

New Physics in Neutrino Oscillations

Jacobo López-Pavón

Workshop on the Standard Model & Beyond | CORFU

6 August 2022

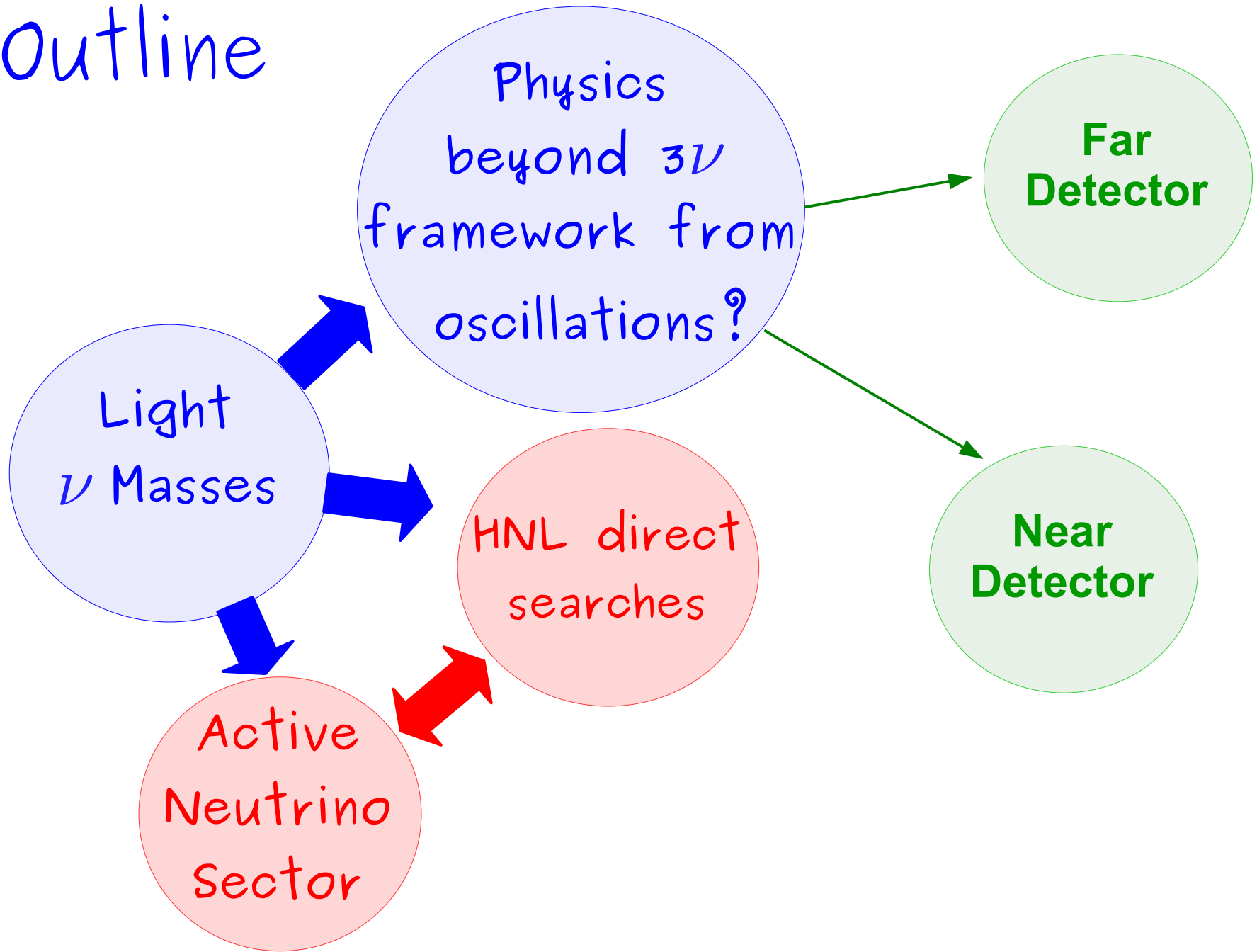


VNIVERSITAT
ID VALÈNCIA

Gen=T
CIDEAGENT/2018/019



Outline



Minimal model: Seesaw Model

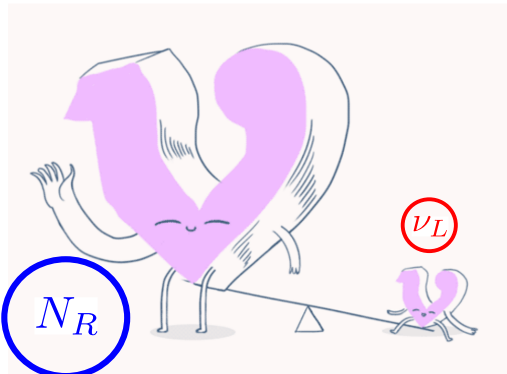
- Simplest extension of SM able to account for neutrino masses. Consists in the addition of **heavy fermion singlets** (N_R) to the SM field content:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_K - \frac{1}{2} \overline{N_i^c} M_{ij} N_j - Y_{i\alpha} \overline{N_i} \tilde{H}^\dagger L_\alpha + h.c.$$



Light
Neutrino
Masses

$$m_\nu = \frac{v^2}{2} Y^T M^{-1} Y$$



Minimal model: Seesaw Model

- Simplest extension of SM able to account for neutrino masses. Consists in the addition of **heavy fermion singlets** (N_R) to the SM field content:

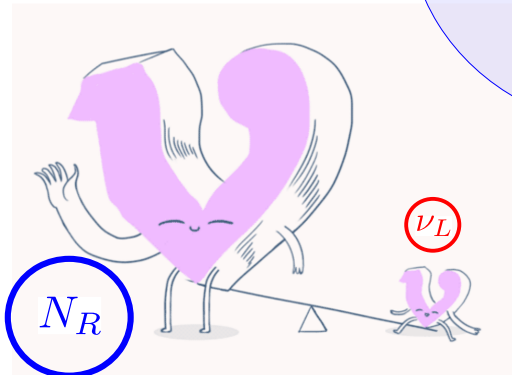
$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_K - \frac{1}{2} \overline{N_i^c} M_{ij} N_j - Y_{i\alpha} \overline{N_i} \tilde{H}^\dagger L_\alpha + h.c.$$

New
Physics
Scale

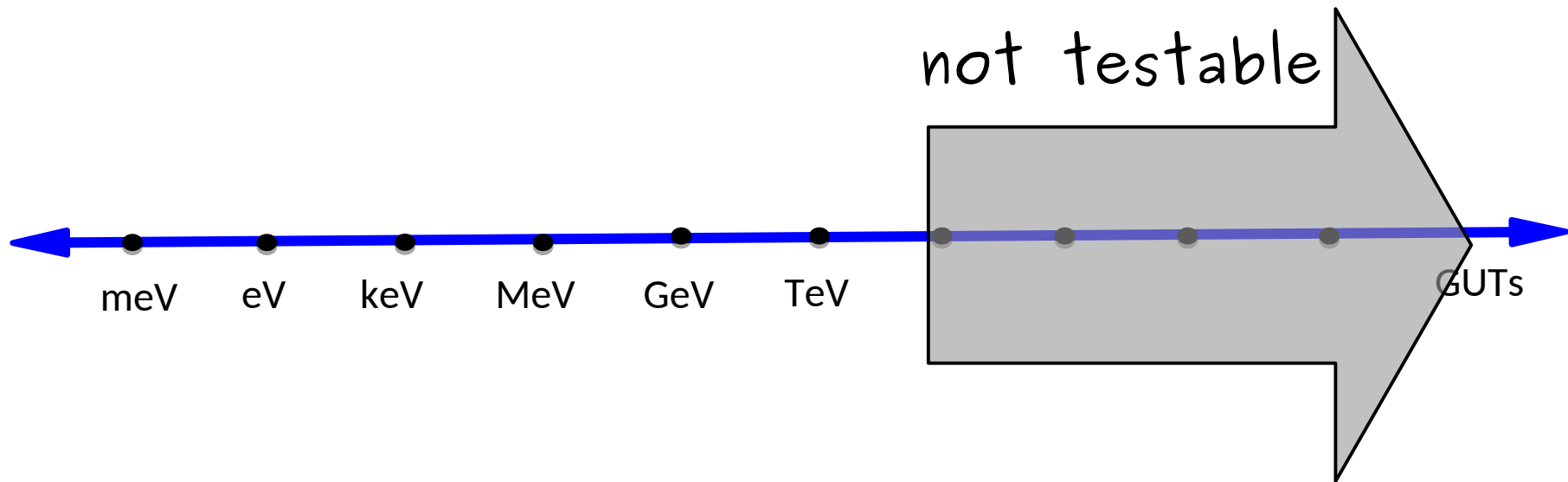
Lepton
Number
Violation

$0\nu\beta\beta$
decay!

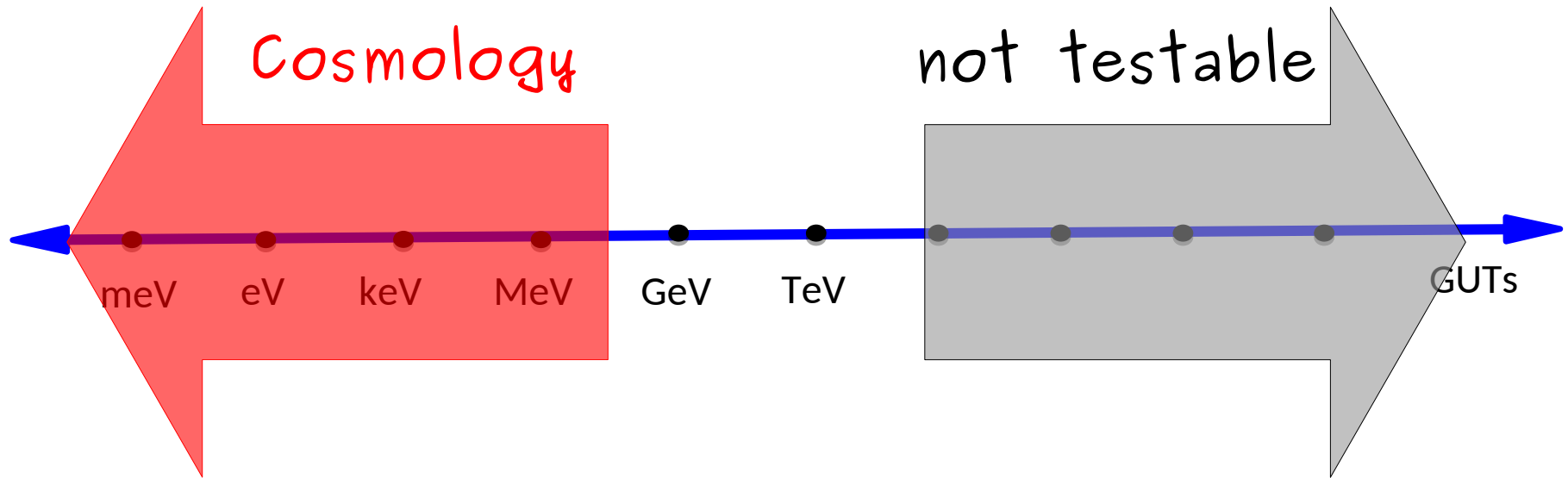
Leptogenesis!



The New Physics Scale

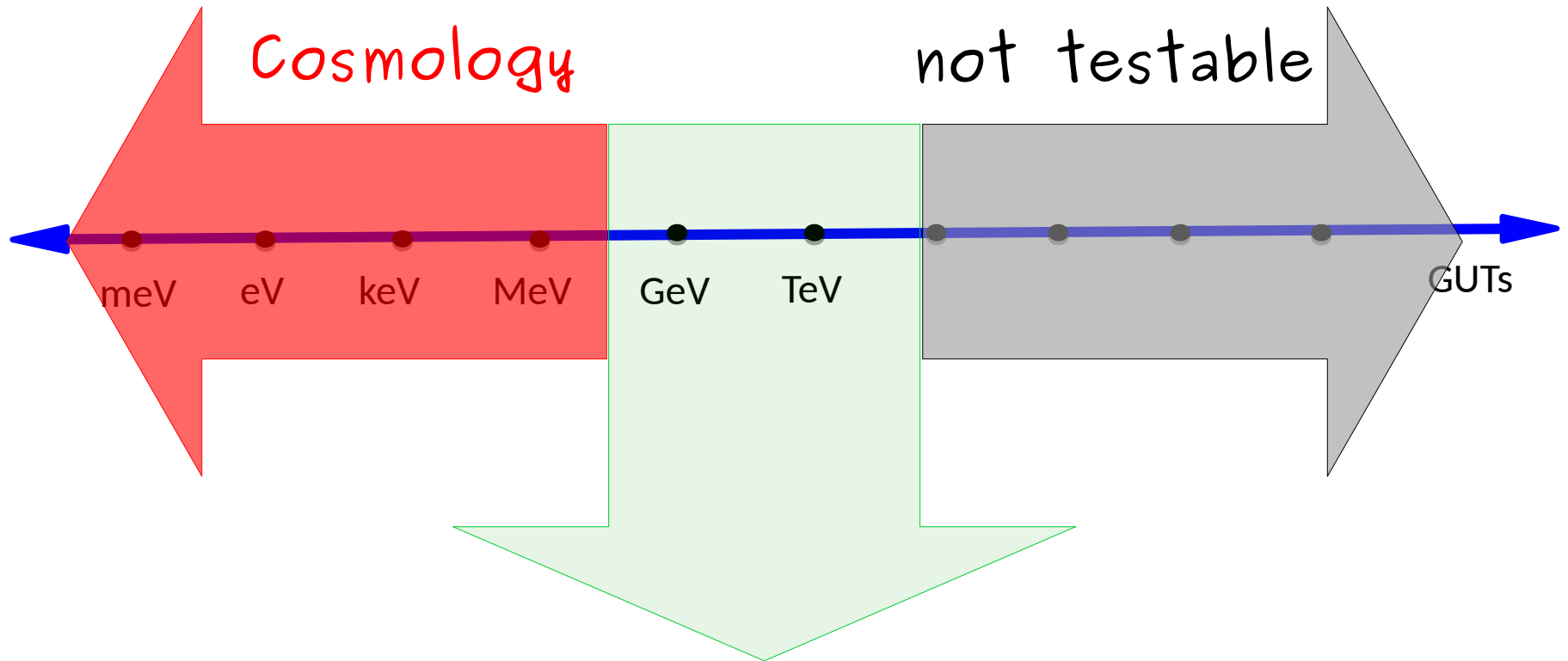


The New Physics Scale



P. Hernandez, M. Kekic, JLP
1311.2614; 1406.2961
Bondarenko, Boyarsky, Klaric,
Mikulenka, Ruchayskiy Syvolap,
Timiryasov 2101.09255

