



# Unified Theories of Flavour



**Steve King,**  
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**2022**



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*European Institute for Sciences and Their Applications*





At Aspen, where we were collaborating on our first paper on “Strong Unification”



At Graham's retirement fest in Oxford

# Stitching the Yukawa Quilt

Graham favoured the symmetric matrices

$$Y^u \propto \begin{pmatrix} 0 & \epsilon^3 & O(\epsilon^3) \\ \cdot & \epsilon^2 & O(\epsilon^2) \\ \cdot & \cdot & 1 \end{pmatrix}, \quad Y^d \propto \begin{pmatrix} 0 & 1.5\bar{\epsilon}^3 & 0.4\bar{\epsilon}^3 \\ \cdot & \bar{\epsilon}^2 & 1.3\bar{\epsilon}^2 \\ \cdot & \cdot & 1 \end{pmatrix} \quad \text{“Hierarchical just like the UK”}$$

$$\epsilon \approx 0.05, \quad \bar{\epsilon} \approx 0.15.$$

Note (1,1) texture zero and symmetric matrices leading to the successful GST relation

$$V_{us} = \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} e^{i\sigma}$$

“Chance to be correct”

# SU(3) Family Symmetry and Pati-Salam Unification

## “Little Flavour Magnets”

Field	SU(3)	SU(4) <sub>PS</sub>	SU(2) <sub>L</sub>	SU(2) <sub>R</sub>
$\psi$	$\mathbf{3}$	$\mathbf{4}$	$\mathbf{2}$	$\mathbf{1}$
$\psi^c$	$\mathbf{3}$	$\overline{\mathbf{4}}$	$\mathbf{1}$	$\mathbf{2}$
$\theta$	$\overline{\mathbf{3}}$	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{2}$
$\overline{\theta}$	$\mathbf{3}$	$\overline{\mathbf{4}}$	$\mathbf{1}$	$\mathbf{2}$
$H$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$
$\Sigma$	$\mathbf{1}$	$\mathbf{15}$	$\mathbf{1}$	$\mathbf{3}$
$\phi_3$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3} \oplus \mathbf{1}$
$\phi_{23}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

Plus shaping  
symmetries and  
extra fields

$$\begin{aligned}
 P_{\text{Yuk}} &\sim \frac{1}{M^2} \psi_i \phi_3^i \psi_j^c \phi_3^j H \\
 &+ \frac{\Sigma}{M^3} \psi_i \phi_{23}^i \psi_j^c \phi_{23}^j H \\
 &+ \frac{1}{M^5} \left( (\epsilon^{ijk} \psi_i^c \overline{\phi_{23,j}} \overline{\phi_{3,k}}) (\psi_l \phi_{23}^l) + (\epsilon^{ijk} \psi_i \overline{\phi_{23,j}} \overline{\phi_{3,k}}) (\psi_l^c \phi_{23}^l) \right) H(\phi_{23}^m \overline{\phi_{3,m}}) \\
 &+ \frac{1}{M^5} (\epsilon^{ijk} \psi_i^c \overline{\phi_{23,j}} \psi_k) H(\phi_{23}^l \overline{\phi_{3,l}})^2 + \frac{1}{M^5} (\epsilon^{ijk} \psi_i^c \overline{\phi_{3,j}} \psi_k) H(\phi_{23}^l \overline{\phi_{23,l}}) (\phi_{23}^m \overline{\phi_{3,m}}) \\
 &+ \frac{1}{M^4} (\psi_i \phi_{23}^i \psi_j^c \phi_3^j + \psi_i \phi_3^i \psi_j^c \phi_{23}^j) H.S \\
 P_{\text{Maj}} &\sim \frac{1}{M} \psi_i^c \theta^i \theta^j \psi_j \\
 &+ \frac{1}{M^{11}} \psi_i^c \phi_{23}^i \psi_j^c \phi_{23}^j (\theta^k \overline{\phi_{23,k}}) (\theta^l \overline{\phi_{3,l}}) (\phi_3 \overline{\phi_{23}})^3 \\
 &+ \frac{1}{M^{13}} (\epsilon^{ijk} \psi_i^c \overline{\phi_{23,j}} \overline{\phi_{3,k}})^2 (\theta^k \overline{\phi_{23,k}}) (\theta^l \overline{\phi_{3,l}}) (\phi_3 \overline{\phi_{23}}) (\phi_{23} \overline{\phi_3})^2
 \end{aligned}$$

# Alignment and Yukawa matrices

$$\langle \phi_3 \rangle = \langle \bar{\phi}_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} a_3^u & 0 \\ 0 & a_3^d \end{pmatrix} \quad \langle \phi_{23} \rangle = \begin{pmatrix} 0 \\ 1 \\ e^{i\theta} \end{pmatrix} b$$

$$\frac{a_3^u}{M^u} = \frac{a_3^d}{M^d} = \sqrt{\bar{\epsilon}}.$$

$$\epsilon \equiv \frac{b}{M^u}, \quad \bar{\epsilon} \equiv \frac{b}{M^d}$$

$$Y^u \approx \begin{pmatrix} 0 & \epsilon^3(g + \frac{h}{3} + \frac{h'}) & \epsilon^3(g - \frac{h}{3})(1 + O(\bar{\epsilon})) \\ \epsilon^3(g' - \frac{h}{3} - \frac{h'}) & \epsilon^2(-\frac{2}{3}) & \epsilon^2(-\frac{2}{3}) + c'\epsilon^3\bar{\epsilon}^{-\frac{1}{2}} \\ \epsilon^3(g' + \frac{h}{3})(1 + O(\bar{\epsilon})) & \epsilon^2(-\frac{2}{3}) + c\epsilon^3\bar{\epsilon}^{-\frac{1}{2}} & 1 + O(\bar{\epsilon}) \end{pmatrix} \bar{\epsilon}, \quad \rightarrow \quad Y^u \propto \begin{pmatrix} 0 & \epsilon^3 & O(\epsilon^3) \\ \cdot & \epsilon^2 & O(\epsilon^2) \\ \cdot & \cdot & 1 \end{pmatrix}$$

$$Y^d \approx \begin{pmatrix} 0 & \bar{\epsilon}^3(g + h + h') & \bar{\epsilon}^3(g - h)(1 + O(\bar{\epsilon})) \\ \bar{\epsilon}^3(g' - h - h') & \bar{\epsilon}^2 & \bar{\epsilon}^2 + c'\bar{\epsilon}^{\frac{5}{2}} \\ \bar{\epsilon}^3(g' + h)(1 + O(\bar{\epsilon})) & \bar{\epsilon}^2 + c\bar{\epsilon}^{\frac{5}{2}} & 1 + O(\bar{\epsilon}) \end{pmatrix} \bar{\epsilon}, \quad \rightarrow \quad Y^d \propto \begin{pmatrix} 0 & 1.5\bar{\epsilon}^3 & 0.4\bar{\epsilon}^3 \\ \cdot & \bar{\epsilon}^2 & 1.3\bar{\epsilon}^2 \\ \cdot & \cdot & 1 \end{pmatrix}$$

$$Y^e \approx \begin{pmatrix} 0 & \bar{\epsilon}^3(g + h + h') & \bar{\epsilon}^3(g - h)(1 + O(\bar{\epsilon})) \\ \bar{\epsilon}^3(g' - h - h') & \bar{\epsilon}^2(3) & \bar{\epsilon}^2(3) + c'\bar{\epsilon}^{\frac{5}{2}} \\ \bar{\epsilon}^3(g' + h)(1 + O(\bar{\epsilon})) & \bar{\epsilon}^2(3) + c\bar{\epsilon}^{\frac{5}{2}} & 1 + O(\bar{\epsilon}) \end{pmatrix} \bar{\epsilon},$$

**GJ for charged leptons**

$$Y^\nu \approx \begin{pmatrix} 0 & \epsilon^3(g + \frac{h}{3} + \frac{h'}) & \epsilon^3(g - \frac{h}{3})(1 + O(\bar{\epsilon})) \\ \epsilon^3(g' - \frac{h}{3} - \frac{h'}) & \epsilon^2(-\alpha) & \epsilon^2(-\alpha) + c'\epsilon^3\bar{\epsilon}^{-\frac{1}{2}} \\ \epsilon^3(g' + \frac{h}{3})(1 + O(\bar{\epsilon})) & \epsilon^2(-\alpha) + c\epsilon^3\bar{\epsilon}^{-\frac{1}{2}} & 1 + O(\bar{\epsilon}) \end{pmatrix} \bar{\epsilon}.$$

**Neutrinos  
c.f. up quarks**

# Neutrino mass and mixing

$$Y^\nu \approx \begin{pmatrix} 0 & \epsilon^3(g + \frac{h}{3} + \frac{h'}{3}) & \epsilon^3(g - \frac{h}{3})(1 + O(\bar{\epsilon})) \\ \epsilon^3(g' - \frac{h}{3} - \frac{h'}{3}) & \epsilon^2(-\alpha) & \epsilon^2(-\alpha) + c'\epsilon^3\bar{\epsilon}^{-\frac{1}{2}} \\ \epsilon^3(g' + \frac{h}{3})(1 + O(\bar{\epsilon})) & \epsilon^2(-\alpha) + c\epsilon^3\bar{\epsilon}^{-\frac{1}{2}} & 1 + O(\bar{\epsilon}) \end{pmatrix} \bar{\epsilon} \rightarrow M_\nu^D \equiv \begin{pmatrix} 0 & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix}$$

$$M_{RR} \approx \begin{pmatrix} \epsilon^6\bar{\epsilon}^3 & 0 & 0 \\ 0 & \epsilon^6\bar{\epsilon}^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} M_3 \rightarrow M_\nu^M \sim \begin{pmatrix} M_1^M & 0 & 0 \\ 0 & M_2^M & 0 \\ 0 & 0 & M_3^M \end{pmatrix} \quad M_1^M < M_2^M \ll M_3^M \approx \frac{\langle \theta \rangle^2}{M_\nu}$$

