0^2}\right)\right)\psi_{l\mathrm{FV}} \longrightarrow \psi_{l\mathrm{FV}}(z) \simeq c_{\mathrm{FV}} \exp\left[\left(1 - \frac{3}{2}\Delta + \left(\frac{\nu^2 - \frac{1}{4}}{2r_0^2} - \frac{21}{8}\right)\Delta^2\right)z\right]$$

$$\longrightarrow \boxed{R_l(\infty) = \Delta^2 e^{D-1}\frac{3}{4}\frac{(l-1)(l+D-1)}{(D-1)^2}}$$

- Negative for $l = 0$ , null for $l = 1$

- High multipoles $\Delta^2\nu^2 \sim O(1)$

$$\psi_{\nu\mathrm{FV}} \simeq e^{k_\nu z}$$

$$k_\nu^2 = 1 + \frac{\Delta^2\nu^2}{r_0^2}$$

$$R_\nu(\infty) = R_{\nu 0}(\infty)e^U\,,$$

$$\longrightarrow \boxed{\ln R_\nu(\infty) = \ln\frac{(k_\nu - 1)(2k_\nu - 1)}{(k_\nu + 1)(2k_\nu + 1)} + 3r_0\left(k_\nu - \sqrt{k_\nu^2 - 1}\right)}$$

- Goes to 1 for large $l$

# THIN WALL PREFACTOR

- Renormalized determinant: $\ln\left(\frac{\det\mathcal{O}}{\det\mathcal{O}_{\rm FV}}\right)=\sum_{\nu=D/2-1}^{\infty}d_\nu\ln R_\nu$ degeneracy $d_\nu=\frac{2\nu\left(\nu+\frac{D}{2}-2\right)!}{(D-2)!\left(\nu-\frac{D}{2}+1\right)!}\simeq\frac{2}{(D-2)!}\nu^{D-2}$

- Expansion of the high multipole result, e.g. up to $O(\nu^{-3})$ :

$$\sum_{\nu\gg1}d_\nu\ln R_{\nu\gg1}\sim-\frac{3r_0(2-r_0)}{(D-2)!\Delta}\sum_{\nu\gg1}\nu^{D-2}\left(\frac{1}{\nu}-\frac{1}{\nu^3}\left(\frac{r_0}{2\Delta}\right)^2\right)$$

divergence for large ν
use same renorm. scheme as before!

- Subtractions are needed $\Sigma_D=\sum_{\nu=\nu_0}^{\infty}\sigma_D=\sum_{\nu=\nu_0}^{\infty}d_\nu\left(\ln R_\nu-\ln R_\nu^a\right)$

inverse powers of ν
[*Dunne, Kirsten*, hep-th/0607066]

- Explicit computation: Euler MacLaurin approximation

$$\Sigma_D\simeq\Sigma_D^{\int}+\Sigma_D^{\rm bnd}+R_p$$

$$\begin{cases}\Sigma_D^{\int}=\int_{\nu_0}^{\infty}{\rm d}\nu\,\sigma_D\\ \Sigma_D^{\rm bnd}=\frac{1}{2}\sigma_D(\nu_0)-\sum_{j=1}^{\lfloor\frac{p}{2}\rfloor}\frac{B_{2j}}{(2j)!}\sigma_D^{(2j-1)}(\nu_0)\end{cases}$$

# THIN WALL DECAY RATE

- Final results:

$$\frac{\Gamma}{\mathcal{V}} = \left(\frac{S_R}{2\pi\hbar}\right)^{\frac{D}{2}} \left|\frac{\det' \mathcal{O}}{\det \mathcal{O}_{\rm FV}}\right|^{-\frac{1}{2}} e^{-\frac{S_R}{\hbar}-S_{\rm ct}} \left(1+\mathcal{O}(\hbar)\right)$$

$$\downarrow$$

$$\boxed{\frac{\Gamma}{\mathcal{V}} \simeq \left(\left(\frac{S}{2\pi}\right)\frac{12}{e^{D-1}}\lambda v^2\right)^{D/2} \exp\left[-S-\frac{1}{\Delta^{D-1}}\begin{cases}\frac{20+9\ln 3}{54}\,, & D=3\,,\\ \frac{45-4\pi\sqrt{3}}{192}\,, & D=4\,,\end{cases}\right]}$$

$$S=\frac{1}{\Delta^{D-1}}\begin{cases}\frac{2^5\pi v}{3^4\sqrt{\lambda}}\left(1-\left(\frac{9\pi^2}{4}-1\right)\Delta^2\right)\,, & D=3\,,\\ \frac{\pi^2}{3\lambda}\left(1-\left(2\pi^2+\frac{9}{2}\right)\Delta^2\right)\,, & D=4\,.\end{cases}$$

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# SUMMARY & OUTLOOK

- We derived a closed-form false vacuum decay rate at one loop for a single real scalar field in the thin wall approximation, fully analytically: in the prefactor, we used the Gel'fand-Yaglom theorem to trade the ratio of determinants for a differential equation, which we then solved separately in the high- and low-multipole regimes.

What's next?

- Thin wall
  - Bounce action at $O(\Delta^4)$
  - Numerics for the prefactor
  - Inclusion of other fluctuations
- Applicability to more realistic BSM theories

- End goal: Gravitational Waves from 1° order phase transitions in the early universe

# THANK YOU!

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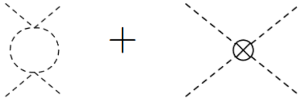
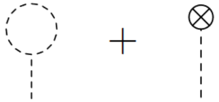
# RENORMALIZATION IN 4D

- Counterterms: start from quartic potential, later check that linear term does not modify the results



$$= 0\,, \qquad \Rightarrow \qquad \delta_\lambda = \frac{1}{(4\pi)^2\, 2\varepsilon} V^{(4)2} \,,$$

$$= 0\,, \qquad \text{iff} \qquad V^{(3)} = \langle\phi\rangle V^{(4)}$$



$$= 0\,, \qquad \delta_{m^2} = \frac{1}{(4\pi)^2\,\varepsilon} V^{(4)}\left(V^{(2)} - \frac{1}{2}V^{(4)}\langle\phi\rangle^2\right) \,,$$

$$= 0$$

$$V_{\text{ct}} = \frac{\delta_{m^2}}{2}\phi^2 + \frac{\delta_\lambda}{4}\phi^4 \quad \left\{ \begin{array}{l} \delta_\lambda = \dfrac{9\lambda^2}{(4\pi)^2\, 2\varepsilon} \\ \delta_{m^2} = -\dfrac{3\lambda^2 v^2}{(4\pi)^2\, 2\varepsilon} \end{array} \right. \quad \Longrightarrow \quad S_{\text{ct}} = \int_D \left(V_{\text{ct}} - V_{\text{ctFV}}\right) = \frac{3\lambda^2}{8\,(4\pi)^2\,\varepsilon}\int_D \left(3\left(\phi^4 - \phi^4_{\text{FV}}\right) - 2v^2\left(\phi^2 - \phi^2_{\text{FV}}\right)\right) \simeq -\frac{3}{16\varepsilon\Delta^3}$$

