Phenomenological implication of modular symmetry

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Workshop on Standard Model and Beyond
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Based on arXiv**2204.12325**, **2112.00493**

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Introduction

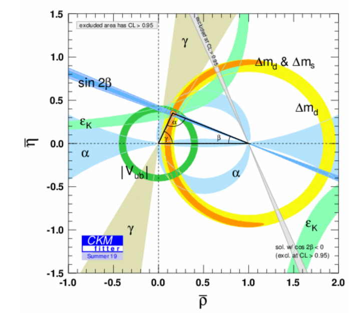
The SM

The origin of flavor
3 generations
hierarchical structure

$M_{u,d,e} \sim$ 

New physics

No significant NP signal
→ NP have highly non-generic flavor structure



Flavor symmetry

Flavor symmetry would play an important role both in the SM and NP

e.g. Discrete flavor symmetry ← Modular flavor symmetry

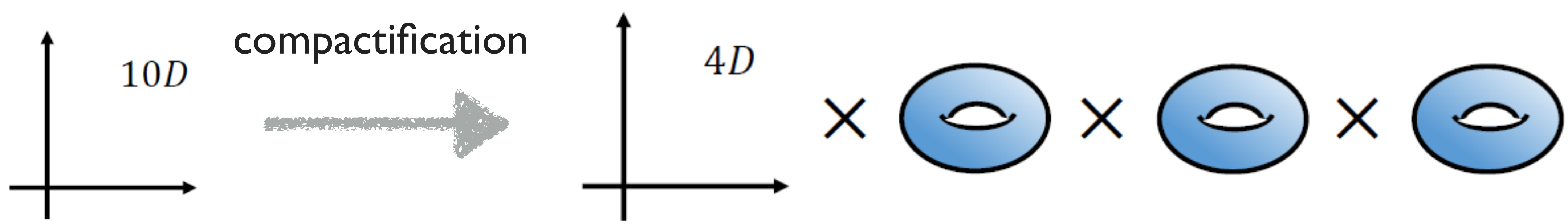
well studied to describe large mixing angle in neutrino

Modular symmetry

see also talk by Sin Kyu Kang

Modular group often appears in the superstring theory

Compactification of the superstring theory

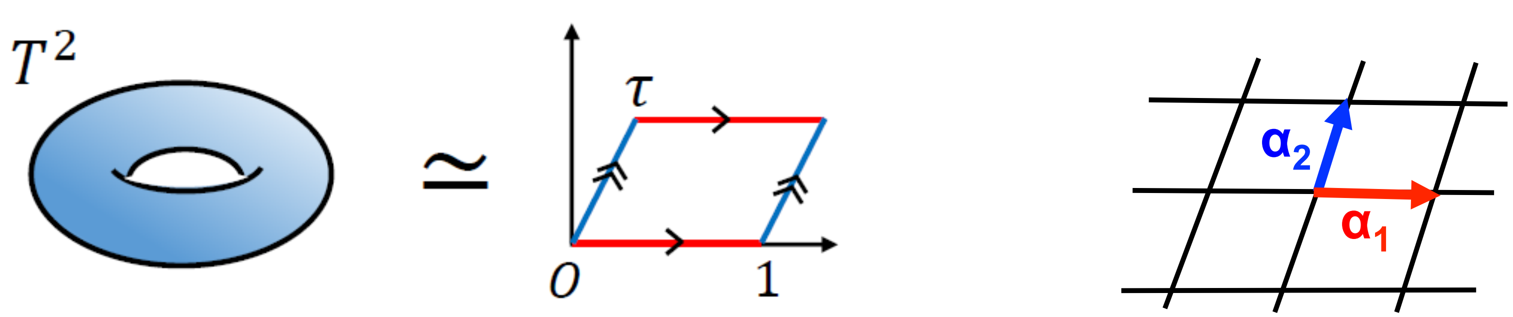


superstring

our universe

2D torus × 3

Two dimensional torus is characterized by modulus τ



$\tau = \alpha_2 / \alpha_1$

2D

complex modulus parameter τ

2D lattice

Modular symmetry

Modular transformation does not change the lattice $\tau = \alpha_2 / \alpha_1$

$$\tau \longrightarrow \gamma\tau = \frac{a\tau + b}{c\tau + d}$$

modular transformation

(2D lattice)' (2D lattice)

$$\begin{pmatrix} \alpha_2' \\ \alpha_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix}$$

$$SL(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, a,b,c,d \in \mathbb{Z}, \quad ad - bc = 1 \right\}$$

The modular group is defined as the transformation group γ, generated by S and T

$$S : \tau \rightarrow -\frac{1}{\tau}$$

duality

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$T : \tau \rightarrow \tau + 1$$

Discrete shift symmetry

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Modular group Γ $\quad \Gamma \simeq \{S, T | S^2 = \mathbb{I}, (ST)^3 = \mathbb{I}\}$

Modular symmetry

$$S : \tau \to -\frac{1}{\tau} \qquad T : \tau \to \tau + 1$$

Modular group Γ $\quad \Gamma \simeq \{S, T | S^2 = \mathbb{I}, (ST)^3 = \mathbb{I}\}$

Quotient group $\Gamma_N \equiv \Gamma / \Gamma(N)$ $\quad \Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}) \,, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$

$$\Gamma_N \equiv \frac{\Gamma}{\Gamma(N)} \simeq \{S, T | S^2 = \mathbb{I}, (ST)^3 = \mathbb{I}, T^N = \mathbb{I}\}$$

isomorphic

Modular symmetry $\simeq$ Discrete symmetry

$N = 2 \qquad \Gamma_2 \simeq S_3$

$N = 3 \qquad \Gamma_3 \simeq A_4$ ← focus on in this work

$N = 4 \qquad \Gamma_4 \simeq S_4$

$N = 5 \qquad \Gamma_5 \simeq A_5$

Modular symmetry

Superstring theory in 10 dimensions

↓ Compactification

4 dimensional theory (SUSY) Γ_N symmetry (modular)

