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Inflation and 4D gravitational low-energy effective action from higher dimensions

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Content of the talk

1. Introduction and motivation
2. Starobinsky inflation and modified gravity in D=4
3. Modified gravity in higher (D>4) dimensions
4. Inflation from higher dimensions via dimensional reduction (naive)
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6. Beyond the Starobinsky model: Starobinsky-Bel-Robinson gravity

notation:

spacetime signature (-,+,⋯,+)

natural units $\hbar = c = 1$

X^A : D-dimensional coordinates

x^α : 4-dimensional coordinates

y^a : (D-4)-dimensional extra coordinates

Starobinsky model of inflation in modified gravity in 4D spacetime

(short review)

The Starobinsky model of inflation is defined by the action (Starobinsky,1980)

$$S_{\text{Star.}} = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} \left(R + \frac{1}{6m^2} R^2 \right) , \tag{1}$$

where we have introduced the reduced Planck mass $M_{\text{Pl}} = 1/\sqrt{8\pi G_{\text{N}}} \approx 2.4 \times 10^{18}$ GeV, and the scalaron (inflaton) mass m as the only parameter. We use the spacetime signature $(-,+,+,+,)$.

In the high-curvature regime, R^2 dominates and leads to a nearly scale-invariant spectrum.

The $(R+R^2)$ gravity model (1) can be considered as the simplest extension of the standard Einstein-Hilbert action in the context of modified $F(R)$ gravity theories with an action

$$S_F = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g}\, F(R) , \tag{2}$$

in terms of the function $F(R)$ of the scalar curvature R.

Equivalence between $f(R)$ gravity and scalar-tensor gravity I

The $F(R)$ gravity action (2) is classically equivalent to

$$S[g_{\mu\nu},\chi] = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\,\left[F'(\chi)(R-\chi)+F(\chi)\right] \tag{3}$$

with the real scalar field χ, provided that $F'' \neq 0$ that we always assume. The primes denote the derivatives with respect to the argument.

The equivalence is easy to verify because the χ-field equation implies $\chi = R$. In turn, the factor F' in front of the R in (3) can be (generically) eliminated by a Weyl transformation of metric $g_{\mu\nu}$, which transforms the action (3) into the action of the scalar field χ minimally coupled to Einstein gravity and having the scalar potential

$$V = \left(\frac{M_{\rm Pl}^2}{2}\right)\frac{\chi F'(\chi) - F(\chi)}{F'(\chi)^2} \quad . \tag{4}$$

Equivalence between $f(R)$ gravity and scalar-tensor gravity II

The kinetic term of χ becomes canonically normalized after the field redefinition $\chi(\varphi)$ as

$$F'(\chi) = \exp\left(\sqrt{\tfrac{2}{3}}\varphi/M_{\rm Pl}\right) \ , \quad \varphi = \frac{\sqrt{3}M_{\rm Pl}}{\sqrt{2}} \ln F'(\chi) \ , \tag{5}$$

in terms of the canonical inflaton field φ, with the total acton

$$S_{\rm quintessence}[g_{\mu\nu},\varphi] = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}\left[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right] \ . \tag{6}$$

The classical and quantum stability conditions of $F(R)$ gravity theory are given by

$$F'(R) > 0 \quad \text{and} \quad F''(R) > 0 \ , \tag{7}$$

and they are obviously satisfied for Starobinsky model (1) for $R > 0$.

The inverse transformation

The inverse transformation reads

$$R = \left[\frac{\sqrt{6}}{M_{\rm Pl}} \frac{dV}{d\varphi} + \frac{4V}{M_{\rm Pl}^2} \right] \exp\left(\sqrt{\tfrac{2}{3}} \varphi / M_{\rm Pl} \right) , \tag{8}$$

$$F = \left[\frac{\sqrt{6}}{M_{\rm Pl}} \frac{dV}{d\varphi} + \frac{2V}{M_{\rm Pl}^2} \right] \exp\left(2\sqrt{\tfrac{2}{3}} \varphi / M_{\rm Pl} \right) . \tag{9}$$

In the case of Starobinsky model (1), one finds the famous potential

$$V(\varphi) = \frac{3}{4} M_{\rm Pl}^2 m^2 \left[1 - \exp\left(-\sqrt{\tfrac{2}{3}} \varphi / M_{\rm Pl} \right) \right]^2 . \tag{10}$$

This scalar potential is bounded from below (non-negative and stable), and it has the absolute minimum at $\varphi = 0$ corresponding to a Minkowski vacuum. The scalar potential (10) also has a plateau of positive height (related to the inflationary energy density), that gives rise to slow roll of inflaton during the inflationary era.