The New Physics Scale

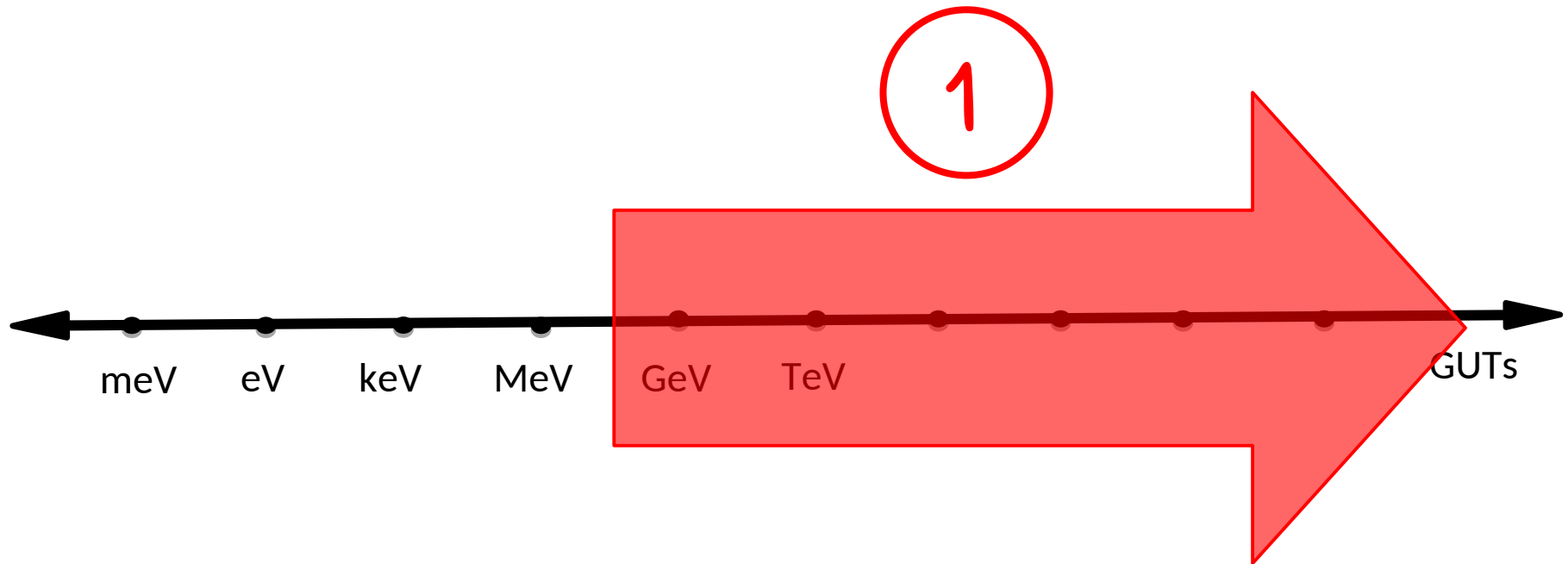


$0\nu\beta\beta$ decay, CLFV, Colliders, Beam-dump..

Are Long Baseline
Neutrino Oscillation experiments
sensitive to
New Physics
beyond 3ν framework

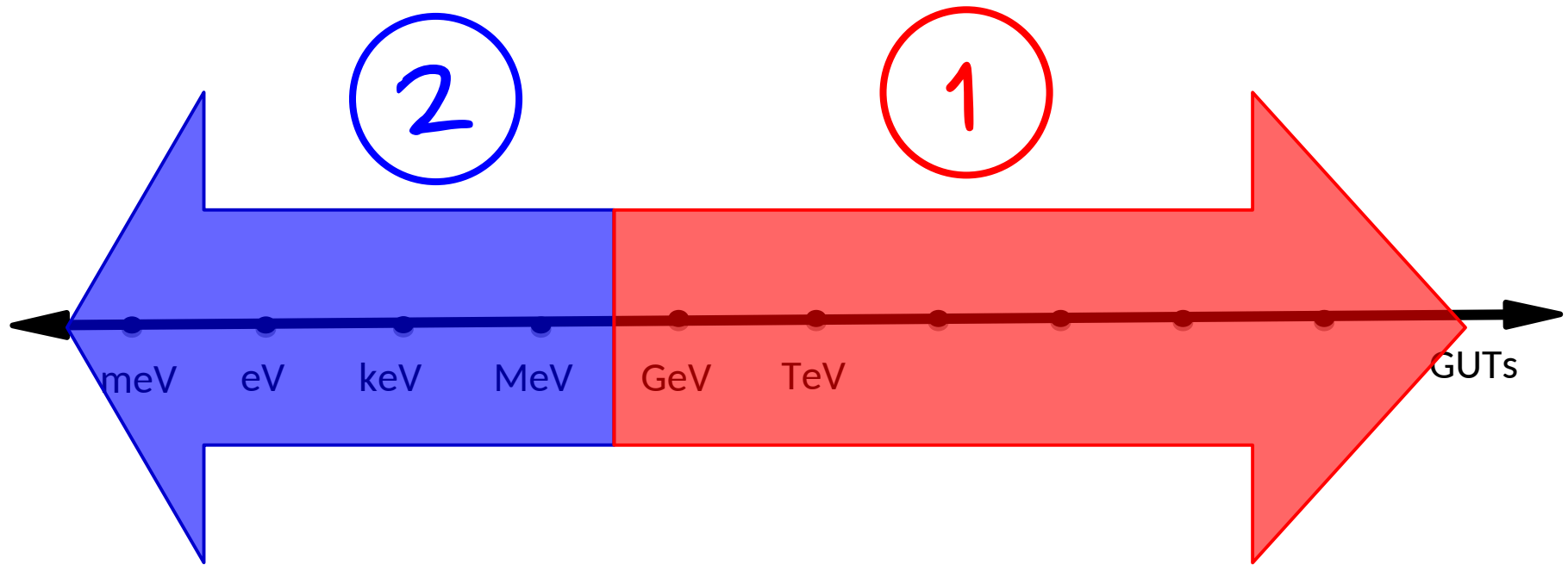


Neutrino Oscillations vs NP scale



Non-Unitary
mixing
(sterile states
integrated out)

Neutrino Oscillations vs NP scale



Kinematically
accessible sterile
neutrinos

Non-Unitary
mixing
(sterile states
integrated out)

Both limits can be studied
in a
unified & model independent way

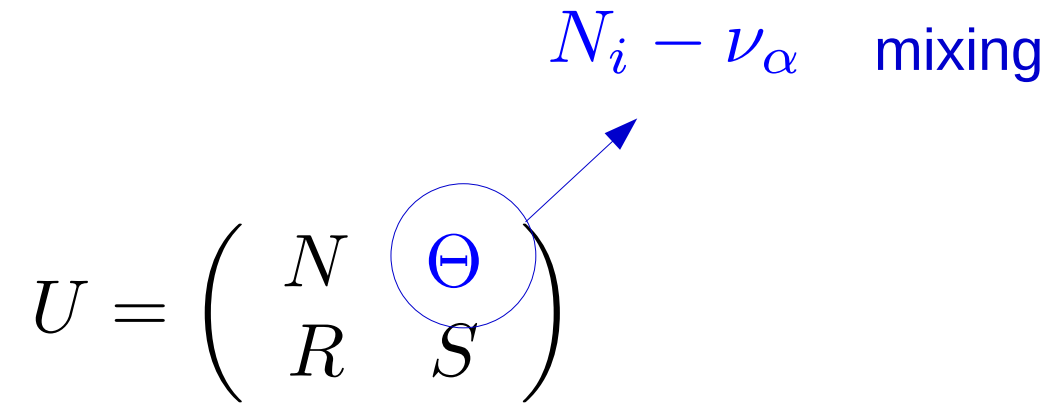
Model Independent Approach

$$U = \begin{pmatrix} N & \Theta \\ R & S \end{pmatrix}$$

Model Independent Approach

$$U = \begin{pmatrix} N & \Theta \\ R & S \end{pmatrix}$$

$N_i - \nu_\alpha$ mixing

The diagram shows a 2x2 matrix U with elements N, R, S, and Theta. The element Theta is circled in blue. A blue arrow points from the circled Theta to the text 'N_i - nu_alpha mixing' located above and to the right of the matrix.

Model Independent Approach

$$U = \begin{pmatrix} N & \Theta \\ R & S \end{pmatrix}$$

$N_i - \nu_\alpha$ mixing

Deviation from unitarity of the PMNS matrix

Langacker, London 1988

Antusch, Biggio, Fernandez-Martinez, Gavela, JLP 2006

General Parameterizations

- Triangular parameterization

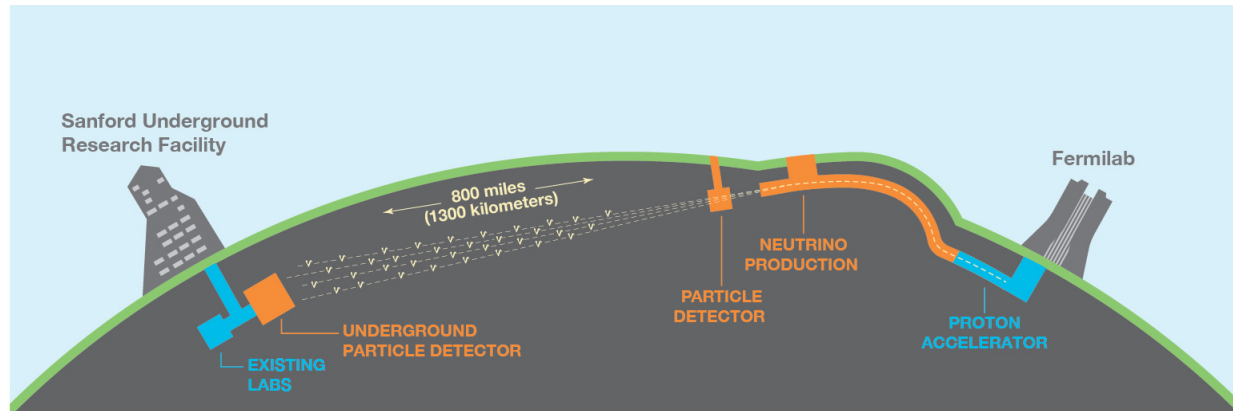
$$N = (I - T)U$$

Deviation from unitarity

$$T = \begin{pmatrix} \alpha_{ee} & 0 & 0 \\ \alpha_{\mu e} & \alpha_{\mu\mu} & 0 \\ \alpha_{\tau e} & \alpha_{\tau\mu} & \alpha_{\tau\tau} \end{pmatrix}$$

Unitary matrix
(standard unitary PMNS
matrix
up to small corrections)

Far Detector vs Near detector



$$N_{\nu_{\alpha} \rightarrow \nu_{\beta}} \sim \frac{\Phi_{\alpha}(E)}{L^2} P_{\alpha\beta}(L/E) \sigma_{\beta}(E) \epsilon_{\beta}(E)$$

- **Sources of systematics**
 - Cross sections
 - Neutrino flux
- **Near detector measurements reduce far detector systematic uncertainties**
- **New Physics at near detector (strongly affected by systematic uncertainties)**

Far Detector

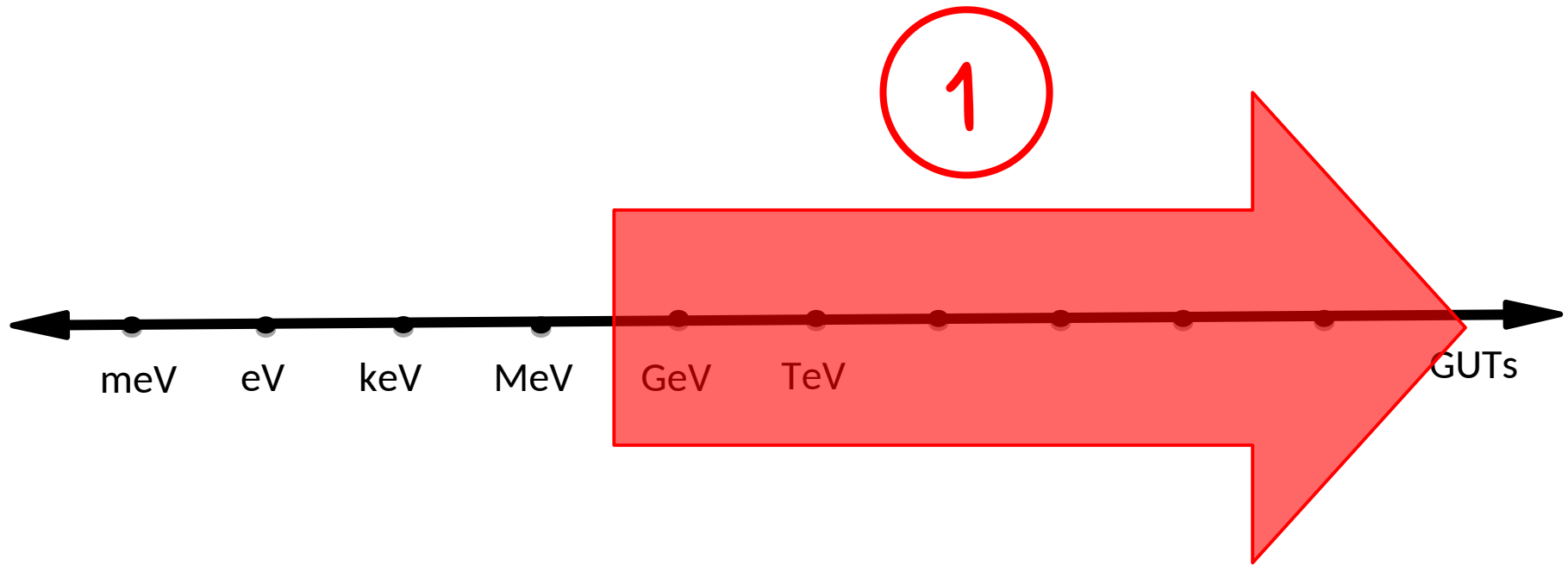
Far Detector

- What is measured in neutrino oscillation experiments

$$\mathcal{P}_{\alpha\beta} = \frac{R_{\beta}}{R_{\alpha}}$$

The diagram shows the equation $\mathcal{P}_{\alpha\beta} = \frac{R_{\beta}}{R_{\alpha}}$. The numerator R_{β} is enclosed in a blue circle, with a blue arrow pointing to the text "Event rate Far Detector". The denominator R_{α} is enclosed in a green circle, with a green arrow pointing to the text "Extrapolation of Near Detector".

