Starobinsky-Type $B-L$ Higgs Inflation Leading Beyond MSSM

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Based on:

- *C.P. and Q. Shafi*, *Eur. Phys. J. C* **78**, no. 1, 13 (2018) [arXiv:1803.00349].

Outline

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Coupling non-Minimally the Inflaton to Gravity

• Our Starting Point is The Action in the **Jordan Frame (JF)** Of A Scalar Field ϕ with Potential $V(\phi)$ non-Minimally Coupled to the Ricci Scalar Curvature, $\mathcal{R}$, Through A Frame Function $f_{\mathcal{R}}(\phi)$. This is:

$$\mathcal{S}=\int d^4x\sqrt{-\mathfrak{g}}\left(-\frac{1}{2}f_{\mathcal{R}}(\phi)\mathcal{R}+\frac{f_{\rm K}(\phi)}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)\right),\quad \text{Where}$$

$\mathfrak{g}$ is the Determinant Of The Background Metric and $f_{\mathcal{R}}(\langle\phi\rangle)\simeq 1$ (in Reduced Planck Units With $m_{\rm P}=1$) to Guarantee the Ordinary **Einstein Gravity** At Low Energy. We Allow for a Kinetic Mixing Through the Function $f_{\rm K}(\phi)$.

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• If we Perform a **Conformal Transformation**[1] According Which We Define the EF Metric:

$$\widehat{g}_{\mu\nu}=f_{\mathcal{R}}\,g_{\mu\nu}\quad\Rightarrow\quad\begin{cases}\sqrt{-\widehat{\mathfrak{g}}}=f_{\mathcal{R}}^2\sqrt{-\mathfrak{g}}\quad\text{and}\quad \widehat{g}^{\mu\nu}=g^{\mu\nu}/f_{\mathcal{R}},\\ \widehat{\mathcal{R}}=\left(\mathcal{R}+3\Box\ln f_{\mathcal{R}}+3g^{\mu\nu}\partial_\mu f_{\mathcal{R}}\partial_\nu f_{\mathcal{R}}/2f_{\mathcal{R}}^2\right)/f_{\mathcal{R}}\end{cases}$$

We End up with the Action $\mathcal{S}$ in the **Einstein Frame (EF)**

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Where we Introduce the **EF Canonically Normalized Field**, $\widehat{\phi}$, and **Potential**, $\widehat{V}$, Defined As Follows:

$$\left(\frac{d\widehat{\phi}}{d\phi}\right)^2=J^2=\frac{f_{\rm K}}{f_{\mathcal{R}}}+\frac{3}{2}\left(\frac{f_{\mathcal{R},\phi}}{f_{\mathcal{R}}}\right)^2\quad\text{and}\quad\widehat{V}(\widehat{\phi})=\frac{V\left(\widehat{\phi}(\phi)\right)}{f_{\mathcal{R}}\left(\widehat{\phi}(\phi)\right)^2}.$$

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• We Observe that $f_{\mathcal{R}}$ Affects **Both** J and $\widehat{V}_{\rm HI}$ whereas $f_{\rm K}$ only J;
• Obviously a clever Choice of V and $f_{\mathcal{R}}$ Can Lead to A **Plateau** Convenient for Drive Inflation;
• The Analysis of Inflation in **the EF** Using The Standard Slow-Roll Approximation is Equivalent With **The Analysis in JF**.

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Inflationary Observational and Theoretical Requirements

In the Era of Precision Cosmology The Inflationary Particle Models Can Be Tightly Restricted Imposing the Constraints:

- The **Number of e-foldings**, $\widehat{N}_\star$, that the Scale $k_\star = 0.05/\mathrm{Mpc}$ Suffers During inflation has to be Sufficient to Resolve the Horizon and Flatness Problems of Standard Big Bang:

$$\widehat{N}_\star = \int_{\widehat{\phi}_{\rm f}}^{\widehat{\phi}_\star} d\widehat{\phi}\, \frac{\widehat{V}}{\widehat{V}_{,\widehat{\phi}}} = \int_{\phi_{\rm f}}^{\phi_\star} d\phi\, J^2\, \frac{\widehat{V}}{\widehat{V}_{,\phi}} \simeq 50-60$$

  Where $\widehat{V}$ is the EF scalar potential of the Inflaton ϕ;
  $\phi_\star$ $[\widehat{\phi}_\star]$ is The Value of ϕ $[\widehat{\phi}]$ **When $k_\star$ Crosses Outside** The Inflationary Horizon;
  $\phi_{\rm f}$ $[\widehat{\phi}_{\rm f}]$ is the Value of ϕ $[\widehat{\phi}]$ at the **End of Inflation** Which Can Be Found From The Condition:

$$\max\{\widehat{\epsilon}(\phi_{\rm f}), |\widehat{\eta}(\phi_{\rm f})|\} = 1, \quad \text{With } \widehat{\epsilon} = \frac{1}{2}\left(\frac{\widehat{V}_{,\widehat{\phi}}}{\widehat{V}}\right)^2 = \frac{1}{2J^2}\left(\frac{\widehat{V}_{,\phi}}{\widehat{V}}\right)^2 \text{ and } \widehat{\eta} = \frac{\widehat{V}_{,\widehat{\phi}\widehat{\phi}}}{\widehat{V}} = \frac{1}{J^2}\left(\frac{\widehat{V}_{,\phi\phi}}{\widehat{V}} - \frac{\widehat{V}_{,\phi}}{\widehat{V}}\frac{J_{,\phi}}{J}\right).$$

- The **Amplitude $A_{\rm s}$ of the Power Spectrum** of the Curvature Perturbations is To Be Normalized With ***Planck*** Data:

$$\sqrt{A_{\rm s}} = \frac{1}{2\sqrt{3}\,\pi}\, \frac{\widehat{V}(\widehat{\phi}_\star)^{3/2}}{|\widehat{V}_{,\widehat{\phi}}(\widehat{\phi}_\star)|} = \frac{|J(\phi_\star)|}{2\sqrt{3}\,\pi}\, \frac{\widehat{V}(\phi_\star)^{3/2}}{|\widehat{V}_{,\phi}(\phi_\star)|} = 4.588\cdot 10^{-5}\,.$$

[2] C.P. Burgess et al. (2009); J.F. Barbon and J.R. Espinosa (2009); R. Lerner and J. McDonald (2010); A. Kehagias et al. (2013).

