Z-boson decay at the NNNLO level

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Outline

1. Standard Model corrections to EWPOs, present status
2. Towards 3-loop SM EWPOs studies
3. Numerical calculations - Methods and tools towards 3-loops
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Published results on EWPOs in the SM @NNLO

Complete corrections $\Delta r, \sin^2\theta^l_{\mathrm{eff}}$:	Freitas, Hollik, Walter, Weiglein: '00
	Awramik,Czakon: '02,Onishchenko,Veretin: '02
	Awramik,Czakon,Freitas,Weiglein: '04
	Awramik,Czakon,Freitas: '06
	Hollik,Meier,Uccirati: '05,'07
	Degrassi,Gambino,Giardino: '14
Fermionic corrections $\sin^2\theta^b_{\mathrm{eff}}, a_f, v_f$:	Awramik,Czakon,Freitas,Kniehl: '09
	Czarnecki,Kühn: '96
	Harlander,Seidensticker,Steinhauser: '98
	Freitas: '13,'14 Freitas: '13,'14
Bosonic corrections: $\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}}$:	Dubovyk, Freitas, JG, Riemann, Usovitsch '16
Bosonic corrections: $\Gamma_Z, R_l, \ldots$:	Dubovyk, Freitas, JG, Riemann, Usovitsch '18,'19
Leading fermionic 3-loops: $M_W, \sin^2\theta^b_{\mathrm{eff}}, \Gamma_Z$:	Chen, Freitas: '20

Higher Order Corrections, Tests

Tables generated with DIZET (v6.45), publically available at the ZFITTER webpage. For LHC needs, it is also used by KKMC generator [S.Jadach et al, 2022].

IHVP and IAMT4 are flags for hadronic vacuum polarization and EW corrections respectively. [A.B. Arbuzov et al, 2006]

1 - [S. Eidelman, F. Jegerlehner, 1995], 5 - [F. Jegerlehner, 2017]

6 - [M. Awramik et al, 2004] , 8 - [I. Dubovyk et al, 2019]

IHVP, IAMT4	1,8	5,6	5,8	$\lvert\delta_{(1,8)-(5,8)}\rvert$	$\lvert\delta_{(5,6)-(5,8)}\rvert$
channel					
$\Gamma_{\nu,\bar{\nu}}$, MeV	167.202	167.202	167.202	0	0
Γ_{e^+,e^-}, MeV	83.977	83.984	83.985	0.008	0.001
Γ_{μ^+,μ^-}, MeV	83.977	83.983	83.985	0.008	0.002
Γ_{τ^+,τ^-}, MeV	83.787	83.794	83.795	0.008	0.001
Γ_{hadron}, MeV	1741.039	1741.268	1741.442	0.403	0.174
Γ_Z, MeV	2494.387	2494.636	2494.814	0.427	0.178

Input parameters dependence

m_t, GeV	$172.76 - 0.30$	172.76	$172.76 + 0.30$	Diff.
$\Gamma_Z(\mu\mu)$, MeV	83.982	83.985	83.987	0.005
Γ_Z, MeV	2494.746	2494.814	2494.883	0.137
$\Gamma_W(l\nu)$, MeV	678.935	678.981	679.027	0.092
Γ_W, MeV	2089.825	2089.967	2090.109	0.284
$\sin^2\theta^l_{eff} \times 10^6$	231508	231500	231491	17

M_H, GeV	$125.25 - 0.17$	125.25	$125.25 + 0.17$	Diff.
$\Gamma_Z(\mu\mu)$, MeV	83.985	83.985	83.985	0
Γ_Z, MeV	2494.818	2494.814	2494.811	0.007
$\Gamma_W(l\nu)$, MeV	678.983	678.981	678.979	0.004
Γ_W, MeV	2089.973	2089.967	2089.961	0.012
$\sin^2\theta^l_{eff} \times 10^6$	231499	231500	231500	1

NNNLO Standard Model Corrections

$Z \to b\bar{b}$			
Number of topologies	1 loop	2 loops	3 loops
	1	5	50
Number of diagrams	15	1114	120187
Fermionic loops	0	150	**17580**
Bosonic loops	15	964	102607
QCD / EW	1 / 14	98 / 1016	10405 / 109782

Table: The number of Z-decay Feynman diagrams needed to be calculated to meet Tera-Z experimental accuracy. Tadpoles, products of lower loop diagrams and symmetrical diagrams are not included.

$\mathcal{O}(10^3)$ 3-loop self-energy integrals to be calculated at first.

