Towards removal of striped phase in matrix model description of fuzzy field theories

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arXiv: 2002.02317 [hep-th] , 2209.00592 [hep-th]

Towards **removal of striped phase** in matrix model description of fuzzy field theories

Towards removal of striped phase in matrix model description of **fuzzy** field theories

Fuzzy spaces

Fuzzy sphere [Hoppe '82; Madore '92; Grosse, Klimčík, Prešnajder '90s]

- Functions on the usual sphere are given by

$$f(\theta,\phi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l} c_{lm}Y_{lm}(\theta,\phi)\ ,$$

  where Y_{lm} are the spherical harmonics

$$\Delta Y_{lm}(\theta,\phi)=l(l+1)Y_{lm}(\theta,\phi)\ .$$

- To describe features at a small length scale we need Y_{lm}'s with a large l.

Fuzzy spaces

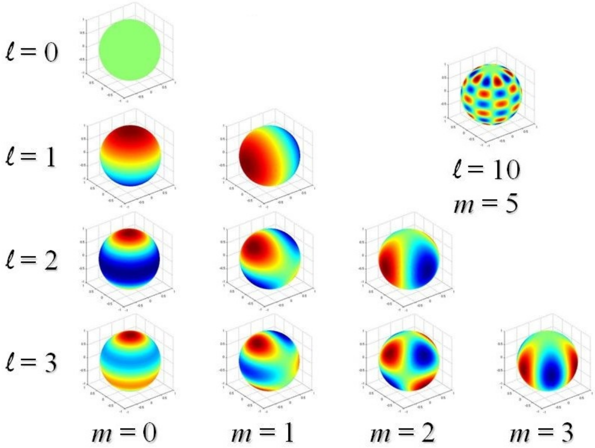


Image taken from http://principles.ou.edu/mag/earth.html

Fuzzy spaces

- If we truncate the possible values of l in the expansion

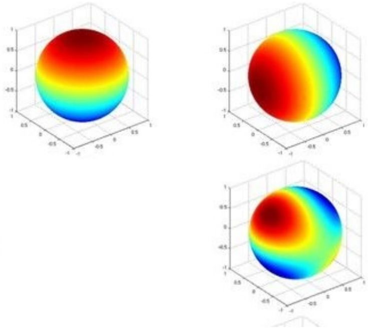
$$f = \sum_{l=0}^{L} \sum_{m=-l}^{l} c_{lm} Y_{lm}(\theta, \phi) \ ,$$

  we will not be able to see any features of functions under certain length scales.
- Points on the sphere (as δ-functions) cease to exist.
- Expressions defined in this way are not closed under multiplication.

Fuzzy spaces

$\ell = 1$

$\ell = 2$



Fuzzy spaces

- Number of independent functions with $l \leq L$ is N^2, the same as the number of $N \times N$ hermitian matrices.
  The idea is to map the former on the latter and borrow a closed product from there.
- In order to do so, we consider a $N \times N$ matrix as a product of two N-dimensional representations $\underline{N}$ of the group $SU(2)$. It reduces to

$$\begin{array}{rcccccccc} \underline{N} \otimes \underline{N} & = & \underline{1} & \oplus & \underline{3} & \oplus & \underline{5} & \oplus & \dots \\ & & \downarrow & & \downarrow & & \downarrow & & \\ & = & \{Y_{0m}\} & \oplus & \{Y_{1m}\} & \oplus & \{Y_{2m}\} & \oplus & \dots \end{array}$$

- We thus have a map $\varphi : Y_{lm} \to M$ and we define the product

$$Y_{lm} \star Y_{l'm'} := \varphi^{-1} \left(\varphi \left(Y_{lm} \right) \varphi \left(Y_{l'm'} \right) \right) \ .$$

Fuzzy spaces

- We have a short distance structure, but the prize we had to pay was a noncommutative product $\star$ of functions. The space, for which this is the algebra of functions, is called the fuzzy sphere.
- Opposing to some lattice discretization this space still possess a full rotational symmetry

$$Y_{lm} \star Y_{l'm'} := \varphi^{-1}\left(\varphi\left(Y_{lm}\right)\varphi\left(Y_{l'm'}\right)\right) \ .$$

- In the limit N or $L \to \infty$ we recover the original sphere.

Fuzzy spaces

- The regular sphere S^2 is given by the coordinates

$$x_i x_i = R^2 \quad , \quad x_i x_j - x_j x_i = 0 \ , \ i,j = 1,2,3 \ ,$$

  which generate the algebra of functions.
- For the fuzzy sphere S^2_N we define

$$\hat{x}_i \hat{x}_i = r^2 \quad , \quad \hat{x}_i \hat{x}_j - \hat{x}_j \hat{x}_i = i\theta\varepsilon_{ijk}\hat{x}_k \ , \ i,j = 1,2,3 \ .$$

- Such $\hat{x}_i$'s generate a different, non-commutative, algebra and S^2_N is an object, which has this algebra as an algebra of functions.

Fuzzy spaces

- The conditions can be realized as an $N = 2s + 1$ dimensional representation of $SU(2)$

$$\hat{x}_i = \frac{2r}{\sqrt{N^2 - 1}} L_i \quad , \quad \theta = \frac{2r}{\sqrt{N^2 - 1}} \sim \frac{2}{N} \quad , \quad \rho^2 = \frac{4r^2}{N^2 - 1} s(s+1) = r^2 \ .$$

- The group $SU(2)$ still acts on $\hat{x}_i$'s and this space enjoys a full rotational symmetry.
- In the limit $N \to \infty$ we recover the original sphere.