- Running: $0 = \mu\dfrac{d}{d\mu}\lambda_{\text{bare}} = \mu\dfrac{d}{d\mu}\left(\mu^\epsilon(\lambda_R + \delta_\lambda)\right) \Longrightarrow \dfrac{1}{\lambda} - \dfrac{1}{\lambda_0} = -\dfrac{9}{(4\pi)^2}\ln\mu/\mu_0$

# CANCELLATION OF $\varepsilon$ , $\mu$ IN $D = 4$

- Running and Renormalized action :

$$0 = \mu \frac{d}{d\mu} \lambda_{\text{bare}} = \mu \frac{d}{d\mu} \left( \mu^{\epsilon} (\lambda_R + \delta_\lambda) \right) \Longrightarrow \frac{1}{\lambda} - \frac{1}{\lambda_0} = -\frac{9}{(4\pi)^2} \ln \mu / \mu_0 \Longrightarrow S_R = \frac{\pi^2}{3\lambda(\mu)\Delta^3} = \frac{\pi^2}{3\Delta^3} \left( \frac{1}{\lambda_0} - \frac{9}{16\pi^2} \ln \boxed{\mu} / \mu_0 \right)$$

- Counterterm action:

$$V_{\text{ct}} = \frac{\delta_{m^2}}{2} \phi^2 + \frac{\delta_\lambda}{4} \phi^4 \left[ \begin{aligned} \delta_\lambda &= \frac{9\lambda^2}{(4\pi)^2 \, 2\varepsilon} \\ \delta_{m^2} &= -\frac{3\lambda^2 v^2}{(4\pi)^2 \, 2\varepsilon} \end{aligned} \right. \Longrightarrow S_{\text{ct}} = \int_D \left( V_{\text{ct}} - V_{\text{ctFV}} \right) = \frac{3\lambda^2}{8 \, (4\pi)^2 \, \varepsilon} \int_D \left( 3 \left( \phi^4 - \phi_{\text{FV}}^4 \right) - 2v^2 \left( \phi^2 - \phi_{\text{FV}}^2 \right) \right) \simeq -\frac{3}{16 \boxed{\varepsilon} \Delta^3}$$

- Renormalized sum in $D = 4$ :

$$\ln \left( \frac{\det \mathcal{O}}{\det \mathcal{O}_{\text{FV}}} \right) = \sum_\nu d_\nu \left( \ln R_\nu - \frac{1}{2\nu} I_1 + \frac{1}{8\nu^3} I_2 \right) - \frac{1}{8} \tilde{I}_2 \longleftarrow \tilde{I}_2 \ni I_2 \left( \frac{1}{\epsilon} + \ln \mu \right) = -\frac{3}{\Delta^3} \left( \frac{1}{\boxed{\epsilon}} + \ln \boxed{\mu} \right)$$

- Cancellation of ε and μ in the decay rate:

$$\ln \frac{\Gamma}{\mathcal{V}} \ni -S_R - S_{\text{ct}} - \frac{1}{2} \left( \Sigma_4 - \frac{\tilde{I}_2}{8} \right)$$

# EXPLICIT COMPUTATION OF RENORMALIZED SUM

- Explicit computation of the sum: Euler MacLaurin approximation $\Sigma_D \simeq \Sigma_D^{\int} + \Sigma_D^{\text{bnd}} + R_p$

$$\Sigma_D^{\int} = \int_{\nu_0}^{\infty} \mathrm{d}\nu\, \sigma_D \qquad\qquad \Sigma_D^{\text{bnd}} = \frac{1}{2}\sigma_D(\nu_0) - \sum_{j=1}^{\lfloor \frac{p}{2} \rfloor} \frac{B_{2j}}{(2j)!}\sigma_D^{(2j-1)}(\nu_0)$$

- $D = 4:$ $\ln\left(\frac{\det \mathcal{O}}{\det \mathcal{O}_{\text{FV}}}\right) = \sum_{\nu} d_\nu \left(\ln R_\nu - \frac{1}{2\nu} I_1 + \frac{1}{8\nu^3} I_2\right) - \frac{1}{8}\tilde{I}_2$

$$\begin{cases} I_1 = \int_0^\infty \mathrm{d}\rho\, \rho \left(V^{(2)} - V_{\text{FV}}^{(2)}\right) \simeq -3\,(2 - r_0)\left(\frac{r_0}{\Delta}\right) \\ I_2 = \int_0^\infty \mathrm{d}\rho\, \rho^3 \left(V^{(2)2} - V_{\text{FV}}^{(2)2}\right) = -3\,(2 - r_0)\left(\frac{r_0}{\Delta}\right)^3 , \\ \tilde{I}_2 = \int_0^\infty \mathrm{d}\rho\, \rho^3 \left(V^{(2)2} - V_{\text{FV}}^{(2)2}\right)\left(\frac{1}{\varepsilon} + \gamma_E + 1 + \ln\left(\frac{\mu\rho}{2}\right)\right) \\ \quad \simeq I_2 \left(\frac{1}{\varepsilon} + \gamma_E + 1 + \ln\left(\frac{\mu r_0}{2\sqrt{\lambda} v \Delta}\right)\right) . \end{cases}$$