**Hierarchical Diagonal RHNs**

**Type I seesaw**  $m_\nu = M_\nu^D (M_\nu^M)^{-1} (M_\nu^D)^T$

$$m_\nu \approx \begin{matrix} \text{Dominant} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & e^2 & ef \\ 0 & ef & f^2 \end{pmatrix} \frac{1}{M_1^M} \end{matrix} + \begin{matrix} \text{Sub-dominant} \\ \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix} \frac{1}{M_2^M} \end{matrix} + \begin{matrix} \text{Decoupled} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c'^2 \end{pmatrix} \frac{1}{M_3^M} \end{matrix}$$

**SRHND**

hep-ph/9806440,  
hep-ph/9912492,  
hep-ph/0204360

$$m_{\nu_3} \sim \frac{e^2 + f^2}{M_{11}^M}, \quad \tan \theta_{23} \sim \frac{e}{f} \quad m_{\nu_2} \sim \frac{a^2}{M_2^M s_{12}^2}, \quad \tan \theta_{12} \sim \frac{\sqrt{2}a}{b-c}, \quad \theta_{13} \lesssim \frac{m_{\nu_2}}{m_{\nu_3}} \quad m_{\nu_1} \sim \frac{c'^2}{M_3^M}$$

**Model predicted**  $\theta_{13} \sim \bar{\epsilon} \sim 0.15$  **way before it was measured**

# How to test such models?

Two options:

## 1. Proton decay and Gravitational Waves

Fu, S.F.K., Marsili, Pascoli, Turner, Zhou 2209.XXXXXXX (tomorrow!)

## 2. At the TeV scale with LHC and B physics

Mario F. Navarro and S.F.K. 2209.XXXXXXX (tomorrow!)

We focus on second option here...



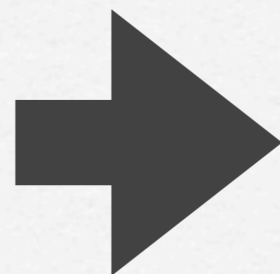
# Twin PS Theory of Flavour with a TeV scale Vector Leptoquark

TeV

Field	$SU(4)_{PS}^I$	$SU(2)_L^I$	$SU(2)_R^I$	$SU(4)_{PS}^{II}$	$SU(2)_L^{II}$	$SU(2)_R^{II}$
$\psi_{1,2,3}$	1	1	1	4	2	1
$\psi_{1,2,3}^c$	1	1	1	$\bar{4}$	1	$\bar{2}$
$\psi_{4,5,6}$	4	2	1	1	1	1
$\psi_{4,5,6}^c$	$\bar{4}$	$\bar{2}$	1	1	1	1
$\psi_{4,5,6}^c$	$\bar{4}$	1	$\bar{2}$	1	1	1
$\psi_{4,5,6}^c$	4	1	2	1	1	1
$\phi$	4	2	1	$\bar{4}$	$\bar{2}$	1
$\bar{\phi}, \bar{\phi}'$	$\bar{4}$	1	$\bar{2}$	4	1	2
$H$	$\bar{4}$	$\bar{2}$	1	4	1	2
$\bar{H}$	4	1	2	$\bar{4}$	$\bar{2}$	1
$\Omega_{15}$	15	1	1	1	1	1

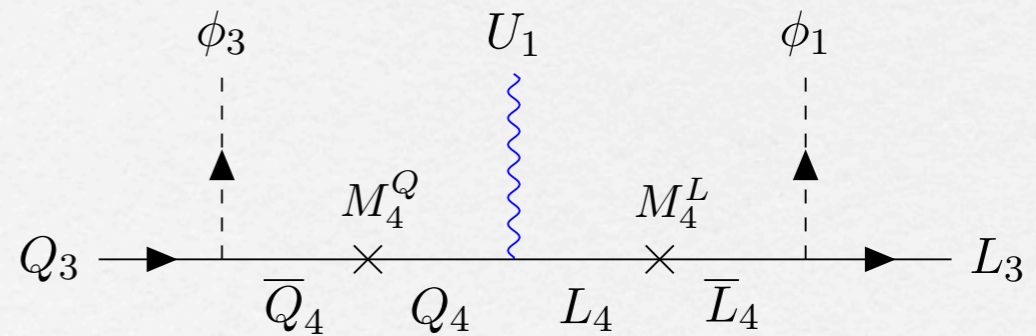
Plus shaping symmetries and extra fields

Chiral families 1,2,3 mix with vector-like families 4,5,6



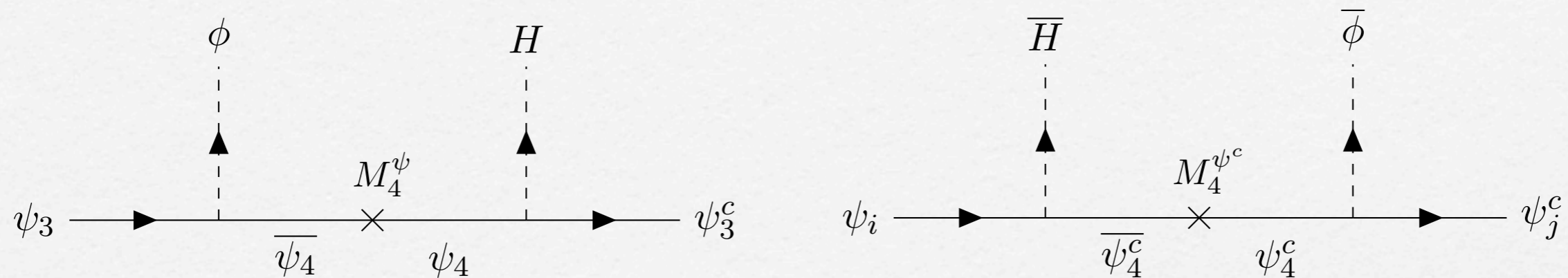
Motivated by B anomalies have  $SU(4)_{PS}$  at TeV

$$G_{422}^I \times G_{422}^{II} \xrightarrow{M_{\text{high}}} G_{4321} \xrightarrow{M_{\text{low}}} G_{321}$$



This mixing generates VLQ couplings (above) and Yukawa matrices

# Yukawa matrices



Looks familiar?

- Hierarchical
- Symmetric
- (1,1) zero
- Personal Higgs

$$M_u \sim \begin{pmatrix} 0 & \bar{m}_u^0 & \bar{m}_u^0 \\ m_u^0 & m_c & m_c \\ m_u^0 & m_c & m_t \end{pmatrix}, \quad M_d \sim \begin{pmatrix} 0 & \bar{m}_d^0 & \bar{m}_d^0 \\ m_d^0 & m_s & m_s \\ m_d^0 & m_s & m_b \end{pmatrix}$$

$$M_\nu^D \sim \begin{pmatrix} 0 & \bar{m}_{\nu e}^D & \bar{m}_{\nu e}^D \\ m_{\nu e}^D & m_{\nu \mu}^D & m_{\nu \mu}^D \\ m_{\nu e}^D & m_{\nu \mu}^D & m_{\nu \tau}^D \end{pmatrix}, \quad M_e \sim \begin{pmatrix} 0 & \bar{m}_e^0 & \bar{m}_e^0 \\ m_e^0 & m_\mu & m_\mu \\ m_e^0 & m_\mu & m_\tau \end{pmatrix}$$

Neutrino mass and mixing from SRHND

# B anomalies in a Twin PS ToF

$$G_{422}^I \times G_{422}^{II} \xrightarrow{M_{\text{high}}} G_{4321} \xrightarrow{M_{\text{low}}} G_{321}$$

TeV scale  $U_1, Z', g'$

Focus on low energy 4321

L.Di Luzio, A.Greljo and M.Nardecchia, Phys. Rev. D 96 (2017) [1708.08450]

Observable	Experiment/constraint
$R_{K^{(*)}} \quad \delta C_L^\mu$	$-0.40_{-0.09}^{+0.08}$ [25]
$R_{D^{(*)}} \quad g_{VL}$	$0.05 \pm 0.02$ [4]
$B_s - \bar{B}_s \quad \delta(\Delta M_s)$	$\lesssim 0.11$ (see Section 2.4.2 and [27])
$\mathcal{B}(\tau \rightarrow 3\mu)$	$< 2.1 \cdot 10^{-8}$ (90% CL) [40]
$\mathcal{B}(\tau \rightarrow \mu\gamma)$	$< 5.0 \cdot 10^{-8}$ (90% CL) [41]
$\mathcal{B}(B_s \rightarrow \tau^\pm \mu^\mp)$	$< 3.4 \cdot 10^{-5}$ (90% CL) [42]
$\mathcal{B}(B^+ \rightarrow K^+ \tau^\pm \mu^\mp)$	$< 2.8 \cdot 10^{-5}$ (90% CL) [43]
$\mathcal{B}(\tau \rightarrow \mu\phi)$	$< 8.4 \cdot 10^{-8}$ (90% CL) [44]
$\mathcal{B}(K_L \rightarrow \mu e)$	$< 4.7 \cdot 10^{-12}$ (90% CL) [45]
$(g_\tau/g_{e,\mu})_{l+\pi+K}$	$1.0003 \pm 0.0014$ [3]
$\mathcal{B}(B_s \rightarrow \tau^+ \tau^-)$	$< 5.2 \times 10^{-3}$ (90% CL) [46]
$\mathcal{B}(B \rightarrow K \tau^+ \tau^-)$	$< 2.25 \times 10^{-3}$ (90% CL) [47]
$\mathcal{B}(B \rightarrow K^{(*)} \nu \bar{\nu}) / \mathcal{B}(B \rightarrow K^{(*)} \nu \bar{\nu})_{\text{SM}}$	$< 3.5$ (3.2) (90% CL) [48, 49]