Fuzzy spaces

- Most importantly nonzero commutators

$$\hat{x}_i\hat{x}_i = \rho^2 \quad , \quad \hat{x}_i\hat{x}_j - \hat{x}_j\hat{x}_i = i\theta\varepsilon_{ijk}\hat{x}_k \ , \ i = 1,2,3 \ .$$

imply uncertainty relations for positions

$$\Delta x_i \Delta x_j \neq 0 \ .$$

- Configuration space is analogous to phase space of quantum mechanics.
- In a similar fashion it is possible to construct an analogous deformation of the plane

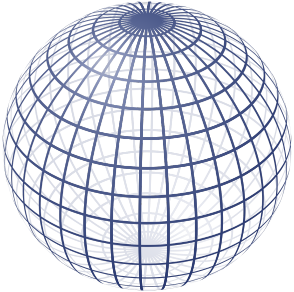
$$\hat{x}_i\hat{x}_j - \hat{x}_j\hat{x}_i = i\theta\varepsilon_{ij} = i\theta_{ij} \ , \ i = 1,2 \ .$$

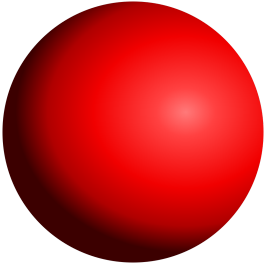
Construction uses the $\star$-product

$$f \star g = f\, e^{\frac{i}{2}\overleftarrow{\partial}\,\theta\,\overrightarrow{\partial}} g = fg + \frac{i\theta^{\mu\nu}}{2}\frac{\partial f}{\partial x^\mu}\frac{\partial g}{\partial x^\nu} + \cdots$$

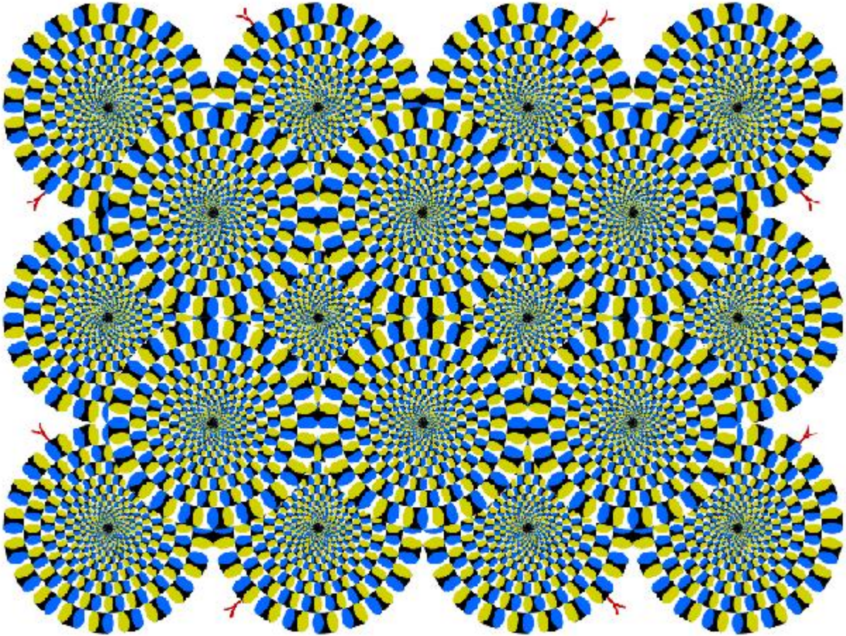
Fuzzy spaces - an alternative construction

- We have divided the sphere into N cells. Function on the fuzzy sphere is given by a matrix M and the eigenvalues of M represent the values of the function on these cells.





- However there are no sharp boundaries between the pieces and everything is blurred, or fuzzy.



Fuzzy spaces

- Regularization of infinities in the standard QFT.
  [Heisenberg ∼'30; Snyder '47, Yang '47]
- Regularization of field theories for numerical simulations.
  [Panero '16]
- An effective description of the open string dynamics in a magnetic background in the low energy limit.
  [Seiberg Witten '99; Douglas, Nekrasov '01]
- Solutions of various matrix formulations of the string theory (IKKT, BFSS, BNM).
  [Steinacker '13]
- Geometric unification of the particle physics and theory of gravity.
  [van Suijlekom '15]
- An effective description of various systems in a certain limit (eg. QHE).
  [Karabali, Nair '06]

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- **Toy models of spaces with discrete quantum structure, which is expected to arise in quantum theory of gravity.**
  [Doplicher, Fredenhagen, Roberts '95; Hossenfelder 1203.6191]

Towards removal of striped phase in matrix model description of fuzzy field theories

Fuzzy scalar field theory

- Commutative euclidean theory of a real scalar field is given by an action

$$S(\Phi) = \int d^2x \left[\frac{1}{2}\Phi\Delta\Phi + \frac{1}{2}m^2\Phi^2 + V(\Phi) \right]$$

  and path integral correlation functions

$$\langle F \rangle = \frac{\int d\Phi\, F(\Phi) e^{-S(\Phi)}}{\int d\Phi\, e^{-S(\Phi)}} \ .$$

- We construct the noncommutative theory as an analogue with
  - field $\rightarrow$ matrix,
  - functional integral $\rightarrow$ matrix integral,
  - spacetime integral $\rightarrow$ trace,
  - derivative $\rightarrow$ L_i commutator.