# EXPLICIT COMPUTATION OF RENORMALIZED SUM

- Divergence in the boundary term for $\nu_0 \sim 1$ :

$$\Sigma_D^{\text{bnd}} = \frac{1}{2}\sigma_D(\nu_0) - \sum_{j=1}^{\lfloor \frac{p}{2} \rfloor} \frac{B_{2j}}{(2j)!}\sigma_D^{(2j-1)}(\nu_0)$$

$$\sigma_4^{(j)}(\nu_0) = 3(-)^{j+1} j!/(8\Delta^3 \nu_0^{j+1})$$

- Split the sum: $\Sigma_D = \Sigma_D^{\text{low}} + \Sigma_D^{\text{high}} = \sum_{\nu=\nu_0}^{\nu_1} \sigma_D + \sum_{\nu=\nu_1+1}^{\infty} \sigma_D$

$$\mathcal{O}(1) = \nu_0 \ll \nu_1 < 1/\Delta$$

- High sum: integral , low sum: explicit calculation

$$y = \frac{\Delta\nu}{r_0}$$

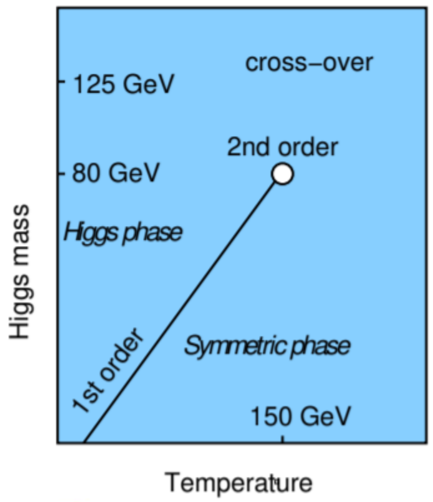
$$\Sigma_4^{\text{high}} \simeq \frac{1}{\Delta^3}\int_{y_1}^{\infty} \mathrm{d}y\, y^2 \left(\ln R_\nu + \frac{3}{2y} - \frac{3}{8y^3}\right) = \frac{3}{8\Delta^3}\left(\frac{9 - 4\sqrt{3}\pi}{36} + \ln 2y_1\right)$$

$$\Sigma_4^{\text{low}} = -\frac{3}{8\Delta^3}\sum_{\nu=1}^{\nu_1}\frac{1}{\nu} = -\frac{3}{8\Delta^3}H_{\nu_1} \simeq -\frac{3}{8\Delta^3}(\ln \nu_1 + \gamma_E)$$

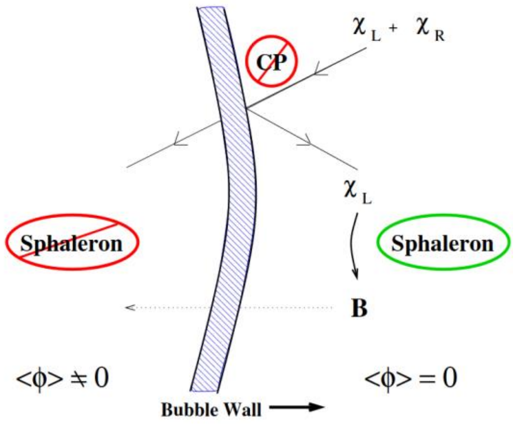
$$\Sigma_4 = \frac{3}{8\Delta^3}\left(\frac{9 - 4\sqrt{3}\pi}{36} - \gamma_E + \ln 2\Delta\right)$$

# ELECTROWEAK PHASE TRANSITION

- Electroweak phase transition (EWPT) $\sim 10^{-11} s$ , $\sim$100 GeV
- Crossover in SM, can be 1° order even in simplest extensions
- Possibility to produce the observed baryon asymmetry $\eta = \dfrac{n_B - n_{\overline{B}}}{n_\gamma} \sim 10^{-9}$ through baryogenesis.
- Sakharov conditions are fulfilled:
  - C and CP violation: particles scatter off the bubble walls, produce asymmetries in front of the walls.
  - Baryon number violation: asymmetries in the symmetric phase bias the EW sphaleron transitions to produce more baryons than antibaryons. Net baryon charge enters into the broken phase.
  - Out of equilibrium: bubble walls and sound shells disturb the symmetric-phase equilibrium state.



[Hindmarsh, Lüben, Lumma, Pauly, 2008.09136]

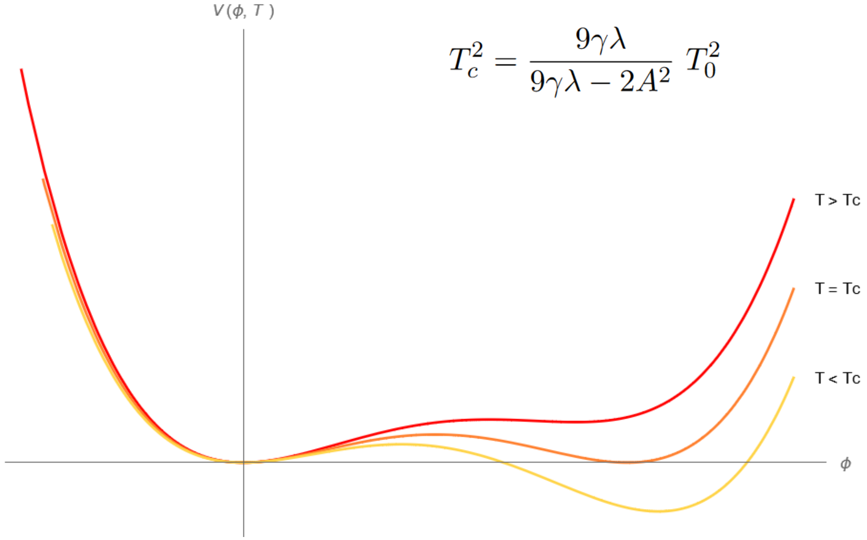


[Morrissey, Ramsey-Musolf, 1206.2942]