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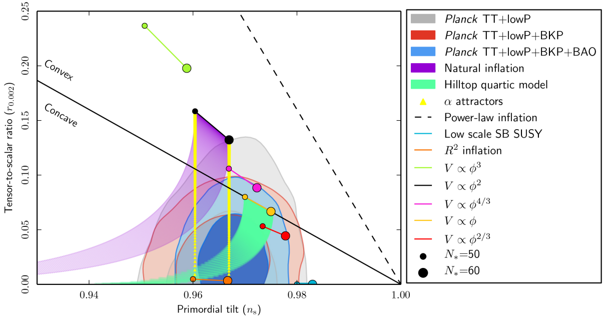
- The Effective Theory has to be Valid. I.e., The Hierarchy Between The **Inflationary Scale**, $\widehat{V}(\phi_*)^{1/4}$, And **The Ultraviolet Cut-off**[2], $\Lambda_{\rm UV} \simeq 1 = m_{\rm P}$, of the Effective Theory has to be the Following:

$$\text{(a)}\ \ \widehat{V}(\phi_*)^{1/4} \le 1 \ \ \text{for} \ \ \text{(b)}\ \ \phi \le 1$$

[2] C.P. Burgess et al. (2009); J.F. Barbon and J.R. Espinosa (2009); R. Lerner and J. McDonald (2010); A. Kehagias et al. (2013).

Constraints for the ΛCDM + r Model From Bicep2/*Keck Array* and *Planck* 2018



• Inflationary Models Which Succeed to Fit the Observational Data on A_s and $\widehat{N}_\star$ Can Be Further Restricted If We Calculate The (Scalar) **Spectral Index** And **Tensor-to-Scalar Ratio** Found Respectively As:

$$n_s = 1 - 6\widehat{\epsilon}(\phi_\star) + 2\widehat{\eta}(\phi_\star) \quad \text{and} \quad r = 16\widehat{\epsilon}(\phi_\star)$$

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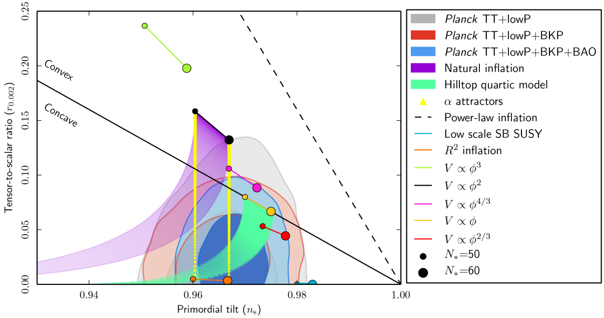

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- The Combined Bicep2/*Keck Array* 2021 and *Planck* 2018 Results Yield

$$n_s = 0.965 \pm 0.009 \;\Rightarrow\; 0.956 \lesssim n_s \lesssim 0.974 \quad \text{and} \quad r \lesssim 0.032 \quad \text{at } 95\%\text{c.l.}$$

- R^2 Inflation (Or **Starobinsky** Inflation) **Predicts $n_s \simeq 0.964$ and $r = 0.003$** for $\widehat{N}_\star \simeq 52$.
- As A Consequence, the Starobinsky Inflation Remains One of the **Most Predictive and Successful** Models.

From Non-Minimal to $\mathcal{R}^2$ Inflation

- The $\mathcal{R}^2$ Inflation can be Introduced as a Type of nMI Employing an **Auxiliary** (Non-Propagating) Field ϕ With

$$f_{\rm K}=0,\quad f_{\mathcal{R}}=1+4c_{\mathcal{R}}\phi \quad \text{And} \quad V=\phi^2$$

Using The Equation Of Motion $\phi=c_{\mathcal{R}}\mathcal{R}$ We Obtain The Action Of The **Original** Model:

$$\mathcal{S}=\int d^4x\sqrt{-\mathfrak{g}}\left(-\frac{1}{2}\mathcal{R}+c_{\mathcal{R}}^2\mathcal{R}^2\right).$$

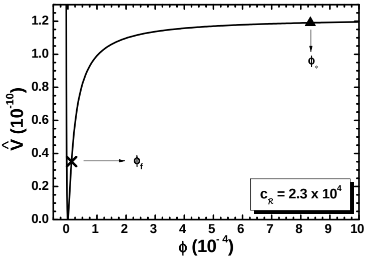
- Applying the Standard Formulae, we Find $J=2\sqrt{6}c_{\mathcal{R}}/f_{\mathcal{R}}$,

$$\widehat{V}=\frac{\phi^2}{f_{\mathcal{R}}^2}\simeq\frac{1}{16c_{\mathcal{R}}^2},\ \widehat{\epsilon}\simeq\frac{1}{12c_{\mathcal{R}}^2\phi^2}\quad \text{and}\quad \widehat{\eta}\simeq\frac{1-4c_{\mathcal{R}}\phi}{12c_{\mathcal{R}}^2\phi^2}.$$

  - Therefore, $\max\{\widehat{\epsilon}(\phi_{\rm f}),|\widehat{\eta}(\phi_{\rm f})|\}=1\ \Rightarrow\ \phi_{\rm f}=\dfrac{1}{2\sqrt{3}c_{\mathcal{R}}}.$

$$\widehat{N}_\star\simeq 3c_{\mathcal{R}}\phi_\star\ \Rightarrow\ \phi_\star=\frac{\widehat{N}_\star}{3c_{\mathcal{R}}}\gg\phi_{\rm f}.\ \text{For}\ \widehat{N}_*\simeq 52\ \text{We Get}$$

  - $A_{\rm s}^{1/2}\simeq\dfrac{\widehat{N}_\star}{12\sqrt{2}\pi c_{\mathcal{R}}}\simeq 4.6\cdot10^{-5}\ \Rightarrow\ \underline{c_{\mathcal{R}}\simeq 2.3\cdot10^4}.$



- $n_{\rm s}\simeq 1-2/\widehat{N}_\star\simeq 0.965,\ \alpha_{\rm s}\simeq -2/\widehat{N}_\star^2\simeq -6.4\cdot10^{-4}$ And $r\simeq 12/\widehat{N}_\star^2\simeq 4\cdot10^{-3}$ **(In Agreement With Observations).**