Fuzzy scalar field theory

- **Commutative**

$$S(\Phi) = \int d^2x \left[\frac{1}{2} \Phi \Delta \Phi + \frac{1}{2} m^2 \Phi^2 + V(\Phi) \right] ,$$

$$\langle F \rangle = \frac{\int d\Phi \, F(\Phi) e^{-S(\Phi)}}{\int d\Phi \, e^{-S(\Phi)}} .$$

- **Noncommutative** (for S_F^2)

$$S(M) = \frac{4\pi R^2}{N} \mathrm{Tr} \left[\frac{1}{2} M \frac{1}{R^2} [L_i, [L_i, M]] + \frac{1}{2} m^2 M^2 + V(M) \right] ,$$

$$\langle F \rangle = \frac{\int dM \, F(M) e^{-S(M)}}{\int dM \, e^{-S(M)}} .$$

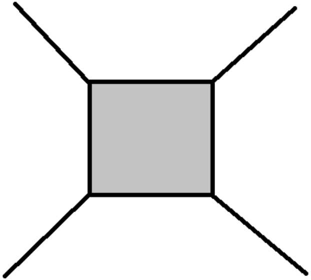
[Balachandran, Kürkçüoğlu, Vaidya '05; Szabo '03; Ydri '16]

Fuzzy scalar field theory - UV/IR mixing

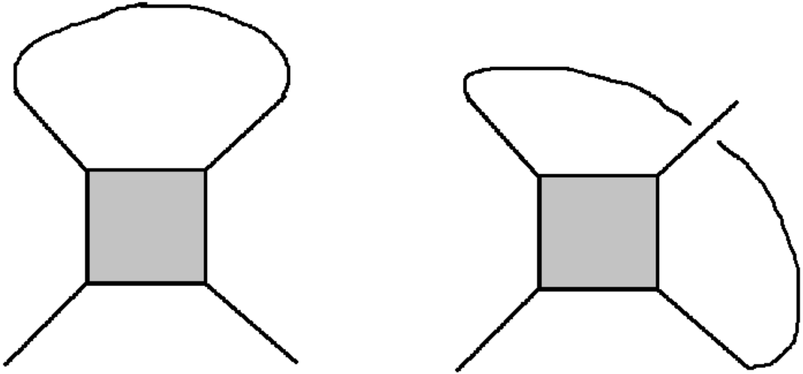
- The key property of the noncommutative field theories is the UV/IR mixing phenomenon, which arises as a result of the nonlocality of the theory.
  [Minwalla, Van Raamsdonk, Seiberg '00; Chu, Madore, Steinacker '01]
- Very energetic fluctuations (UV physics) have consequences at large distances (IR physics).
- In terms of diagrams different properties of planar and non-planar ones.
  The (matrix) vertex is not invariant under permutation of incoming momenta.

Fuzzy scalar field theory - UV/IR mixing

$$M=\sum_{l=0}^{N-1}\sum_{m=-l}^{l}c_{lm}T_{lm}\ ,\ \ \mathrm{Tr}\left(M^4\right)=\sum_{l_{1\dots4}}\sum_{m_{1\dots4}}c_{l_1,m_1}c_{l_2,m_2}c_{l_3,m_3}c_{l_4,m_4}\mathrm{Tr}\left(T_{l_1,m_1}T_{l_2,m_2}T_{l_3,m_3}T_{l_4,m_4}\right)$$



Fuzzy scalar field theory - UV/IR mixing



Fuzzy scalar field theory - UV/IR mixing

[Chu, Madore, Steinacker '01]

$$I^{NP} - I^{P} = \sum_{j=0}^{N-1} \frac{2j+1}{j(j+1)+m^2} \left[(-1)^{l+j+N-1} \left\{ \begin{array}{ccc} l & s & s \\ j & s & s \end{array} \right\} - 1 \right]$$

- This difference is finite in $N \to \infty$ limit.
- $N \to \infty$ limit of the effective action is different from the standard S^2 effective action. Regularization of the field theory by NC space is anomalous.
- In the planar limit $S^2 \to \mathbb{R}^2$ one recovers singularities and the standard UV/IR-mixing.
- The space (geometry) forgets where it came from, but the field theory (physics) remembers its fuzzy origin.

$$S = \int d^2x \left(\frac{1}{2}\partial_\mu \phi \star \partial^\mu \phi + \frac{m^2}{2} \phi \star \phi + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right)$$

[Minwalla, Van Raamsdonk, Seiberg '00]

- Planar contribution

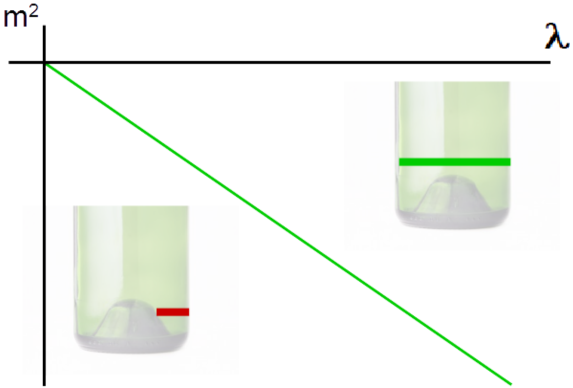
$$I_P = \frac{\lambda}{4!}\int \frac{d^2k}{(2\pi)^2}\frac{2}{k^2+m^2}\ .$$

- Non-planar contribution

$$I_{NP} = \frac{\lambda}{4!}\int \frac{d^2k}{(2\pi)^2}\frac{\exp(ik_\mu\theta^{\mu\nu}p_\nu)}{k^2+m^2} = \frac{\lambda}{96\pi}\log\frac{\Lambda_{\text{eff}}^2}{m^2}+\cdots\ ,\ \Lambda_{\text{eff}}^2 = \frac{1}{1/\Lambda^2+|\theta^{\mu\nu}p_\nu|^2}\ .$$

Towards removal of **striped phase** in matrix model description of fuzzy field theories

$$S[\phi] = \int d^2x \left(\frac{1}{2}\partial_i\phi\partial_i\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 \right)$$