① Non-Unitary Mixing



Non-Unitary
mixing
(sterile states
integrated out)

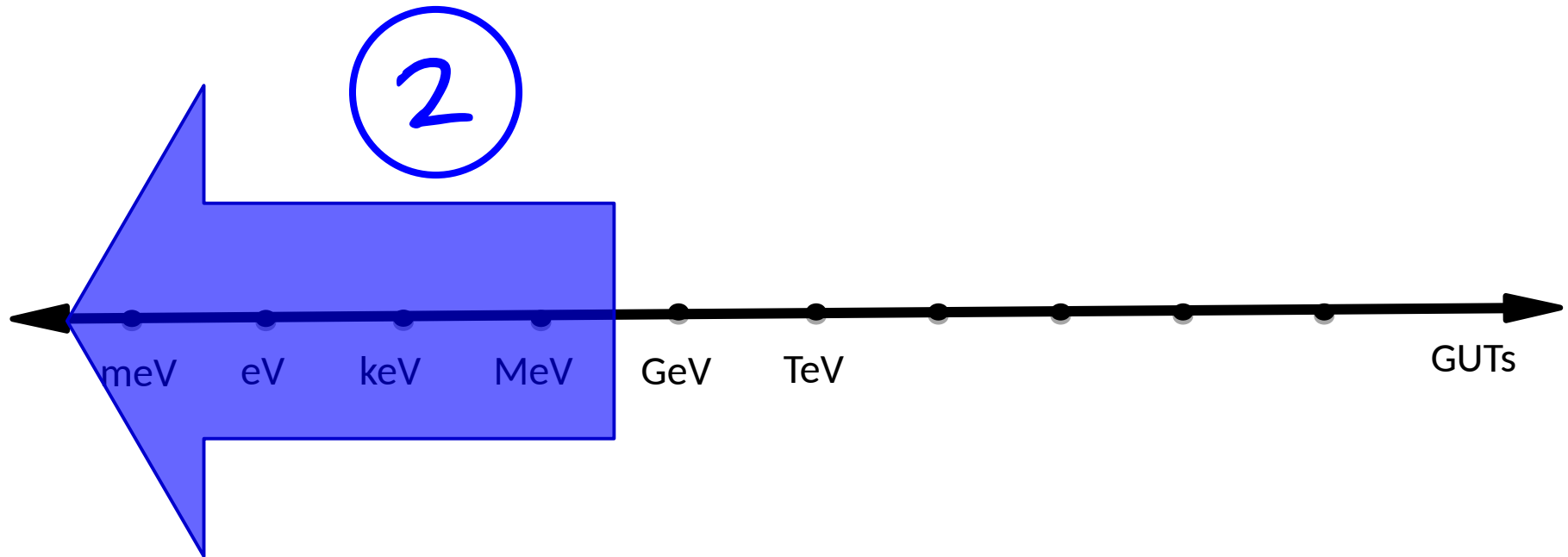
① Non-Unitary Mixing

- What is measured in neutrino oscillation experiments

$$\mathcal{P}_{\alpha\beta} = \frac{|(N \exp(-iHL) N^\dagger)_{\beta\alpha}|^2}{[(NN^\dagger)_{\alpha\alpha}]^2}.$$

- When $NN^\dagger = I \longrightarrow \mathcal{P}_{\alpha\beta} = P_{\alpha\beta}$ (SM limit recovered)

② Kinematically accessible sterile ν



Kinematically
accessible sterile
neutrinos

② Kinematically accessible sterile ν

1. The light-heavy oscillations averaged out at the near detector.
Identical to the heavy non-unitarity case

Blennow, Coloma, Fernandez-Martinez, Hernandez-Garcia, JLP 1609.08637

② Kinematically accessible sterile ν

1. The light-heavy oscillations averaged out at the near detector.
Identical to the heavy non-unitarity case

Blennow, Coloma, Fernandez-Martinez, Hernandez-Garcia, JLP 1609.08637

2. The oscillation frequency dictated by the light-heavy frequency matches the near detector distance.
Oscillations could be observed at the near detector

Coloma, JLP, Rosauero-Alcaraz, **Urrea** 2105.11466

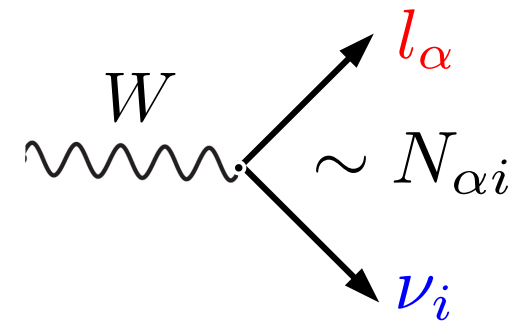
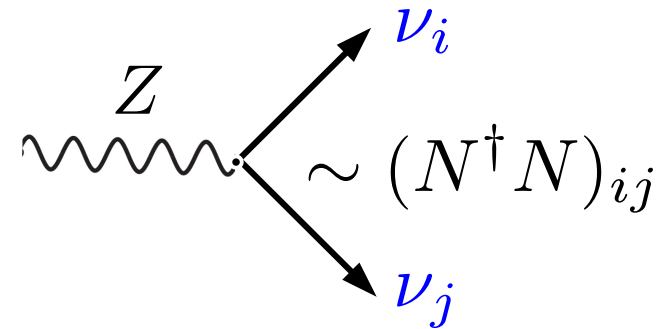
② Kinematically accessible sterile ν

1. The light-heavy oscillations averaged out at the near detector.
Identical to the heavy non-unitarity case

Low Scale
Non-Unitarity

Present Bounds

	Non-Unitarity ($M > \text{EW}$)
α_{ee}	$1.3 \cdot 10^{-3}$
$\alpha_{\mu\mu}$	$2.2 \cdot 10^{-4}$
$\alpha_{\tau\tau}$	$2.8 \cdot 10^{-3}$
$ \alpha_{\mu e} $	$6.8 \cdot 10^{-4} (2.4 \cdot 10^{-5})$
$ \alpha_{\tau e} $	$2.7 \cdot 10^{-3}$
$ \alpha_{\tau\mu} $	$1.2 \cdot 10^{-3}$



EW & CLFV
precision
data

Fernandez-Martinez, Hernandez-Garcia, JLP
1605.08774
Blennow, Coloma, Fernandez-Martinez,
Hernandez-Garcia, JLP 1609.08637

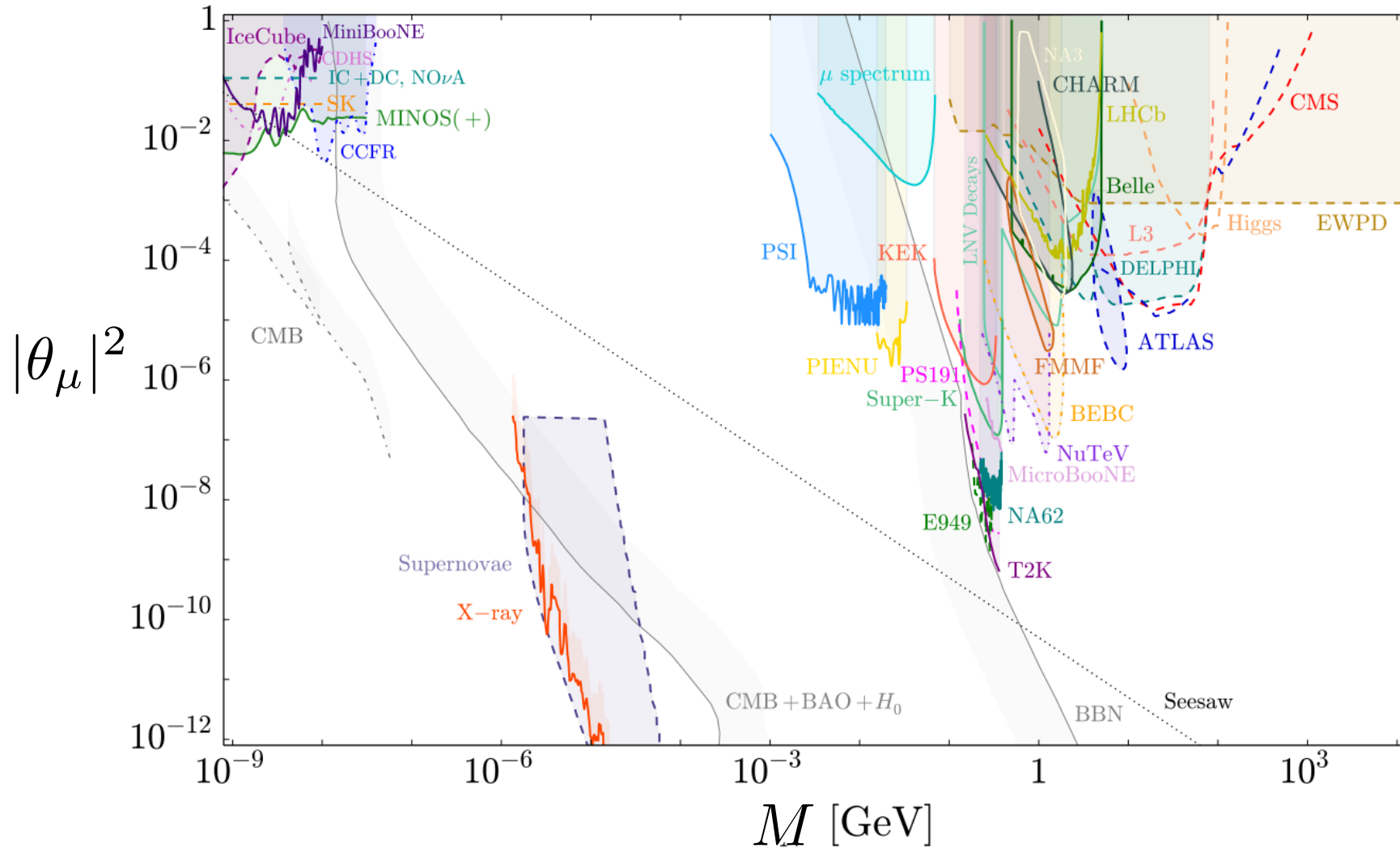
Present Bounds

	Non-Unitarity ($M > \text{EW}$)	Non-Unitarity ($M \gtrsim 1 \text{ eV}$)	
α_{ee}	$1.3 \cdot 10^{-3}$	BUGEY	$2.4 \cdot 10^{-2}$
$\alpha_{\mu\mu}$	$2.2 \cdot 10^{-4}$		SK
$\alpha_{\tau\tau}$	$2.8 \cdot 10^{-3}$	SK	$1.0 \cdot 10^{-1}$
$ \alpha_{\mu e} $	$6.8 \cdot 10^{-4} (2.4 \cdot 10^{-5})$		$2.5 \cdot 10^{-2}$ NOMAD
$ \alpha_{\tau e} $	$2.7 \cdot 10^{-3}$		$6.9 \cdot 10^{-2}$
$ \alpha_{\tau\mu} $	$1.2 \cdot 10^{-3}$		$1.2 \cdot 10^{-2}$ NOMAD

Fernandez-Martinez, Hernandez-Garcia, JLP
 1605.08774
 Blennow, Coloma, Fernandez-Martinez,
 Hernandez-Garcia, JLP 1609.08637

$$\alpha_{\alpha\beta} \leq 2\sqrt{\alpha_{\alpha\alpha}\alpha_{\beta\beta}}$$

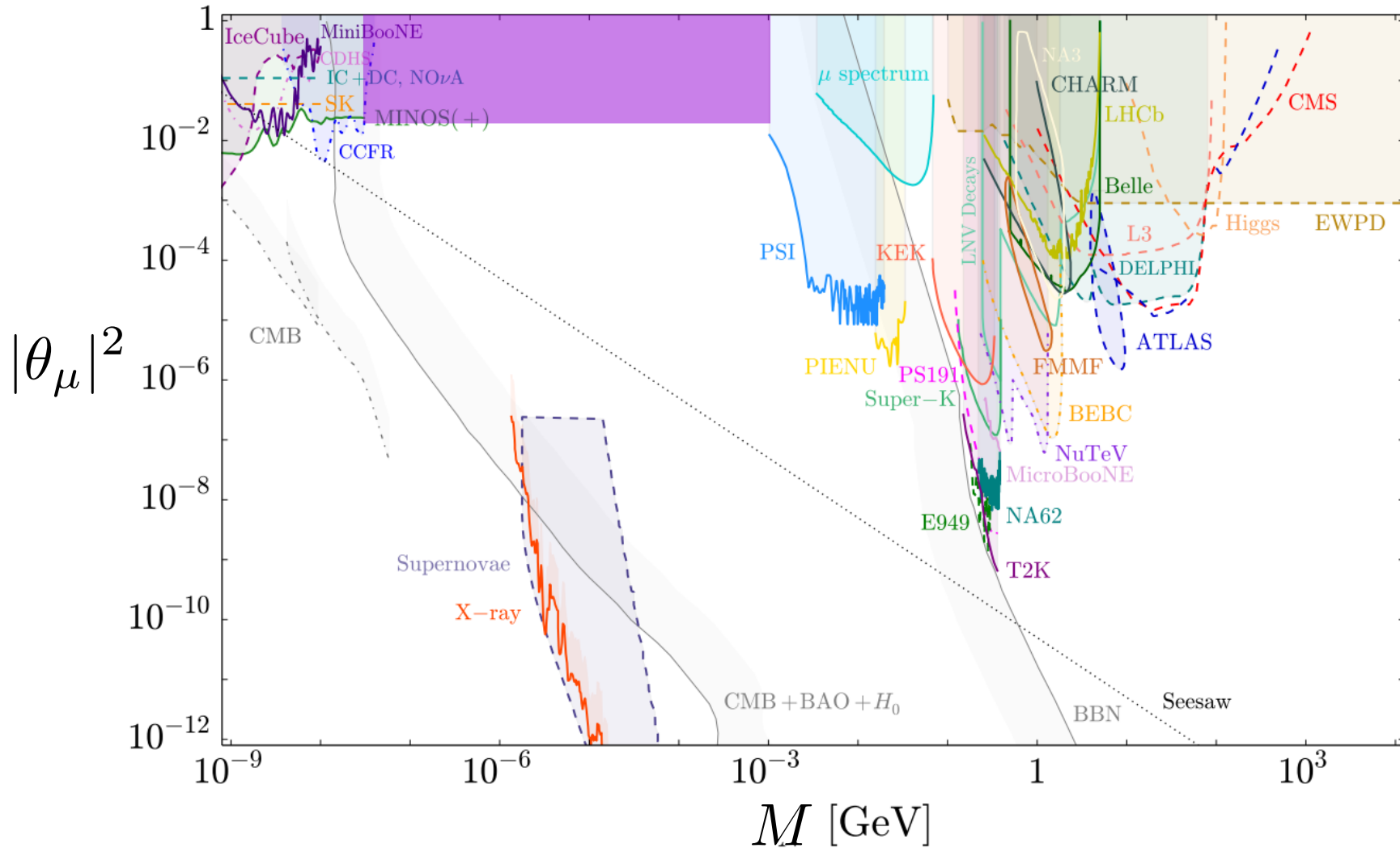
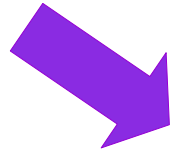
Present Bounds



$$|\alpha_{\beta\beta}|^2 = \frac{1}{2} \sum_i |\theta_{\alpha i}|^2$$

Figure from Bolton, Deppisch, Bhupal Dev 1912.03058

Present Bounds



$$|\alpha_{\beta\beta}|^2 = \frac{1}{2} \sum_i |\theta_{\alpha i}|^2$$

Deep Underground Neutrino Experiment

DUNE

Sanford Underground Research Facility
Lead, South Dakota

Fermilab
Batavia, Illinois

20 miles

800 miles

Sanford Underground Research Facility

(Proposed)