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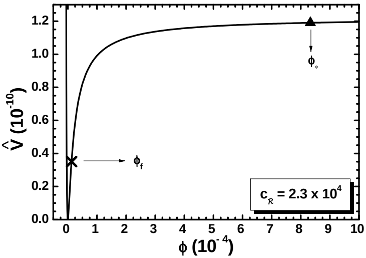
$$\widehat{N}_\star\simeq 3c_{\mathcal{R}}\phi_\star\ \Rightarrow\ \phi_\star=\frac{\widehat{N}_\star}{3c_{\mathcal{R}}}\gg\phi_{\rm f}.$$ For $\widehat{N}_*\simeq 52$ We Get

- $$A_{\rm s}^{1/2}\simeq\frac{\widehat{N}_\star}{12\sqrt{2}\pi c_{\mathcal{R}}}\simeq 4.6\cdot 10^{-5}\ \Rightarrow\ \underline{c_{\mathcal{R}}\simeq 2.3\cdot 10^4}.$$



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- There is **No Problem with Perturbative Unitarity**, Since We Obtain $\Lambda_{\rm UV}=1$ If we Perform Expansions Around $\langle\phi\rangle=0$:

$$J^2\dot{\phi}^2=\left(1-2\sqrt{\frac{2}{3}}\frac{\widehat{\phi}}{m_{\rm P}}+2\frac{\widehat{\phi}^2}{m_{\rm P}^2}-\cdots\right)\dot{\widehat{\phi}}^2 \quad \text{and} \quad \widehat{V}=\frac{\widehat{\phi}^2}{24c_{\mathcal{R}}^2}\left(1-2\sqrt{\frac{2}{3}}\frac{\widehat{\phi}}{m_{\rm P}}+2\frac{\widehat{\phi}^2}{m_{\rm P}^2}-\cdots\right)$$ With $\widehat{\phi}=2\sqrt{3}c_{\mathcal{R}}\phi$

- The **Mass of the Inflaton** at the Vacuum is: $\widehat{m}_{\delta\phi}=\left\langle\widehat{V}_{,\widehat{\phi}\widehat{\phi}}\right\rangle^{1/2}=\left\langle\widehat{V}_{,\phi\phi}/J^2\right\rangle^{1/2}=1/2\sqrt{3}c_{\mathcal{R}}\simeq 1.25\cdot 10^{-5}$ (i.e. $3\cdot 10^{13}$ GeV).

Induced-Gravity Inflation (IGI)

- It Would Be Certainly Beneficial to Obtain STI Avoiding Drastic Deviations from Einstein Gravity, at Least at Present.

This Can be Achieved Introducing the Idea of **Induced Gravity**.

- IGI Can Be Realized Employing an **Real-Propagating** Field ϕ If We Adopt The Following Ingredients:

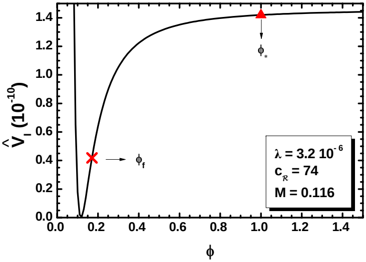
$$\text{(a)}\ f_{\rm K}=1,\quad \text{(b)}\ f_{\mathcal R}=c_{\mathcal R}\phi^2\ \text{ and }\ \text{(c)}\ V=\lambda\left(\phi^2-M^2\right)^2/4\,.$$

- **Recovering Einstein Gravity** at the Vacuum Implies

$$f_{\mathcal R}(\langle\phi\rangle)=1\ \Rightarrow\ \langle\phi\rangle\overset{\rm (b)}{=}1/\sqrt{c_{\mathcal R}}\overset{\rm (c)}{\simeq}M.$$

- For $c_{\mathcal R}\gg 1$ and Defining $f_\phi=1-c_{\mathcal R}\phi^2$ We Find $J=\sqrt{6}/\phi$,

$$\widehat V_{\rm I}=\frac{\lambda f_\phi^2}{4f_{\mathcal R}^2}\simeq\frac{\lambda}{4c_{\mathcal R}^2},\ \widehat\epsilon\simeq\frac{4}{3f_\phi^2}\ \text{ and }\ \widehat\eta\simeq\frac{4(1+f_\phi)}{3f_\phi^2}$$



- Also, $\widehat N_\star\simeq 3c_{\mathcal R}\phi_\star^2/4\ \Rightarrow\ \phi_\star=2\sqrt{\widehat N_\star/3c_{\mathcal R}}\gg\phi_{\rm f}=\sqrt{(1+2/\sqrt3)/c_{\mathcal R}}$. Imposing $\phi_\star\leq 1\ \Rightarrow\ c_{\mathcal R}\geq 4\widehat N_\star/3c_{\mathcal R}\simeq 74$

For $\widehat N_\star\simeq 52,\ A_{\rm s}^{1/2}\simeq\dfrac{\sqrt\lambda\widehat N_\star}{6\sqrt{2}\pi c_{\mathcal R}}\simeq 4.6\cdot10^{-5}\ \Rightarrow\ \underline{c_{\mathcal R}\simeq 41850\sqrt\lambda}$ and $\widehat m_{\delta\phi}=\left\langle\widehat V_{,\widehat\phi\widehat\phi}\right\rangle^{1/2}=\sqrt\lambda/\sqrt3 c_{\mathcal R}\simeq\underline{1.25\cdot10^{-5}}$.

- $n_{\rm s}\simeq 1-2/\widehat N_\star\simeq 0.962,\ \alpha_{\rm s}\simeq -2/\widehat N_\star^2\simeq -7\cdot10^{-4},\ r\simeq 12/\widehat N_\star^2\simeq 4\cdot10^{-3}$ **(:Identically With The Starobinsky Model)**

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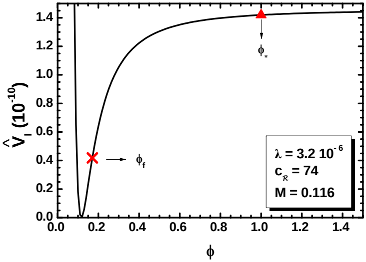
$$\widehat{V}_{\rm I}=\frac{\lambda f_\phi^2}{4f_{\mathcal{R}}^2}\simeq\frac{\lambda}{4c_{\mathcal{R}}^2},\ \widehat{\epsilon}\simeq\frac{4}{3f_\phi^2}\ \ \text{and}\ \ \widehat{\eta}\simeq\frac{4(1+f_\phi)}{3f_\phi^2}$$



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- The Model is **Unitarity Safe**, Since We Obtain $\underline{\Lambda_{\rm UV}=1}$ If We Perform an Expansion About $\widehat{\delta\phi}=\phi-M\simeq 0$:

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SUGRA (i.e. Supergravity) Potential

- It Would Be Interesting If ϕ may be Promoted to **A Gauge non-Singlet** Field and M (or $c_{\mathcal{R}}$) Is Related to the Scale of **MSSM Gauge Unification**. To This End, We Work Within SUGRA Where The Gauge Hierarchy Problem is Elegantly Addressed.