Starobinsky inflation and CMB (Planck)

The Starobinsky model (1) is in very good agreement with the Planck data. The Planck (2018) satellite mission measurements of the Cosmic Microwave Background (CMB) radiation give the scalar perturbations tilt as $n_s \approx 1 + 2\eta_V - 6\varepsilon_V \approx 0.9649 \pm 0.0042$ (68%CL) and restrict the tensor-to-scalar ratio as $r \approx 16\varepsilon_V < 0.064$ (95%CL). The Starobinsky inflation yields $r \approx 12/N_e^2 \approx 0.004$ and $n_s \approx 1 - 2/N_e$, where N_e is the e-foldings number between 50 and 60, with the best fit at $N_e \approx 55$.

The Starobinsky model (1) is geometrical (based on gravity only), while its (mass) parameter m is fixed by the observed CMB amplitude (COBE, WMAP) given by $\log(10^{10} A_s) = 2.975 \pm 0.056$ (68%CL) (or $A_s \approx 1.96 \cdot 10^{-9}$) as

$$m \approx 3 \cdot 10^{13}\ \text{GeV} \quad \text{or} \quad \frac{m}{M_{\text{Pl}}} \approx 1.3 \cdot 10^{-5}\,. \tag{14}$$

A numerical analysis of (11) with the potential (10) yields (with $N_e \approx 55$)

$$\sqrt{\frac{2}{3}}\varphi_*/M_{\text{Pl}} \approx \ln\left(\frac{4}{3}N_e\right), \qquad \sqrt{\frac{2}{3}}\varphi_{\text{end}}/M_{\text{Pl}} \approx \ln\left[\frac{2}{11}(4+3\sqrt{3})\right]$$

4D inflation from Modified gravity in higher dimensions D>4

We consider a higher dimensional modified gravity with extra dimensions compactified on a sphere as a toy model.

①We derive the inflaton potential from the higher dimensional modified gravity

Ketov, Nakada (2017)

② The model is extended by adding a (p-1)-form gauge field to achieve modulus stabilization by a flux

cf. : S. P. Otero, F. G. Pedro and C. Wieck(2017)

③We studied in detail the D=8 model.

: Ketov, Nakada (2017

$$S_D = \frac{M_D^{D-2}}{2} \int d^D X \sqrt{-g} [R + \gamma_D R^n - 2\Lambda_D \quad ①$$

$$+ g^{A_1 B_1} g^{A_2 B_2} \cdots g^{A_p B_p} F_{A_1 A_2 \cdots A_p} F_{B_1 B_2 \cdots B_p}] \quad ②$$

$$M_D^{D-2} = \frac{1}{\kappa^2}$$

where the presence of $F = dA$, A is (p-1) –form gauge field is motivated by supergravity

Legendre-Weyl transformation in D dimensions

①Introduce new field B: $R + \gamma R^n \longrightarrow (1+B)R - \left(\frac{1}{\gamma n}\right)^{\frac{1}{n-1}} \left(\frac{n-1}{n}\right) B^{\frac{n}{n-1}}$

$$S = \frac{1}{2\kappa^2}\int d^Dx\sqrt{-g_D}\left[(1+B)R - \left(\frac{1}{\gamma n}\right)^{\frac{1}{n-1}} \left(\frac{n-1}{n}\right) B^{\frac{n}{n-1}} - 2\Lambda\right]$$

EOM of B:

$$B = \gamma n R^{n-1}$$

→This action is equivalent then

②Weyl transformation: $g_{AB} = \Omega^{-2}\tilde{g}_{AB}, \quad \sqrt{-g} = \Omega^{-D}\sqrt{-\tilde{g}}$

choose $\Omega^{D-2} = e^{(D-2)f} = 1 + B$

where $f = \ln\Omega$

$$f_A = \frac{\partial_A \Omega}{\Omega}$$

③by field redefinition: $\phi = \sqrt{\frac{(D-1)(D-2)}{\kappa^2}}\, f$

We get the standard action of Einstein gravity minimally coupled to the canonical **scalar:**

$$S = \frac{1}{2\kappa^2}\int d^Dx\sqrt{-\tilde{g}_D}\tilde{R} + \int d^Dx\sqrt{-\tilde{g}_D}\left[-\frac{1}{2}\tilde{g}^{AB}\partial_A\phi\partial_B\phi - V(\phi)\right]$$