Key features of our analysis

- GIM-like (3 VLFs)
- PS unification
- Fermion masses (2nd, 3rd family)
- Fermiophobic construction

# Input parameters

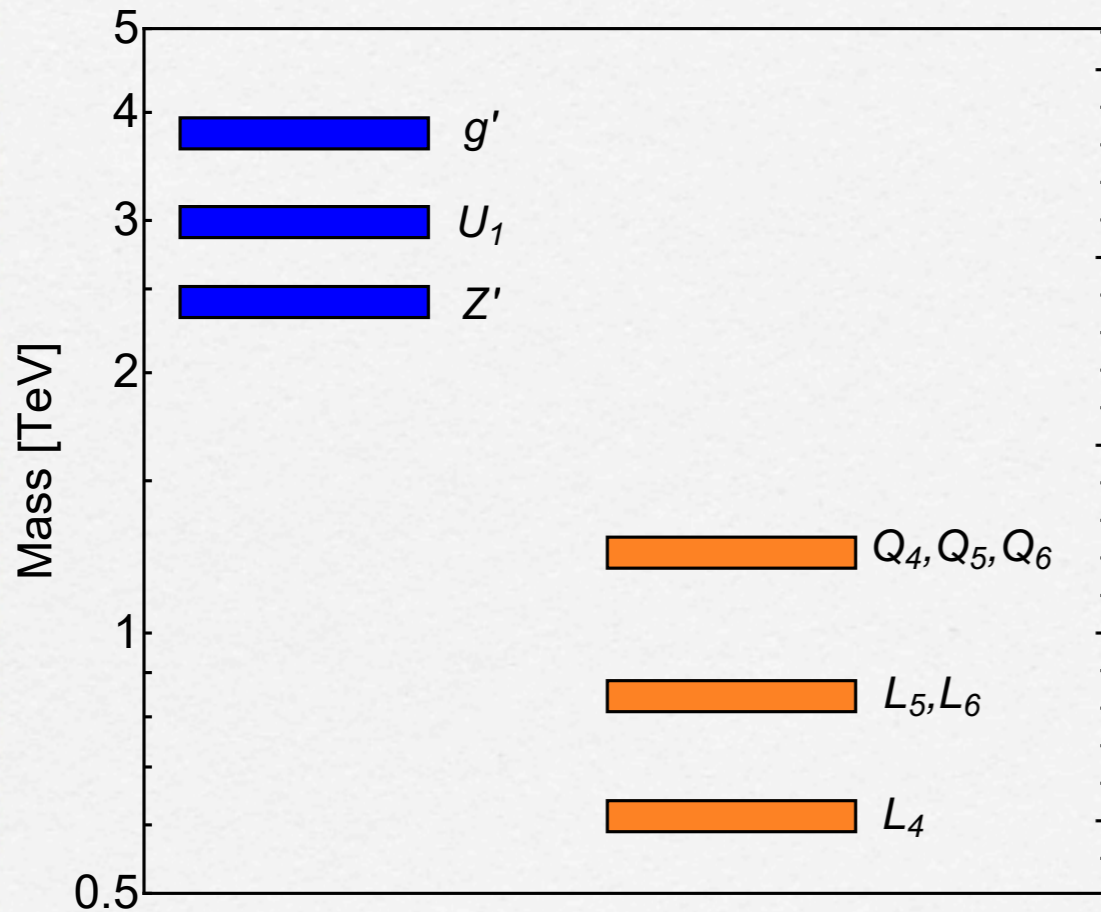
	$\psi_1^c$	$\psi_2^c$	$\psi_3^c$	$\psi_4^c$	$\psi_5^c$	$\psi_6^c$	$\bar{\psi}_4$	$\bar{\psi}_5$	$\bar{\psi}_6$
$\psi_1$	0	0	0	0	0	0	0	0	$x_{16}^\psi \phi$
$\psi_2$	0	0	0	$y_{24}^\psi \bar{H}$	$y_{25}^\psi \bar{H}$	0	0	$x_{25}^\psi \phi$	0
$\psi_3$	0	0	0	$y_{34}^\psi \bar{H}$	$y_{35}^\psi \bar{H}$	0	$x_{34}^\psi \phi$	$x_{35}^\psi \phi$	0
$\psi_4$	0	0	$y_{43}^\psi H$	0	0	0	$\bar{M}_{44}^\psi$	$M_{45}^\psi$	0
$\psi_5$	0	0	$y_{53}^\psi H$	0	0	0	$M_{54}^\psi$	$\bar{M}_{55}^\psi$	0
$\psi_6$	0	0	0	0	0	0	0	0	$\bar{M}_{66}^\psi$
$\bar{\psi}_4^c$	0	$x_{42}^{\psi^c} \bar{\phi}'$	$x_{43}^{\psi^c} \bar{\phi}$	$M_{44}^{q^c, e^c}$	$M_{45}^{\psi^c}$	0	0	0	0
$\bar{\psi}_5^c$	0	$x_{52}^{\psi^c} \bar{\phi}'$	$x_{53}^{\psi^c} \bar{\phi}$	$M_{54}^{\psi^c}$	$M_{44}^{q^c, e^c}$	0	0	0	0
$\bar{\psi}_6^c$	$x_{61}^{\psi^c} \bar{\phi}$	0	0	0	0	$M_{66}^{q^c, e^c}$	0	0	0

- Zeroes enforced by symmetries

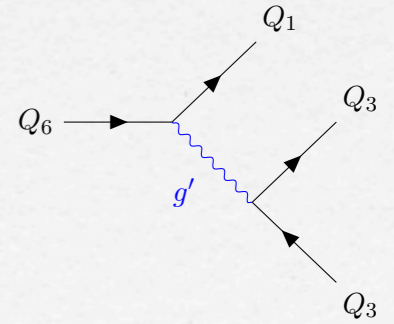
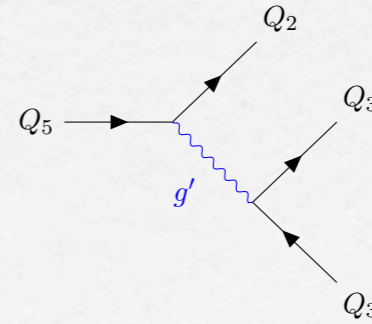
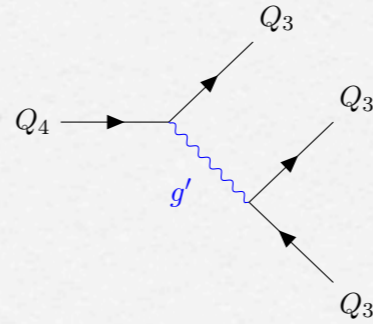
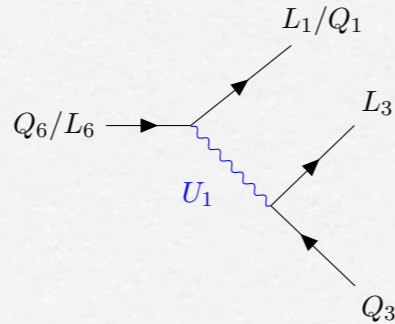
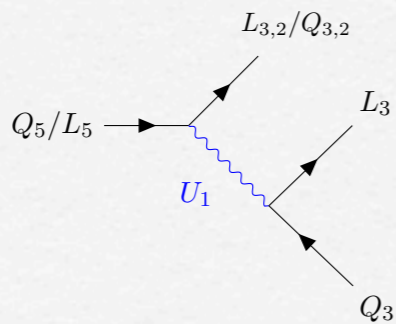
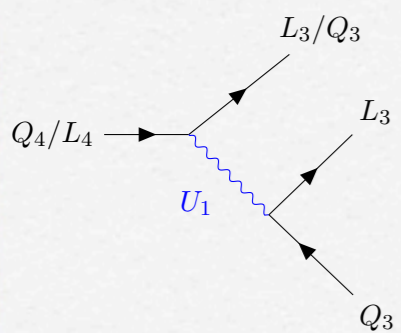
# Benchmark and output

benchmark (BP)				Output			
$g_4$	3.5	$\lambda_{15}^{44}$	-0.5	$s_{34}^Q$	0.978	$M_{g'}$	3782.86 GeV
$g_{3,2,1}$	1, 0.65, 0.36	$\lambda_{15}^{55}, \lambda_{15}^{66}$	2.5, 0.8	$s_{34}^L$	0.977	$M_{Z'}$	2414.32 GeV
$x_{34}^\psi$	2	$x_{42}^{\psi^c}$	0.4	$s_{25}^Q = s_{16}^Q$	0.1986	$s_{23}^u$	0.042556
$x_{25}^\psi = x_{16}^\psi$	0.41	$x_{43}^{\psi^c}$	1	$s_{25}^L = s_{16}^Q$	0.1455	$s_{23}^d$	0.001497
$M_{44}^\psi$	320 GeV	$M_{44}^{\psi^c}$	5 TeV	$s_{\theta_{LQ}}$	0.7097	$s_{23}^e$	-0.111
$M_{55}^\psi$	780 GeV	$y_{53,43,34,24}^\psi$	-0.3, 1, 1, 1	$\widetilde{M}_4^Q$	1226.82 GeV	$V_{cb}$	0.04106
$M_{66}^\psi$	930 TeV	$\langle H_t \rangle$	177.2 GeV	$\widetilde{M}_5^Q$	1238.69 GeV	$m_t$	172.91 GeV
$M_{45}^\psi$	-700 GeV	$\langle H_c \rangle$	26.8 GeV	$\widetilde{M}_4^L$	614.038 GeV	$m_c$	1.270 GeV
$M_{54}^\psi$	50 GeV	$\langle H_b \rangle$	4.25 GeV	$\widetilde{M}_5^L$	845.263 GeV	$m_b$	4.180 GeV
$\langle \phi_3 \rangle$	0.6 TeV	$\langle H_s \rangle$	2.1 GeV	$\widetilde{M}_6^Q$	1235.73 GeV	$m_s$	0.0987 GeV
$\langle \phi_1 \rangle$	0.3 TeV	$\langle H_\tau \rangle$	1.75 GeV	$\widetilde{M}_6^L$	842.39 GeV	$m_\tau$	1.7765 GeV
$v_{15}$	0.4 TeV	$\langle H_\mu \rangle$	4.5788 GeV	$M_{U_1}$	2986.99 GeV	$m_\mu$	105.65 MeV