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- The Relevant Part of the Einstein Frame Action In Four Dimensional, $\mathcal{N}=1$ SUGRA is (z^α are Scalar Complex Fields):

$$\mathcal{S}=\int d^4x\sqrt{-\widehat{g}}\left(-\frac{1}{2}\widehat{\mathcal{R}}+K_{\alpha\bar\beta}\widehat{g}^{\mu\nu}D_\mu z^\alpha D_\nu z^{*\bar\beta}-\widehat{V}\right)\quad \text{Where}\quad \widehat{V}=\widehat{V}_{\rm F}+\widehat{V}_{\rm D}$$

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How We can Apply These General Ideas to IGHI?

Selecting Conveniently the Superpotential and Kähler Potentials

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Charge Assignments

Superfields:	S	Φ	$\bar\Phi$
$U(1)_R$	1	0	0
$U(1)_{B-L}$	0	1	-1

At The SUSY Vacuum $\langle S\rangle=0, |\langle\Phi\rangle|=|\langle\bar\Phi\rangle|=M/2$,
Since in The SUSY Limit, After IGHI, We Expect to Get

$$V_{\rm eff}\simeq\lambda^2e^K\left|\bar\Phi\Phi-\frac{1}{4}M^2\right|^2+\frac{g^2}{2}\left(\Phi K_\Phi-\bar\Phi K_{\bar\Phi}\right)^2+|S|^2(\cdots)$$

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• **Possible Kähler Potentials**

• IGHI can be Obtained Selecting the Following K's Which Are **Quadratic and Invariant** Under $U(1)_{B-L}$ and R Symmetries:

$$K_1 = -3\ln\left(c_{\mathcal{R}}\left(F_{\mathcal{R}} + F_{\mathcal{R}}^*\right) - \frac{|\Phi|^2 + |\bar{\Phi}|^2}{3} + F_{1S}\right), \quad K_2 = -2\ln\left(c_{\mathcal{R}}(F_{\mathcal{R}} + F_{\mathcal{R}}^*) - \frac{|\Phi|^2 + |\bar{\Phi}|^2}{2}\right) + F_{2S}\left(|S|^2\right)$$

Where We Use **Integer** Prefactors for the Logarithmic Terms (To Avoid **Tuning**) and We Choose The Functions[3]

$$F_{\mathcal{R}} = \bar{\Phi}\Phi, \quad F_{1S} = -\ln\left(1 + |S|^2/3\right) \quad \text{And} \quad F_{2S} = N_S\ln(1 + |S|^2/N_S) \quad \text{With} \quad N_S > 0$$

Note that $c_{\mathcal{R}}(F_{\mathcal{R}} + F_{\mathcal{R}}^*)$ Dominates $f_{\mathcal{R}}$ and $|\Phi|^2 + |\bar{\Phi}|^2$ $f_{\rm K}$, Whereas $F_{1,2S}$ Assures $m_S^2 > 0$ & $m_S^2 > \widehat{H}_{\rm HI}^2$ During IGHI.

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- Given that K's Have the form $K = -N\ln f_{\mathcal{R}}$, Imposing the **Induced-Gravity Requirement** We Obtain:

$$M = \sqrt{\frac{2N}{Nc_{\mathcal{R}} - 1}} \quad \text{Where} \quad N = \begin{cases} 3 & \text{for } K = K_1, \\ 2 & \text{for } K = K_2. \end{cases}$$

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The Relevant Super- & Kähler Potentials

- Promoting To Local The Already Existing $U(1)_{B-L}$ Global Symmetry of the MSSM, We Obtain a **Superpotential** Invariant under the $G_{\rm SM} \times U(1)_{B-L}$ Gauge Group Which Respects Also Three Other Global Symmetries (R, B, L):

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d^c_i	$(\mathbf{3},\mathbf{2},1/3,-1/3)$	1	−1/3	0
Q_i	$(\bar{\mathbf{3}},\mathbf{2},1/6,-1/3)$	1	1/3	0
Higgs Fields				
H_d	$(\mathbf{1},\mathbf{2},-1/2,0)$	0	0	0
H_u	$(\mathbf{1},\mathbf{2},1/2,0)$	0	0	0
S	$(\mathbf{1},\mathbf{1},0,0)$	4	0	0
$\bar\Phi$	$(\mathbf{1},\mathbf{1},0,2)$	0	0	−2
Φ	$(\mathbf{1},\mathbf{1},0,-2)$	0	0	2

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• The Above W May Cooperate With **One Of The Kähler Potentials** K_1 and K_2, Mentioned Above, if We Replace

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Inflationary Potential

- If We Use The Parametrization: $\Phi = \phi e^{i\theta}\cos\theta_\Phi/\sqrt{2}$ and $\bar\Phi = \phi e^{i\bar\theta}\sin\theta_\Phi/\sqrt{2}$ and $X^\beta = \left(x^\beta + i\bar{x}^\beta\right)/\sqrt{2}$,

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- The Only **Surviving Term** of $V_{\rm F}$ Along the Path in Eq. (I) is

$$\widehat{V}_{\rm HI} = e^K K^{SS^*}\,|W_{{\rm HI},S}|^2 \simeq \frac{\lambda^2\phi^4}{16 f_{\mathcal R}^N}\cdot\begin{cases} f_{\mathcal R} & \text{for } K = K_1,\\ 1 & \text{for } K = K_2,\end{cases}$$