↓ Expectation value of modulus τ breaks the symmetry

Γ_N and SUSY breaking scales are not determined

Low scale phenomenology

T. Kobayashi, H. Otsuka [2108.02700]

SUSY breaking terms are invariant (covariant) under modular transformation in moduli-mediated SUSY breaking scenario

We can consider modular invariant SMEFT by supposing modular forms to be **spurion**

A_4 modular symmetry

Non-Abelian discrete symmetry A_4 group could be adjusted to family symmetry:

The minimum group containing triplet

Irreducible representations: 1, 1'', 1', 3 ← e_R , μ_R , τ_R , (e_L, μ_L, τ_L)

	L_L	$(e^c_R,\ \mu^c_R,\ \tau^c_R)$	H_d	$Y(\tau_e)$
$SU(2)$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
A_4	$\mathbf{3}$	$(\mathbf{1},\ \mathbf{1}'',\ \mathbf{1}')$	$\mathbf{1}$	$\mathbf{3}$
k	2	$(0,\ 0,\ 0)$	0	2

modular form

Effective theories with Γ_N symmetry

modular form

$$\mathscr{L}_{\text{eff}} \supset Y(\tau)_{ij} H \phi^{(I)} \phi^{(J)}$$

chiral superfield with modular weight k transforms as

$$\phi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \phi^{(I)}$$

$\phi^{(I)}$, $f(\tau)$:representation of Γ_N

$\rho(\gamma)$, $\rho^{(I)}(\gamma)$: unitary rep. matrix

Holomorphic functions which transform under modular trans., are called modular form with weight k

$$Y(\tau) \to (c\tau + d)^k \rho(\gamma) Y(\tau)$$

A_4 modular symmetry

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k	2	(0, 0, 0)	0	2

modular form

Effective theories with Γ_N symmetry

modular form

$$\mathscr{L}_{\text{eff}} \supset Y(\tau)_{ij} H \phi^{(I)} \phi^{(J)}$$

Automorphy factor $(c\tau + d)^k (c\tau + d)^{-k_I} (c\tau + d)^{-k_J} = (c\tau + d)^{k - k_I - k_J}$

vanishes if k = kI + kJ

Modular forms are explicitly given if weight k is fixed.

On the other hand, chiral superfields are not modular forms and we have no restriction on the possible value of weight kI, a priori.

A_4 modular symmetry

F. Feruglio [1706.08749]

The holomorphic and anti-holomorphic modular forms with weight 2 compose the A_4 triplet

$$Y_{\mathbf{3}}^{(2)}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix}, \qquad \overline{Y_{\mathbf{3}}^{(2)}(\tau)} \equiv Y_{\mathbf{3}}^{(2)*}(\tau) = \begin{pmatrix} Y_1^*(\tau) \\ Y_3^*(\tau) \\ Y_2^*(\tau) \end{pmatrix}$$

Y_i (i=1,2,3) is a function of the modulus τ

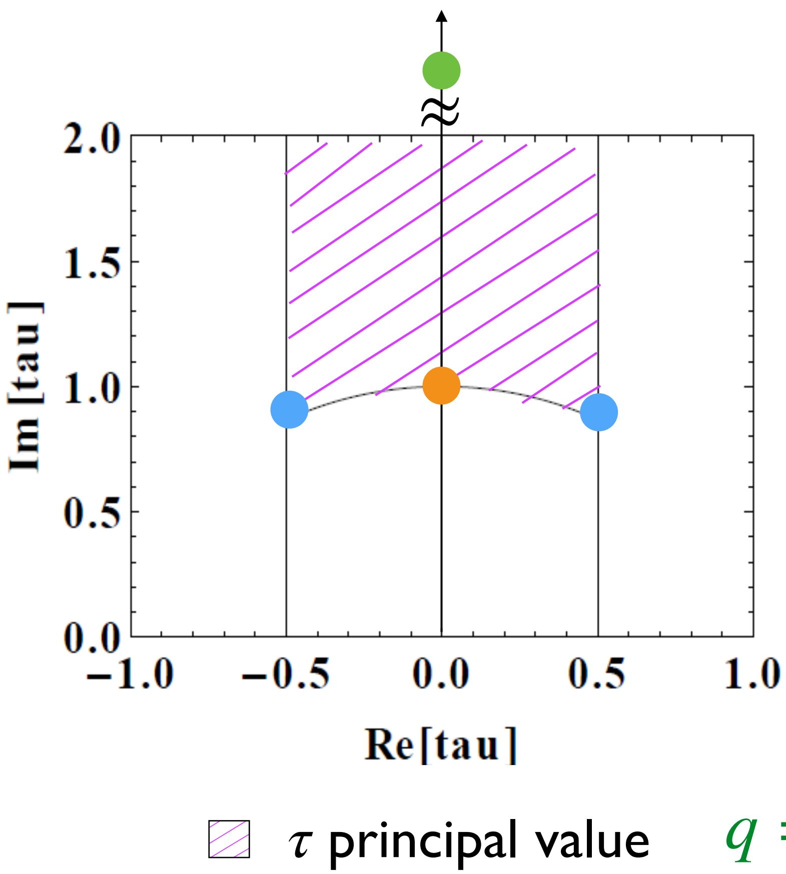
$$\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + 12q^3 + \dots \\ -6q^{1/3}(1 + 7q + 8q^2 + \dots) \\ -18q^{2/3}(1 + 2q + 5q^2 + \dots) \end{pmatrix} \qquad q = e^{2\pi i\tau}$$