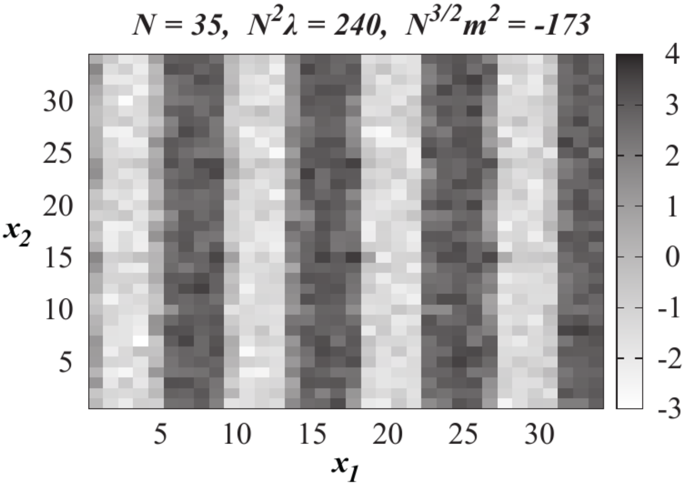
[Glimm, Jaffe '74; Glimm, Jaffe, Spencer '75; Chang '76]
[Loinaz, Willey '98; Schaich, Loinaz '09]

Phases of fuzzy field theories

- The phase diagram of noncommutative field theories has one more phase. It is a non-uniform order phase, or a striped phase.
  [Gubser, Sondhi '01; Chen, Wu '02]
- In this phase, the field does not oscillate around one given value in the whole space. Translational symmetry is broken.
- This has been established in numerous numerical works for variety different spaces.
  [Martin '04; García Flores, Martin, O'Connor '06, '09; Panero '06, '07; Ydri '14; Bietenholz, F. Hofheinz, Mejía-Díaz, Panero '14; Mejía-Díaz, Bietenholz, Panero '14; Medina, Bietenholz, D. O'Connor '08; Bietenholz, Hofheinz, Nishimura '04; Lizzi, Spisso '12; Ydri, Ramda, Rouag '16; Kováčik, O'Connor '18]
  [Panero '15]

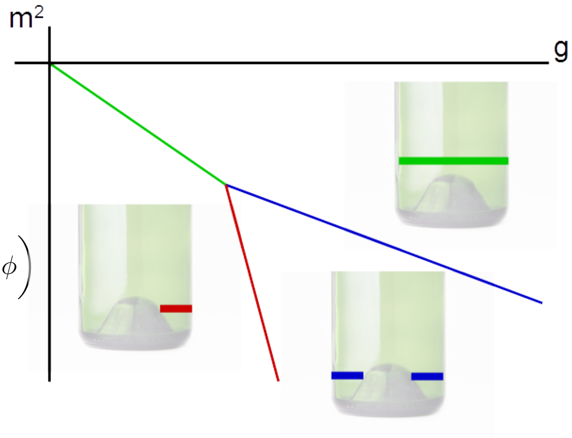
Phases of fuzzy field theories

[Mejía-Díaz, Bietenholz, Panero '14] for $\mathbb{R}^2_\theta$



$$S[M] = \mathrm{Tr}\left(\frac{1}{2}M[L_i,[L_i,M]] + \frac{1}{2}m^2M^2 + gM^4\right)$$

$$S = \int d^2x \left(\frac{1}{2}\partial_\mu\phi \star \partial^\mu\phi + \frac{m^2}{2}\phi\star\phi + \frac{\lambda}{4!}\phi\star\phi\star\phi\star\phi\right)$$



Towards removal of striped phase in **matrix model description** of fuzzy field theories

Random matrices

[M.L. Mehta '04; B. Eynard, T. Kimura, S. Ribault '15; G. Livan, M. Novaes, P. Vivo '17]

- Matrix model = ensemble of random matrices.
- An important example - ensemble of $N \times N$ hermitian matrices with

$$P(M) = e^{-N\mathrm{Tr}(V(M))}\ , \ \text{usually}\ V(x) = \frac{1}{2} r\, x^2 + g\, x^4$$

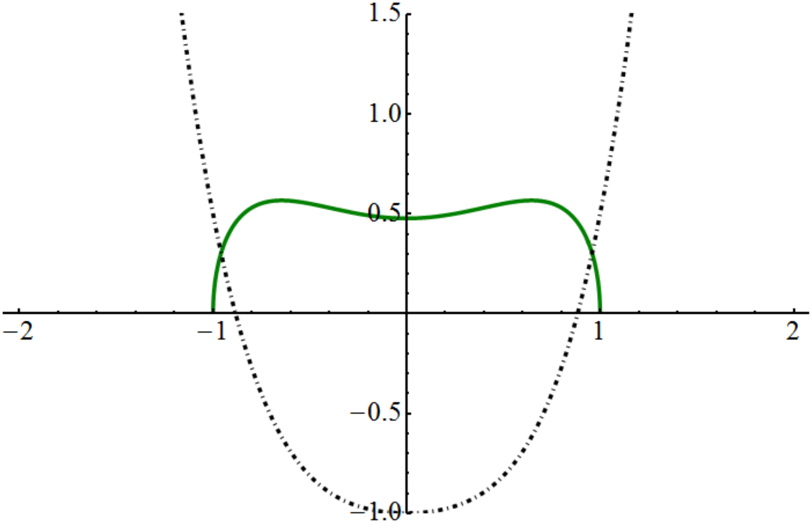
  and

$$dM = \left[\prod_{i=1}^{N} M_{ii}\right]\left[\prod_{i<j} Re\, M_{ij}\, Im\, M_{ij}\right] .$$