Fermilab

Sanford Underground Research Facility

Fermilab

ν_μ
 ν_e
 ν_τ

ν_μ

UNDERGROUND PARTICLE DETECTOR

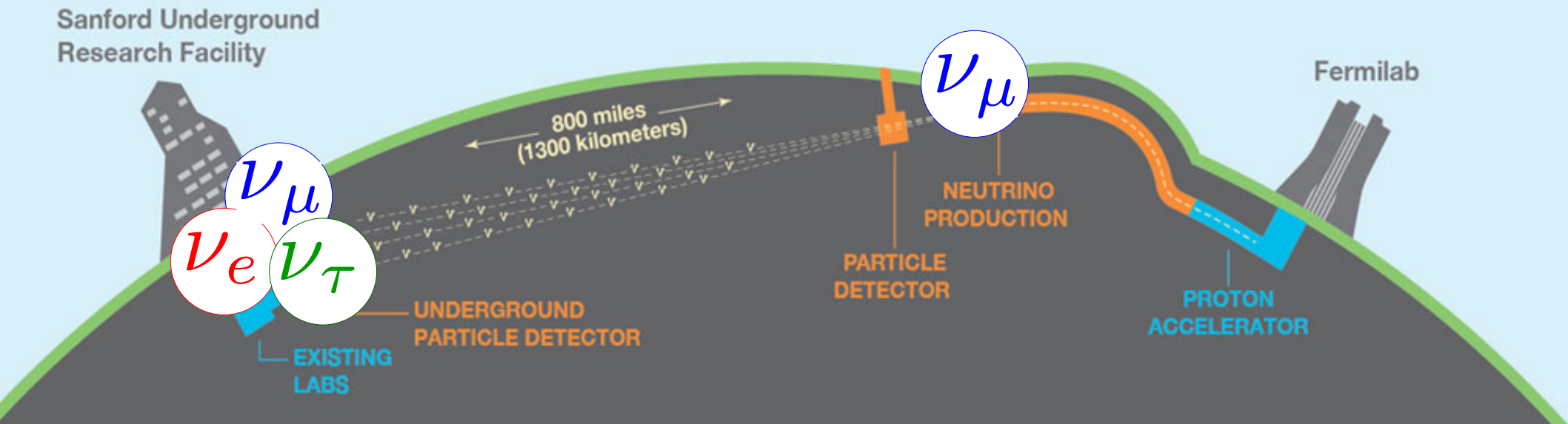
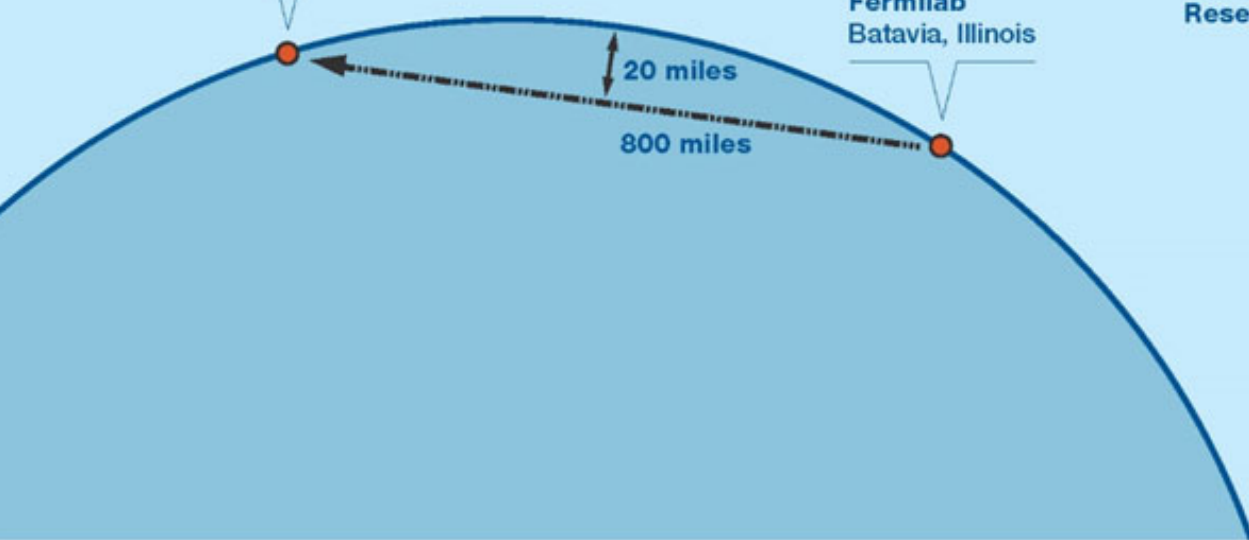
PARTICLE DETECTOR

NEUTRINO PRODUCTION

PROTON ACCELERATOR

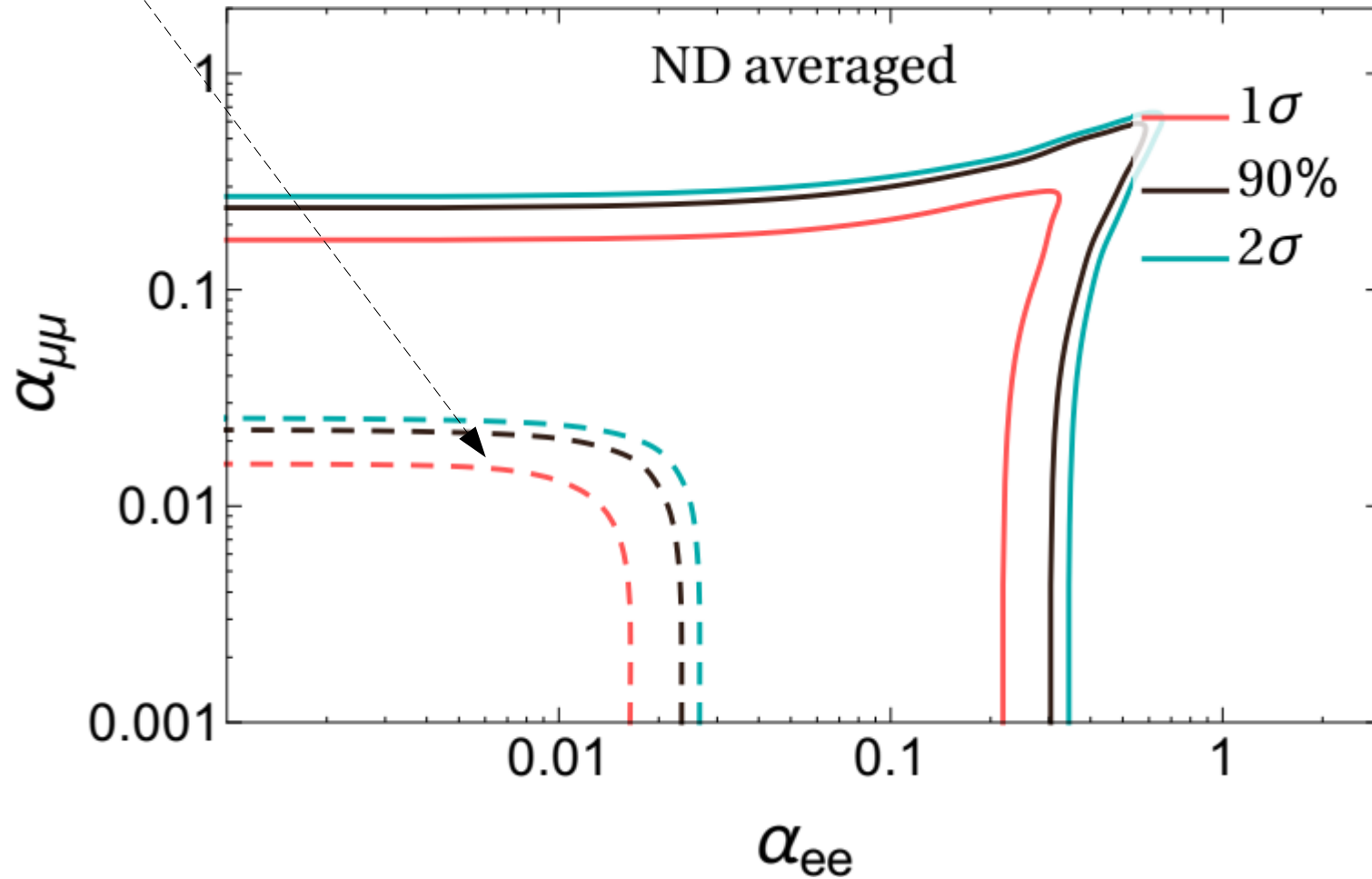
EXISTING LABS

800 miles
(1300 kilometers)



Far Detector

Prior
(present bounds)



Near Detector

Coloma, JLP, Rosauro-Alcaraz, **Urrea** 2105.11466.

See also Escrihuela, Forero, Miranda, Tortola, Valle arXiv:1503.08879 for other Near Detector configurations (without including tau detection).

High Scale Non-Unitarity

- What is measured in Near Detector

$$\mathcal{P}_{\alpha\beta} = |(NN^\dagger)_{\beta\alpha}|^2 = |\alpha_{\alpha\beta}|^2$$

zero
distance
effect!

$$\mathcal{P}_{\alpha\alpha} = |(NN^\dagger)_{\alpha\alpha}|^2 = 1 - 4\alpha_{\alpha\alpha}$$

Sterile Neutrinos: 3+1

- What is measured in Near Detector

$$\mathcal{P}_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2 \sin^2 \frac{\Delta m_{41}^2 L}{4E}$$

$$\mathcal{P}_{\alpha\alpha} = 1 - 4|U_{\alpha 4}|^2 \sin^2 \frac{\Delta m_{41}^2 L}{4E}$$

Averaged-out regime

- What is measured in Near Detector

$$\Delta m_{41}^2 \gtrsim 100 \text{ eV}^2$$

$$\langle \mathcal{P}_{\alpha\beta} \rangle = 2|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

zero
distance
effect!

$$\langle \mathcal{P}_{\alpha\alpha} \rangle = 1 - 2|U_{\alpha 4}|^2$$

Averaged-out regime

- What is measured in Near Detector

$$\Delta m_{41}^2 \gtrsim 100 \text{ eV}^2$$

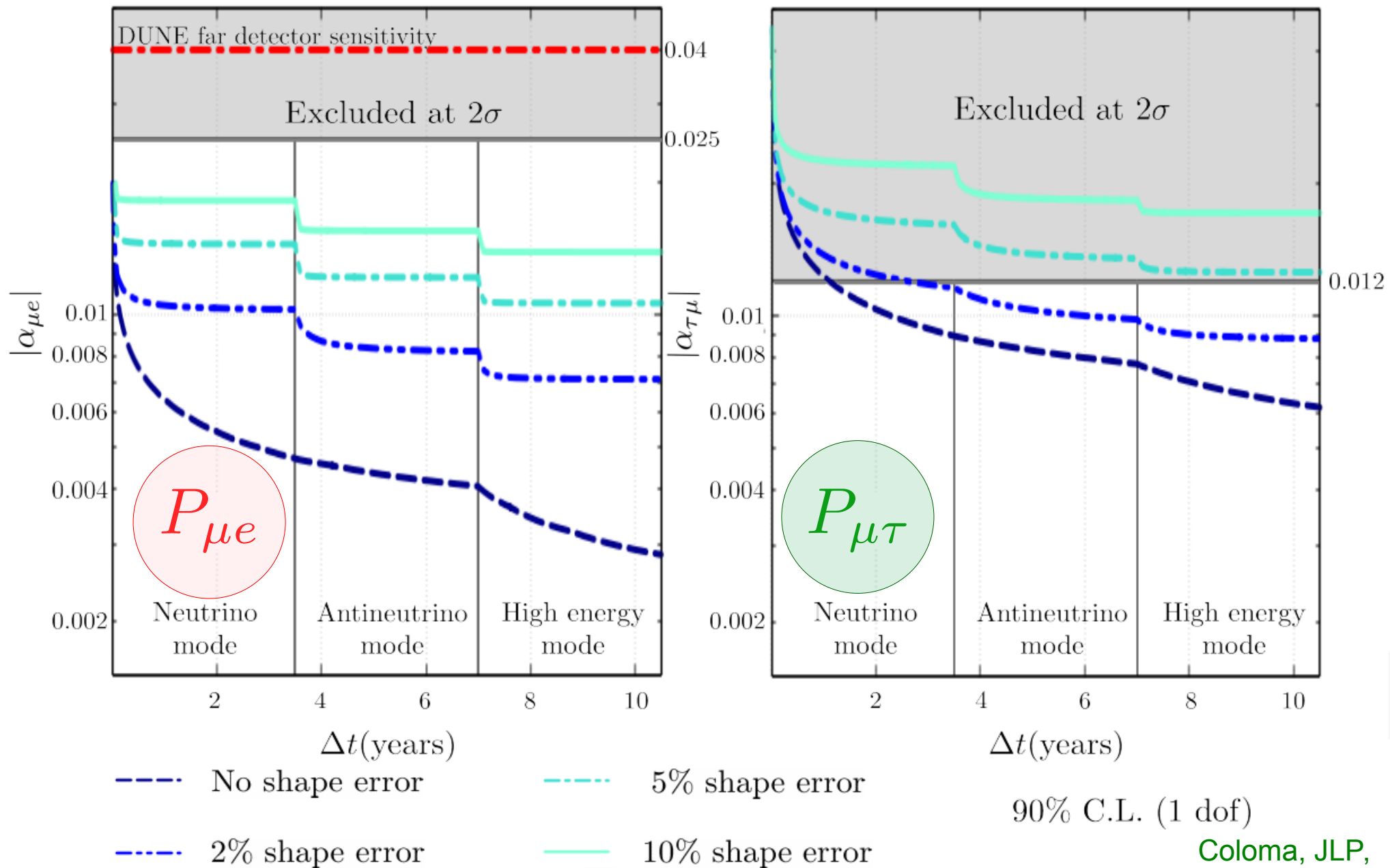
$$\langle \mathcal{P}_{\alpha\beta} \rangle = 2|\alpha_{\alpha\beta}|^2$$