Once τ is determined, the Yukawa is fixed

Modular forms with higher weights k=4, 6 ... are constructed by them

A_4 modular symmetry

Fixed point for τ from the view point of the vacuum stability study



$q = e^{2\pi i \tau}$

- $\tau = \omega$ (ST symmetry) $\omega = \frac{-1 + i\sqrt{3}}{2}$
- $\tau = i$ (S symmetry) ← focus on in this talk
  (the successful lepton & quark mass matrix has been reproduced)
- $\tau = i\infty$ (T symmetry)

At exact fixed point, CP is not violated

→ need small deviation from these point : $\tau =$ (fixed point) $+\epsilon$

phenomenologically $\mathcal{O}(\epsilon) \sim 10^{-2}$

Modular symmetry in the SMEFT

String Ansatz

T. Kobayashi, H. Otsuka [2108.02700]

String compactifications leads to 4-dim low energy field theories with the specific structure

Through String Ansatz, higher-dimensional operators are related with 3-point couplings

$$y^{(4)}_{ijk\ell} = \sum_m y^{(3)}_{ijm} y^{(3)}_{mk\ell}$$

m is virtual mode H

→

SMEFT operator

e.g. $Q^{(1)}_{qq}$ $\quad (\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$

$Q^{(1)}_{\ell q}$ $\quad (\bar{\ell}_p \gamma_\mu \ell_r)(\bar{q}_s \gamma^\mu q_t)$

Strategy

[In this talk]

- write down fermionic SMEFT operator so as to be invariant at A_4 and modular symmetry

  focus on $(\bar{L}R)$ bilinear structure in lepton sector

- expand modular forms $Y(\tau)$ at three fixed point, and then include small deviation : $\tau =$ (fixed point) $+\epsilon$

  - $\tau = \omega$ (ST symmetry)
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  - $\tau = i\infty$ (T symmetry)

  focus on $\tau = i$ case

- diagonalize the mass matrix and move to mass eigenstate basis

- pheno. study

  $(g-2)_\mu$ & Lepton flavor violation

$(\bar{L}R)$ structure in the modular symmetry

	L_L	$(e_R^c,\ \mu_R^c,\ \tau_R^c)$	H_d	$Y(\tau_e)$
$SU(2)$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
A_4	$\mathbf{3}$	$(\mathbf{1},\ \mathbf{1}'',\ \mathbf{1}')$	$\mathbf{1}$	$\mathbf{3}$
k	2	(0, 0, 0)	0	2

※ γ_μ structure Γ is omitted

$$[\bar{L}_R L_L]$$

$$A_4: \quad \{1, 1'', 1'\} \otimes 3$$

$$k_I: \quad 0 \quad -2$$

not invariant both A_4 and modular

$(\bar{L}R)$ structure in the modular symmetry

	L_L	$(e_R^c,\ \mu_R^c,\ \tau_R^c)$	H_d	$Y(\tau_e)$
$SU(2)$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
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k	2	(0, 0, 0)	0	2

modular form

※ γ_μ structure Γ is omitted

$[\bar{L}_R L_L]$ → $[\bar{L}_R Y(\tau_q) L_L]$

$A_4:\quad \{1,1'',1'\} \otimes 3 \qquad \{1,1'',1'\} \otimes 3 \otimes 3$

$k_I:\quad 0 \quad -2 \qquad 0 \quad 2 \quad -2$

not invariant both A_4 and modular

invariant

$(\bar{L}R)$ structure in the modular symmetry

	L_L	$(e_R^c,\ \mu_R^c,\ \tau_R^c)$	H_d	$Y(\tau_e)$
$SU(2)$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
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k	2	(0, 0, 0)	0	2

modular form

※ γ_μ structure Γ is omitted

$[\bar{L}_R L_L] \rightarrow [\bar{L}_R Y(\tau_q) L_L]$

decomposition

$$\{1,1'',1'\} \otimes \underbrace{3 \otimes 3}_{} = 1 \oplus 1'' \oplus 1' \oplus 3s \oplus 3a \quad \text{(i)}$$

$1 \otimes 1 = 1$ and $1' \otimes 1'' = 1$ (ii)

The generators of A4 triplet

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix},$$

$$\omega = e^{i\frac{2}{3}\pi}$$

(i)

$$Y(\tau) \otimes L_L = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}_{\mathbf{3}}$$

A_4 multiplication rule

$$= (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_{\mathbf{1}} + (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_{\mathbf{1}'} + (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_{\mathbf{1}''}$$
$$+ (\cdots)_{\mathbf{3s}} + (\cdots)_{\mathbf{3a}}$$

(ii) $\bar{L}_R \otimes (Y(\tau) \otimes L_L)$

$= \bar{e}_R \otimes (Y(\tau) \otimes L_L) \xrightarrow{1 \otimes (1 \oplus 1' \oplus 1'') \to 1 \otimes 1} = \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_{\mathbf{1}}$

$+\bar{\mu}_R \otimes (Y(\tau) \otimes L_L) \xrightarrow{1' \otimes (1 \oplus 1' \oplus 1'') \to 1' \otimes 1''} +\bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_{\mathbf{1}''}$

$+\bar{\tau}_R \otimes (Y(\tau) \otimes L_L) \xrightarrow{1'' \otimes (1 \oplus 1' \oplus 1'') \to 1'' \otimes 1'} +\bar{\tau}_R \otimes (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_{\mathbf{1}'}$

$(\bar{L}R)$ structure in the modular symmetry

$$[\bar{L}_R \otimes Y(\tau) \otimes L_L]_{\mathbf{1}} = \alpha_e\, \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_{\mathbf{1}} + \beta_e\, \bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_{\mathbf{1}''}$$

$$+\gamma_e\, \bar{\tau}_R \otimes (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_{\mathbf{1}'}$$

$$= (\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R) \begin{pmatrix} \alpha_e & 0 & 0 \\ 0 & \beta_e & 0 \\ 0 & 0 & \gamma_e \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$$