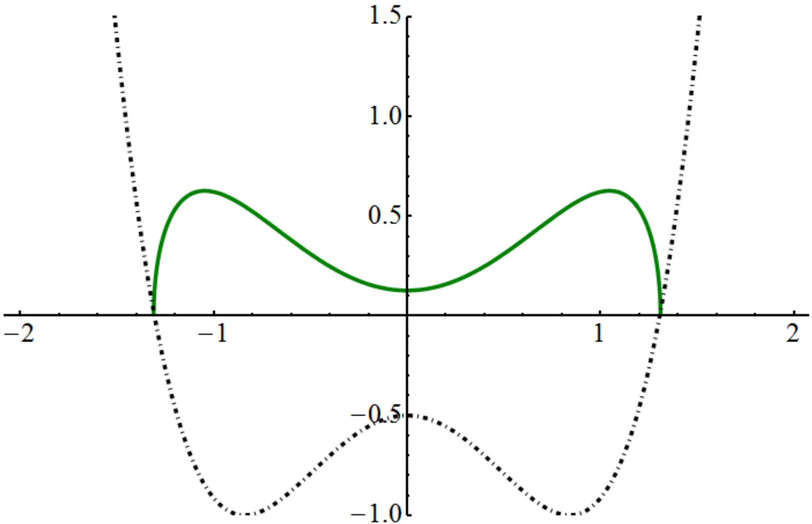
- Both the measure and the probability distribution are invariant under $M \to UMU^\dagger$ with $U \in SU(N)$.
- Requirement of such invariance is very restrictive. One is usually interested in the distribution of eigenvalues.

Random matrices - quartic potential

$V(x) = rx^2/2 + gx^4$ and $r > 0$

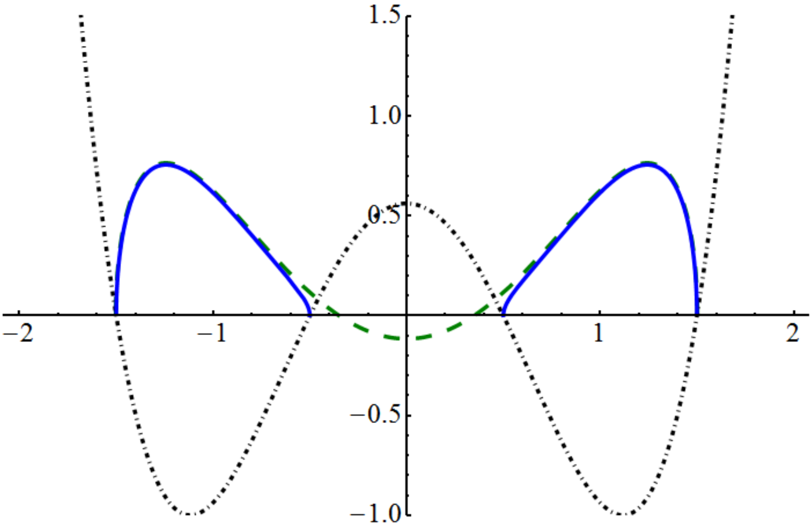


$V(x) = rx^2/2 + gx^4$ and $r < 0$

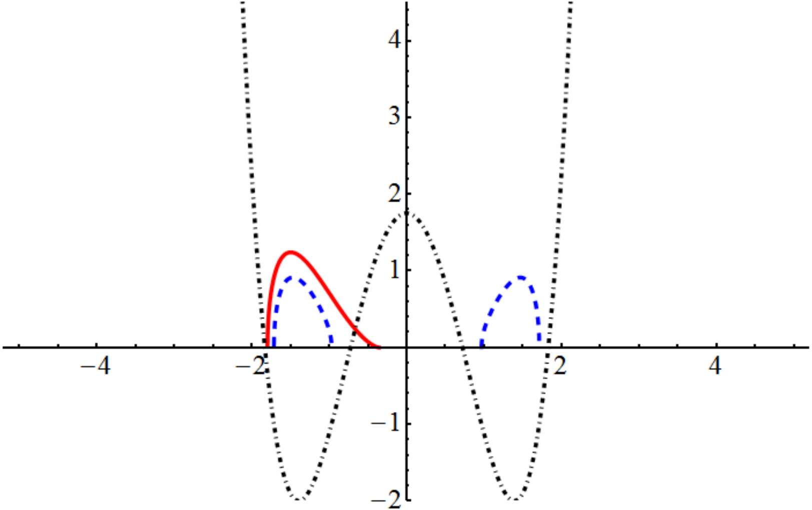


Random matrices - quartic potential

$V(x) = rx^2/2 + gx^4$ and $r < -4\sqrt{g}$



$V(x) = rx^2/2 + gx^4$ and $r << 0$



- Recall the action of the fuzzy scalar field theory

$$S(M) = \frac{1}{2}\mathrm{Tr}\left(M[L_i,[L_i,M]]\right) + \frac{1}{2}m^2\,\mathrm{Tr}\left(M^2\right) + g\,\mathrm{Tr}\left(M^4\right)\ .$$

This is a particular case of a matrix model since we need

$$\int dM\, F(M) e^{-S(M)}\ .$$

- The large N limit of the model with the kinetic term is not well understood. The key issue being that diagonalization $M = U\,\mathrm{diag}(\lambda_1,\ldots,\lambda_N)U^\dagger$ no longer straightforward.
- Integrals like

$$\langle F\rangle \sim \int\left(\prod_{i=1}^N d\lambda_i\right)\int dU\ \ F(\lambda_i,U)\ e^{-N^2\left[\frac{1}{2}m^2\frac{1}{N}\sum\lambda_i^2+g\frac{1}{N}\sum\lambda_i^4-\frac{2}{N^2}\sum_{i<j}\log|\lambda_i-\lambda_j|\right]}$$

$$\times\ e^{-\frac{1}{2}\mathrm{Tr}\left(U\Lambda U^\dagger[L_i,[L_i,U\Lambda U^\dagger]]\right)}\ .$$