Methods and tools

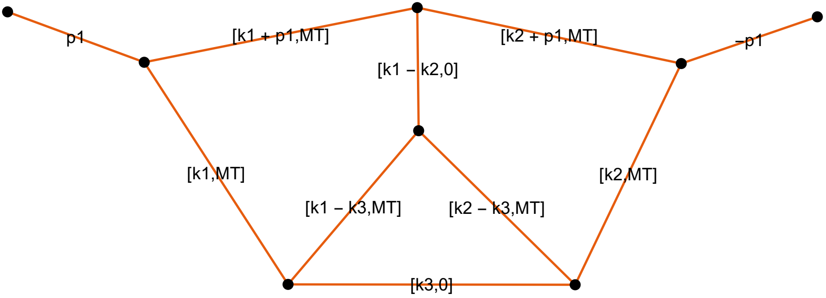
- Sector decomposition (SD) method:
  - FIESTA5 [2012], [A.V.Smirnov]
  - pySecDec [2022], [Expansion by regions with pySecDec],
- The Mellin-Barnes (MB) method:
  - MB [M.Czakon, 2006]
  - MBnumerics [J.Usovitsch, I.Dubovyk, T.Riemann, 2015] - Minkowskian kinematics
- Differential equations (DEqs) method:
  - `DiffExp` [F. Moriello, 2019; M. Hidding, 2021],
  - `AMFlow` [X. Liu, Y.-Q. Ma, 2022],
  - `SeaSyde` [T. Armadillo, R. Bonciani, S. Devoto, N. Rana, A. Vi, 2022]

NNLO Z-pole SM completed: 10^{-8} accuracy achieved for most of Feynman integrals. Final result with 10^{-4} accuracy.

SD + MB - not enough at NNNLO, can only cover a small part of calculations

pySecDec

- Minkowskian kinematics ($M_Z^2 = 1 = s$)

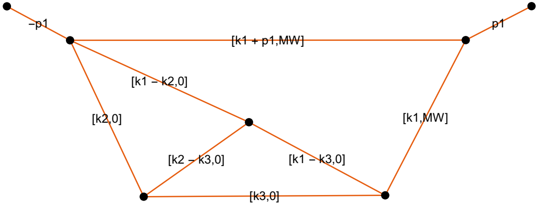


$$0.025510249860 \pm 6.23 \times 10^{-10}.$$

MB

- Minkowskian kinematics ($M_Z^2 = 1 = s$)

$$I \sim \int_{-i\infty}^{+i\infty} \prod_{i=1}^{4} dz_i \left(-\frac{M_W^2}{s}\right)^{z_3} \frac{\Gamma(-\epsilon - z_1)\Gamma(-z_1)\Gamma(1+2\epsilon+z_1)}{\Gamma(1-2\epsilon)\Gamma(1-3\epsilon-z_1)} \dots$$

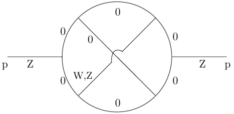


Method	Result	Absolute error
MBnumerics	-18.779406962-6.390785027 i	$10^{-9} + 10^{-9}$ i
pySecDec	-18.787167067-6.384327811 i	0.0093+0.0097 i

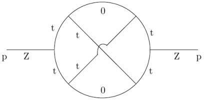
MIs with high accuracy, results*

*Results for 3-loop EWPOs at the e^+e^- Z-resonance peak,

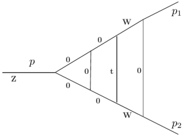
I. Dubovyk, A. Freitas, J. Gluza, KG, M. Hidding, J. Usovitsch, ‘Evaluation of multi-loop multi-scale Feynman integrals for precision physics’, 2201.02576



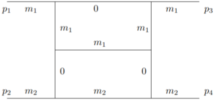
lhNp1



taNp1



vtwPl

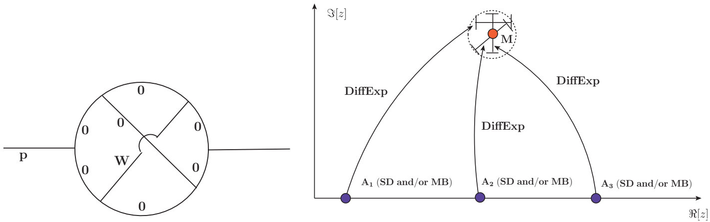


box2l

$$
\begin{aligned}
I_{\mathbf{box2l}}[2,1,1,1,1,1,1,0,0,s,t,m_1^2,m_2^2] \quad = \quad & +0.000328707579/\epsilon^2 \\
- \quad & (0.0014129475 - 0.0020653306\ i)/\epsilon \\
- \quad & (0.005702737 - 0.000485980\ i) + \mathcal{O}(\epsilon), \\
& 55 \text{ MIs}, s = 2, t = 5, m_1^2 = 4, m_2^2 = 16.
\end{aligned}
$$

DEqs

- Minkowskian kinematics ($M_Z^2 = 1 = s$)



$$I_{pySecDec} = 0.460 - 19.164 \cdot I \pm (0.298 + 0.281 \cdot I)$$

$$I_{DEqs} = 0. - 19.12623029881384 4 \cdot I$$

$$I_{AMflow} = 0. - 19.1262302990801 \cdot I$$

Summary and outlook

- Progress in development of methods and tools to tackle 3-loop Z resonance phycics
- Knowledge of the higher order SM radiative corrections in connection with future precision experimental measurements will be instrumental for BSM searching.