# BRIEF OVERVIEW ON BSM THEORIES

- Many different ways to introduce a potential barrier:
  - Tree level effects: $V(\phi,\chi) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 - \frac{M^2}{2}\chi^2 + \frac{\eta}{4}\chi^4 + \frac{\gamma}{2}\phi^2\chi^2$
  - SMEFT: add higher-dimensional operators: $V_{\text{tree}}(\phi) = \mu_h^2\,\phi^\dagger\phi + \lambda\,(\phi^\dagger\phi)^2 + \frac{1}{M^2}(\phi^\dagger\phi)^3$
  - Thermal effects: $V(\phi,T) = \frac{1}{2}\left(T^2 - T_0^2\right)\gamma\phi^2 - \frac{1}{3}AT\phi^3 + \frac{1}{4}\lambda\phi^4$
  - 1-loop effects (e.g. Coleman-Weinberg):

$$V_{CW}^i = (-1)^F g_i \frac{m_i^4}{64\pi^2}\left(\log\left[\frac{m_i^2}{\mu_R^2}\right] - c_i\right)$$



$$T_c^2 = \frac{9\gamma\lambda}{9\gamma\lambda - 2A^2}\,T_0^2$$

# STANDARD MODEL + SINGLET

*[Preliminary]*

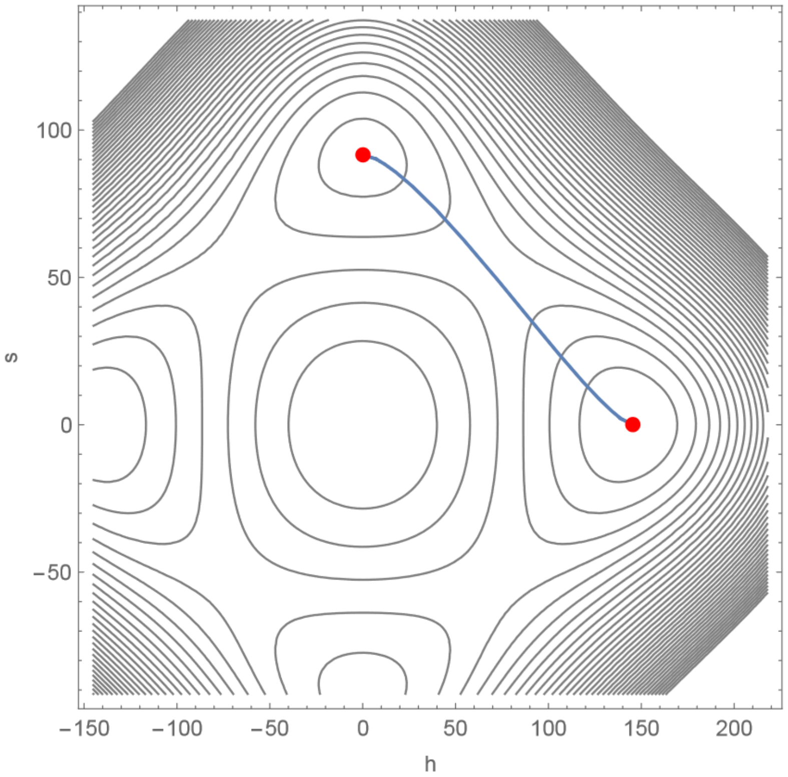
- Tree level potential: $V_{tree}(h, s) = -\frac{1}{2}\mu_h^2 h^2 + \frac{1}{2}\mu_s^2 s^2 + \frac{1}{4}\lambda_h h^4 + \frac{1}{4}\lambda_s s^4 + \frac{1}{4}\lambda h^2 s^2$
- High-temperature effective potential:

$$V(h, s, T) = \frac{1}{2}(c_h h^2 + c_s s^2)T^2 \qquad \left\{ \begin{aligned} c_h &= \frac{2M_W^2 + M_Z^2 + m_h^2 + 2m_t^2}{4v^2} + \frac{\lambda}{4!} \\ c_s &= \frac{2\lambda + 3\lambda_s}{12} \end{aligned} \right.$$

- 1° order PT with Higgs as TV:

$$\left\{ \begin{aligned} &\mu_h^2 = \lambda_h v_h^2(T) \ , \quad \mu_s^2(T) = -\lambda_s v_s^2(T) \ , \quad \lambda_h = \frac{m_h^2}{2v^2} \\ &v_h^2(T) = v^2 \left(1 - \frac{2c_h T^2}{m_h^2}\right) \ , \quad v_s^2(T) = \frac{1}{\lambda_s}\left(\frac{m_h}{2v} v_h^2(T)\sqrt{2\lambda_s} + c_s T_C^2 - c_s T^2\right) \end{aligned} \right.$$

Non-trivial bounce



# STANDARD MODEL + SINGLET

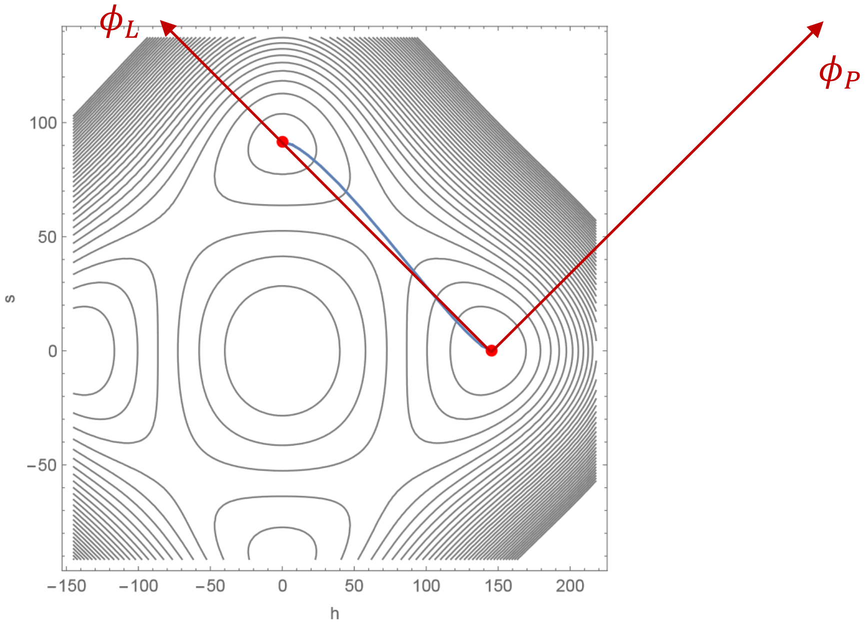
*[Preliminary]*

- Idea: translate to single field problem

Perform a global rotation $(h, s) \rightarrow (\phi_P, \phi_L)$

$$\phi_P = (h - v_h(T)) \cos\alpha + s \sin\alpha$$

$$\phi_L = -(h - v_h(T)) \sin\alpha + s \cos\alpha$$



- Insert the inverse relations into the SM+singlet potential

(first approximation: $\phi_P \approx 0$) ➡ Single field potential V($\phi_L$)

- Map the Thin Wall parameters by matching V($\phi_L$) and the TW potential:

$$\Delta \rightarrow \Delta(\alpha; T; SM)\ , \quad \lambda \rightarrow \lambda(\alpha; T; SM)\ , \quad v \rightarrow v(\alpha; T; SM)$$