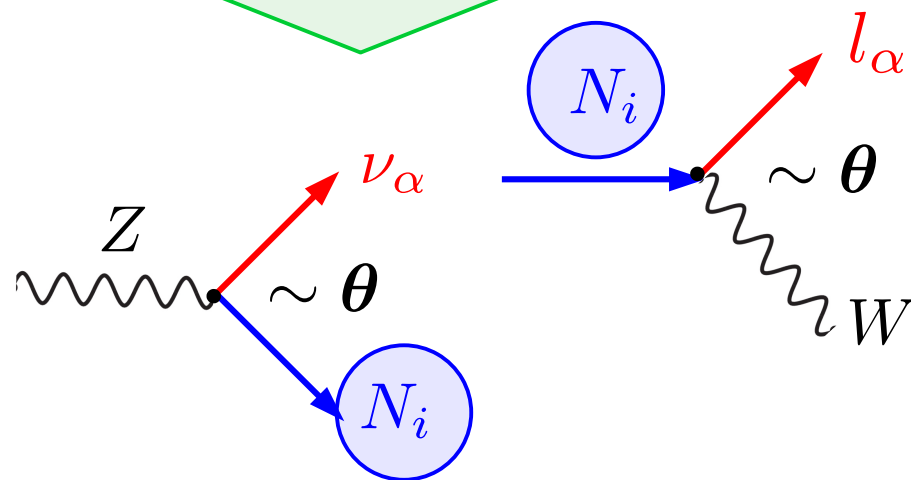
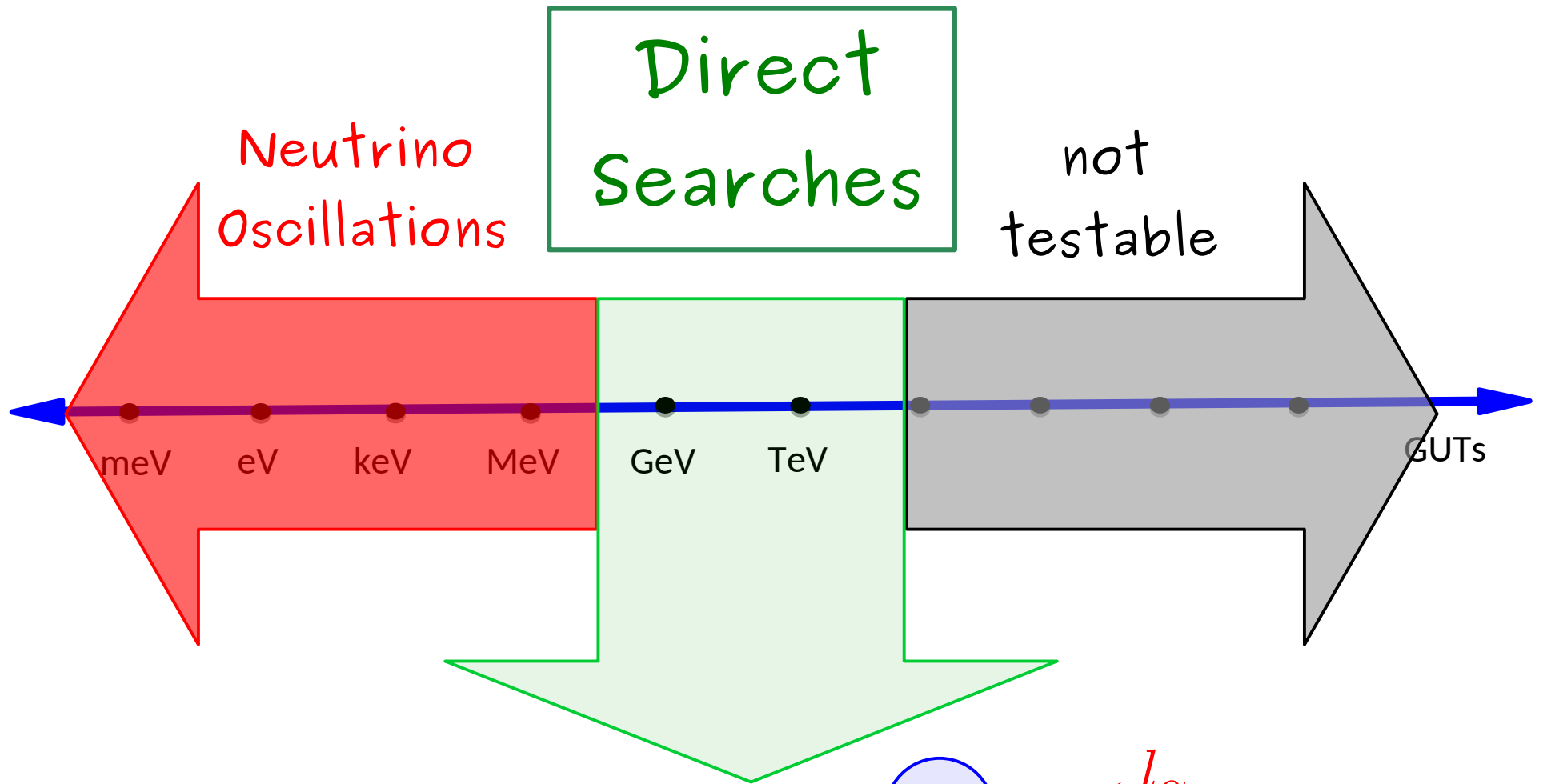
zero
distance
effect!

$$\langle \mathcal{P}_{\alpha\alpha} \rangle = 1 - 4|\alpha_{\alpha\alpha}|$$

Low Scale
Non-Unitarity

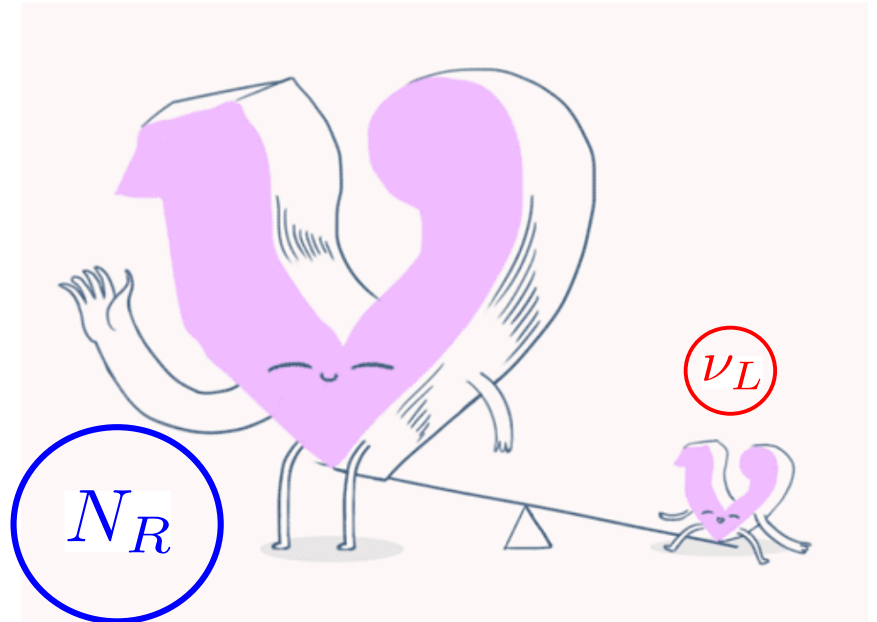
Low Scale Non-Unitarity





Light Neutrino mass generation

- Generation of light neutrino masses imposes **constraints on mixing between HNLs and active neutrinos from *light neutrino sector***

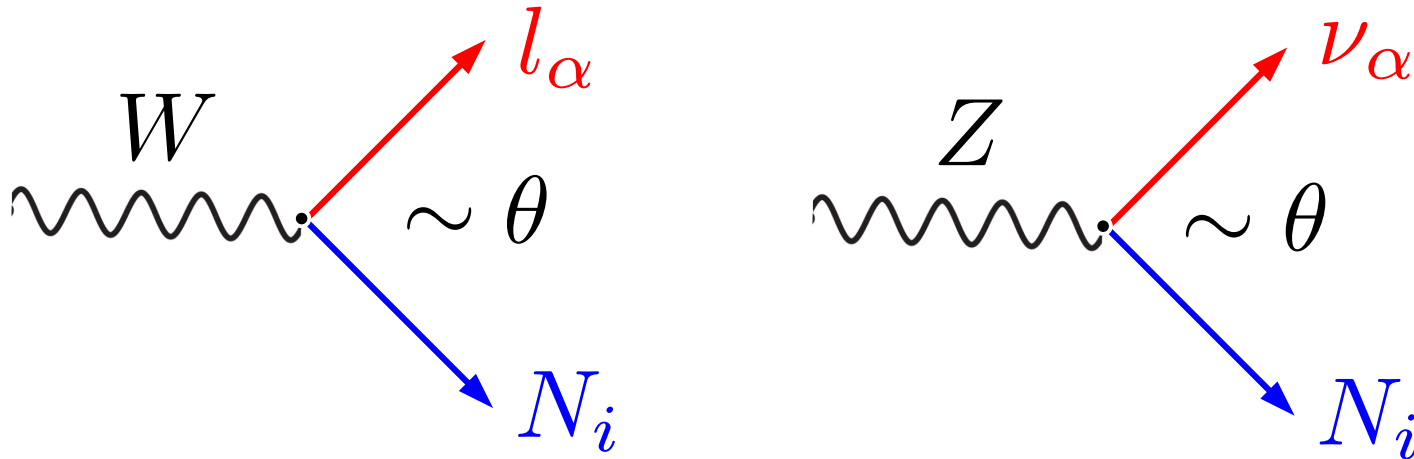


$$m_\nu = \frac{v^2}{2} Y^T M^{-1} Y = \boxed{\theta M \theta^T} = \boxed{U m U^T}$$