The inflaton potential

$$2\kappa^2 V(\phi) = \left(\frac{1}{\gamma n}\right)^{\frac{1}{n-1}} \left(\frac{n-1}{n}\right) \left[e^{(D-2)\kappa\phi/\sqrt{(D-1)(D-2)}} - 1\right]^{\frac{n}{n-1}} \times$$
$$\times\, e^{-D\kappa\phi/\sqrt{(D-1)(D-2)}} + 2\Lambda e^{-D\kappa\phi/\sqrt{(D-1)(D-2)}}$$

$$\tilde{V}(\tilde{\phi}) = \left[e^{(D-2)\tilde{\phi}} - 1\right]^{\frac{n}{n-1}} e^{-D\tilde{\phi}} + \lambda e^{-D\tilde{\phi}}$$

where

$$\tilde{V}(\tilde{\phi}) = \frac{2V_4(\phi_4)}{\left(\frac{1}{\gamma n}\right)^{\frac{1}{n-1}} \left(\frac{n-1}{n}\right)}, \quad \tilde{\phi} = \frac{\phi_4}{\sqrt{(D-1)(D-2)}}, \quad \lambda = \left(\frac{n}{n-1}\right)\left(\frac{1}{\gamma n}\right)^{-\frac{1}{n-1}} 2\Lambda$$

with the compactification and dimensional reduction

$$\int d^D x = V_{D-4} \int d^4 x\,, \quad \phi = \phi_4/\sqrt{V_{D-4}}\,, \quad \kappa = \kappa_4\sqrt{V_{D-4}}\,, \quad V = V_4/V_{D-4}$$

A requirement for the potential

①to have a plateau at $\phi \sim \infty$

$$\left[e^{(D-2)\tilde{\phi}}\right]^{\frac{n}{n-1}} e^{-D\tilde{\phi}} = 1 \quad \Rightarrow \quad n = \frac{D}{2}$$

The potential becomes

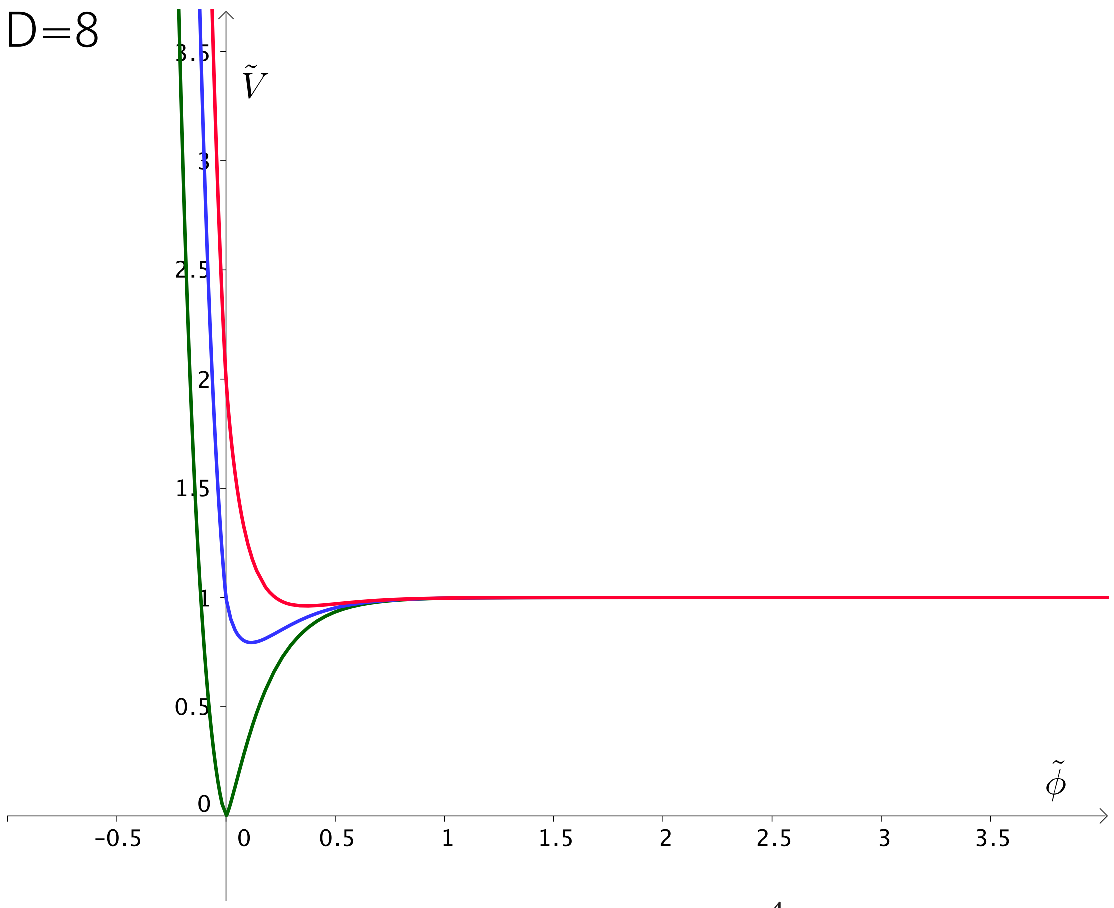
$$\tilde{V}(\tilde{\phi}) = \left[1 - e^{-(D-2)\tilde{\phi}}\right]^{\frac{D}{D-2}} + \lambda e^{-D\tilde{\phi}}$$