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$$\left(K_{\alpha\bar\beta}\right) = {\rm diag}\left(M_\pm, K_{SS^*}\right) \quad\text{with}\quad M_\pm = \frac{1}{f_{\mathcal R}^2}\begin{pmatrix}\kappa & \bar\kappa\\ \bar\kappa & \kappa\end{pmatrix},$$

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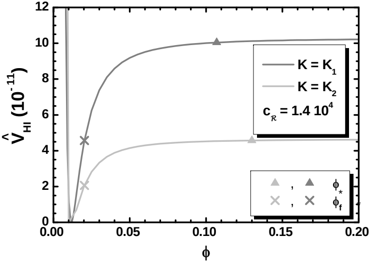

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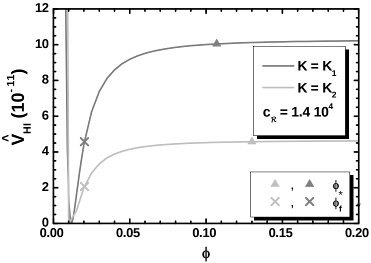

- The EF Canonically Normalized Fields, Which Are Denoted By Hat, Can Be Obtained As Follows:

$$\frac{d\widehat\phi}{d\phi} = J = \sqrt{\kappa_+},\ \ \widehat\theta_+ = \frac{J\phi\theta_+}{\sqrt{2}},\ \ \widehat\theta_- = \sqrt{\frac{\kappa_-}{2}}\phi\theta_-,\ \ \text{and}\ \ \widehat\theta_\Phi = \phi\sqrt{\kappa_-}\left(\theta_\Phi - \frac{\pi}{4}\right),\ \ \left(\widehat{x}^\beta, \widehat{\bar{x}}^\beta\right) = \left(x^\beta, \bar{x}^\beta\right),$$

Where $\theta_\pm = (\theta\pm\bar\theta)/\sqrt{2}$, $\kappa_+ = Nc_{\mathcal{R}}f_{\mathcal{R}}^{-1}$ and $\kappa_- = f_{\mathcal{R}}^{-1}$.

- We Can Check the Stability of the Trajectory in Eq. (I) w.r.t the Fluctuations Of The Various Fields, i.e.

$$\left.\frac{\partial V}{\partial\widehat{z}^\alpha}\right|_{\rm Eq.\ (I)} = 0 \ \ \text{and}\ \ \widehat{m}^2_{z^\alpha} > 0 \ \ \text{Where}\ \ \widehat{m}^2_{z^\alpha} = {\rm Egv}\left[\widehat{M}^2_{\alpha\beta}\right] \ \ \text{With}\ \ \widehat{M}^2_{\alpha\beta} = \left.\frac{\partial^2 V}{\partial\widehat{z}^\alpha\partial\widehat{z}^\beta}\right|_{\rm Eq.\ (I)} \ \ \text{and}\ \ z^\alpha = \theta_-, \theta_+, \theta_\Phi, x^\beta, \bar{x}^\beta.$$

Stability and Radiative Corrections

The Mass Spectrum Along The Inflationary Trajectory

Fields	Eingestates	Masses Squared		
			$K=K_1$	$K=K_2$
14 Real Scalars	$\widehat{\theta}_+$	$\widehat{m}^2_{\theta+}$	$4\widehat{H}^2_{\rm HI}$	$6\widehat{H}^2_{\rm HI}$
	$\widehat{\theta}_\Phi$	$\widehat{m}^2_{\theta_\Phi}$	M^2_{BL}	M^2_{BL}
	$\widehat{s},\widehat{\bar{s}}$	$\widehat{m}^2_s$	$\widehat{H}^2_{\rm HI}(c_{\mathcal{R}}\phi^2-9)$	$6\widehat{H}^2_{\rm HI}/N_S$
	$\widehat{h}_\pm,\widehat{\bar{h}}_\pm$	$\widehat{m}^2_{h\pm}$	$3\widehat{H}^2_{\rm HI}c_{\mathcal{R}}\left(\phi^2/6\pm2\lambda_\mu/\lambda\right)$	$3\widehat{H}^2_{\rm HI}\left(1+1/N_S\pm4\lambda_\mu/\lambda\phi^2\right)$
	$\widehat{\tilde{\nu}}^c_i,\widehat{\bar{\tilde{\nu}}}^c_i$	$\widehat{m}^2_{i\tilde{\nu}^c}$	$3\widehat{H}^2_{\rm HI}c_{\mathcal{R}}\left(\phi^2/6+8\lambda^2_{iN^c}/\lambda^2\right)$	$3\widehat{H}^2_{\rm HI}\left(1+1/N_S+16\lambda^2_{iN^c}/\lambda^2\phi^2\right)$
1 Gauge Boson	A_{BL}	M^2_{BL}	$2Ng^2/\left(Nc_{\mathcal{R}}-1\right)$	
7 Weyl Spinors	$\widehat{\psi}_\pm$	$\widehat{m}^2_{\psi\pm}$	$12\widehat{H}^2_{\rm HI}/c^2_{\mathcal{R}}\phi^4$	
	$\lambda_{BL},\widehat{\psi}_{\Phi-}$	M^2_{BL}	$2Ng^2/\left(Nc_{\mathcal{R}}-1\right)$	
	$\widehat{N}^c_i$	$\widehat{m}^2_{iN^c}$	$48\widehat{H}^2_{\rm HI}c_{\mathcal{R}}\lambda^2_{iN^c}/\lambda^2\phi^2$	

- We can Obtain $\forall\,\alpha,\ \widehat{m}^2_{\chi^\alpha}>0$. Especially

$$\widehat{m}^2_s>0\ \Leftrightarrow\ N_S<6\ \text{and}\ \widehat{m}^2_{h-}>0\ \Leftrightarrow\ \lambda_\mu\leq\lambda\phi^2/4N\ [\lambda_\mu\lesssim\lambda\phi^2(1+1/N_S)/4]\sim10^{-5}\ \text{for}\ K=K_1[K=K_2].$$

- We can Obtain $\forall\,\alpha,\ \widehat{m}^2_{\chi^\alpha}>\widehat{H}^2_{\rm HI}$ and So Any Inflationary Perturbations Of The Fields Other Than ϕ Are Safely Eliminated;
- $M_{BL}\neq0$ Signals the Fact that That $U(1)_{B-L}$ Is Broken and so, **No Topological Defects** are Produced;
- We **Determine** $c_{\mathcal{R}}$ Demanding That The Unification Scale $M_{\rm GUT}\simeq2/2.43\times10^{-2}$ is Identified with M_{BL} at the Vacuum, I.e.,

$$2Ng^2/(Nc_{\mathcal{R}}-1)=M^2_{\rm GUT}\ \Rightarrow\ c_{\mathcal{R}}=1/N+2g^2/M^2_{\rm GUT}\simeq1.451\cdot10^4\ \text{with}\ g\simeq0.7\ \text{(GUT Gauge Coupling)}.$$

- The One-Loop **Radiative Corrections** à la Coleman-Weinberg to $\widehat{V}_{\rm HI}$ Can Be Kept Under Control.