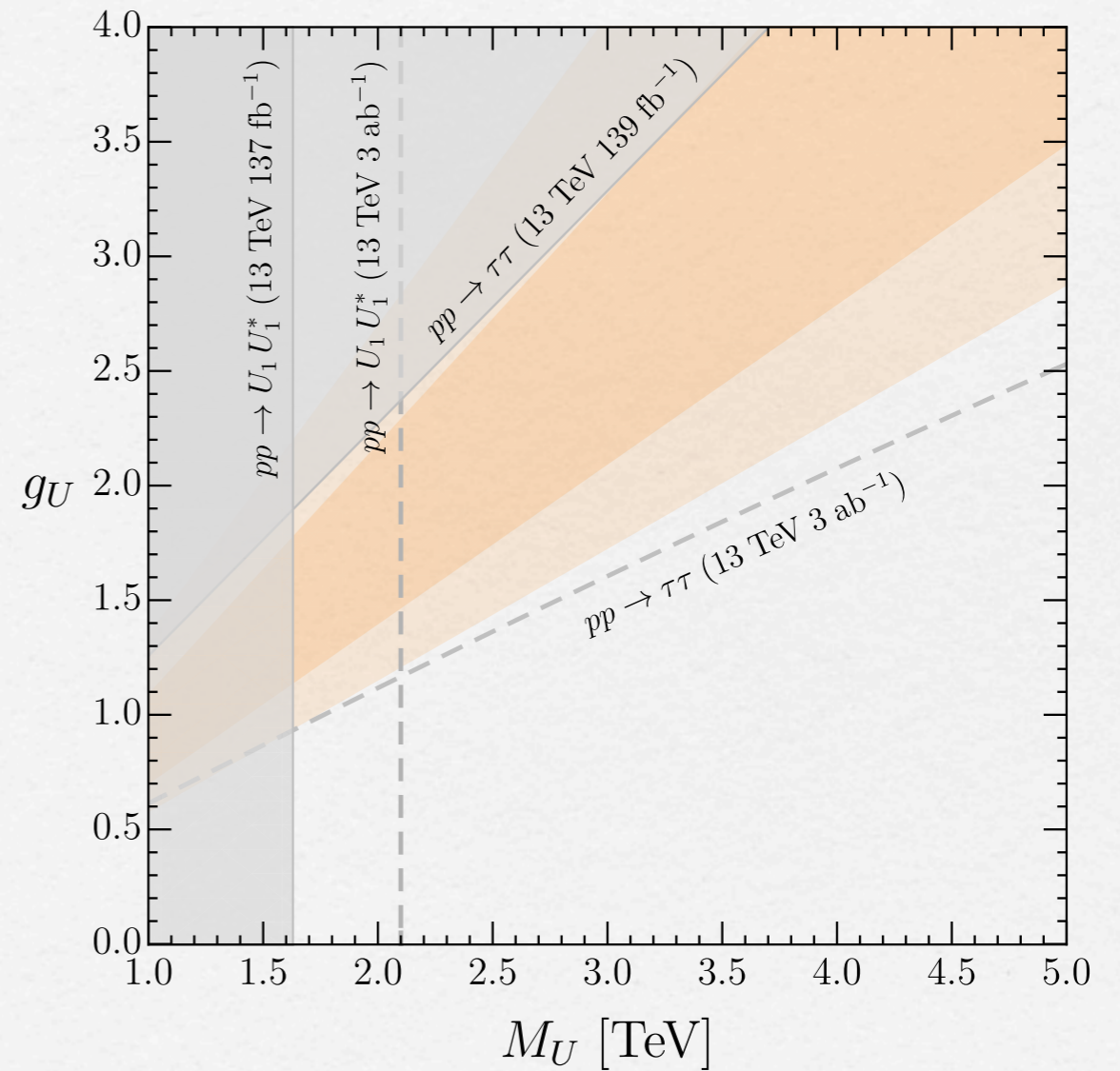
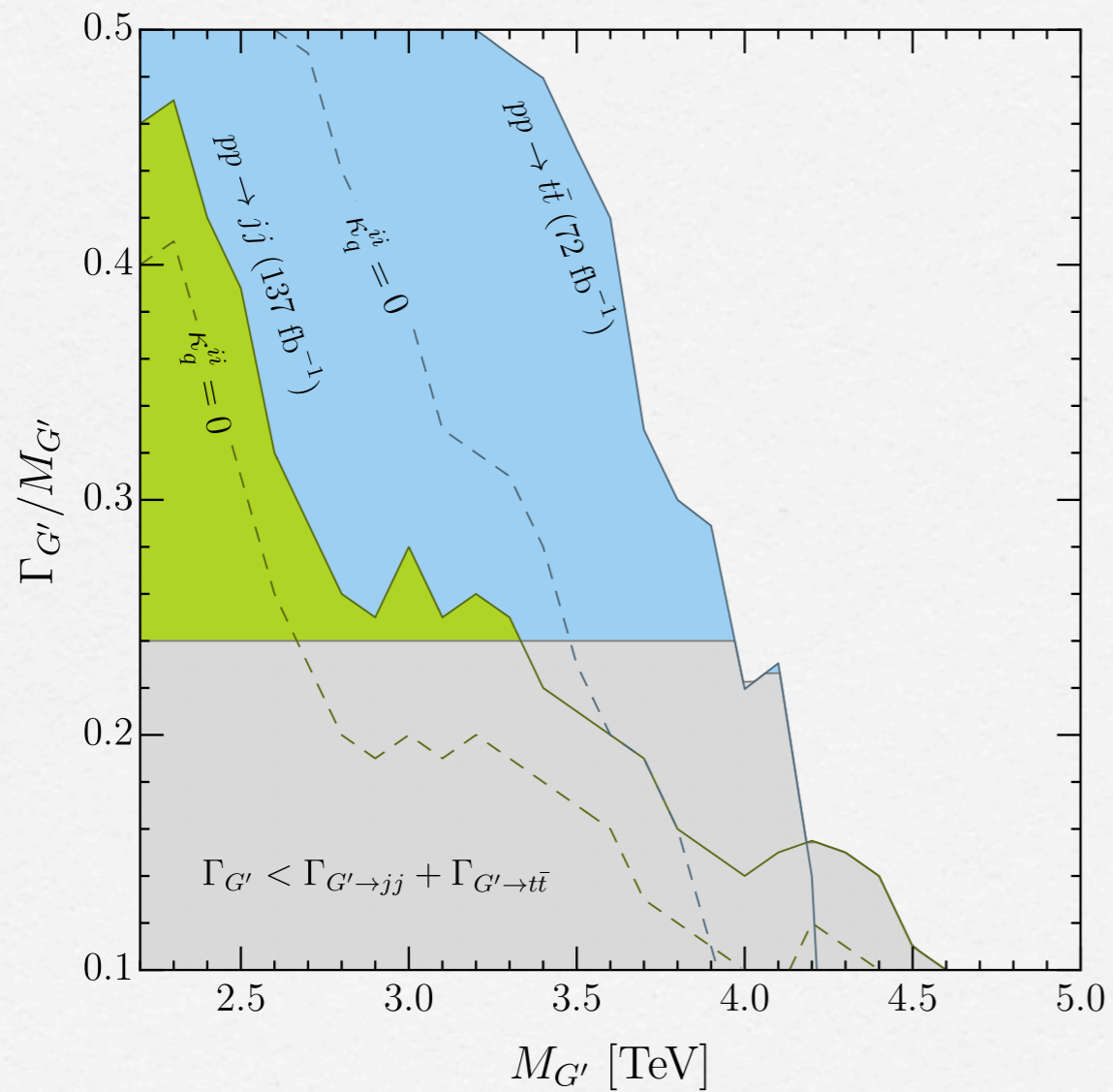
# Benchmark Spectrum