②global definition of R and ϕ ⇒ D is multiple of 4

③demand a stable flat direction in V and positive inflaton mass in D=4 ⇒ p=n & $\Lambda_D > 0$

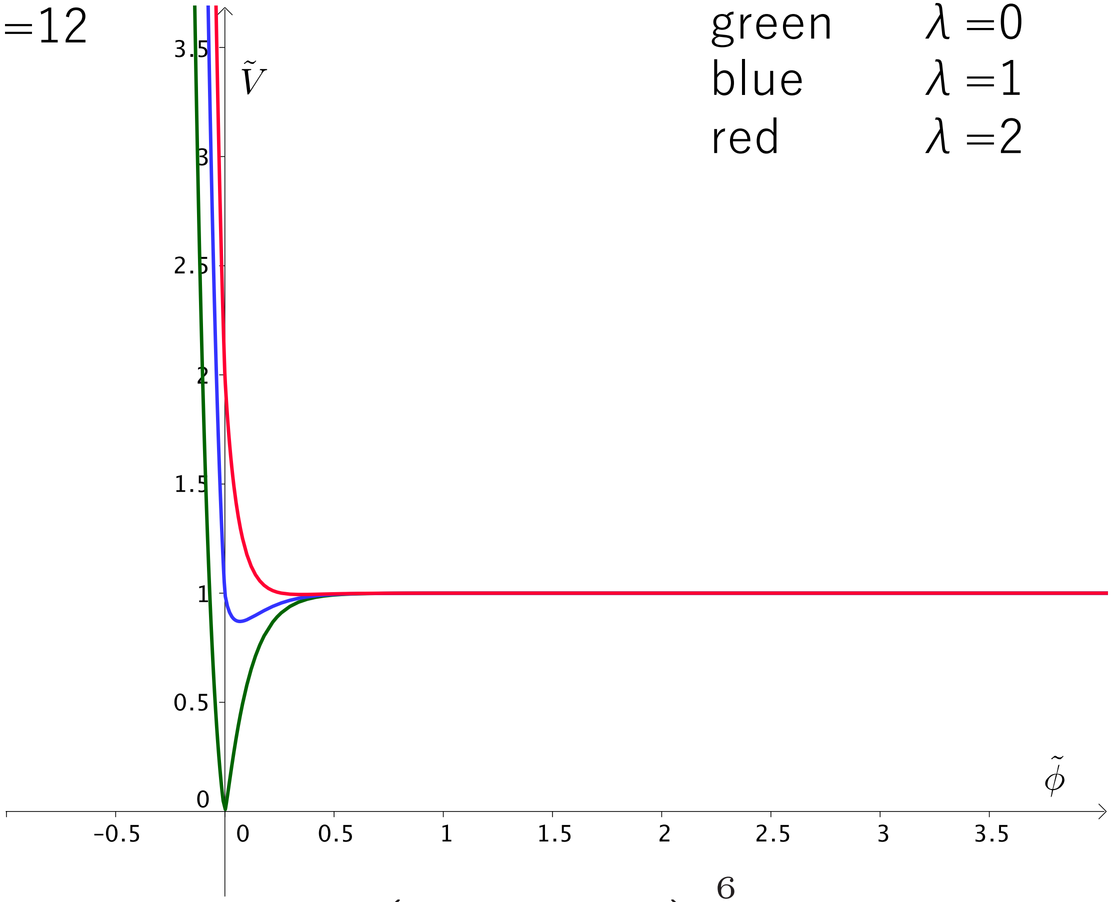
The inflaton potential in D=8 and D=12

D=8



$$\tilde{V}(\tilde{\phi}) = \left(1 - e^{-6\tilde{\phi}}\right)^{\frac{4}{3}} + \lambda e^{-8\tilde{\phi}}$$