Same structure with mass matrix :

$$M_e = v_d \begin{pmatrix} \alpha_{e(m)} & 0 & 0 \\ 0 & \beta_{e(m)} & 0 \\ 0 & 0 & \gamma_{e(m)} \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix}_{RL}$$

if mode m is only higgs

$$\alpha_d = c\alpha_{d(m)}, \qquad \beta_d = c\beta_{d(m)}, \qquad \gamma_d = c\gamma_{d(m)}$$

→ flavor changing like $\mu \to e$ never happen

$(\bar{L}R)$ structure in the modular symmetry

$$[\bar{L}_R \otimes Y(\tau) \otimes L_L]_{\mathbf{1}} = \alpha_e \, \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_{\mathbf{1}} + \beta_e \, \bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_{\mathbf{1}''}$$

$$+ \gamma_e \, \bar{\tau}_R \otimes (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_{\mathbf{1}'}$$

$$= (\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R) \begin{pmatrix} \alpha_e & 0 & 0 \\ 0 & \beta_e & 0 \\ 0 & 0 & \gamma_e \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$$

Same structure with mass matrix :

$$M_e = v_d \begin{pmatrix} \alpha_{e(m)} & 0 & 0 \\ 0 & \beta_{e(m)} & 0 \\ 0 & 0 & \gamma_{e(m)} \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix}_{RL}$$

if there are additional unknown modes (e.g. multi Higgs modes), it causes flavor violations

Suppose unknown mode contribution being small and couplings are Higgs-like

$$\alpha_d - \alpha_{d(m)} \ll \alpha_d, \qquad \beta_d - \beta_{d(m)} \ll \beta_d, \qquad \gamma_d - \gamma_{d(m)} \ll \gamma_d.$$

$(\bar{L}R)$ structure in the modular symmetry

$$[\bar{L}_R \otimes Y(\tau) \otimes L_L]_{\mathbf{1}} = \alpha_e\, \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_{\mathbf{1}} + \beta_e\, \bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_{\mathbf{1}''}$$
$$+ \gamma_e\, \bar{\tau}_R \otimes (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_{\mathbf{1}'}$$

Same

$$\frac{\tilde{\beta}_e}{\tilde{\beta}_{e(m)}} = \frac{\tilde{\beta}_{e(m)} + c_\beta}{\tilde{\beta}_{e(m)}} = 1 + \frac{c_\beta}{\tilde{\beta}_{e(m)}} \equiv 1 + \delta_\beta\,,$$
$$\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}} = \frac{\tilde{\alpha}_{e(m)} + c_\alpha}{\tilde{\alpha}_{e(m)}} = 1 + \frac{c_\alpha}{\tilde{\alpha}_{e(m)}} \equiv 1 + \delta_\alpha\,,$$
$$\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}} = \frac{\tilde{\gamma}_{e(m)} + c_\gamma}{\tilde{\gamma}_{e(m)}} = 1 + \frac{c_\gamma}{\tilde{\gamma}_{e(m)}} \equiv 1 + \delta_\gamma\,,$$

δ’s are very small

if there are additional unknown modes (e.g. multi Higgs modes), it causes flavor violations

Suppose unknown mode contribution being small and couplings are Higgs-like

$$\alpha_d - \alpha_{d(m)} \ll \alpha_d, \qquad \beta_d - \beta_{d(m)} \ll \beta_d, \qquad \gamma_d - \gamma_{d(m)} \ll \gamma_d.$$

Strategy

In this talk

- write down fermionic SMEFT operator so as to be invariant at A_4 and modular symmetry

  focus on $(\bar{L}R)$ bilinear structure in lepton sector

- expand modular forms $Y(\tau)$ at three fixed point, and then include small deviation : $\tau =$ (fixed point) $+\epsilon$

  - $\tau = \omega$ (ST symmetry)
  - $\tau = i$ (S symmetry)
  - $\tau = i\infty$ (T symmetry)

  focus on $\tau = i$ case

- diagonalize the mass matrix and move to mass eigenstate basis

- pheno. study

  $(g-2)_{e,\mu}$ & Lepton flavor violation

at $\tau = i$ (S symmetry); Diagonalization

Results of $(\bar{L}R)$ structure in interaction basis

$\bar{R}L$ $\bar{L}R$	$\bar{\mu}_R\Gamma\tau_L$ $\bar{\mu}_L\Gamma\tau_R$	$\bar{e}_R\Gamma\tau_L$ $\bar{e}_L\Gamma\tau_R$	$\bar{e}_R\Gamma\mu_L$ $\bar{e}_L\Gamma\mu_R$
Coeff.	$\beta_e\, Y_3(\tau_e)$ $\gamma_e\, Y_2^*(\tau_e)$	$\alpha_e\, Y_2(\tau_e)$ $\gamma_e\, Y_3^*(\tau_e)$	$\alpha_e\, Y_3(\tau_e)$ $\beta_e\, Y_2^*(\tau_e)$

Insert holomorphic modular forms of weight 2 at $\tau = i$

$$Y(\tau_e = i) = Y_1(i)\begin{pmatrix} 1 \\ 1-\sqrt{3} \\ -2+\sqrt{3} \end{pmatrix}$$

Transrate

$$D_L \to D_L^S \equiv U_S\, D_L\,, \qquad \bar{D}_L \to \bar{D}_L^S \equiv \bar{D}_L U_S^\dagger\,,$$
$$E_L \to E_L^S \equiv U_S\, E_L\,, \qquad \bar{E}_L \to \bar{E}_L^S \equiv \bar{E}_L\, U_S^\dagger\,,$$

The flavor structure of the FC bilinear operators at $\tau = i$

$\tau = i + \epsilon$, then the left-handed fields are not yet the mass eigenstate, but close to it

using approximate behaviors

$$\frac{Y_2(\tau)}{Y_1(\tau)} \simeq (1+\epsilon_1)\,(1-\sqrt{3})\,, \quad \frac{Y_3(\tau)}{Y_1(\tau)} \simeq (1+\epsilon_2)\,(-2+\sqrt{3})\,, \quad \epsilon_1 = \frac{1}{2}\epsilon_2 \simeq 2.05\, i\, \epsilon$$