↓ ↓

HNL sector Light-active neutrino sector

Constraint on HNL mixing from active sector



Casas-Ibarra

$$\theta = iU m^{1/2} R^\dagger M^{-1/2}$$

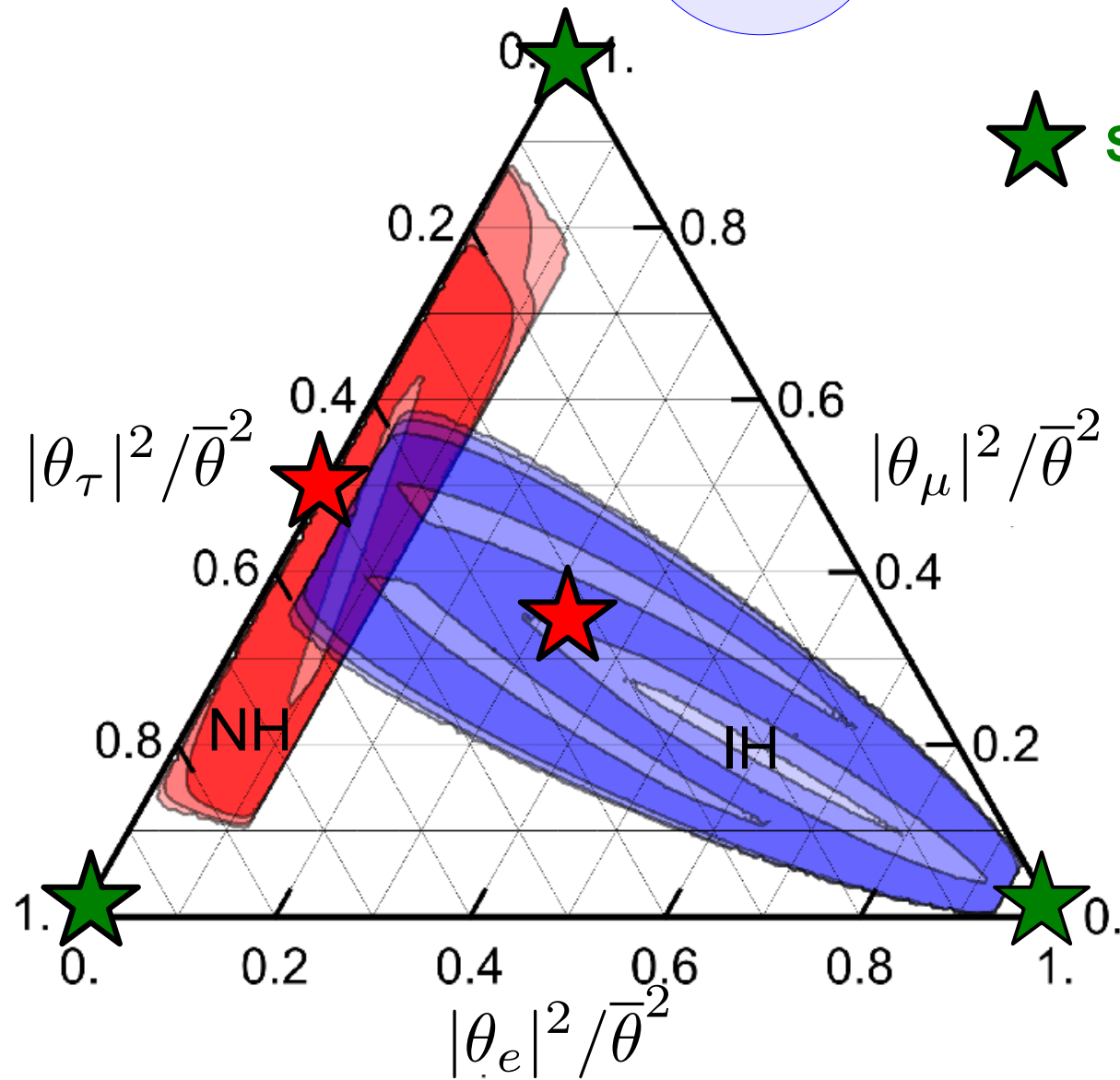
Active Sector

- 3x3 PMNS mixing matrix
- light neutrino masses

HNL Sector

- Complex $3 \times N_R$ orthogonal matrix
- HNL masses

Minimal model $N_R=2$: Flavor structure

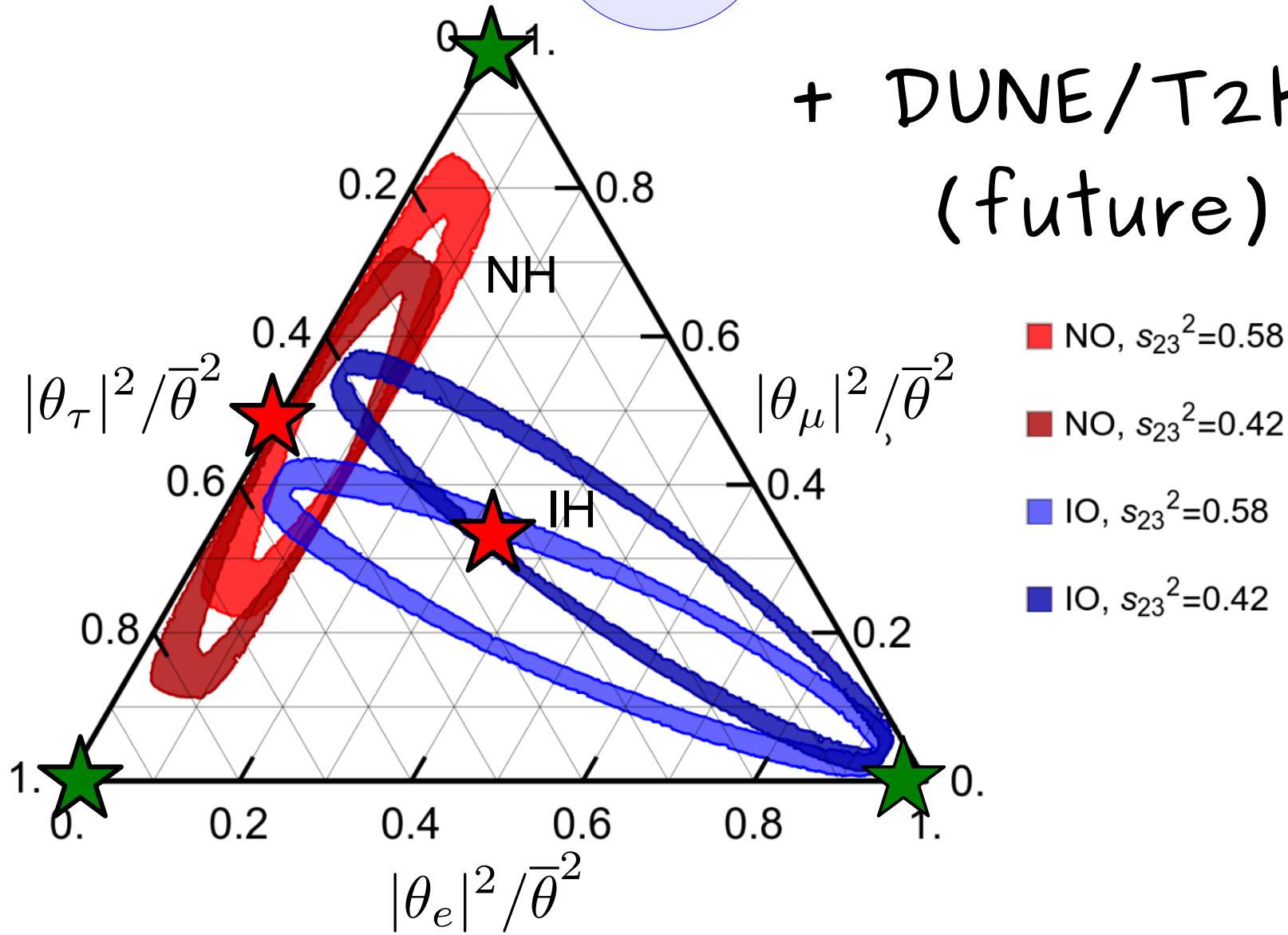


★ **Single flavored benchmarks**
 $(1,0,0)$, $(0,1,0)$, $(0,0,1)$

★ **NEW 2021**
 $(0, 1/2, 1/2)$
 $(1/3, 1/3, 1/3)$

Minimal model $N_R=2$: Flavor Structure

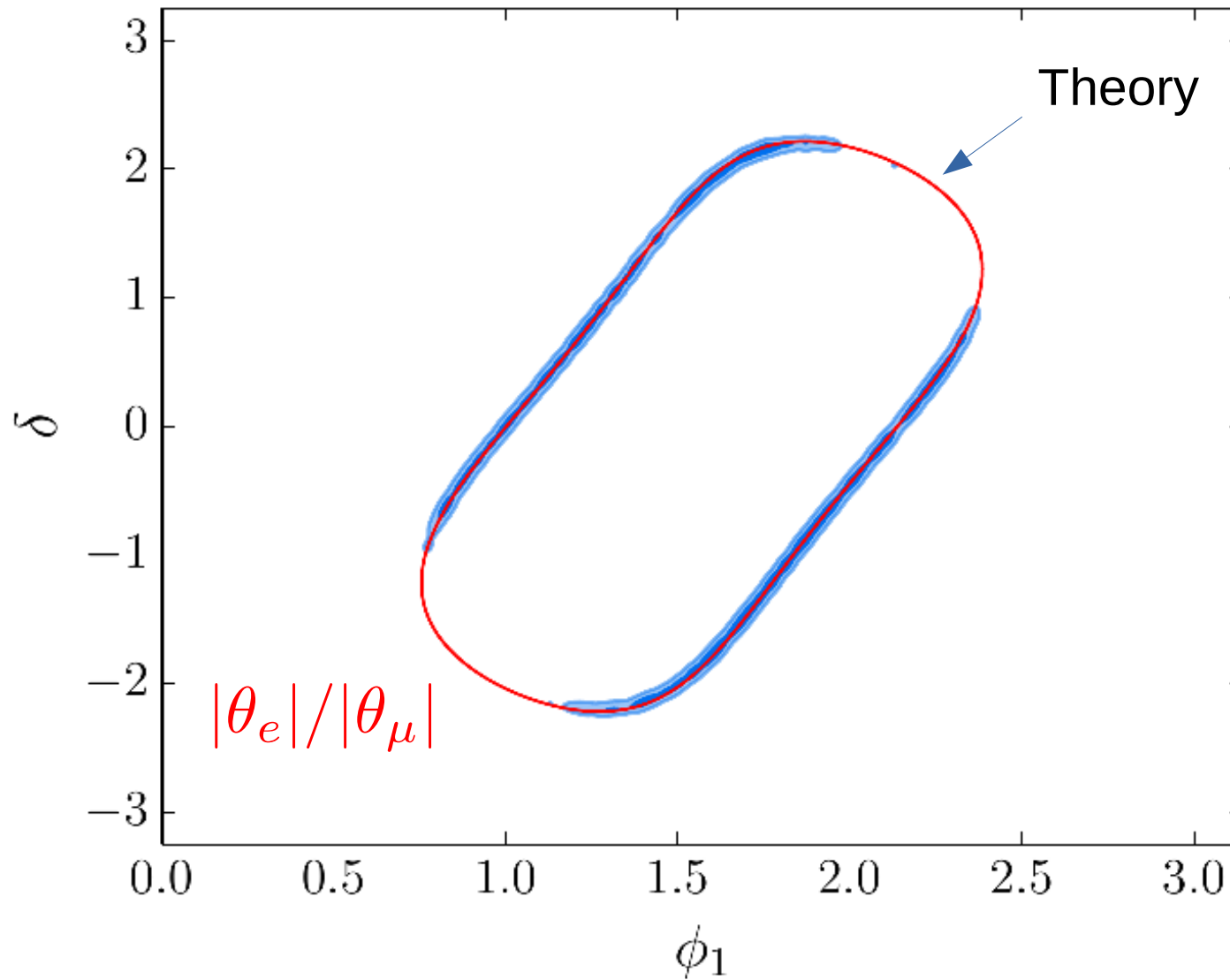
+ DUNE/T2HK...
(future)



DUNE forecast assuming $\delta = -\pi/2$

Abdullahi et al arXiv:2203.08039
Drewes, Klaric, JLP arXiv: 2207.02742

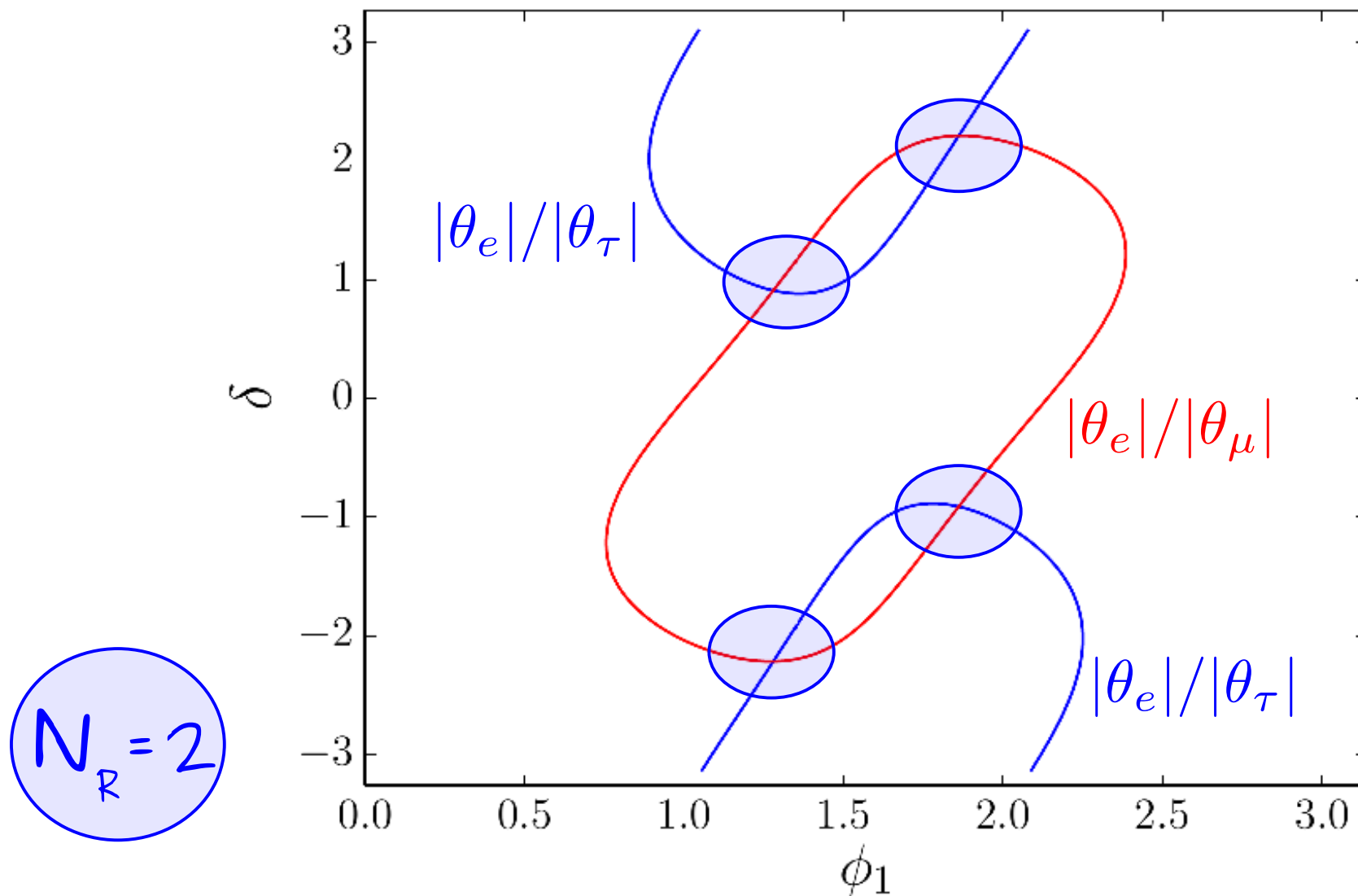
PMNS CP-phases from HNLs searches



Hernandez, Kekic, JLP, Racker, Salvado 1606.06719
Caputo, Hernandez, Kekic, JLP, Salvado 1611.05000

$N_R = 2$

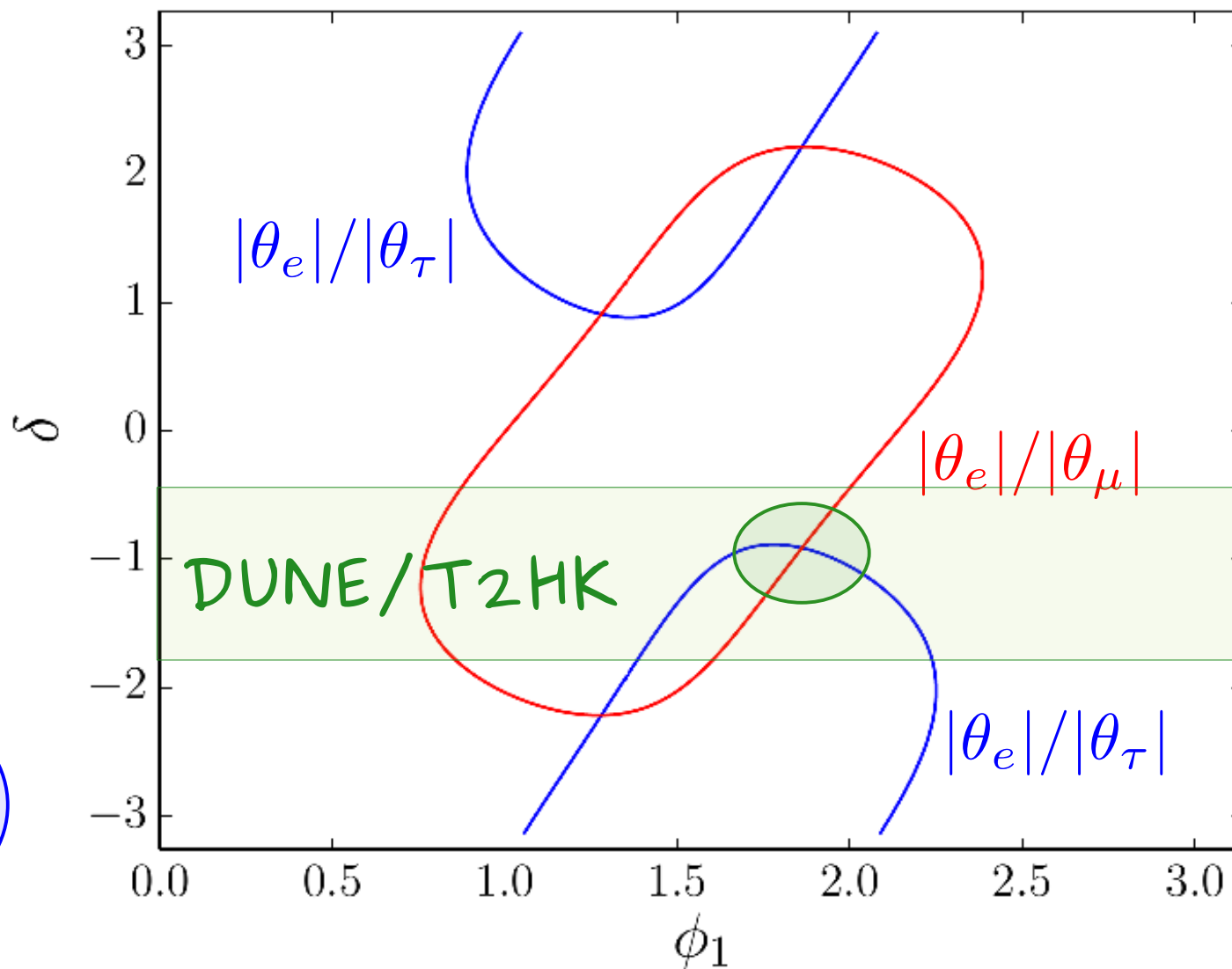
PMNS CP-phases from HNLs searches



- **Measurement of mixing with tau neutrinos would allow to break degeneracies.**