D=12



green λ =0
blue λ =1
red λ =2

$$\tilde{V}(\tilde{\phi}) = \left(1 - e^{-10\tilde{\phi}}\right)^{\frac{6}{5}} + \lambda e^{-12\tilde{\phi}}$$

These potentials are already suitable for 4D inflation,

with $n = D/2$ and $\Lambda_D > 0$, if we assume dimensional reduction: i.e. all fields in D dimensions independent upon extra coordinates(y) of the hidden flat space.

Then we find the tensor-to-scalar ratio as $$r = \frac{8(D-1)}{(D-2)N_e^2}$$

However, this does not lead to stabilization of extra dimensions.

→Dimensional reduction has to be replaced by spontaneous compactification, with moduli stabilization of extra dimensions.

⇒ adding $F_p = dA_{p-1}$ with flux $\neq 0$, and $p = n$.

cf. : S. P. Otero, F. G. Pedro and C. Wieck(2017)

The D=8 model and its D=4 spontaneous compactification

$$S = \frac{M_8^6}{2}\int d^8X\sqrt{-g_8}\left[R_8 + \gamma_8 R_8^4 - 2\Lambda_8 - g^{A_1B_1}g^{A_2B_2}g^{A_3B_3}g^{A_4B_4}F_{A_1A_2A_3A_4}F_{B_1B_2B_3B_4}\right]$$

Legendre-Weyl transformation

$$S_{\rm dual} = \frac{M_8^6}{2}\int d^8X\sqrt{-\tilde{g}_8}\Big[\tilde{R}_8 - 42\tilde{g}^{AB}\partial_A f\partial_B f - M_8^2\tilde{V}(f) - \tilde{g}^{A_1B_1}\tilde{g}^{A_2B_2}\tilde{g}^{A_3B_3}\tilde{g}^{A_4B_4}F_{A_1A_2A_3A_4}F_{B_1B_2B_3B_4}\Big]$$

where

$$\tilde{V}(f) = a^{-2}(1-e^{-6f})^{\frac{4}{3}} + 2e^{-8f}\tilde{\Lambda}_8$$

with

$$\gamma_8 \equiv M_8^{-6}\tilde{\gamma}_8 \quad , \quad \Lambda_8 \equiv M_8^2\tilde{\Lambda}_8 \quad , \quad \frac{3}{4}\left(\frac{1}{4\tilde{\gamma}_8}\right)^{\frac{1}{3}} \equiv a^{-2}$$

Flux compactification

We assume 4 extra dimensions are compactified on sphere S^4, Freund-Rubin (1980)
and use this ansatz for 8D metric with the warp factor $e^{2\chi}$:

$$ds_8^2 = \tilde{g}_{AB}(X)dX^A dX^B = g_{\alpha\beta}(x)dx^\alpha dx^\beta + e^{2\chi(x)} g_{ab}(y)dy^a dy^b$$

this means

$$M^4 \times e^{2\chi} S^4$$

and we define the 4-form gauge field strength flux F by integrating over S^4

$$\int d^4y\sqrt{g_y}g^{a_1b_1}\cdots g^{a_4b_4}F_{a_1\cdots a_4}F_{b_1\cdots b_4} = M_8^{-2}F^2 \,, \qquad F = const. \neq 0$$

The total potential becomes :