Okada and Tanimoto [2009.14242]

These approximate forms are agreement with exact numerical values within $0.1\,\%$ for $|\epsilon| \leq 0.05$

Mass eigenstate basis at $\tau = i$ and $\tau = i + \epsilon$

at $\tau = i$ (S symmetry); Diagonalization

Mass eigenstate basis at $\tau = i$ and $\tau = i + \epsilon$

$\bar{\mu}_R\Gamma\tau_L$ $\bar{\mu}_L\Gamma\tau_R$	$\bar{e}_R\Gamma\tau_L$ $\bar{e}_L\Gamma\tau_R$	$\bar{e}_R\Gamma\mu_L$ $\bar{e}_L\Gamma\mu_R$
$\frac{\sqrt{3}}{2}(\tilde{\alpha}_e + 2s^e_{R23}\tilde{\gamma}_e)$ V $(\sqrt{3}s^e_{23L} + s^e_{12L}\lvert\epsilon_1^*\rvert)\tilde{\gamma}_e - \frac{3}{2}s^e_{R23}\tilde{\alpha}_e$	$\frac{\sqrt{3}}{2}(\tilde{\beta}_e - s^e_{12R}\tilde{\alpha}_e + 2(s^e_{R13} - s^e_{R12}s^e_{R23})\tilde{\gamma}_e)$ V $(\sqrt{3}s^e_{13L} + \lvert\epsilon_1^*\rvert)\tilde{\gamma}_e$	$\frac{3}{2}(\tilde{\beta}_e + s^e_{12R}\tilde{\alpha}_e)$ V $\frac{1}{2}(3s^e_{12L} - \sqrt{3}s^e_{13L} + 2\lvert\epsilon_1^*\rvert)\tilde{\alpha}_e$

$$s^e_{L12} \simeq -|\epsilon_1^*|\,, \qquad s^e_{L23} \simeq -\frac{\sqrt{3}}{4}\frac{\tilde{\alpha}^2_{e(m)}}{\tilde{\gamma}^2_{e(m)}}, \qquad s^e_{L13} \simeq -\frac{\sqrt{3}}{3}|\epsilon_1^*|\,,$$

$$s^e_{R12} \simeq -\frac{\tilde{\beta}_{e(m)}}{\tilde{\alpha}_{e(m)}}\,, \qquad s^e_{R23} \simeq -\frac{1}{2}\frac{\tilde{\alpha}_{e(m)}}{\tilde{\gamma}_{e(m)}}\,, \qquad s^e_{R13} \simeq -\frac{1}{2}\frac{\tilde{\beta}_{e(m)}}{\tilde{\gamma}_{e(m)}}$$

$$\tilde{\alpha}_{e(m)} = (6-3\sqrt{3})Y_1(i)\alpha_{e(m)},\ \tilde{\beta}_{e(m)} = (6-3\sqrt{3})Y_1(i)\beta_{e(m)} \text{ and } \tilde{\gamma}_{e(m)} = (6-3\sqrt{3})Y_1(i)\gamma_{e(m)}$$

$\tau, \alpha_e, \beta_e, \gamma_e$: Best fit values of parameters in A4 modular invariant model to realize lepton mass matrix, neutrino data

Okada and Tanimoto [2012.01688]

$$\tau = -0.080 + 1.007\,i\,, \quad |\epsilon_1| = 0.165\,, \quad \frac{\tilde{\alpha}_{e(m)}}{\tilde{\gamma}_{e(m)}} \simeq \frac{\tilde{\alpha}_e}{\tilde{\gamma}_e} = 6.82\times 10^{-2}\,, \quad \frac{\tilde{\beta}_{e(m)}}{\tilde{\alpha}_{e(m)}} \simeq \frac{\tilde{\beta}_e}{\tilde{\alpha}_e} = 1.50\times 10^{-2}$$

→ predict flavor observables

Strategy

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  focus on $(\bar{L}R)$ bilinear structure in lepton sector

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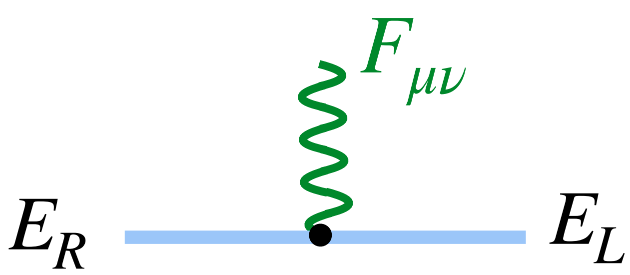
- pheno. study

  $(g-2)_\mu$ & Lepton flavor violation

$(g-2)_\mu$ & $\mu \to e\gamma$

Dipole operator



$$\mathcal{L}_{\rm dipole} = \frac{1}{\Lambda^2}\left(\mathcal{C}'_{\underset{LR}{e\gamma}}\mathcal{O}_{\underset{LR}{e\gamma}} + \mathcal{C}'_{\underset{RL}{e\gamma}}\mathcal{O}_{\underset{RL}{e\gamma}}\right)$$

$$\mathcal{O}_{\underset{LR}{e\gamma}} = \frac{v}{\sqrt{2}}\overline{E}_L\sigma^{\mu\nu}E_R F_{\mu\nu}$$

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

$(g-2)_\mu$

$$\Delta a_\mu = \frac{4m_\mu}{e}\frac{v}{\sqrt{2}}\frac{1}{\Lambda^2}\mathrm{Re}\left[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}\right]$$

$$\Delta a_\mu = a_\mu^{\rm Exp} - a_\mu^{\rm SM} = (251 \pm 59) \times 10^{-11}$$