Inflationary Dynamics & Predictions

- The **Slow-Roll Parameters** Are Determined Using the Standard Formulae Employing The Canonically Normalized $\widehat{\phi}$:

$$\widehat{\epsilon} \simeq 16\frac{\tilde{f}_{\rm W}^2}{Nc_{\mathcal{R}}^4\phi^8} \quad \text{and} \quad \widehat{\eta} = 8\frac{2-\tilde{f}_{\rm W}}{N\tilde{f}_{\rm W}^2} \quad \text{With} \quad \tilde{f}_{\rm W} = c_{\mathcal{R}}\phi^2 - 2\cdot$$

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- The **Number of e-Foldings** That $k_\star = 0.05/{\rm Mpc}$ Experiences During IGHI Is Calculated to be

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- The Observables Are **Predicted** To be Identical With those Obtained in the Original STI

$$n_{\rm s} \simeq 1 - \frac{2}{\widehat{N}_\star} = 0.963\,, \;\; r \simeq \frac{4N}{\widehat{N}_\star^2} = 0.0032\;[0.0022] \;\; \text{and} \;\; \alpha_{\rm s} \simeq -\frac{2}{\widehat{N}_\star^2} - \frac{7N}{2\widehat{N}_\star^3} = -0.005 \;\; \text{for} \;\; K = K_1 \;\; [K_2].$$

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• Although $c_{\mathcal{R}}$ Is Large, **No Problem** With The Perturbative Unitarity Emerges Since The Expansions Abound $\langle\phi\rangle = 0$ Are $c_{\mathcal{R}}$ **Independent**:

$$J^2\dot{\phi}^2 \simeq \left(1-\sqrt{\frac{2}{N}}\widehat{\delta\phi}+\frac{3}{2N}\widehat{\delta\phi}^2-\sqrt{\frac{2}{N^3}}\widehat{\delta\phi}^3+\cdots\right)\dot{\widehat{\delta\phi}}^2 \;\;\text{and}\;\; \widehat{V}_{\rm HI} \simeq \frac{\lambda^2\widehat{\delta\phi}^2}{2c_{\mathcal{R}}^2}\left(1-\frac{2N-1}{\sqrt{2N}}\widehat{\delta\phi}+\frac{8N^2-4N+1}{8N}\widehat{\delta\phi}^2+\cdots\right).$$

Perturbative Reheating

- At the SUSY Vacuum, The Inflaton And The RHNs, N_i^c, Acquire Masses $\widehat{m}_{\delta\phi}$ and M_{iN^c} Respectively Given by

$$\widehat{m}_{\delta\phi} \simeq \frac{\lambda m_{\rm P}}{\sqrt{c_{\mathcal{R}}\,(Nc_{\mathcal{R}}-1)}} \simeq 2.8\cdot 10^{13}~{\rm GeV} \quad \text{and} \quad M_{iN^c} = \lambda_{iN^c} M\,,$$

Where **We Restore $m_{\rm P}$** in the Formulas.

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- The Inflaton Can Decay Perturbatively Into:
  - **A Pair of RHNs** (N_j^c) With Majorana Masses M_{jN^c} Through The Following Decay Width

$$\widehat{\Gamma}_{\delta\phi\to N_i^c} = \frac{g_{iN^c}^2}{16\pi}\widehat{m}_{\delta\phi}\left(1-\frac{4M_{iN^c}^2}{\widehat{m}_{\delta\phi}^2}\right)^{3/2} \quad \text{With} \quad g_{iN^c} = (N-1)\frac{\lambda_{iN^c}}{\langle J\rangle} \quad \text{Arising from} \quad \mathcal{L}_{\widehat{\delta\phi}\to N_i^c} = g_{iN^c}\,\widehat{\delta\phi}\;N_i^cN_i^c\,.$$

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  - **MSSM (s)-Particles XYZ** Through The Following c_+-Dependent 3-Body Decay Width

$$\widehat{\Gamma}_{\delta\phi\to XYZ} = g_y^2\frac{14}{512\pi^3}\frac{\widehat{m}_{\delta\phi}^3}{m_{\rm P}^2} \quad \text{With} \quad g_y = y_3\left(\frac{Nc_{\mathcal{R}}-1}{2c_{\mathcal{R}}}\right)^{1/2} \quad \text{and} \quad y_3 = h_{t,b,\tau}(\widehat{m}_{\delta\phi}) \simeq 0.5\,.$$

  This Decay Arises From $\mathcal{L}_{\widehat{\delta\phi}\to XYZ} = -\lambda_y(\widehat{\delta\phi}/m_{\rm P})\,(X\psi_Y\psi_Z + Y\psi_X\psi_Z + Z\psi_X\psi_Y) + {\rm h.c.}$

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- The Reheating Temperature, $T_{\rm rh}$, is given by

$$T_{\rm rh} = \left(72/5\pi^2 g_*\right)^{1/4}\widehat{\Gamma}_{\delta\phi}^{1/2}m_{\rm P}^{1/2} \quad \text{with} \quad \widehat{\Gamma}_{\delta\phi} = \widehat{\Gamma}_{\delta\phi\to N_i^c} + \widehat{\Gamma}_{\delta\phi\to H} + \widehat{\Gamma}_{\delta\phi\to XYZ}, \quad \text{with} \quad g_* \simeq 228.75\,.$$