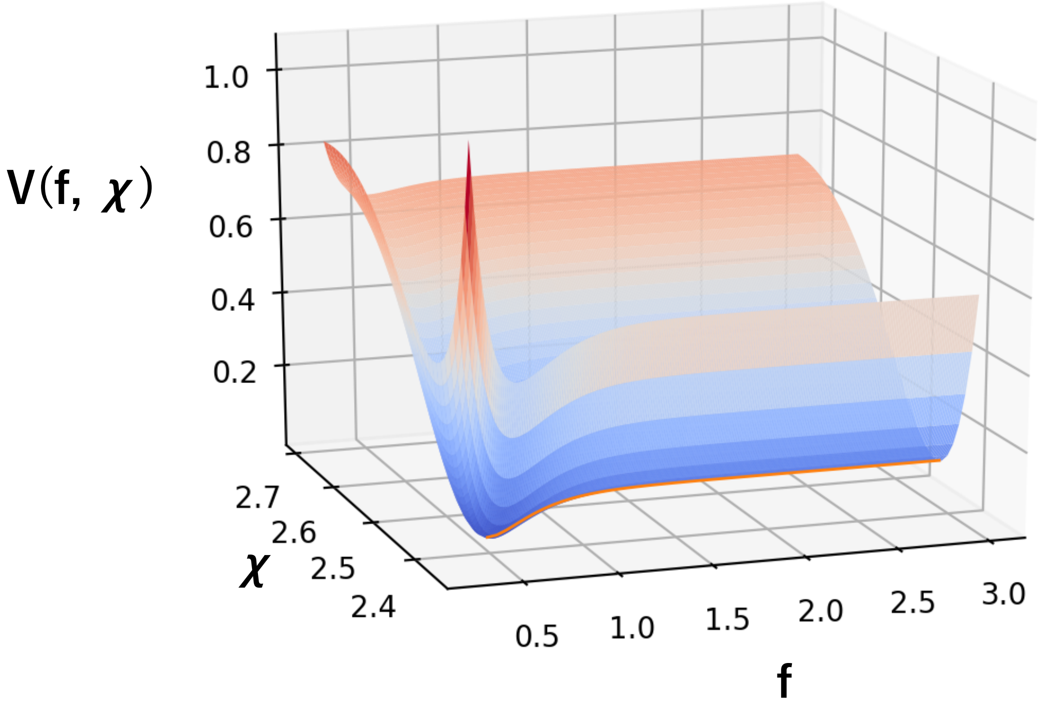
$$M_{Pl}^{-4}V(\chi, f) = \left[a^{-2}(1-e^{-6f})^{\frac{4}{3}} + 2\tilde{\Lambda}_8 e^{-8f}\right]e^{-4\chi} - 2e^{-6\chi} + F^2 e^{-12\chi}$$

where

$$\gamma_8 \equiv M_8^{-6}\tilde{\gamma}_8 \,, \quad \Lambda_8 \equiv M_8^2\tilde{\Lambda}_8 \,, \quad \frac{3}{4}\left(\frac{1}{4\tilde{\gamma}_8}\right)^{\frac{1}{3}} \equiv a^{-2}$$

Study of the potential for two scalar fields

$$V(\chi, f) = \left[a^{-2}(1-e^{-6f})^{\frac{4}{3}} + 2\tilde{\Lambda}_8 e^{-8f}\right] e^{-4\chi} - 2e^{-6\chi} + F^2 e^{-12\chi}$$



$$F^2 = 10^6,\ \tilde{\gamma}_8 = 6\times 10^4,\ \tilde{\Lambda}_8 \approx 0.0174$$

f direction: Starobinsky-type inflation

χ direction: modulus stabilized

in D=8 : $\alpha = \sqrt{\frac{6}{7}}$, $N_e \approx 55$

$$\Rightarrow \begin{cases} r \sim 0.0037 \\ n_S \sim 1 - \frac{2}{N_e} \approx 0.96 \end{cases}$$

Planck observation: $\begin{cases} r < 0.064 \\ n_S = 0.9649 \pm 0.0042 \end{cases}$

The masses derived from study of scalar potential

$$\left.\frac{\partial V}{\partial f}\right|_{f=f_0} = \left.\frac{\partial V}{\partial \chi}\right|_{\chi=\chi_0} = V|_{f=f_0,\ \chi=\chi_0} = 0 \ .$$

$$\begin{cases} e^{6f_0} = 1 + (2\tilde{\Lambda}_8 a^2)^3 \\ e^{6\chi_0} = 2F^2 \\ \frac{2}{3}\tilde{\Lambda}_8 = \left(\frac{1}{16F^2 - 256\tilde{\gamma}_8}\right)^{1/3} \end{cases}$$

mass

$$m^2_{\hat{f}_0} = \left.\frac{\partial^2 V}{\partial f^2}\right|_{f=f_0} \frac{1}{42 M^2_{\rm Pl}} = \frac{M^2_{\rm Pl}}{56 F^2}\left(\frac{F^2}{\tilde{\gamma}_8} - 16\right)$$