FNAL, BNL

$$\rightarrow \quad \frac{1}{\Lambda^2}\mathrm{Re}\left[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}\right] \approx 1.0 \times 10^{-5}\ \mathrm{TeV}^{-2}$$

Lepton flavor violation $\mu \to e\gamma$

$$\mathcal{B}(\ell_r \to \ell_s\gamma) = \frac{m_{\ell_r}^3 v^2}{8\pi\Gamma_{\ell_r}}\frac{1}{\Lambda^4}\left(\left|\mathcal{C}'_{\underset{rs}{e\gamma}}\right|^2 + \left|\mathcal{C}'_{\underset{sr}{e\gamma}}\right|^2\right)$$

$$\mathcal{B}(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13}\ (90\%\ \mathrm{C.L.})$$

MEG

$$\rightarrow \quad \frac{1}{\Lambda^2}\left|\mathcal{C}'_{\underset{e\mu(\mu e)}{e\gamma}}\right| < 2.1 \times 10^{-10}\ \mathrm{TeV}^{-2}$$

strong flavor alignment

$$\left|\frac{\mathcal{C}'_{\underset{e\mu(\mu e)}{e\gamma}}}{\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}}\right| < 2.1 \times 10^{-5}$$

specific flavor structure

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

Isidori, Pages and Wilsch [2111.13724]

$(g-2)_\mu$ & $\mu \to e\gamma$

Wilson coefficients in A4 modular symmetry in mass basis

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

$$\mathcal{C}'_{\underset{ee}{e\gamma}} = 3\,(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*|\,, \quad \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} = \frac{3}{2}\,(1-\sqrt{3})\tilde{\alpha}_e\,, \quad \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} = \sqrt{3}\,(1-\sqrt{3})\tilde{\gamma}_e$$

$$\mathcal{C}'_{\underset{\tau\mu}{e\gamma}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\alpha}_e\left(1-\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\frac{\tilde{\alpha}_{e(m)}}{\tilde{\alpha}_e}\right)\,,$$

$$\mathcal{C}'_{\underset{\tau e}{e\gamma}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\beta}_e\left(1+\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}-2\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\right)$$

$$\mathcal{C}'_{\underset{\mu e}{e\gamma}} = \frac{3}{2}(1-\sqrt{3})\tilde{\beta}_e\left(1-\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\right)\,,$$

flavor alignment

$$\left|\frac{\mathcal{C}'_{\underset{e\mu}{e\gamma}}}{\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}}\right| = \frac{\tilde{\beta}_e}{\tilde{\alpha}_e}\left|1-\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\right| < 2.1\times 10^{-5}$$

$$\frac{\tilde{\beta}_e}{\tilde{\beta}_{e(m)}} = \frac{\tilde{\beta}_{e(m)}+c_\beta}{\tilde{\beta}_{e(m)}} = 1+\frac{c_\beta}{\tilde{\beta}_{e(m)}} \equiv 1+\delta_\beta\,,$$
$$\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}} = \frac{\tilde{\alpha}_{e(m)}+c_\alpha}{\tilde{\alpha}_{e(m)}} = 1+\frac{c_\alpha}{\tilde{\alpha}_{e(m)}} \equiv 1+\delta_\alpha\,,$$
$$\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}} = \frac{\tilde{\gamma}_{e(m)}+c_\gamma}{\tilde{\gamma}_{e(m)}} = 1+\frac{c_\gamma}{\tilde{\gamma}_{e(m)}} \equiv 1+\delta_\gamma\,,$$

$$\rightarrow \quad \left|1-\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\right| \simeq |\delta_\beta-\delta_\alpha| < 1.4\times 10^{-3}$$

without tuning between $\delta_{\alpha,\beta}$, $\quad |\delta_\alpha| < \mathcal{O}(10^{-3})\,, \qquad |\delta_\beta| < \mathcal{O}(10^{-3})$

$\tau \to \mu\gamma$ & $\tau \to e\gamma$ & $\mu \to e\gamma$

Wilson coefficients in A4 modular symmetry in mass basis

$$\mathcal{C}'_{\substack{e\gamma \\ LR}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma \\ ee}} & \mathcal{C}'_{\substack{e\gamma \\ e\mu}} & \mathcal{C}'_{\substack{e\gamma \\ e\tau}} \\ \mathcal{C}'_{\substack{e\gamma \\ \mu e}} & \mathcal{C}'_{\substack{e\gamma \\ \mu\mu}} & \mathcal{C}'_{\substack{e\gamma \\ \mu\tau}} \\ \mathcal{C}'_{\substack{e\gamma \\ \tau e}} & \mathcal{C}'_{\substack{e\gamma \\ \tau\mu}} & \mathcal{C}'_{\substack{e\gamma \\ \tau\tau}} \end{pmatrix}$$

$$\mathcal{C}'_{\substack{e\gamma \\ ee}} = 3\,(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*|\,, \qquad \mathcal{C}'_{\substack{e\gamma \\ \mu\mu}} = \frac{3}{2}\,(1-\sqrt{3})\tilde{\alpha}_e\,, \qquad \mathcal{C}'_{\substack{e\gamma \\ \tau\tau}} = \sqrt{3}\,(1-\sqrt{3})\tilde{\gamma}_e$$

$$\mathcal{C}'_{\substack{e\gamma \\ \tau\mu}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\alpha}_e\left(1-\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\frac{\tilde{\alpha}_{e(m)}}{\tilde{\alpha}_e}\right)\,,$$

$$\mathcal{C}'_{\substack{e\gamma \\ \tau e}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\beta}_e\left(1+\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}-2\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\right)$$

$$\mathcal{C}'_{\substack{e\gamma \\ \mu e}} = \frac{3}{2}(1-\sqrt{3})\tilde{\beta}_e\left(1-\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\right)\,,$$

the case that the additional unknown mode of m is the Higgs-like mode ($\delta_\alpha \sim \delta_\beta \sim \delta_\gamma$)

$$\frac{\mathcal{C}'_{\substack{e\gamma \\ \tau e}}}{\mathcal{C}'_{\substack{e\gamma \\ \mu e}}} = \frac{1}{\sqrt{3}} \times \mathcal{O}(1) \qquad\qquad \frac{\mathcal{C}'_{\substack{e\gamma \\ \tau e}}}{\mathcal{C}'_{\substack{e\gamma \\ \tau\mu}}} = \frac{\tilde{\beta}_e}{\tilde{\alpha}_e} \times \mathcal{O}(1) \sim 10^{-2}$$

$$\mathcal{B}(\tau \to \mu\gamma) : \mathcal{B}(\tau \to e\gamma) : \mathcal{B}(\mu \to e\gamma) \sim \; 10^4 : 1 : 10$$