Second moment approximation

- For the free theory $g = 0$ the kinetic term just rescales the eigenvalues. [Steinacker '05]
- There is a unique parameter independent effective action that reconstructs this rescaling. [Polychronakos '13]

$$S_{eff}(\lambda_i) = \frac{1}{2}\log\left(\frac{c_2}{1-e^{-c_2}}\right) + \mathcal{R}\ .$$

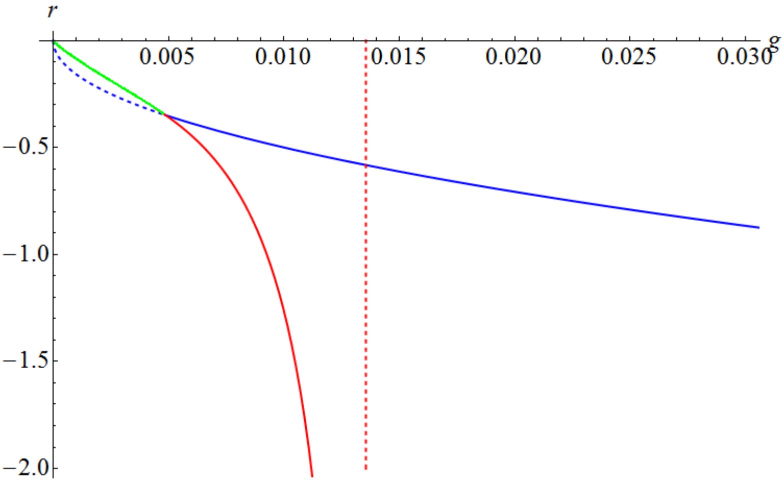
Can be generalized to more a more complicated kinetic term $\mathcal{K}$.

- Introducing the asymmetry $c_2 \to c_2 - c_1^2$ we obtain a matrix model

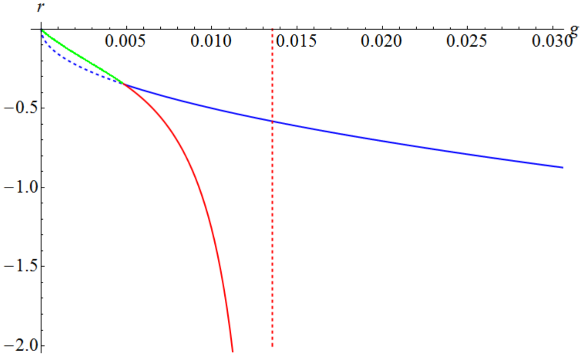
$$S(M) = \frac{1}{2}F(c_2 - c_1^2) + \frac{1}{2} r \,\mathrm{Tr}\left(M^2\right) + g\,\mathrm{Tr}\left(M^4\right) \quad , \quad F(t) = \log\left(\frac{t}{1-e^{-t}}\right)\ .$$

[Šubjaková, JT PoS CORFU2019]

Second moment approximation



[JT '18; Šubjaková, JT '20]


r
g
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0.025
0.030
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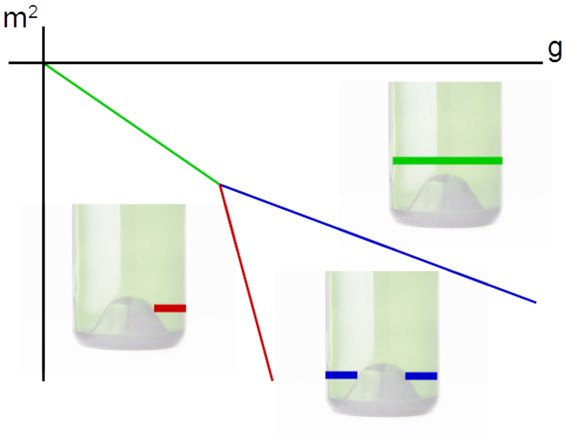
−1.0
−1.5
−2.0


m^2
g

Towards removal of striped phase in matrix model description of fuzzy field theories

Removal of stripes – fuzzy sphere

- We would like to analyse the more complicated model

$$S = \mathrm{Tr}\left(\frac{1}{2}M[L_i,[L_i,M]] + 12gMQM + \frac{1}{2}rM + gM^4\right) \ ,$$

where

$$QT_{lm} = \underbrace{-\left(\sum_{j=0}^{N-1}\frac{2j+1}{j(j+1)+r}\left[(-1)^{l+j+N-1}\left\{\begin{array}{ccc} l & s & s \\ j & s & s \end{array}\right\} - 1\right]\right)}_{Q(l)} T_{lm} \ .$$

- This removes the UV/IR mixing in the theory, essentially by removing the problematic part by brute force.

  [Dolan, O'Connor, Prešnajder '01]

Removal of stripes – fuzzy sphere

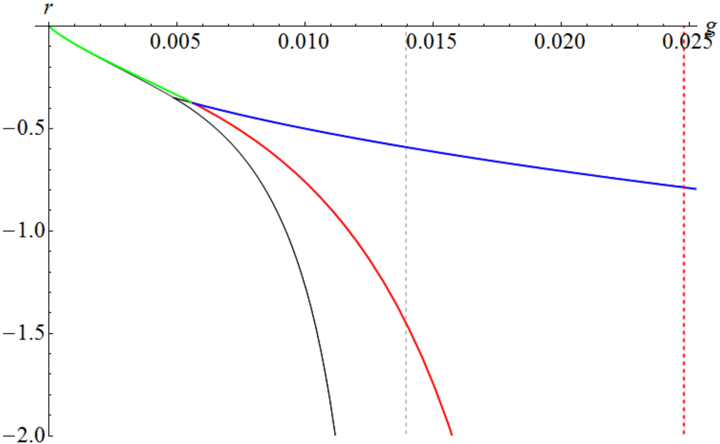
- Operator Q can be expressed as a power series in $C_2 = [L_i, [L_i, \cdot]]$

$$Q = q_1 C_2 + q_2 C_2^2 + \dots \ .$$

- As a starting point, it is interesting to see the phase structure of such simplified model. [O'Connor, Säman '07]
- This is the case of

$$\mathcal{K} = (1 + ag) C_2 \quad \text{or} \quad \mathcal{K} = (1 + ag) C_2 + bg \, C_2^2 \ .$$