# GRAVITATIONAL WAVES PROPAGATION

- Einstein equations: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$

linearized metric: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ , $|h_{\mu\nu}| \ll 1$

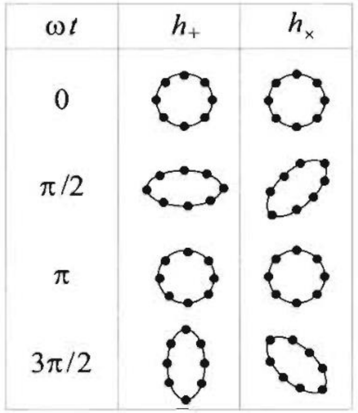
«trace reversed»: $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$

- Symmetry of GR: general coordinate transformations
- Lorentz gauge: $\partial^\nu \bar{h}_{\mu\nu} = 0$ ➡ Linearized equations (outside the source): $\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} = 0$
- Transverse-traceless gauge: $h^{0\mu} = 0$ , $h^i_i = 0$ , $\partial^j h_{ij} = 0$

$$\Big\downarrow$$

$$\boxed{h^{TT}_{ij}(t,z) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij} \cos\omega(t - z/c)}$$

- Interaction with test masses:



# ENERGY-MOMENTUM TENSOR OF GRAVITATIONAL WAVES

- To obtain stress-energy tensor of GWs: generic background $g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + h_{\mu\nu}(x)\ , \quad |h_{\mu\nu}| \ll 1$
- Einstein equations: $R_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$ separation of scales: $R_{\mu\nu} = \bar{R}_{\mu\nu} + R^{(1)}_{\mu\nu} + R^{(2)}_{\mu\nu} + \cdots$
- Low-frequency part:

$$\bar{R}_{\mu\nu} = -\left[R^{(2)}_{\mu\nu}\right]^{\text{low}} + \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\text{low}}$$

$$\Longrightarrow \bar{R}_{\mu\nu} = -\langle R^{(2)}_{\mu\nu}\rangle + \frac{8\pi G}{c^4}\langle T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\rangle \equiv -\langle R^{(2)}_{\mu\nu}\rangle + \frac{8\pi G}{c^4}\left(\bar{T}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{T}\right)$$

- Define:

$$t_{\mu\nu} = -\frac{c^4}{8\pi G}\langle R^{(2)}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}R^{(2)}\rangle \Longrightarrow \bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \frac{8\pi G}{c^4}\left(\bar{T}_{\mu\nu} + t_{\mu\nu}\right)$$

«Coarse-grained» Einstein equations: dynamics of the low-frequency part of the metric

- Energy-momentum tensor of GWs: $\boxed{t^{GW}_{\mu\nu} = -\frac{c^4}{32\pi G}\langle\partial_\mu h_{\alpha\beta}\partial_\nu h^{\alpha\beta}\rangle}$

# GRAVITATIONAL WAVES POWER SPECTRUM

- Energy momentum tensor of GWs: $T^{\mathrm{gw}}_{\mu\nu} = \frac{1}{32\pi G}\langle \partial_\mu h_{ij} \partial_\nu h_{ij} \rangle$

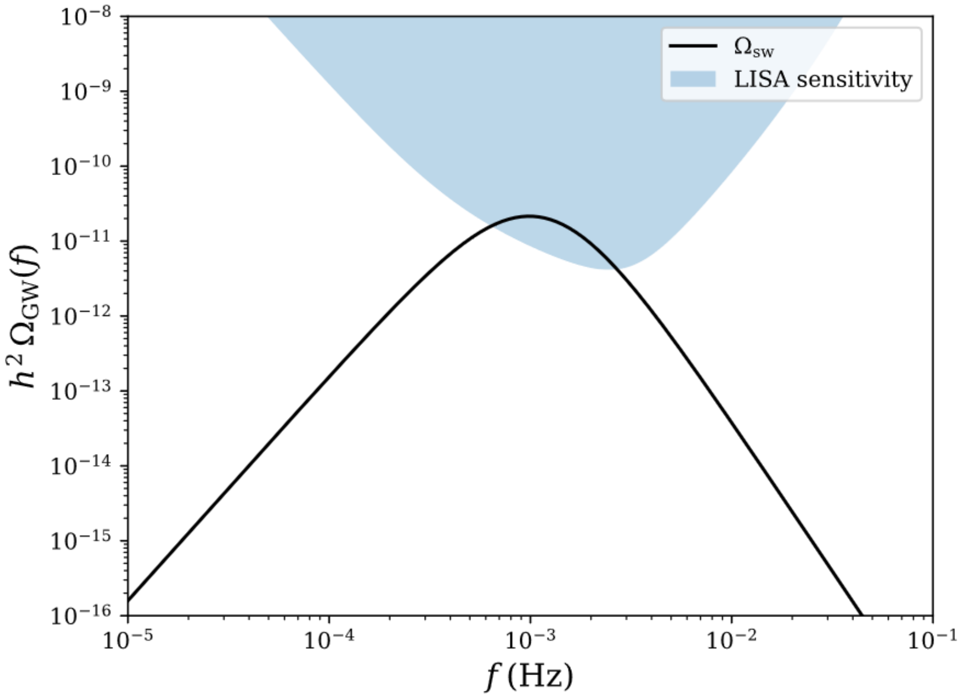
- Energy density: $\rho_{\mathrm{gw}} = \frac{1}{32\pi G}\langle \dot{h}_{ij}^2 \rangle$