$$\Rightarrow \quad \frac{F^2}{\tilde{\gamma}_8} > 16$$

$$m^2_{\hat{\chi}_0} = \left.\frac{\partial^2 V}{\partial \chi^2}\right|_{\chi=\chi_0} \frac{1}{12 M^2_{\rm Pl}} = \frac{M^2_{\rm Pl}}{F^2}$$

$$m^2_{\hat{\chi}_c} = \left.\frac{\partial^2 V}{\partial \chi^2}\right|_{\chi=\chi_c} = 8e^{-6\chi_c}\left(9 - 4a^{-2}e^{2\chi_c}\right)$$

$m_{\hat{f}_0} < m_{\hat{\chi}_0}$ and positivity of $m^2_{\hat{\chi}_c}$ $\Rightarrow$ $\frac{F^2}{\tilde{\gamma}_8} < 54$

Physical properties of D=4 scalar potential

Our potential has a stable Minkowski vacuum in 4D
and a plateau with a positive height (needed for inflation), while

$\tilde{\Lambda}_8 > 0$, $0 < \chi_c - \chi_0 \ll 1$ χ_c : the critical point at the $f \to \infty$

So, the potential describes a single field inflation consistent with observations. (WMAP, Planck)

To get the hierarchy of scales, we need

$M_{inf.} \ll M_{KK} \ll M_{Pl}$ imply $F^2 \gg 1$ where $M_{KK} \approx e^{-\chi_0} M_{Pl}$

and

$m_{f_0} < m_{\chi_0}$ leads to $\frac{F^2}{\tilde{\gamma}_8} < 72$.

Beyond Starobinsky gravity

Question: What could be the next term in the effective quantum gravity beyond the R^2 term in $4D$, when starting from M-theory in $11D$ after its compactification to $4D$?

The bosonic terms of M-theory in the leading order beyond $11D$ supergravity are (M.Green, P. Vanhove, 1997; A.Tseytlin, 2000)

$$\begin{aligned} S_{\mathrm{M}} = \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-g}\left[R-\frac{1}{2\cdot 4!}F^2-\frac{1}{6\cdot 3!\cdot(4!)^2}\varepsilon_{11}CFF\right] \\ -\frac{T_2}{(2\pi)^4\cdot 3^2\cdot 2^{13}}\int d^{11}x\sqrt{-g}\left(J_{11}-\frac{1}{2}E_8\right)+T_2\int C\wedge X_8 \quad , \end{aligned} \tag{1}$$

$11D$ notation I

where κ_{11} is the gravitational constant, T_2 is the M2-brane tension,

$$T_2 = \left(\frac{2\pi^2}{\kappa_{11}^2}\right)^{1/3} \quad , \tag{2}$$

C is the 3-form gauge field, $F = d \wedge C$ is the four-form gauge field strength, R is the gravitational scalar, ε_{11} stands for the Levi-Civita symbol, and (J, E_8, X_8) are the *quartic* polynomials with respect to the Riemann curvature, all in $11D$. The J_{11} is given by

$$J_{11} = 3 \cdot 2^8 \left(R^{mijn} R_{pijq} R_m{}^{rsp} R^q{}_{rsn} + \frac{1}{2} R^{mnij} R_{pqij} R_m{}^{rsp} R^q{}_{rsn} \right) \ , \tag{3}$$

$11D$ notation II

the E_8 is Euler density in $8D$,

$$E_8 = \frac{1}{3!}\varepsilon^{abcm_1 n_1 \ldots m_4 n_4}\varepsilon_{abcm'_1 n'_1 \ldots m'_4 n'_4} R^{m'_1 n'_1}{}_{m_1 n_1} \cdots R^{m'_4 n'_4}{}_{m_4 n_4} \tag{4}$$

and the X_8 is given by the gravitational 8-form

$$X_8 = \frac{1}{192 \cdot (2\pi^2)^4}\left[\mathrm{tr}\widehat{R}^4 - \frac{1}{4}(\mathrm{tr}\widehat{R}^2)^2\right] \quad , \tag{5}$$

where $\widehat{R}$ stands for the spacetime curvature 2-form, and the traces are taken with respect to (implicit) Lorentz indices in $11D$. All (latin) vector indices take values $i, j, k, \ldots = 0, 1, 2, \ldots, 10$, while they are all suppressed in Eq. (1).