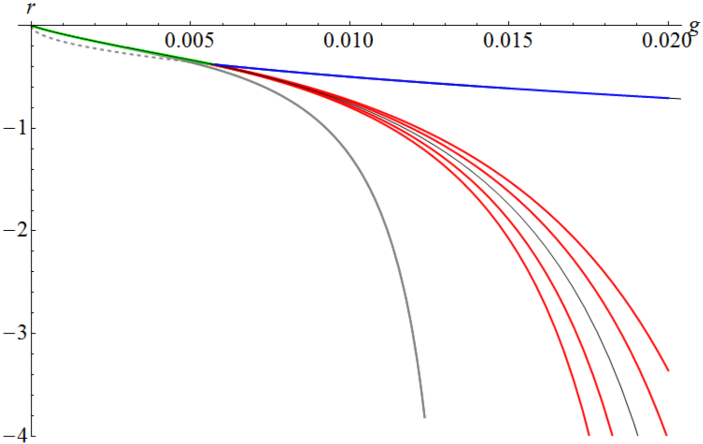
[Šubjaková, JT '20]



$$a = 3e^{3/2} \ , \ \ b = 0 \ .$$

REMOVAL OF STRIPES – FUZZY SPHERE

[Šubjaková, JT '20]



$$a = 3e^{3/2}\ ,\ \ b = -4, -2, 0, 2, 4\ .$$

- Grosse-Wulkenhaar model ['00's]

$$S_{GW} = \int d^2x \left(\frac{1}{2} \partial_\mu \phi \star \partial^\mu \phi + \frac{1}{2} \Omega^2 (\tilde{x}_\mu \phi) \star (\tilde{x}^\mu \phi) + \frac{m^2}{2} \phi \star \phi + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right) ,$$

$$\tilde{x}_\mu = 2(\theta^{-1})_{\mu\nu} x^\nu \ .$$

- This model is renormalizable.
- Described by a matrix model in terms of truncated Heisenberg algebra. [Burić, Wohlgenannt '10]

Removal of stripes – GW model

- The NC plane coordinates can be realized by

$$X = \frac{1}{\sqrt{2}} \begin{pmatrix} & +\sqrt{1} & & & \\ +\sqrt{1} & & +\sqrt{2} & & \\ & +\sqrt{2} & & \ddots & \\ & & \ddots & & \ddots \\ & & & \ddots & \end{pmatrix} , \quad Y = \frac{i}{\sqrt{2}} \begin{pmatrix} & -\sqrt{1} & & & \\ +\sqrt{1} & & -\sqrt{2} & & \\ & +\sqrt{2} & & \ddots & \\ & & \ddots & & \ddots \\ & & & \ddots & \end{pmatrix} ,$$

then

$$[X, Y] = i \ .$$

- This algebra is then truncated to a finite dimension.

Removal of stripes – GW model

- Define finite matrices

$$X = \frac{1}{\sqrt{2}} \begin{pmatrix} & +\sqrt{1} & & & \\ +\sqrt{1} & & +\sqrt{2} & & \\ & +\sqrt{2} & & \ddots & \\ & & \ddots & & \sqrt{N-1} \\ & & & \sqrt{N-1} & \end{pmatrix} \ , \ Y = \dots \ ,$$

  which gives

$$[X, Y] = i(1 - Z) \ , \ Z = \mathrm{diag}\,(0, \dots, N) \ .$$

- Original algebra is recovered in the $N \to \infty$ limit or under the $Z = 0$ condition.
- The kinetic term becomes

$$\frac{1}{2} \partial_\mu \phi \star \partial^\mu \phi \to [X, M][X, M] + [Y, M][Y, M] \ .$$

Removal of stripes – GW model

- The harmonic potential becomes

$$\frac{1}{2}\Omega^2(\tilde{x}_\mu\phi)\star(\tilde{x}^\mu\phi)\to RM^2\ ,$$

  where R is a fixed external matrix

$$R=\frac{15}{2}-4Z^2-8\left(X^2+Y^2\right)=\frac{31}{2}-16\,\mathrm{diag}\,(1,2,\ldots,N-1,8N)\ .$$

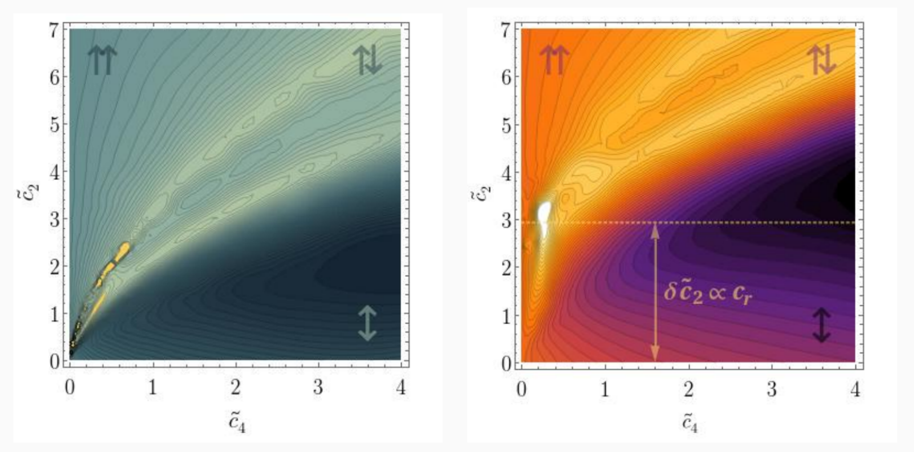
- Interpretation of coupling to the curvature of the space.
- We are thus left with a matrix model with action

$$S=\mathrm{Tr}\,(M[X,[X,M]]+M[Y,[Y,M]])-g_r\mathrm{Tr}\left(RM^2\right)-g_2\mathrm{Tr}\left(M^2\right)+g_4\mathrm{Tr}\left(M^4\right)\ .$$