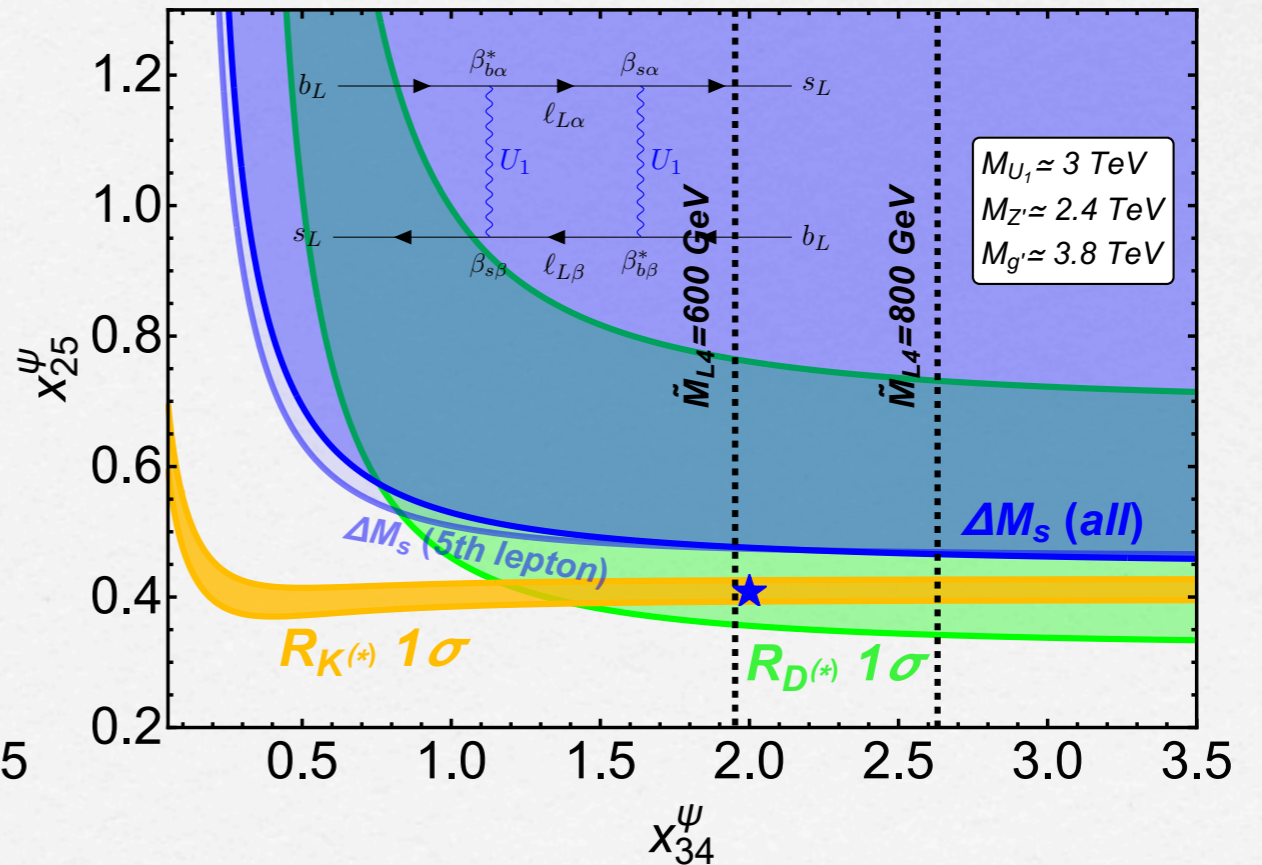
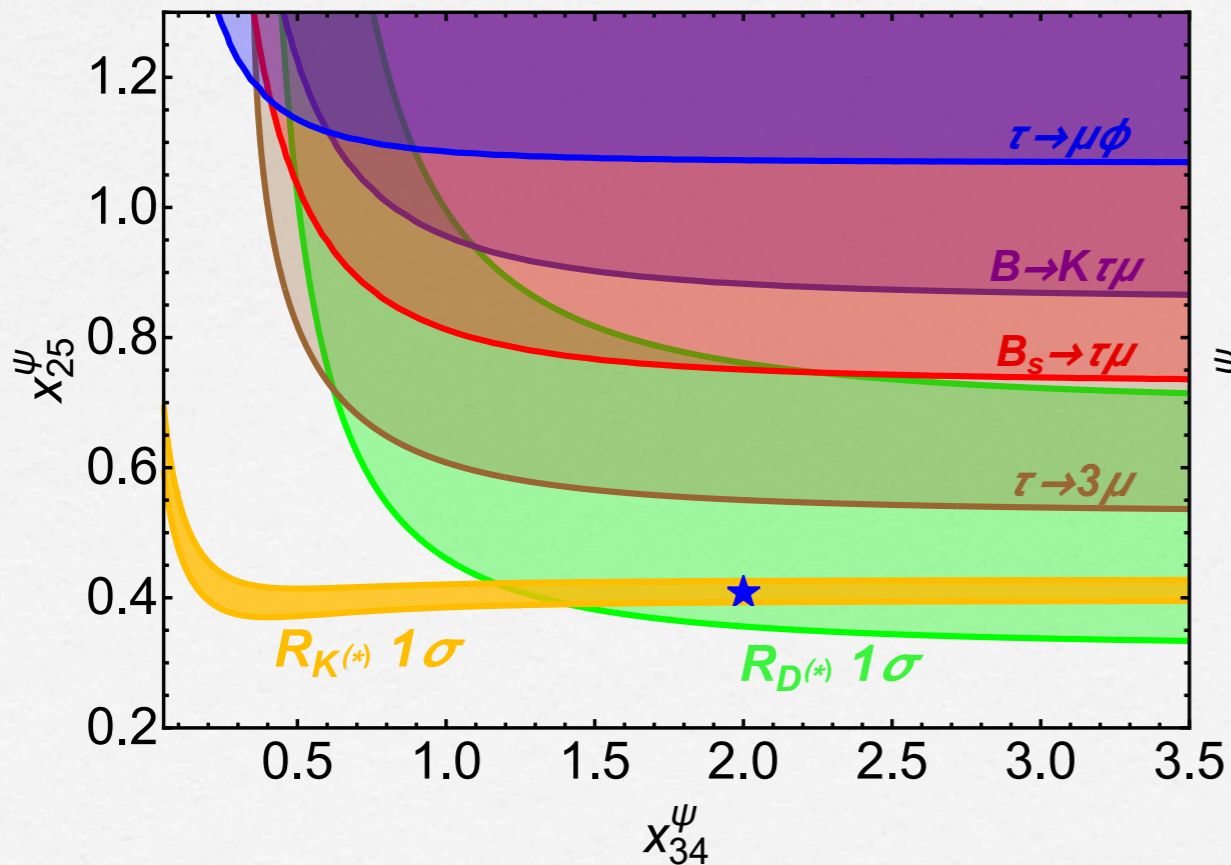
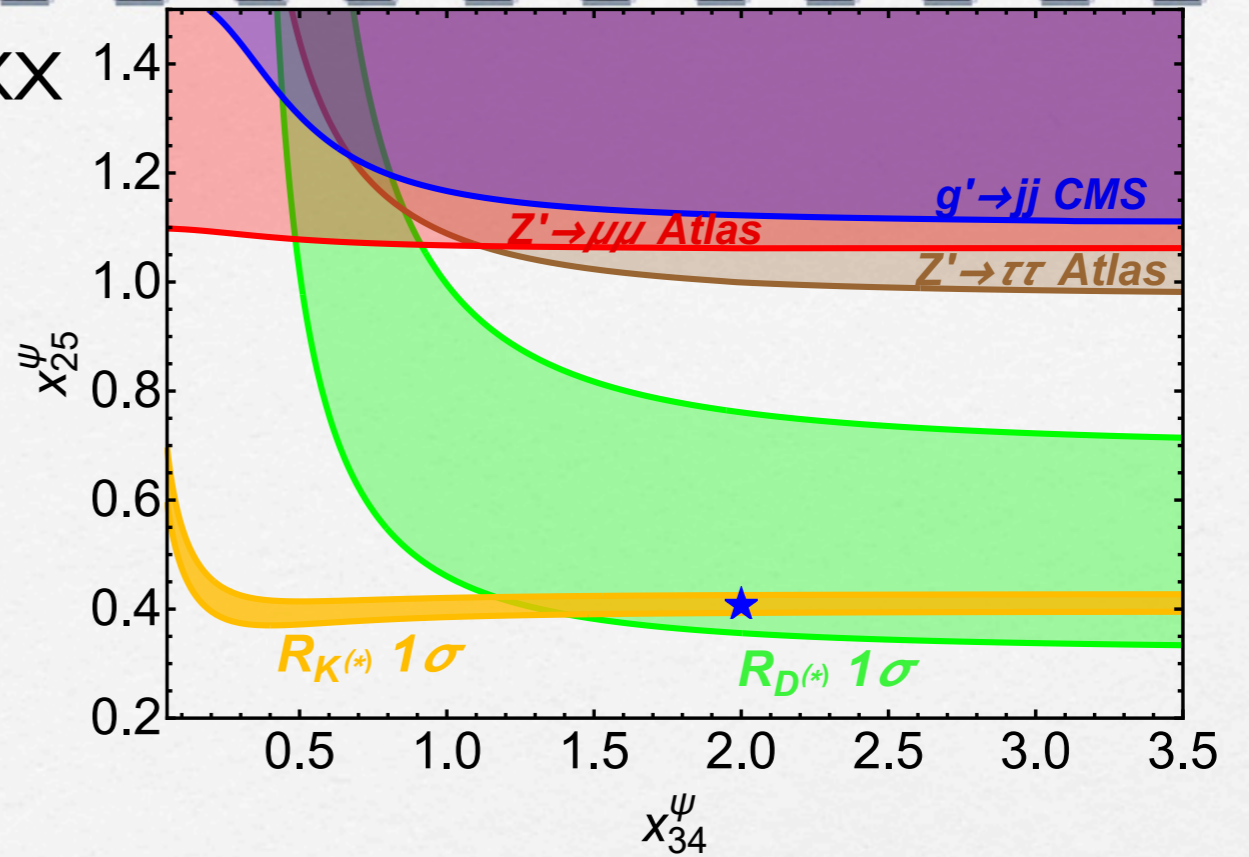
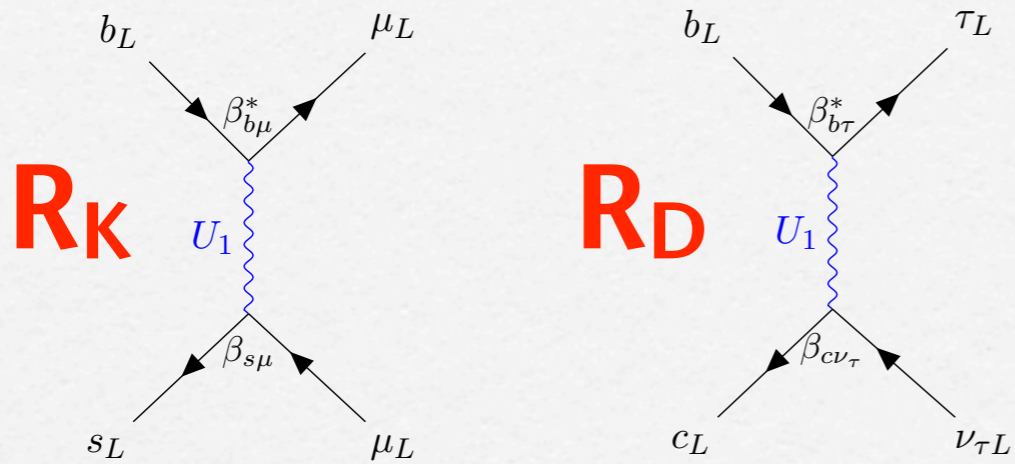
Particle	Decay mode	$\mathcal{B}(\text{BP})$	$\Gamma/M$
$U_1$	$Q_3 L_5 + Q_5 L_3$	$\sim 0.47$	0.32
	$Q_3 L_3$	$\sim 0.22$	
	$Q_5 L_5$	$\sim 0.24$	
	$Q_i L_a + Q_a L_i$	$\sim 0.07$	
$g'$	$Q_3 Q_3$	$\sim 0.3$	0.5
	$Q_5 Q_5$	$\sim 0.3$	
	$Q_6 Q_6$	$\sim 0.3$	
	$Q_1 Q_6 + Q_2 Q_5 + Q_3 Q_4$	$\sim 0.1$	
$Z'$	$L_5 L_5$	$\sim 0.29$	0.24
	$L_6 L_6$	$\sim 0.29$	
	$L_3 L_3$	$\sim 0.27$	
	$Q_3 Q_3 + Q_5 Q_5 + Q_6 Q_6$	$\sim 0.09$	
	$L_1 L_6 + L_2 L_5 + L_3 L_4$	$\sim 0.06$	