Down to four spacetime dimensions

The M-theory (1) can be warp-compactified on the product $S^3 \times S^4$ down to $4D$ via $8D$, in the presence of fluxes needed for moduli stabilization (M.Douglas, S.Kachru,2006). We apply dimensional reduction to the action (1) and ignore details of compactification together with all moduli. Then only one term survives (SVK, M.Iihoshi,2007),

$$S_4 = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R - \kappa^6 \beta J_4 \right) \qquad (6)$$

where all quantities are now in $4D$ with $\kappa = 1/M_{\rm Pl}$, and β is the new dimensionless coupling constant whose value is supposed to be determined by compactification. The J_4 in $4D$ has the same structure as in $11D$, but with $i, j, k, \ \dots = 0, 1, 2, 3$,

$$J_4 = R^{mijn} R_{pijq} R_m{}^{rsp} R^q{}_{rsn} + \frac{1}{2} R^{mnij} R_{pqij} R_m{}^{rsp} R^q{}_{rsn} \quad . \qquad (7)$$

Structure of $4D$ terms I

The peculiar structure and physical meaning of the quartic curvature terms (7) can be revealed via their connection to the *Bel-Robinson* (BR) tensor in $4D$ (L.Bel,1959; I.Robinson,1959; Deser,1999). The BR tensor is defined by

$$T^{iklm} \equiv R^{ipql} R^{k}{}_{pq}{}^{m} + {}^{*}R^{ipql*} R^{k}{}_{pq}{}^{m} \tag{8}$$

by analogy with the energy-momentum tensor of the Maxwell theory of electromagnetism,

$$T_{ij}^{\text{Maxwell}} = F_{ik} F_{j}{}^{k} + {}^{*}F_{ik}{}^{*}F_{j}{}^{k} \quad , \qquad F_{ij} = \partial_i A_j - \partial_j A_i \quad , \tag{9}$$

where the superscript $(*)$ means the dual tensor in $4D$. We also have

$${}^{*}R_{iklm} = \frac{1}{2} E_{ikpq} R^{pq}{}_{lm} \quad . \tag{10}$$

Structure of $4D$ terms II

where $E_{iklm} = \sqrt{-g}\,\varepsilon_{iklm}$ is Levi-Civita tensor in $4D$.

Then we find

$$T^2_{ijkl} = 8J_4 = -\frac{1}{4}({}^*R^{\,2}_{ijkl})^2 + \frac{1}{4}({}^*R_{ijkl}R^{ijkl})^2 = \frac{1}{4}(P_4^2 - E_4^2)\ , \qquad (11)$$

where we have introduced the (standard) Euler and Pontryagin topological densities in $4D$,

$$E_4 = \frac{1}{4}\varepsilon_{ijkl}\varepsilon^{mnpq}R^{ij}{}_{mn}R^{kl}{}_{pq} = {}^*R_{ijkl}{}^*R^{ijkl} = \text{Gauss} - \text{Bonnet} - \text{term}\ , \qquad (12)$$

and

$$P_4 = {}^*R_{ijkl}R^{ijkl}\ , \qquad (13)$$

Conclusion I

Our Starobinsky-like models of inflation in $4D$ derived from *higher* (D) spacetime dimensions are based on the $(R+R^n)$ gravity with $n = D/2$ in the presence of the n-form field F after *flux compactification* with a *warp* factor. It yields the dimensional dependence of the cosmological observables as

$$\alpha = \sqrt{\frac{D-2}{D-1}} \quad \text{and} \quad r = \frac{8(D-1)}{(D-2)N_e^2} \quad . \tag{14}$$

Conclusion II

Combining the insights from Starobinsky inflation and M-theory leads to the $4D$ gravitational low-energy effective action in the form

$$
\begin{aligned}
S_{\mathrm{SBR}}[g_{ij}] &= \frac{M_{\mathrm{Pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\left[R+\frac{1}{6m^2}R^2-\frac{\beta}{M_{\mathrm{Pl}}^6}J_4\right] \ , \\
&= \frac{M_{\mathrm{Pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\left[R+\frac{1}{6m^2}R^2-\frac{\beta}{8M_{\mathrm{Pl}}^6}T^2\right] \ , \\
&= \frac{M_{\mathrm{Pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\left[R+\frac{1}{6m^2}R^2-\frac{\beta}{32M_{\mathrm{Pl}}^6}(P_4^2-E_4^2)\right] \ .
\end{aligned}
\tag{15}
$$

Thank you very much for your attention!