Removal of stripes – GW model

[Prekrat, Todorović-Vasović, Ranković '21; Prekrat '21]

- Numerical investigation of this matrix model leads to



Removal of stripes – GW model

[Prekrat, Todorović-Vasović, Ranković, Kováčik, JT '22]

- We concentrate on the effect of the curvature term and discard the kinetic term

$$S(M) = \mathrm{Tr}\,(M\mathcal{K}M) - \mathrm{Tr}\left(g_r R M^2\right) - g_2\,\mathrm{Tr}\left(M^2\right) + g_4\,\mathrm{Tr}\left(M^4\right)\ .$$

- This leads to the angular integral

$$\int dU\, e^{g_r \mathrm{Tr}\left(URU^\dagger \Lambda^2\right)}\ ,$$

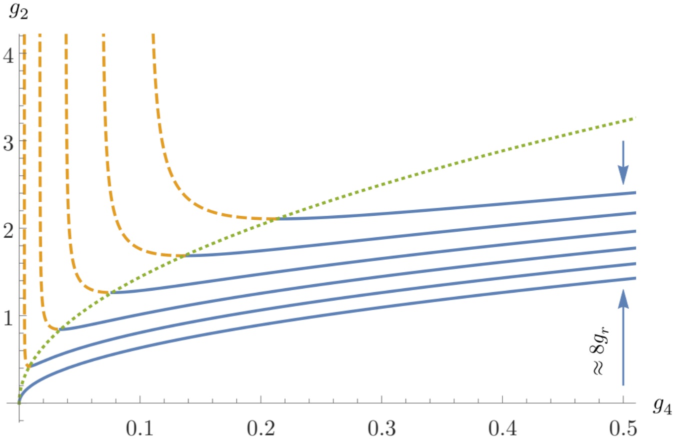
which gives up to g_r^4

$$S(\Lambda) = N\,\mathrm{Tr}\left(-g_2\Lambda^2 + 8g_r\Lambda^2 + g_4\Lambda^4 - \frac{32}{3}g_r^2\Lambda^4\right) + \frac{1024}{45}g_r^4\Lambda^8 + \\ + \frac{32}{3}g_r^2\left(\mathrm{Tr}\left(\Lambda^2\right)\right)^2 + \frac{1024}{15}g_r^4\left(\mathrm{Tr}\left(\Lambda^4\right)\right)^2 - \frac{4096}{45}g_r^4\,\mathrm{Tr}\left(\Lambda^6\right)\mathrm{Tr}\left(\Lambda^2\right)\ .$$

- This is a multitrace matrix model which can be analyzed.

Removal of stripes – GW model

[Prekrat, Todorović-Vasović, Ranković, Kováčik, JT '22]



Removal of stripes – GW model

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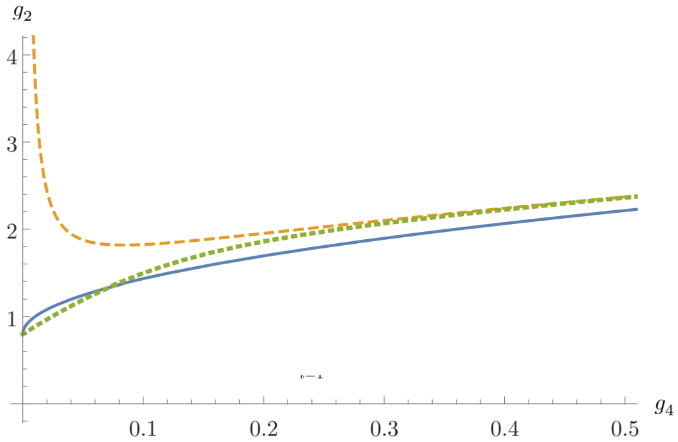
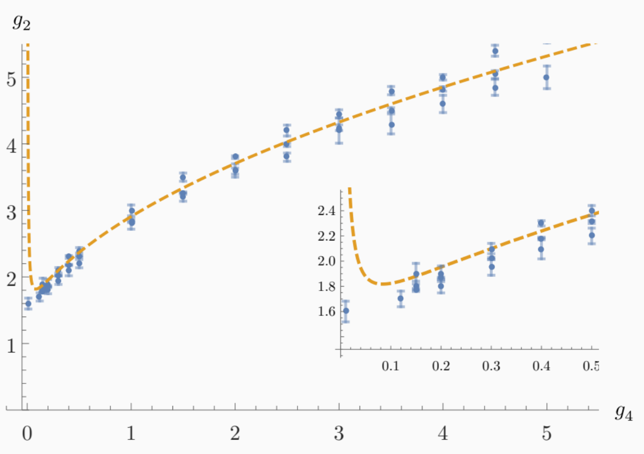


[Prekrat, Todorović-Vasović, Ranković, Kováčik, JT '22]



Take home message

Take home message and 2do list

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- The UV/IR-mixing is exhibited as a non-local, or striped, phase of the model.
- In models describing theories free of the UV/IR-mixing this phase is reasonably assumed to be removed.

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Thank you for your attention!