# LHC limits on $g'$ and $U_1$

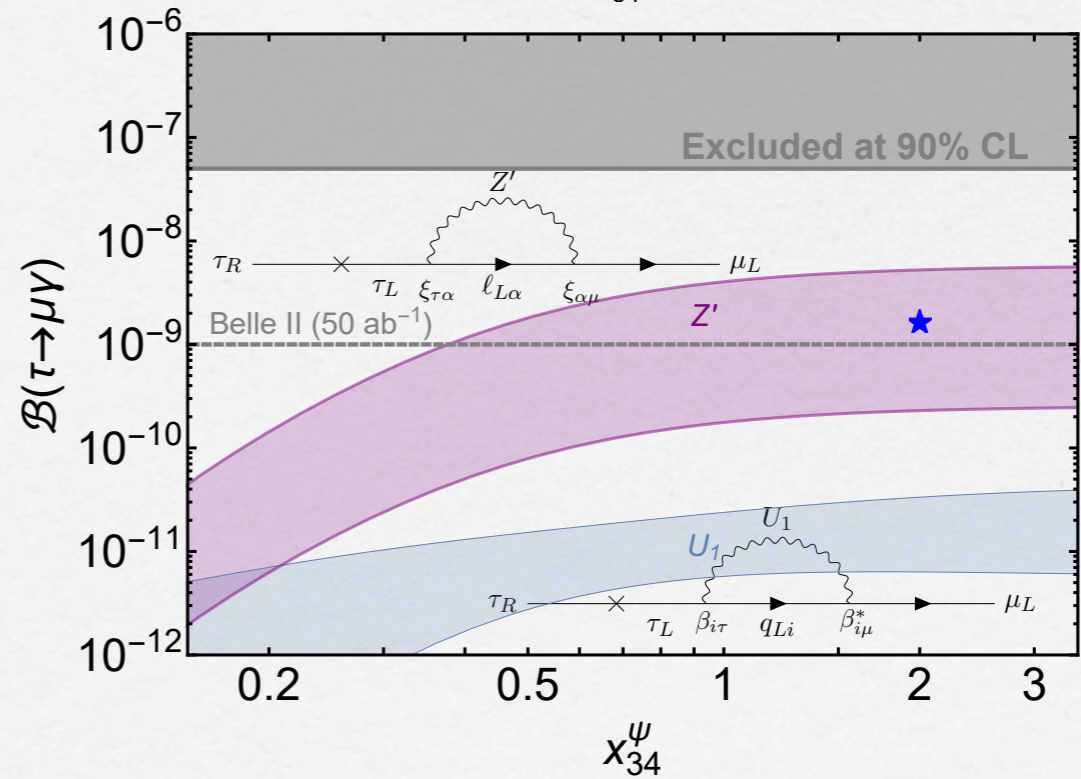
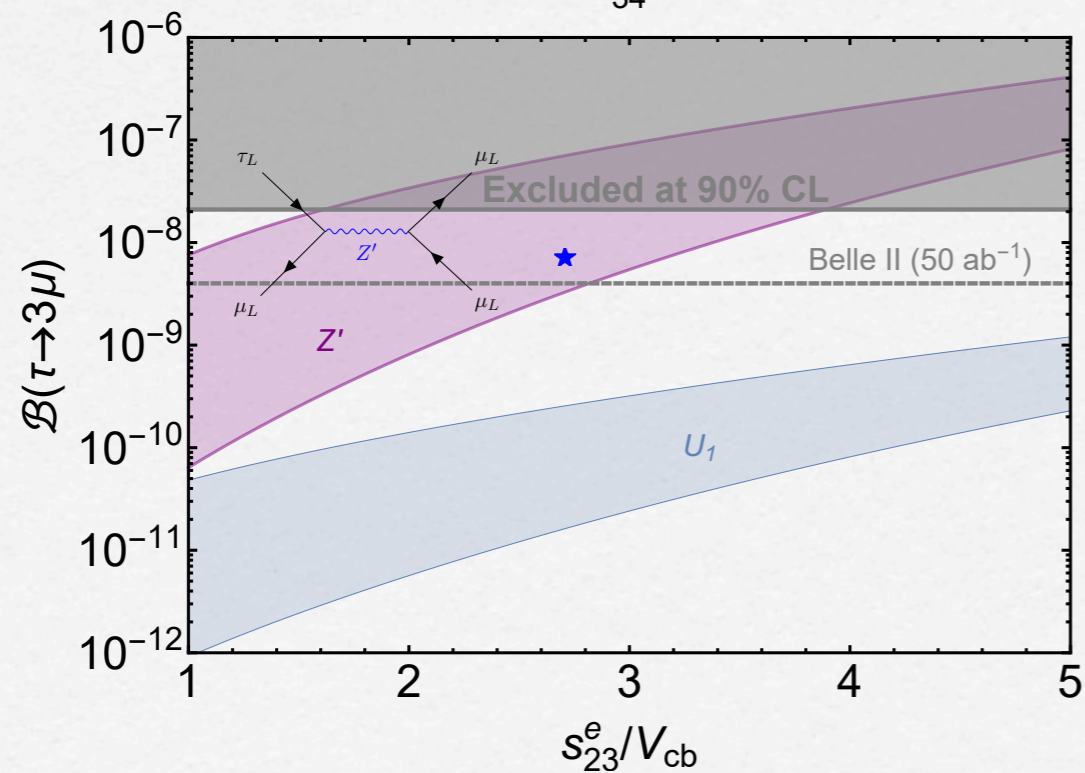
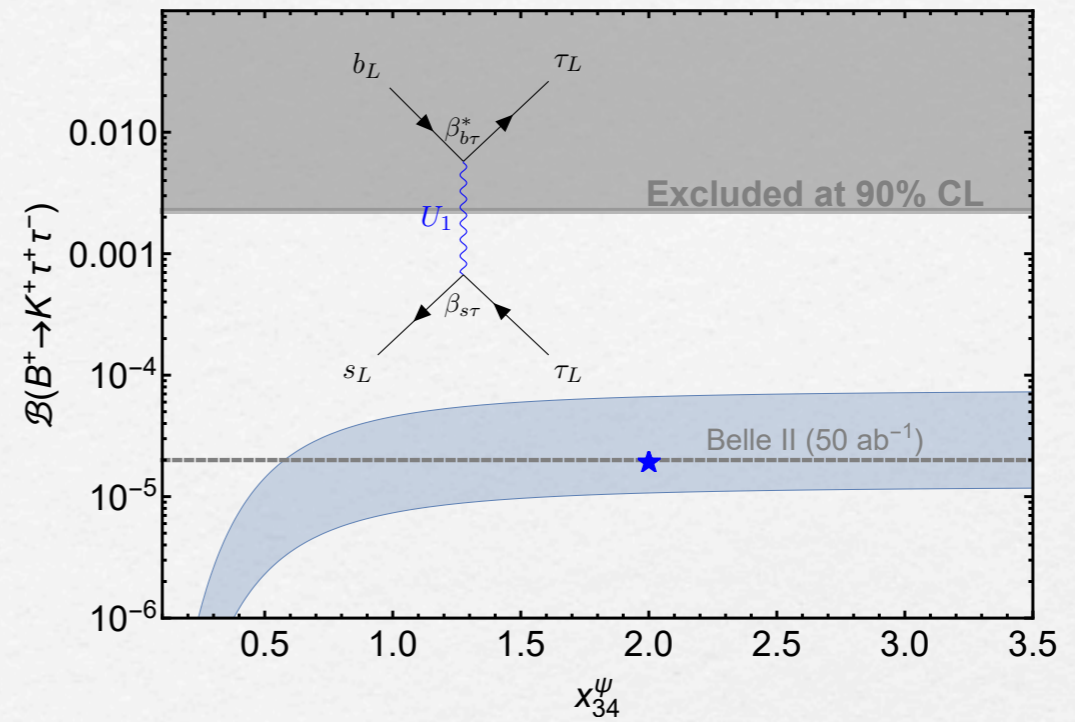
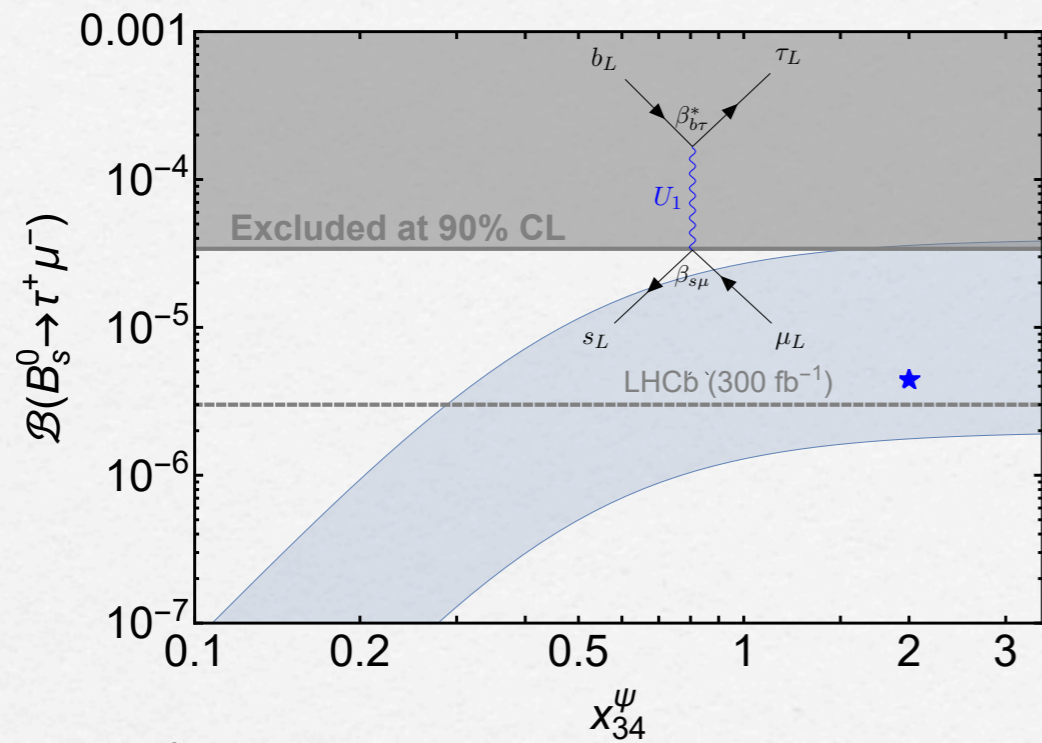