Since the present upper bounds of B(τ → eγ) and B(τ → μγ) are 3.3×10^{-8} and 4.4×10^{-8}, respectively, we expect the experimental test of this prediction for τ → μγ in the future

Summary

We discuss Modular flavor symmetry in the SMEFT

predictions for $(g-2)_\mu$ & Lepton flavor violation

we have also studied lepton EDM

We should check whether our results is model dependent

other models with S_4, A_5 …

Approach to other flavor phenomena in the quark sector

$b \to s\gamma$ ...

Backup

$(\bar{L}R)$ structure in the modular symmetry

	L_L	$(e^c_R,\ \mu^c_R,\ \tau^c_R)$	H_d	$Y(\tau_e)$
$SU(2)$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
A_4	$\mathbf{3}$	$(\mathbf{1},\ \mathbf{1}'',\ \mathbf{1}')$	$\mathbf{1}$	$\mathbf{3}$
k	2	(0, 0, 0)	0	2

Representation of down-type quark and charged leptons

Left

$$E_L = \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}, \quad \bar{E}_L = \begin{pmatrix} \bar{e}_L \\ \bar{\tau}_L \\ \bar{\mu}_L \end{pmatrix}$$

A_4

Right

$$(e^c_R,\ \mu^c_R,\ \tau^c_R) = (1,\ 1'',\ 1')$$

$$(e_R,\ \mu_R,\ \tau_R) = (1,\ 1',\ 1'')$$

Lepton flavor violation : Modular vs. $U(2)$

$U(2)$ flavor symmetry

Faroughy, Isidori, Wilsch, Yamamoto '20

$U(2)^3 = U(2)^q \times U(2)^u \times U(2)^d$ flavor symmetry is good approximation in the SM Yukawa

acting on 1st & 2nd generations only

exact symmetry for $m_u, m_d, m_c, m_s = 0$

$$\psi = (\psi_1, \psi_2, \psi_3)$$

U(2) doublet singlet

Yukawa in U(2)

Spurions : $V_q \sim (2,1,1)$, $\Delta_u \sim (2,\bar{2},1)$, $\Delta_d \sim (2,1,\bar{2})$

$$Y_e = y_\tau \begin{pmatrix} \Delta_e & x_\tau V_\ell \\ 0 & 1 \end{pmatrix}, \qquad Y_u = y_t \begin{pmatrix} \Delta_u & x_t V_q \\ 0 & 1 \end{pmatrix}, \qquad Y_d = y_b \begin{pmatrix} \Delta_d & x_b V_q \\ 0 & 1 \end{pmatrix}$$

$y_{\tau,t,b}$ and $x_{\tau,t,b}$: $\mathcal{O}(1)$ free complex parameters

Transformation for spurions

$$V_{q(\ell)} = e^{i\bar{\phi}_{q(\ell)}} \begin{pmatrix} 0 \\ \epsilon_{q(\ell)} \end{pmatrix}, \quad \Delta_e = O_e^\intercal \begin{pmatrix} \delta'_e & 0 \\ 0 & \delta_e \end{pmatrix}, \quad \Delta_u = U_u^\dagger \begin{pmatrix} \delta'_u & 0 \\ 0 & \delta_u \end{pmatrix}, \quad \Delta_d = U_d^\dagger \begin{pmatrix} \delta'_d & 0 \\ 0 & \delta_d \end{pmatrix}$$

$$O_e = \begin{pmatrix} c_e & s_e \\ -s_e & c_e \end{pmatrix}, \qquad U_q = \begin{pmatrix} c_q & s_q\, e^{i\alpha_q} \\ -s_q\, e^{-i\alpha_q} & c_q \end{pmatrix}$$

$$\epsilon_i = \mathcal{O}(y_t |V_{ts}|) = \mathcal{O}(10^{-1})$$

$$\delta_i = \mathcal{O}\left(\frac{y_c}{y_t}, \frac{y_s}{y_b}, \frac{y_\mu}{y_\tau}\right) = \mathcal{O}(10^{-2})$$

$$\delta'_i = \mathcal{O}\left(\frac{y_u}{y_t}, \frac{y_d}{y_b}, \frac{y_e}{y_\tau}\right) = \mathcal{O}(10^{-3})$$

Spurion order count

$$1 \gg \epsilon_i \gg \delta_i \gg \delta'_i > 0$$

Lepton flavor violation : Modular vs. $U(2)$

$\mu \to e\gamma$ etc. in $U(2)$

Faroughy, Isidori, Wilsch, Yamamoto '20

	$\mu \to e\gamma$	$\tau \to \mu\gamma$	$\tau \to e\gamma$
O^D_{RL}	$(\rho_1 s_e \delta'_e)^* [\bar{e}_R \sigma^{\mu\nu} \mu_L]$	$(\sigma_1 \epsilon_\ell \delta_e)^* [\bar{\mu}_R \sigma^{\mu\nu} \tau_L]$	$(\sigma_1 \epsilon_\ell s_e \delta'_e)^* [\bar{e}_R \sigma^{\mu\nu} \tau_L]$
O^D_{LR}	$-\rho_1 s_e \delta_e [\bar{e}_L \sigma^{\mu\nu} \mu_R]$	$\beta_1 \epsilon_\ell [\bar{\mu}_L \sigma^{\mu\nu} \tau_R]$	-