Leptogenesis and $\widetilde{G}$ Abundance

- The **Out-Of-Equilibrium Decay** of N_i^c can Generate an L Asymmetry Which Can Be Converted to the **B Yield**:

$$Y_B = -0.35\ 2\ \frac{5}{4}\ \frac{T_{\rm rh}}{\widehat{m}_{\delta\phi}}\ \frac{\widehat{\Gamma}_{\delta\phi\to N_i^c}}{\widehat{\Gamma}_{\delta\phi}}\varepsilon_i \ \text{ Where }\ \varepsilon_i = \sum_{j\neq i}\frac{\mathrm{Im}\left[(m_{\rm D}^\dagger m_{\rm D})_{ij}^2\right]}{8\pi\langle H_u\rangle^2(m_{\rm D}^\dagger m_{\rm D})_{ii}}\Big(F_{\rm S}\left(x_{ij},y_i,y_j\right)+F_{\rm V}(x_{ij})\Big).$$

Here $x_{ij} := M_{jN^c}/M_{iN^c}$ and $y_i := \Gamma_{iN^c}/M_{iN^c} = (m_{\rm D}^\dagger m_{\rm D})_{ii}/8\pi\langle H_u\rangle^2$ and $\widehat{m}_{\delta\phi} < 2M_{iN^c}$ For Some i with $i=1,2,3$. Also $F_{\rm V}$ and $F_{\rm S}$ Represent, Respectively, The Contributions From **Vertex And Self-Energy** Diagrams.

- $m_{i\rm D}$ are the Dirac Masses Which May Be Diagonalized In the **Weak (primed) Basis**

$$U^\dagger m_{\rm D} U^{c\dagger} = d_{\rm D} = \mathrm{diag}\left(m_{1\rm D}, m_{2\rm D}, m_{3\rm D}\right) \ \text{ Where }\ L' = LU \ \text{ and }\ N^{c\prime} = U^c N^c.$$

And Are Related to M_{iN^c} via the **Type I Seesaw** Formula

$$m_\nu = -m_{\rm D}\, d_{N^c}^{-1}\, m_{\rm D}^{\mathsf T}, \ \text{ Where }\ d_{N^c} = \mathrm{diag}\left(M_{1N^c}, M_{2N^c}, M_{3N^c}\right) \ \text{ with }\ M_{1N^c}\leq M_{2N^c}\leq M_{3N^c} \ \text{ Real and Positive.}$$

- Replacing $m_{\rm D}$ in the See-Saw Formula We Extract The Mass Matrix of Light Neutrinos In The Weak Basis

$$\bar m_\nu = U^\dagger m_\nu U^* = -d_{\rm D} U^c d_{N^c}^{-1} U^{c\mathsf T} d_{\rm D},$$

Which Can Be Diagonalized by the Unitary **PMNS Matrix** U_ν Parameterized As Follows:

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12}-s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12}-s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12}-c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12}-c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}\cdot\begin{pmatrix} e^{-i\varphi_1/2} & & \\ & e^{-i\varphi_2/2} & \\ & & 1 \end{pmatrix},$$

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- The **Thermally Produced $\widetilde{G}$ Yield** At The Onset of **Big-Bang Nucleosynthesis (BBN)** Is Estimated To Be:

$$Y_{\widetilde{G}} \simeq 1.9 \cdot 10^{-22}\, T_{\rm rh}/\mathrm{GeV}.$$

Post-Inflationary Requirements

The **Achievement Of Baryogenesis** via non-Thermal Leptogenesis Can be Characterized Successful If:
(i) We Obtain the **Observationally Required B Yield** Which is $Y_B = (8.697 \pm 0.054) \cdot 10^{-11}$ at 95% c.l.

[4] M. Kawasaki, K. Kohri, and T. Moroi (2005); J.R. Ellis, K.A. Olive, and E. Vangioni (2005).

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(ii) **Constraints on M_{iN^c} Are Satisfied.** We have To Avoid Any Erasure Of The Produced Y_L; Ensure That The ϕ Decay To N_i^c Is Kinematically Allowed; and M_{iN^c} are Theoretically Acceptable, We Have To Impose The Constraints:

$$(a)\;\; M_{1N^c} \gtrsim 10T_{\rm rh},\;\; (b)\;\; \widehat{m}_{\delta\phi} \geq 2M_{1N^c}\;\; \text{and}\;\; (c)\;\; M_{iN^c} \lesssim 7.1M \;\Leftrightarrow\; \lambda_{iN^c} \lesssim 3.5.$$

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(i) We Obtain the **Observationally Required B Yield** Which is $Y_B = (8.697 \pm 0.054) \cdot 10^{-11}$ at 95% c.l.

(ii) **Constraints on M_{iN^c} Are Satisfied.** We have To Avoid Any Erasure Of The Produced Y_L; Ensure That The ϕ Decay To N_i^c Is Kinematically Allowed; and M_{iN^c} are Theoretically Acceptable, We Have To Impose The Constraints:

$$(a)\;\; M_{1N^c} \gtrsim 10 T_{\rm rh},\;\; (b)\;\; \widehat{m}_{\delta\phi} \geq 2M_{1N^c} \;\;\text{and}\;\; (c)\;\; M_{iN^c} \lesssim 7.1 M \;\Leftrightarrow\; \lambda_{iN^c} \lesssim 3.5.$$

(iii) **$\widetilde{G}$ Constraint Is Under Control.** Assuming Unstable $\widetilde{G}$, We Impose an Upper Bound[4] on $Y_{\widetilde{G}}$ In Order to Avoid Problems With the BBN:

$$Y_{3/2} \lesssim \begin{cases} 10^{-14} \\ 10^{-13} \\ 10^{-12} \end{cases} \Rightarrow\; T_{\rm rh} \lesssim \begin{cases} 5.3 \cdot 10^{7}\ \text{GeV} \\ 5.3 \cdot 10^{8}\ \text{GeV} \\ 5.3 \cdot 10^{9}\ \text{GeV} \end{cases} \;\;\text{for } \widetilde{G} \text{ Mass}\;\; m_{3/2} \simeq \begin{cases} 0.69\ \text{TeV}, \\ 10.6\ \text{TeV}, \\ 13.5\ \text{TeV}. \end{cases}$$

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(iv) Be In Agreement With the **Light Neutrino Data**.