# $R_K, R_D$ anomalies

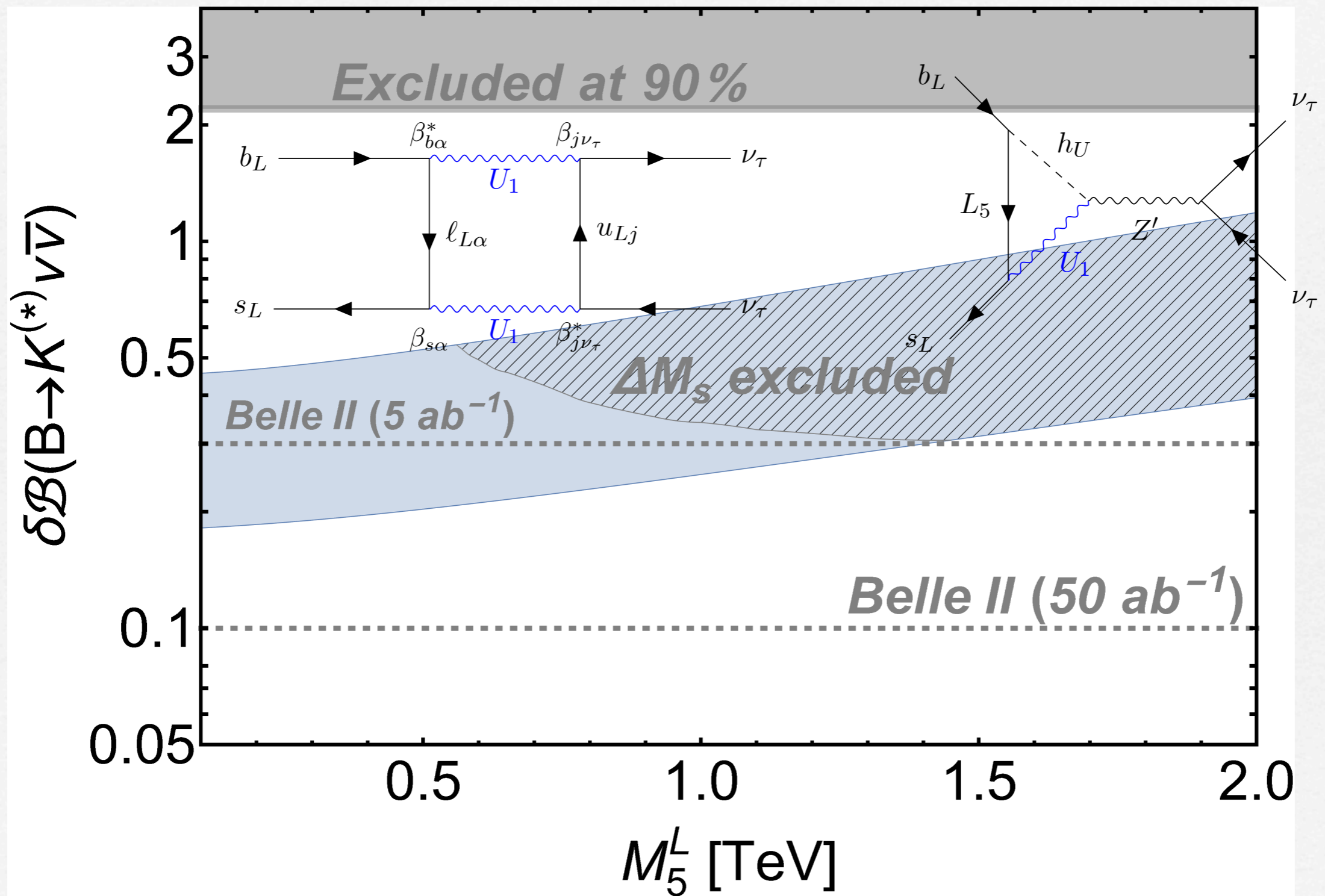




# Predictions for LHCb and Belle II



# Prediction for $B \rightarrow K \nu \nu$ at Belle II



# Conclusion

- $SU(3) \times PS$  model still valid today,  $\theta_{13}$  prediction confirmed
- Such high scale models difficult to test directly (PD, GWs ?)
- It is possible that Nature has given us a lucky break and provided us with a TeV scale PS symmetry
- $(PS)^2$  with TeV scale PS symmetry reproduces similar mass matrices to Graham's and explains  $R_K, R_D$
- This allows the theory to be probed by experiments such as LHC, LHCb, Belle II, providing a testable theory of flavour

# Standard Model

## Yukawa matrices (basis dependent)

$$-\mathcal{L}_Y = (Y_u)_{ij} \bar{q}_{iL} \tilde{\phi} u_{iR} + (Y_d)_{ij} \bar{q}_{iL} \phi d_{iR} + (Y_\ell)_{ij} \bar{l}_{iL} \phi e_{iR} + \text{H.c.}$$

## Mass matrices (basis dependent)

$$m_u \equiv \frac{v}{\sqrt{2}} Y_u, \quad m_d \equiv \frac{v}{\sqrt{2}} Y_d, \quad m_\ell \equiv \frac{v}{\sqrt{2}} Y_\ell.$$