$$\Omega_{\mathrm{gw}} = \frac{\rho_{\mathrm{gw}}}{\rho_{\mathrm{tot}}}$$

$$H^2 = \frac{8\pi G \rho_{\mathrm{tot}}}{3}$$

+ frequency space

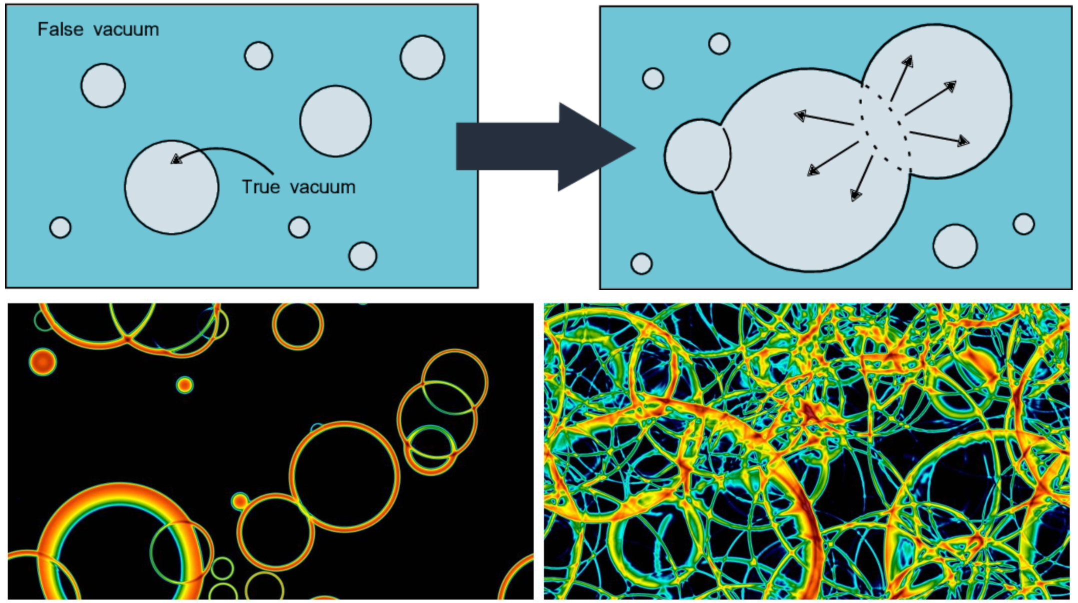
Power spectrum



[LISA Cosmology Working Group, 1910.13125]

# EARLY UNIVERSE – GWs

- Signature of a 1° order PT: Gravitational Waves
- Sources of GWs:
  - Bubble collisions
  - Sound waves in the fluid
  - Turbulence in the fluid



[*Weir*, 1705.01783]

- Early universe opaque to light, but transparent to GWs!
- 1° order PTs particularly interesting at Electroweak to TeV scales:
  - Stochastic background of GWs has peaks at frequencies accessible by future experiments
  - New physics related to e.g. baryogenesis

# EARLY UNIVERSE – P.T. PARAMETERS

- GW production depends on 4 parameters, determine length scale, amplitude, and lifetime. These parameters are in principle computable from the Lagrangian of a specific theory: potential to probe physics beyond the Standard Model!

- Bubble wall velocity: speed of the phase interface after nucleation in the rest frame of the plasma far from the wall, it impacts the energy budget.

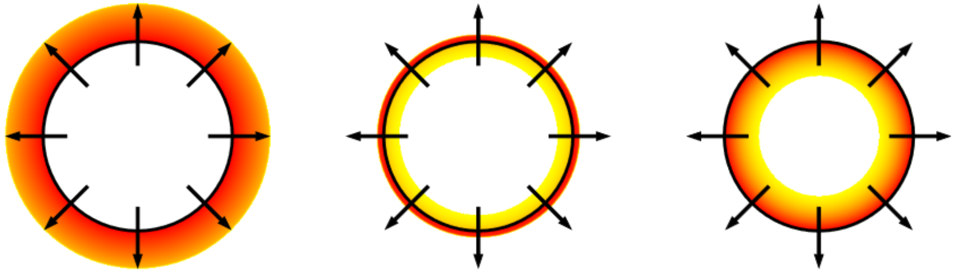
- Percolation temperature (or Hubble rate at percolation): successful completion of the PT. As the bubbles grow and more nucleate, the fraction of the Universe in the metastable phase decreases rapidly, leading to bubble percolation.

- Nucleation rate parameter at this temperature: determines the mean bubble separation (sound shells of this size are expected to carry the majority of the energy of the transition).

- Strength parameter at the nucleation temperature ($T$ at which, on average, 1 bubble nucleates per Hubble horizon): related to the scalar potential energy released during the PT.

# EARLY UNIVERSE – P.T. PARAMETERS

- **Bubble wall velocity**: out-of-equilibrium calculation, combination of Boltzmann and scalar field equations.



subsonic deflagration
$v_\mathrm{w} \leq c_s$

supersonic deflagration
$c_s < v_\mathrm{w} < c_\mathrm{J}$

detonation
$c_\mathrm{J} \leq v_\mathrm{w}$

[Hindmarsh, Lüben, Lumma, Pauly, 2008.09136]

- **Nucleation rate** parameter at percolation: its derivative can be thought of as inverse duration of the PT. From this we can calculate the mean bubble separation

$$\beta \equiv \frac{\mathrm{d}}{\mathrm{d}t} \log\left(\frac{\Gamma(t)}{\mathcal{V}}\right)\Bigg|_{t=t_f}$$

$$R_* \propto \frac{\mathrm{Max}(v_w, c_s)}{\beta}$$

- **Strength parameter** at the nucleation temperature: the expanding bubble converts potential energy of the scalar field into kinetic energy and heat.

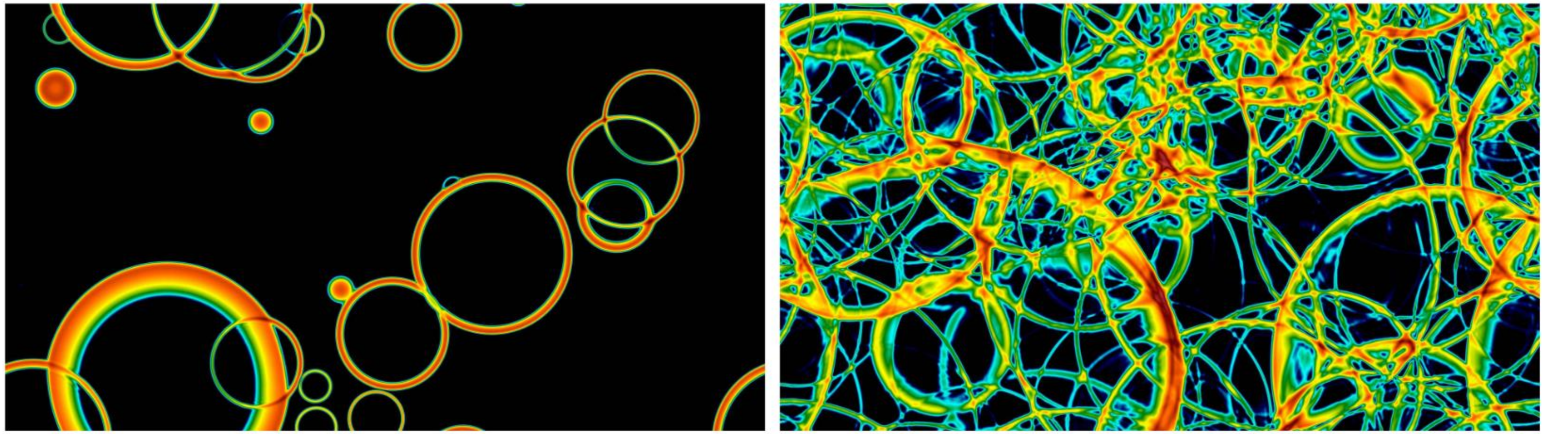
$$\alpha_e \equiv \frac{4}{3}\frac{\Delta e(T_\mathrm{n})}{w_+(T_\mathrm{n})}$$