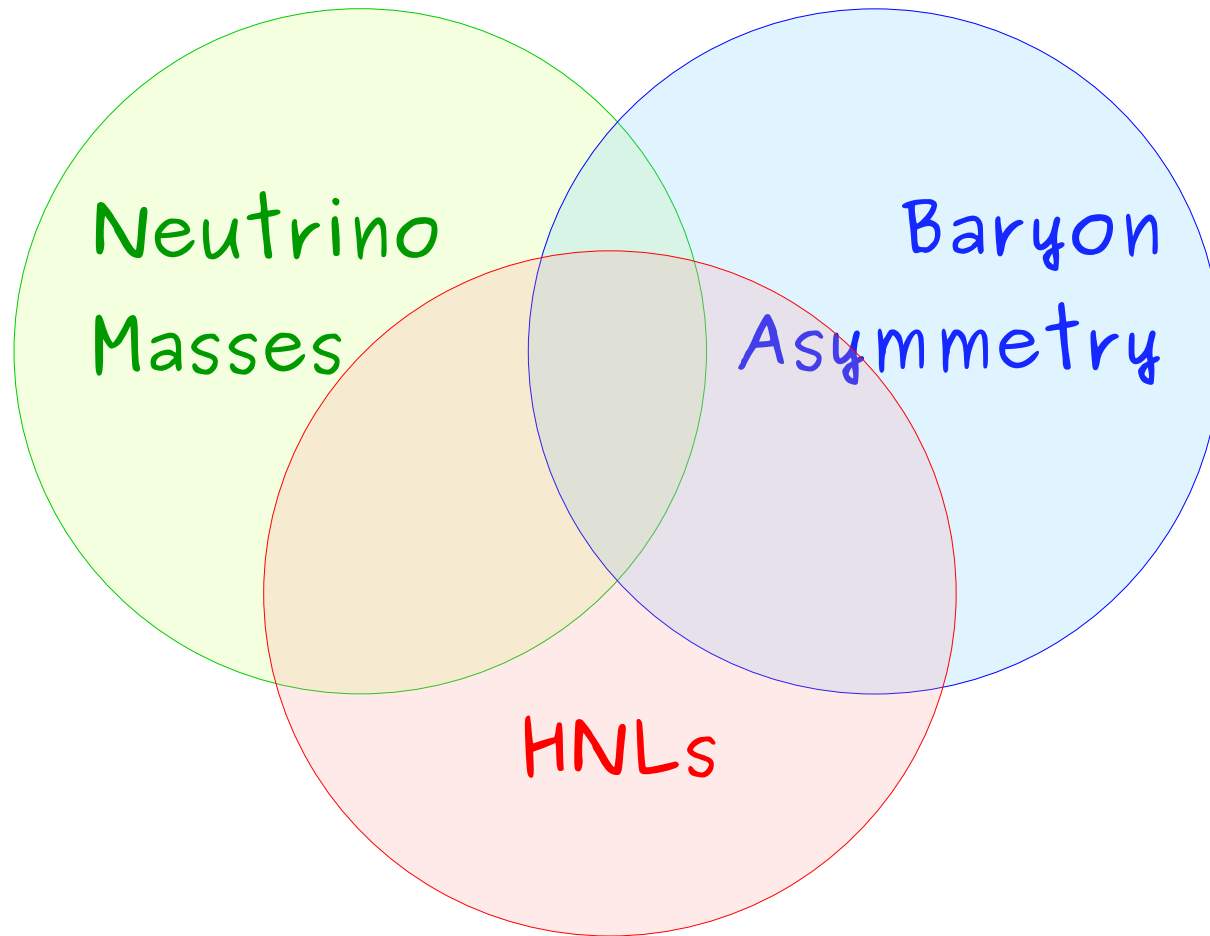
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PMNS CP-phases from HNLs searches



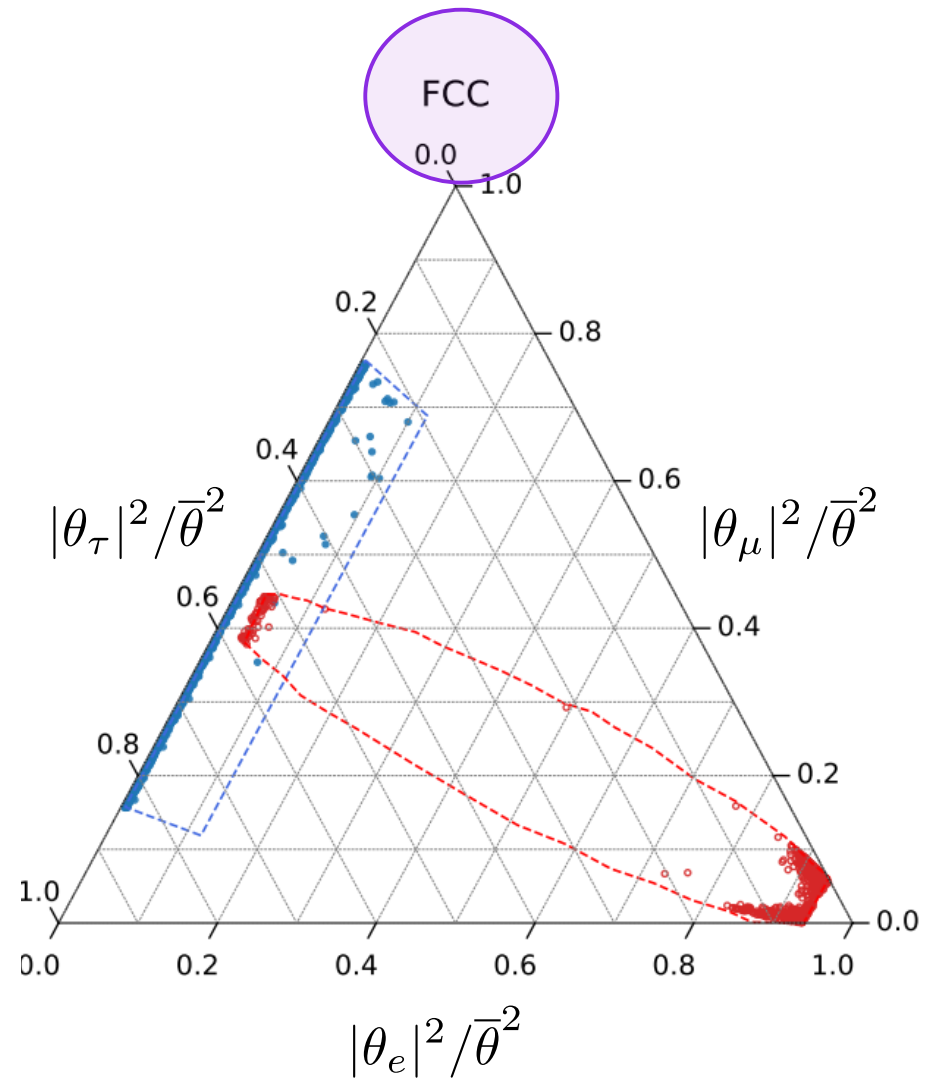
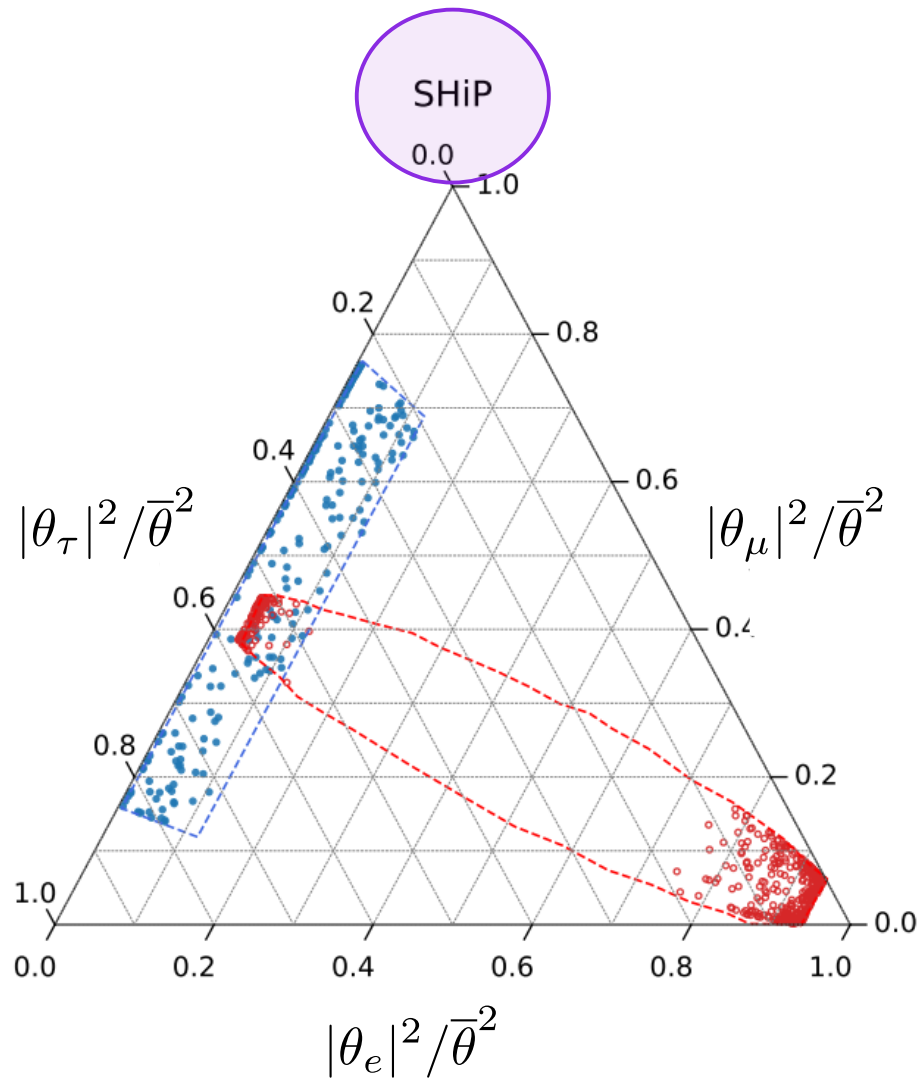
$N_R = 2$