## Fermion masses and mixing (observables)

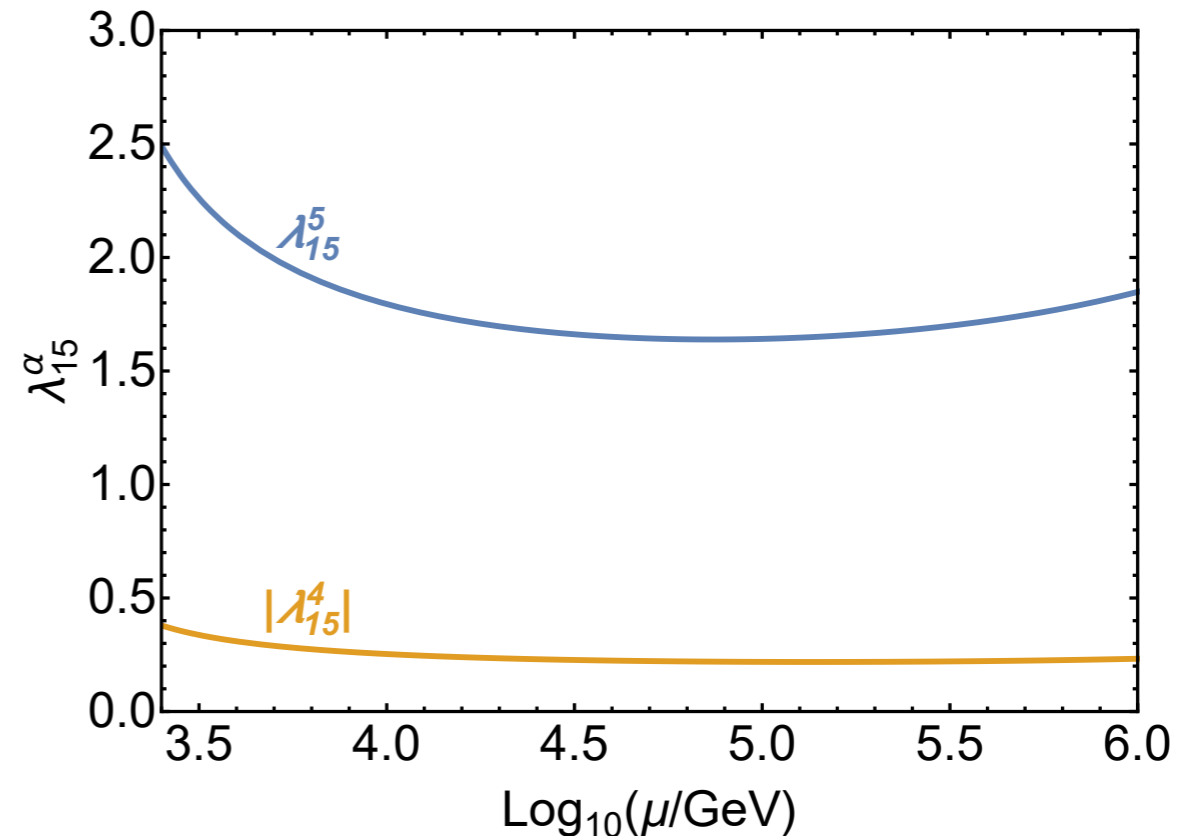
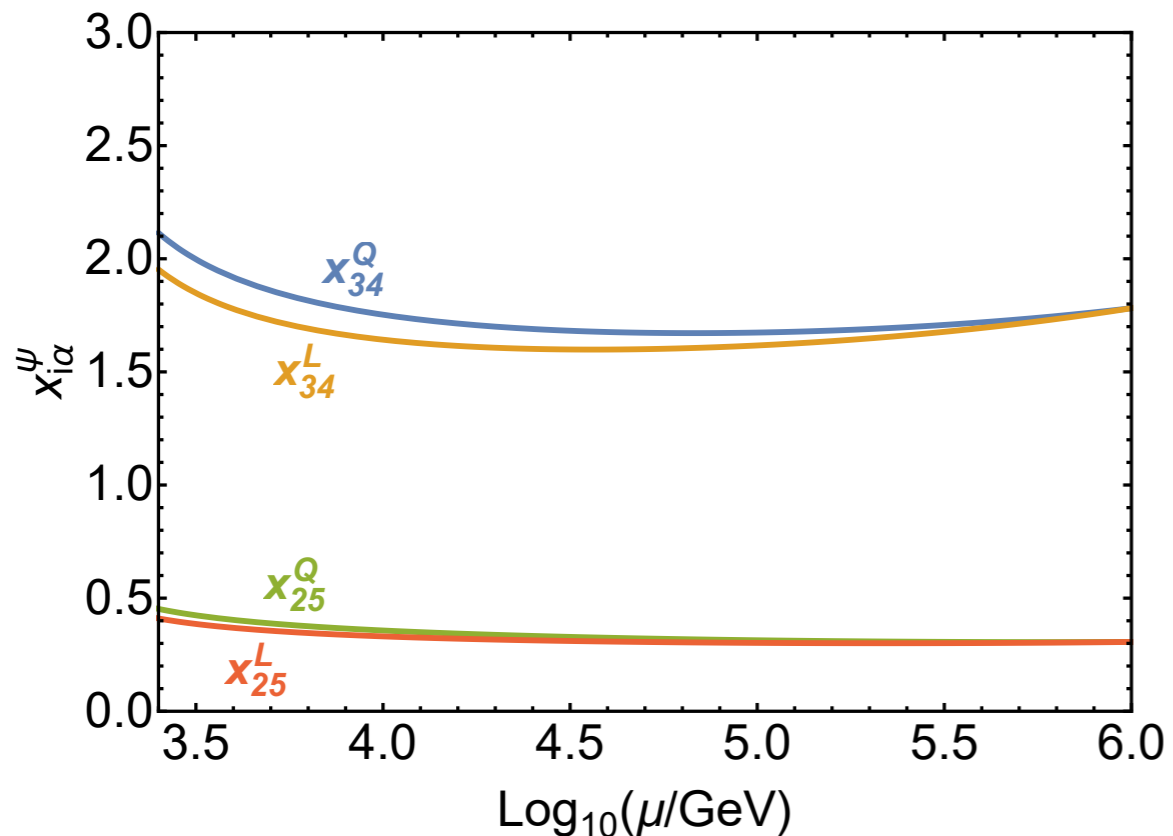
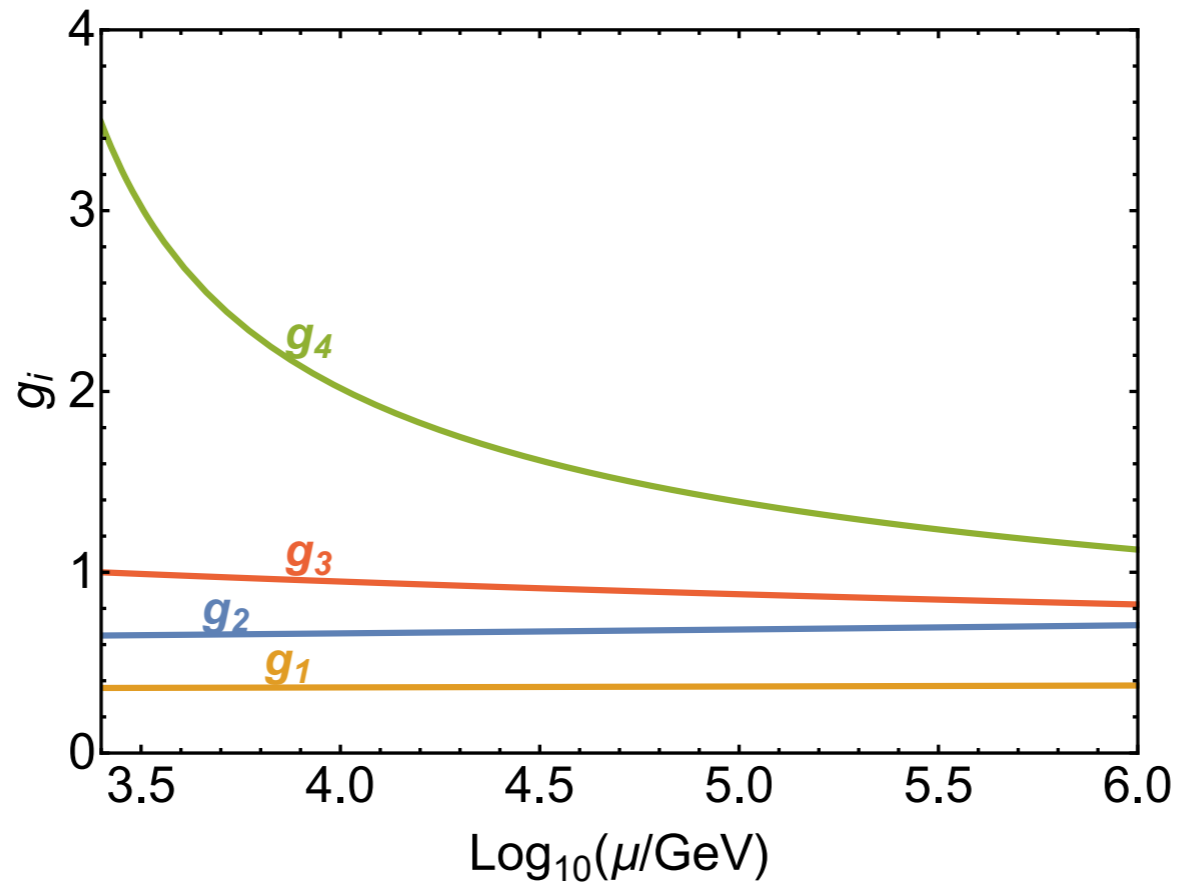
$$m_u \longrightarrow U_L^{u\dagger} m_u U_R^u = \text{diag}(m_u, m_c, m_t), \quad V \equiv U_L^{u\dagger} U_L^d.$$

$$m_d \longrightarrow U_L^{d\dagger} m_d U_R^d = \text{diag}(m_d, m_s, m_b),$$

$$m_\ell \longrightarrow U_L^{e\dagger} m_\ell U_R^e = \text{diag}(m_\ell, m_\mu, m_\tau).$$

(basis independent)

# Perturbativity



# LHC production