Spurion order count

$$1 \gg \epsilon_i \gg \delta_i \gg \delta'_i > 0$$

Predictions at $U(2)$ case

$$\mathrm{BR}(\tau \to \mu\gamma) \gg \mathrm{BR}(\mu \to e\gamma) \gg \mathrm{BR}(\tau \to e\gamma)$$

Predictions at $\tau = i$ case

$$\mathcal{B}(\tau \to \mu\gamma) : \mathcal{B}(\tau \to e\gamma) : \mathcal{B}(\mu \to e\gamma) \sim \;\; 10^4 \; : \; 1 \; : \; 10$$

Class 5–7: Fermion Bilinears operators ($\bar{\psi}\psi$)

$(\bar{L}R)$				
	5: $\psi^2 H^3 +$ h.c.		6: $\psi^2 X H +$ h.c.	
$(\bar{\ell}e)$	Q_{eH}	$(H^\dagger H)(\bar{\ell}_p e_r H)$	Q_{eW}	$(\bar{\ell}_p \sigma^{\mu\nu} e_r)\tau^I H W^I_{\mu\nu}$
			Q_{eB}	$(\bar{\ell}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
$(\bar{q}u)$	Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \tilde{H})$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\tilde{H} G^A_{\mu\nu}$
			Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \tilde{H} W^I_{\mu\nu}$
			Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tilde{H} B_{\mu\nu}$
$(\bar{q}d)$	Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G^A_{\mu\nu}$
			Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H W^I_{\mu\nu}$
			Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$

$\tau \to \mu\gamma$ & $\tau \to e\gamma$ & $\mu \to e\gamma$

Wilson coefficients in A4 modular symmetry in mass basis

$$\mathcal{C}'_{\underset{ee}{e\gamma}} = 3\,(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*|\,, \qquad \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} = \frac{3}{2}\,(1-\sqrt{3})\tilde{\alpha}_e\,, \qquad \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} = \sqrt{3}\,(1-\sqrt{3})\tilde{\gamma}_e$$

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

$$\mathcal{C}'_{\underset{\tau\mu}{e\gamma}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\alpha}_e\left(1-\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\frac{\tilde{\alpha}_{e(m)}}{\tilde{\alpha}_e}\right),$$

$$\mathcal{C}'_{\underset{\tau e}{e\gamma}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\beta}_e\left(1+\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}-2\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\right)$$

$$\mathcal{C}'_{\underset{\mu e}{e\gamma}} = \frac{3}{2}(1-\sqrt{3})\tilde{\beta}_e\left(1-\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\right),$$

the case is that unknown mode of m is the flavor blind one ($c_\alpha = c_\beta = c_\gamma = c$)

$$\frac{\mathcal{C}'_{\underset{\tau e}{e\gamma}}}{\mathcal{C}'_{\underset{\mu e}{e\gamma}}} \simeq \frac{1}{\sqrt{3}}\,, \qquad \frac{\mathcal{C}'_{\underset{\tau e}{e\gamma}}}{\mathcal{C}'_{\underset{\tau\mu}{e\gamma}}} \simeq \frac{\tilde{\beta}_e\delta_\beta}{\tilde{\alpha}_e\delta_\alpha} \simeq \frac{\tilde{\beta}_e\frac{c}{\tilde{\beta}_{e(m)}}}{\tilde{\alpha}_e\frac{c}{\tilde{\alpha}_{e(m)}}} \simeq 1$$

$$\mathcal{B}(\tau\to\mu\gamma) : \mathcal{B}(\tau\to e\gamma) : \mathcal{B}(\mu\to e\gamma) \sim 1:1:10$$

$(g-2)_e$

$$\mathcal{C}'_{\substack{e\gamma \\ ee}} = 3\,(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*|\,, \qquad \mathcal{C}'_{\substack{e\gamma \\ \mu\mu}} = \frac{3}{2}\,(1-\sqrt{3})\tilde{\alpha}_e\,, \qquad \mathcal{C}'_{\substack{e\gamma \\ \tau\tau}} = \sqrt{3}\,(1-\sqrt{3})\tilde{\gamma}_e$$

$$\frac{\mathcal{C}'_{\substack{e\gamma \\ ee}}}{\mathcal{C}'_{\substack{e\gamma \\ \mu\mu}}} = 2\frac{\tilde{\beta}_e}{\tilde{\alpha}_e}|\epsilon_1^*| \simeq 4.9\times 10^{-3}\,, \qquad \frac{\mathcal{C}'_{\substack{e\gamma \\ \mu\mu}}}{\mathcal{C}'_{\substack{e\gamma \\ \tau\tau}}} = \frac{\sqrt{3}}{2}\frac{\tilde{\alpha}_e}{\tilde{\gamma}_e} \simeq 5.9\times 10^{-2}\,,$$

$$\Delta a_e = \frac{4m_e}{e}\frac{v}{\sqrt{2}}\frac{1}{\Lambda^2}\mathrm{Re}\,[\mathcal{C}'_{\substack{e\gamma \\ ee}}] \simeq 5.8\times 10^{-14}$$

This result is agreement with the naive mass scaling $\Delta a_\ell \propto m_\ell^2$

observations

$$\Delta a_e^{Cs} = a_e^{\mathrm{Exp}} - a_e^{\mathrm{SM,CS}} = (-8.8\pm 3.6)\times 10^{-13}\;,$$
$$\Delta a_e^{Rb} = a_e^{\mathrm{Exp}} - a_e^{\mathrm{SM,Rb}} = (4.8\pm 3.0)\times 10^{-13}\;.$$

Wait for future measurements !