# EARLY UNIVERSE – GW PRODUCTION

- Simulations for bubble expansion and collision. 3 stages of GW production:
- Bubbles collision and merger: short duration (usually subdominant);
- Acoustic stage: shells of fluid kinetic energy continue to expand into the plasma as sound waves, overlap and source gravitational waves (believed to be dominant);
- Turbulent phase: non-linearity in the fluid equations becomes important, the previous phases might produce turbulence (not well-understood).

- Example of simulation:

[Weir, 1705.01783 ]



# COUPLED FIELD – FLUID MODEL

- Potential & eq. of state $V(\phi,T)=\frac{1}{2}\left(T^2-T_0^2\right)\gamma\phi^2-\frac{1}{3}AT\phi^3+\frac{1}{4}\lambda\phi^4$ , $\epsilon(T,\phi)=3aT^4+V(\phi,T)-T\frac{\partial V}{\partial T}$

$$p(T,\phi)=aT^4-V(\phi,T)$$

- Energy-momentum tensor:

$$T^{\mu\nu}=\partial^\mu\phi\partial^\nu\phi-\tfrac{1}{2}g^{\mu\nu}(\partial\phi)^2+[\epsilon+p]\,U^\mu U^\nu+g^{\mu\nu}p$$

$$\left\{\begin{aligned}&[\partial_\mu T^{\mu\nu}]_{\text{field}}=(\partial_\mu\partial^\mu\phi)\partial^\nu\phi-\frac{\partial V}{\partial\phi}\partial^\nu\phi=\delta^\nu\\&[\partial_\mu T^{\mu\nu}]_{\text{fluid}}=\partial_\mu[(\epsilon+p)U^\mu U^\nu]-\partial^\nu p+\frac{\partial V}{\partial\phi}\partial^\nu\phi=-\delta^\nu\end{aligned}\right.\qquad\longleftarrow\qquad\delta^\nu=\eta U^\mu\partial_\mu\phi\partial^\nu\phi$$

[Hindmarsh, Huber, Rummukainen, Weir, 1504.03291]

- Numerical simulations: $U^i=WV^i$ , $E=W\epsilon$ , $Z_i=W(\epsilon+p)U_i$

$$\left\{\begin{aligned}&-\ddot\phi+\nabla^2\phi-\frac{\partial V}{\partial\phi}=\eta W(\dot\phi+V^i\partial_i\phi)\\&\dot E+\partial_i(EV^i)+p[\dot W+\partial_i(WV^i)]-\frac{\partial V}{\partial\phi}W(\dot\phi+V^i\partial_i\phi)\\&\qquad\qquad=\eta W^2(\dot\phi+V^i\partial_i\phi)^2\\&\dot Z_i+\partial_j(Z_iV^j)+\partial_ip+\frac{\partial V}{\partial\phi}\partial_i\phi=-\eta W(\dot\phi+V^j\partial_j\phi)\partial_i\phi\end{aligned}\right.$$

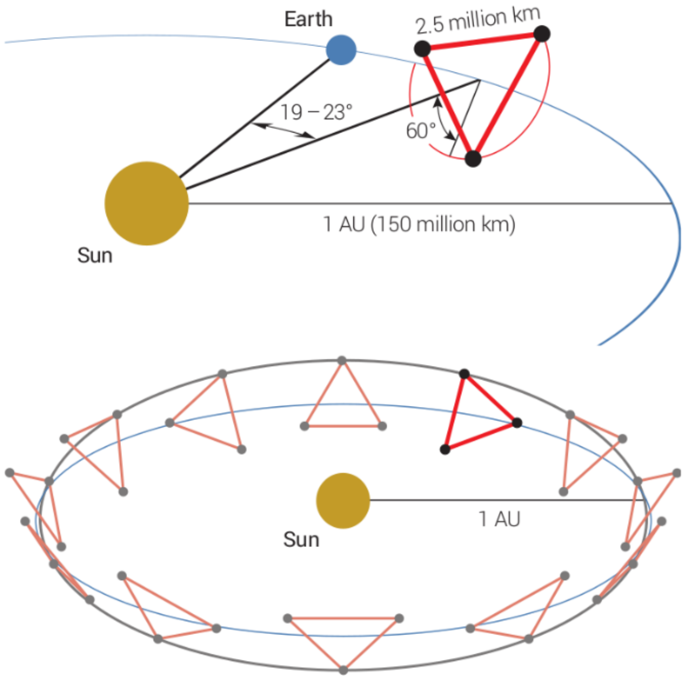
➡ GWs: $\tau^\phi_{ij}=\partial_i\phi\partial_j\phi,\quad\tau^{\text{f}}_{ij}=W^2(\epsilon+p)V_iV_j$

$$h_{ij}(\mathbf{k},t)=(16\pi G)\lambda_{ij,kl}(\mathbf{k})\int_0^t dt'\frac{\sin[k(t-t')]}{k}\tau_{kl}(\mathbf{k},t')$$

# LISA MISSION

- Laser Interferometer Space Antenna
- ESA – expected to launch in 2030s
- 3 satellites orbiting Earth, arms of 2.5Gm
- Lasers and photodetectors which detect small changes in separation through time delays of signals