Parameter	Best Fit Value (2021) Normal Hierarchy	Best Fit Value (2021) Inverted Hierarchy
$\Delta m^2_{21}/10^{-3}{\rm eV}^2$	7.5	
$\Delta m^2_{31}/10^{-3}{\rm eV}^2$	2.55	2.45
$\sin^2\theta_{12}/0.1$	3.18	
$\sin^2\theta_{13}/0.01$	2.2	2.225
$\sin^2\theta_{23}/0.1$	5.74	5.78
δ/π	1.08	1.58

- The Masses, $m_{i\nu}$, of ν_i Are Calculated as Follows:

$$m_{2\nu} = \sqrt{m^2_{1\nu} + \Delta m^2_{21}} \;\;\text{and}$$

$$\begin{cases} m_{3\nu} = \sqrt{m^2_{1\nu} + \Delta m^2_{31}}, & \text{for Normally Ordered (NO) } m_\nu\text{'s} \\ \quad\text{or} & \\ m_{1\nu} = \sqrt{m^2_{3\nu} + \left|\Delta m^2_{31}\right|}, & \text{for Invertedly Ordered (IO) } m_\nu\text{'s} \end{cases}$$

- $\sum_i m_{i\nu} \leq 0.12\ [0.15]\ {\rm eV}$ at 95% c.l. For NO [IO] m_ν's.

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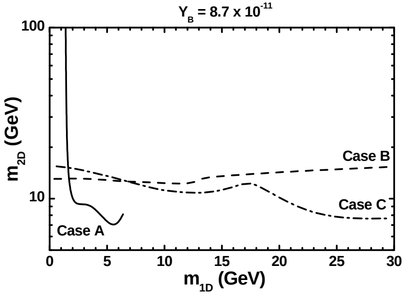
Combining Inflationary and Post-Inflationary Requirements

- Enforcing the Post-Inflationary Constraints, We Can Obtain **Predictions for m_{iD}'s or M_{iN^c}** Employing as **Free Parameters** $m_{r\nu}, \varphi_1$ and φ_2, (Where $m_{r\nu}$ is A **Reference Scale** for the Neutrino Masses).
- **All** the Requirements can be Met Along the Lines Presented in the $m_{1D}-m_{2D}$ Plane for $\lambda_\mu = 10^{-6}$.



CASES :	A	B	C
Hierarchy :	NO	NO	IO
$m_{r\nu}$ / eV	0.001	0.05	0.005
$\Sigma_i m_{i\nu}$ / eV	0.06	0.074	0.1
m_{3D} / GeV	100	100	33
φ_1	$-\pi/2$	$\pi/6$	$\pi/4$
φ_2	0	0	$-\pi/3$
$M_{1N^c}/10^{12}$ GeV	0.9 - 2.2	0.01 - 6.4	0.01 - 3.8
$M_{2N^c}/10^{12}$ GeV	2 - 447	5 - 78	4.3 - 11
$M_{3N^c}/10^{15}$ GeV	2.3 - 9.5	0.58	0.1

- We take $m_{r\nu}=m_{1\nu}$ for NO ν_i's and $m_{r\nu}=m_{3\nu}$ for IO ν_i's.
- **The Inflaton Decays into the Lightest and Next-to-Lightest of RHN** Since $2M_{iN^c} > \widehat{m}_{\delta\phi}$ for $i=3$.
- Y_B Is Equal to its Central Value and **the $\widetilde{G}$ Constraint is Under Control** for $m_{3/2}\sim 10$ TeV Since We Obtain

$$1.4 \lesssim Y_{\widetilde{G}}/10^{-13} \lesssim 1.7 \quad \text{with} \quad 7.5 \lesssim T_{\rm rh}/10^8\,{\rm GeV} \lesssim 9,$$

Where The Lowest Values Obtained For Case A.

Generation of the μ-Term of MSSM Applying the Mechanism of G. Dvali, G. Lazarides and Q. Shafi (1999)

- The Origin of the μ Term Can be Explained If We Combine the Terms $W_{\rm HI} + W_{\mu} = \lambda S\left(\bar\Phi\Phi - M^2/4\right) + \lambda_\mu S\, H_u H_d\,.$

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$$V_{\rm soft} = \left(\lambda A_\lambda S\bar\Phi\Phi + \lambda_\mu A_\mu S H_u H_d - {\rm a}_S S\lambda M^2/4 + {\rm h.c.}\right) + m_{\bar\alpha}^2\left|z^{\bar\alpha}\right|^2 \quad \text{with} \quad z^{\bar\alpha} = \Phi, \bar\Phi, S, H_u, H_d$$

where $m_\alpha, A_\lambda, A_\mu$ and ${\rm a}_S$ are Soft SUSY Breaking Mass Parameters Of the Order of Gravitino Mass $m_{3/2}$.

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Where the Prefactor is Absorbed Since Successful IGHI Needs **$\lambda_\mu\lesssim2\cdot10^{-5}$ For Stability Reasons**.
- The Allowed λ_μ Values Render Our Models Compatible With The **Best-Fit Points** in the CMSSM[5] Setting, E.g.,

$m_0=m_{3/2}$ and $|A_\lambda|=|{\rm a}_S|=|A_0|$ – Regions (I) & (IV) are More Favored From the $\widetilde G$ Constraint.

CMSSM Region ($m_h\simeq125$ GeV & $\Omega_\chi h^2\lesssim0.12$)		$\lvert A_0\rvert$ (TeV)	m_0 (TeV)	$\lvert\mu\rvert$ (TeV)	${\rm a}_{3/2}$	λ_μ (10^{-6})	
						$K=K_1$	$K=K_2$
(I)	A/H Funnel	9.9244	9.136	1.409	1.086	**0.963**	**1.184**
(II)	$\tilde\tau_1-\chi$ Coannihilation	1.2271	1.476	2.62	0.831	***14.48***	**17.81**
(III)	$\tilde t_1-\chi$ Coannihilation	9.965	4.269	4.073	2.33	**2.91**	**3.41**
(IV)	$\tilde\chi_1^\pm-\chi$ Coannihilation	9.2061	9.000	0.983	1.023	**0.723**	**0.89**

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- It Allows for **Baryogenesis** via non-TL Compatible With $\widetilde{G}$ **Constraints and Neutrino Data**. In particular We may have $m_{3/2} \gtrsim 10$ TeV, With The Inflaton Decaying Mainly to N_1^c and N_2^c – We Obtain M_{iN^c} in the Range $(10^{10} - 10^{15})$ GeV.

• It Remains to Introduce a Consistent **Soft SUSY Breaking Sector** in the Theory.

Thank You!