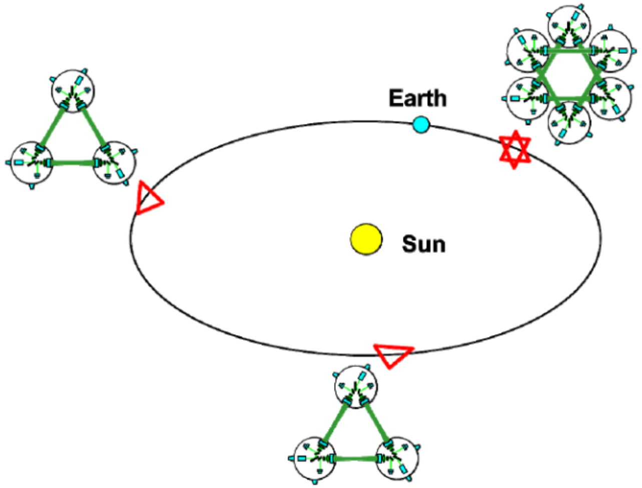
- Most sensitive in the range $10^{-3} - 10^{-2}\ Hz$

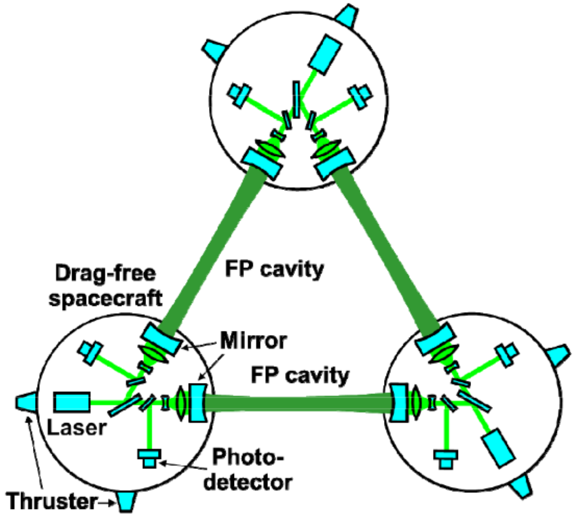


[Amaro-Seoane et al., 1702.00786]

# DECIGO MISSION

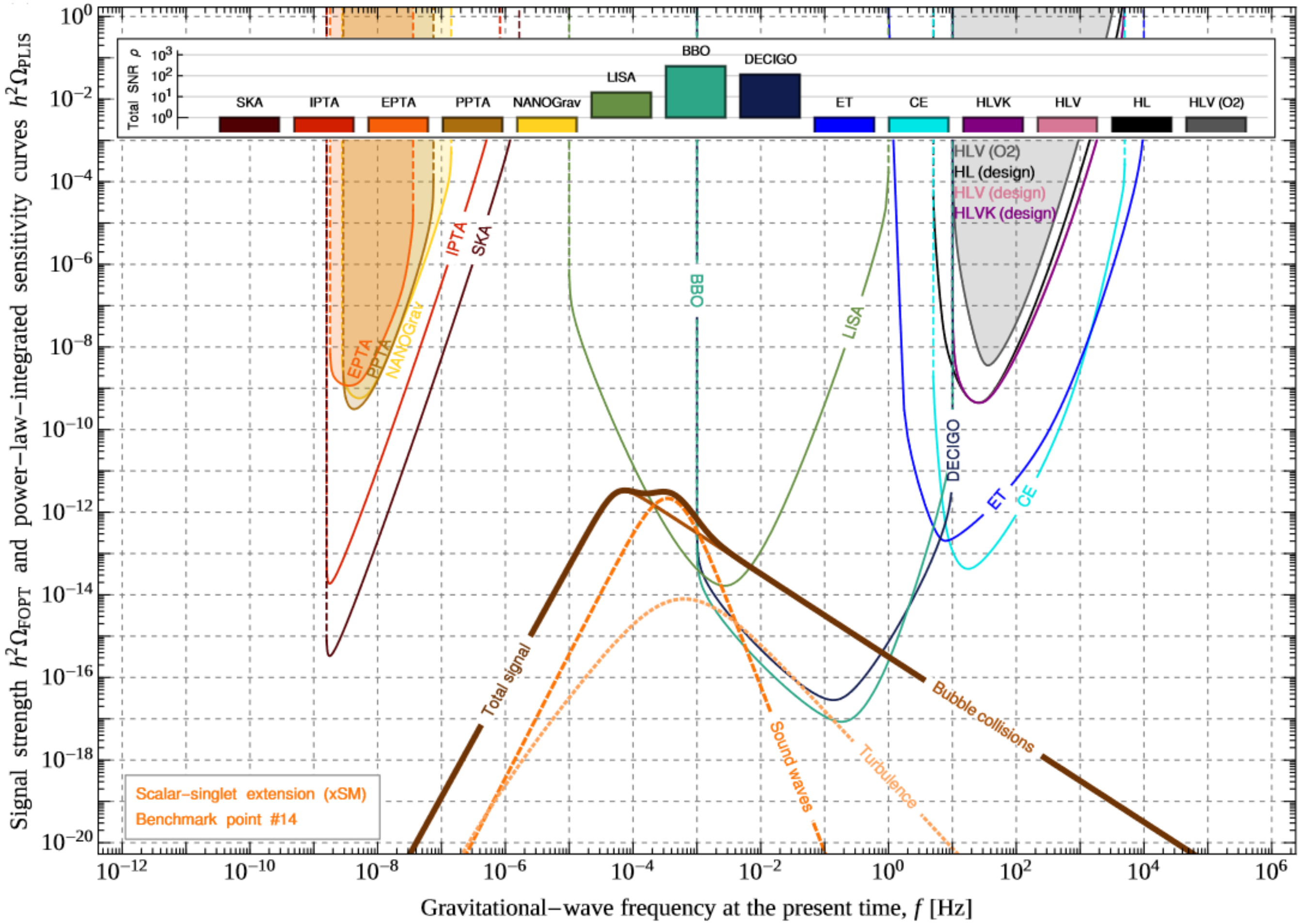
- Deci-hertz Interferometer Gravitational Wave Observatory
- Japanese project – expected to launch in 2030s
- Four clusters of observatories placed in the heliocentric orbit.
- Each cluster: three spacecraft, which form three Fabry-Perot Michelson interferometers with an arm length of 1,000 km
- Most sensitive in the range $0.1 - 10\ Hz$





[Kawamura, Ando, Seto, Sato, Musha et al., 2006.13545]

# AN EXAMPLE OF SIGNAL



[Schmitz, 2002.04615 ]