- **Potential determination of the PMNS Majorana phase!**



See talk by Apostolos Pilaftsis

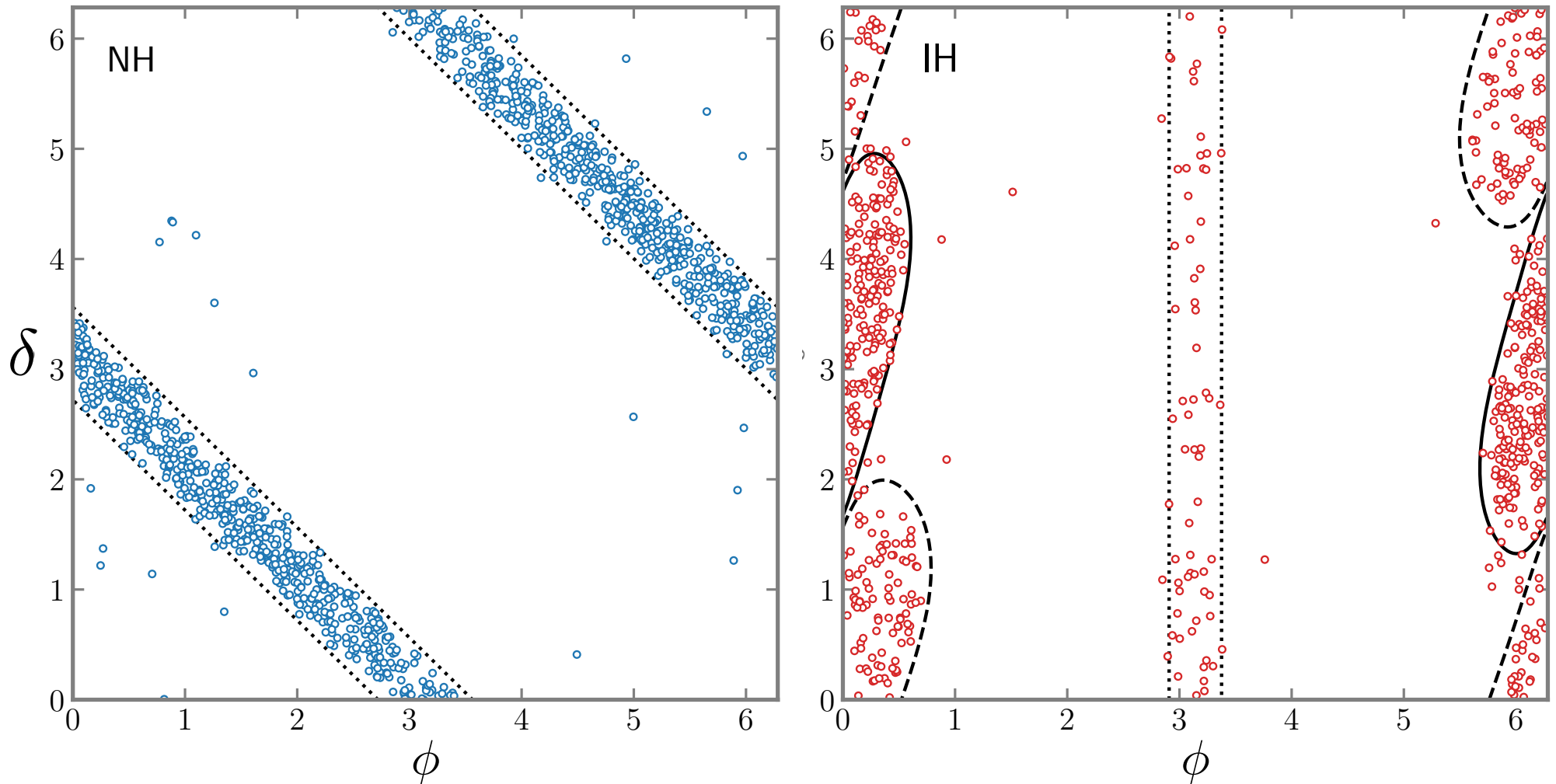
Low Scale Leptogenesis: Flavor



$$\Delta M/M = 10^{-2}$$

$$N_R = 2$$

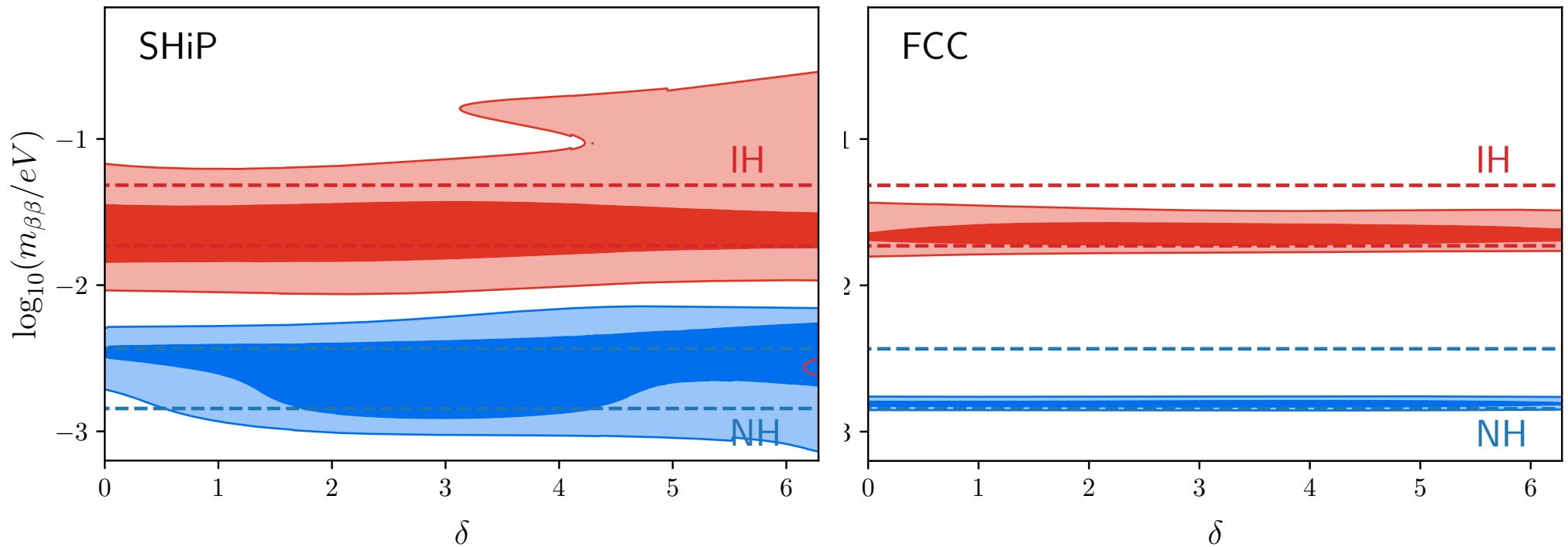
Low Scale Leptogenesis: CP phases



$$\Delta M/M = 10^{-2}$$

$$N_R = 2$$

Low Scale Leptogenesis: $0\nu\beta\beta$ decay



$$\Delta M/M = 10^{-2}$$

$$N_R = 2$$

Conclusions

- Near detectors in future neutrino oscillation experiments can play a relevant role in testing the robustness of the 3-neutrino picture

Low scale Non-Unitarity, sterile neutrino oscillations, NSI (Non-Unitarity results can be easily mapped to NSI framework, see 2105.11466)

- *Keeping under control shape uncertainties is a key issue.*

Joint experimental and theoretical effort required to reduce systematics. *Independent measurements of the cross sections would give very relevant information of the energy dependence (see ν STORM proposal)*

- Minimal neutrino mass model

measurement of HNLs mass & mixing would allow to

test the mechanisms generating neutrino masses and Baryon asymmetry and to indirectly measure the PMNS phases.

Thank you!

FIPs 2022

Workshop on Feebly-Interacting Particles

17-21 October 2022
CERN



FIPs in colliders
extracted beams /
fixed-target experiments
neutrino experiments
astroparticle physics / cosmology
direct and indirect
dark matter detectors
axion / ALP experiments
ultra-light particle searches
and beyond

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Maxim Pospelov
Philip Schuster
Mikhail Shaposhnikov
Jessie Shelton
Yevgeny Stadnik
Stefan Ulmer



indico.cern.ch/e/FIPs2022

Approximated LNC

$$M_\nu = \begin{pmatrix} \overline{\nu^c} & \overline{N}_1 & \overline{N}_2 & L \\ 1 & -1 & 1 & \\ 0 & Y_1^T v/\sqrt{2} & \epsilon Y_2^T v/\sqrt{2} & 1 \\ Y_1 v/\sqrt{2} & \mu' & \Lambda & -1 \\ \epsilon Y_2 v/\sqrt{2} & \Lambda & \mu & 1 \end{pmatrix} \begin{matrix} \nu \\ N_1^c \\ N_2^c \end{matrix}$$

- Light nu masses suppressed with LNV parameters

$$m_\nu = \mu \frac{v^2}{2\Lambda^2} Y_1^T Y_1 + \frac{v^2}{2\Lambda} \epsilon Y_2^T Y_1 + \frac{v^2}{2\Lambda} Y_1^T \epsilon Y_2$$

- Quasi-Dirac heavy neutrinos with large mixings:

$$M_2 \approx M_1 \approx \Lambda \quad \Delta M \approx \mu' + \mu \quad \theta \sim Y_1 v/\Lambda$$

Direct searches of HNLs

- Direct detection requires:

$$N_R = 2$$

$$\theta_{\alpha i} \gg \sqrt{m/M} \iff R_{ij} \gg 1$$

$$\iff \theta_{\alpha i}^2 \propto e^{-2\theta i} e^{2\gamma} f(\delta, \phi_1, M_j)$$

$$\bullet |\theta_{e1}|^2 / |\theta_{\mu 1}|^2 \simeq |\theta_{e2}|^2 / |\theta_{\mu 2}|^2 \simeq$$

$$(1 + s_{\phi_1} \sin 2\theta_{12})(1 - \theta_{13}^2) + \frac{1}{2} r^2 s_{12} (c_{12} s_{\phi_1} + s_{12})$$

$$\frac{(1 - \sin 2\theta_{12} s_{\phi_1} (1 + \frac{r^2}{4}) + \frac{r^2 c_{12}^2}{2}) c_{23}^2 + \theta_{13} (c_{\phi_1} s_{\delta} - \cos 2\theta_{12} s_{\phi_1} c_{\delta}) \sin 2\theta_{23} + \theta_{13}^2 (1 + \sin 2\theta_{12}) s_{23}^2 s_{\phi_1}}{}$$

Sensitivity to
PMNS CP-phases!
 δ, ϕ_1

Predicting γ_B in minimal model $N_R=2$

- Neutrinoless double beta decay effective mass in the IH case

$$\begin{aligned}
 |m_{\beta\beta}|_{IH} &\simeq \\
 &\simeq \sqrt{\Delta m_{atm}^2} \left[\boxed{c_{13}^2 \left(c_{12}^2 + e^{2i\phi_1} s_{12}^2 \left(1 + \frac{r^2}{2} \right) \right)} \right. \\
 &\quad \left. - \boxed{f(A) e^{2i\theta} e^{2\gamma} (c_{12} - ie^{i\phi_1} s_{12})^2 \frac{(0.9 \text{ GeV})^2}{2M_1^2} \frac{\Delta M}{M_1}} \right]
 \end{aligned}$$

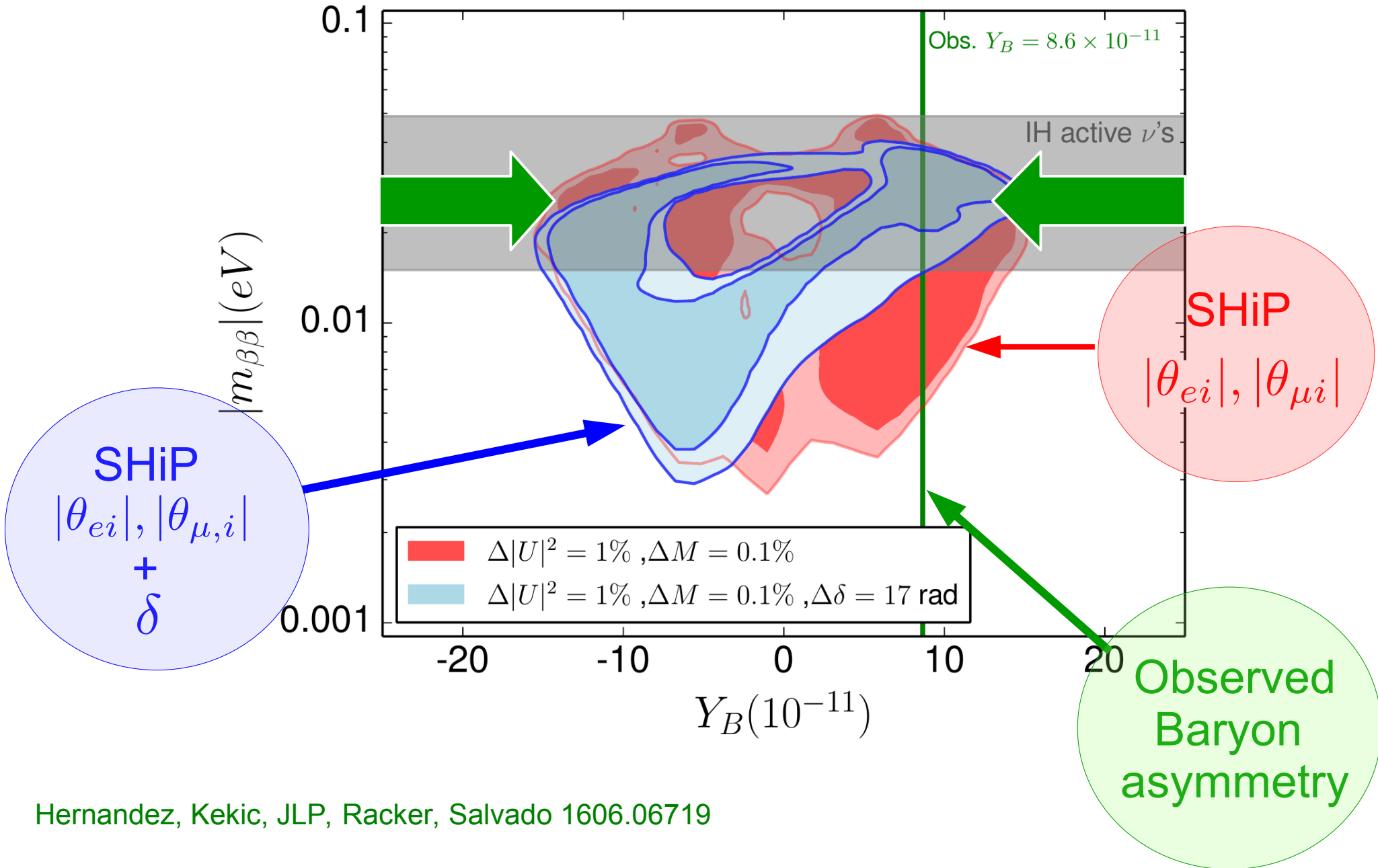
LIGHT NEUTRINO contribution

HEAVY NEUTRINO contribution

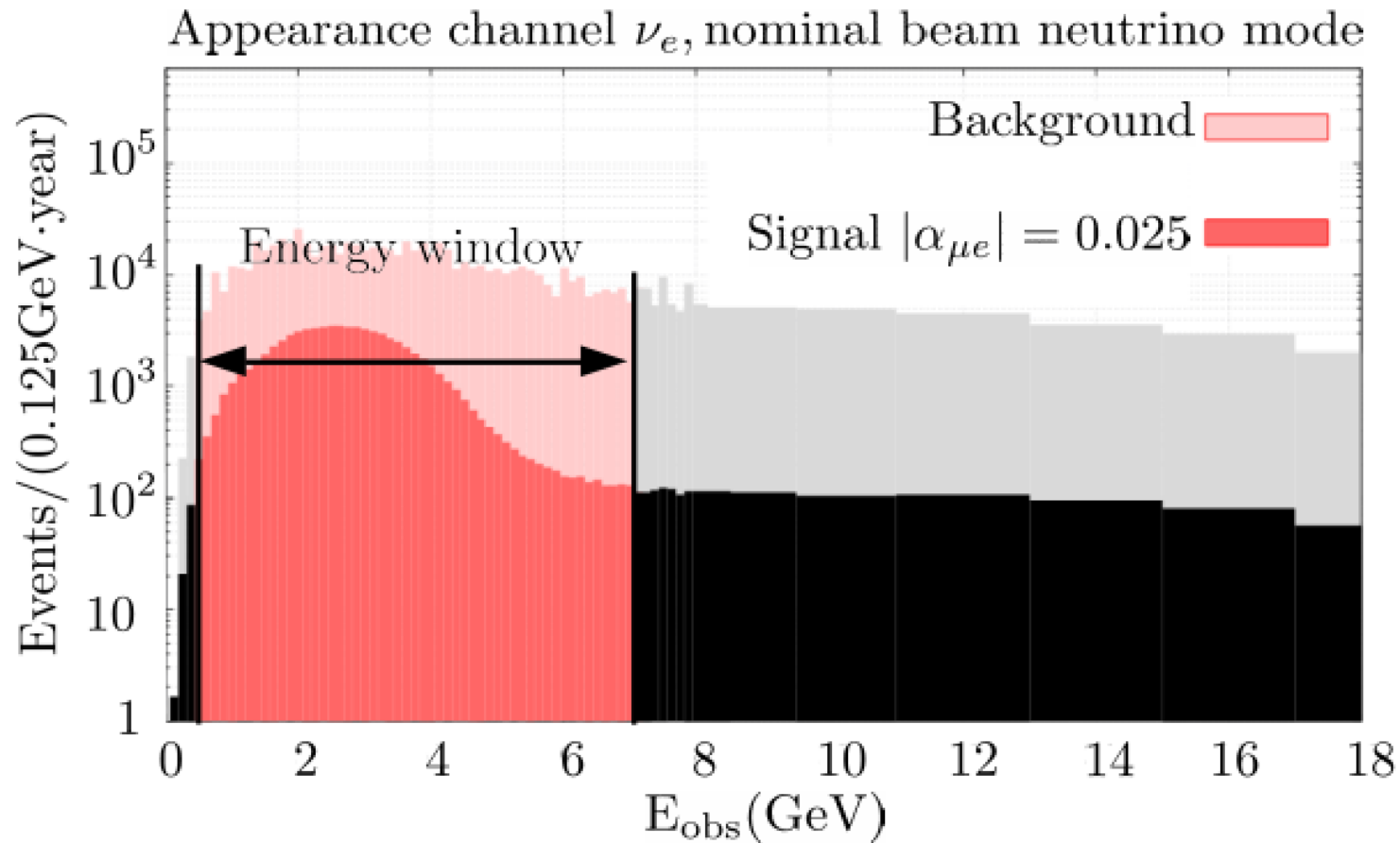
- Heavy neutrino contribution can be sizable for $M \sim O(\text{GeV})$

GeV Scale Leptogenesis

$N_R = 2$

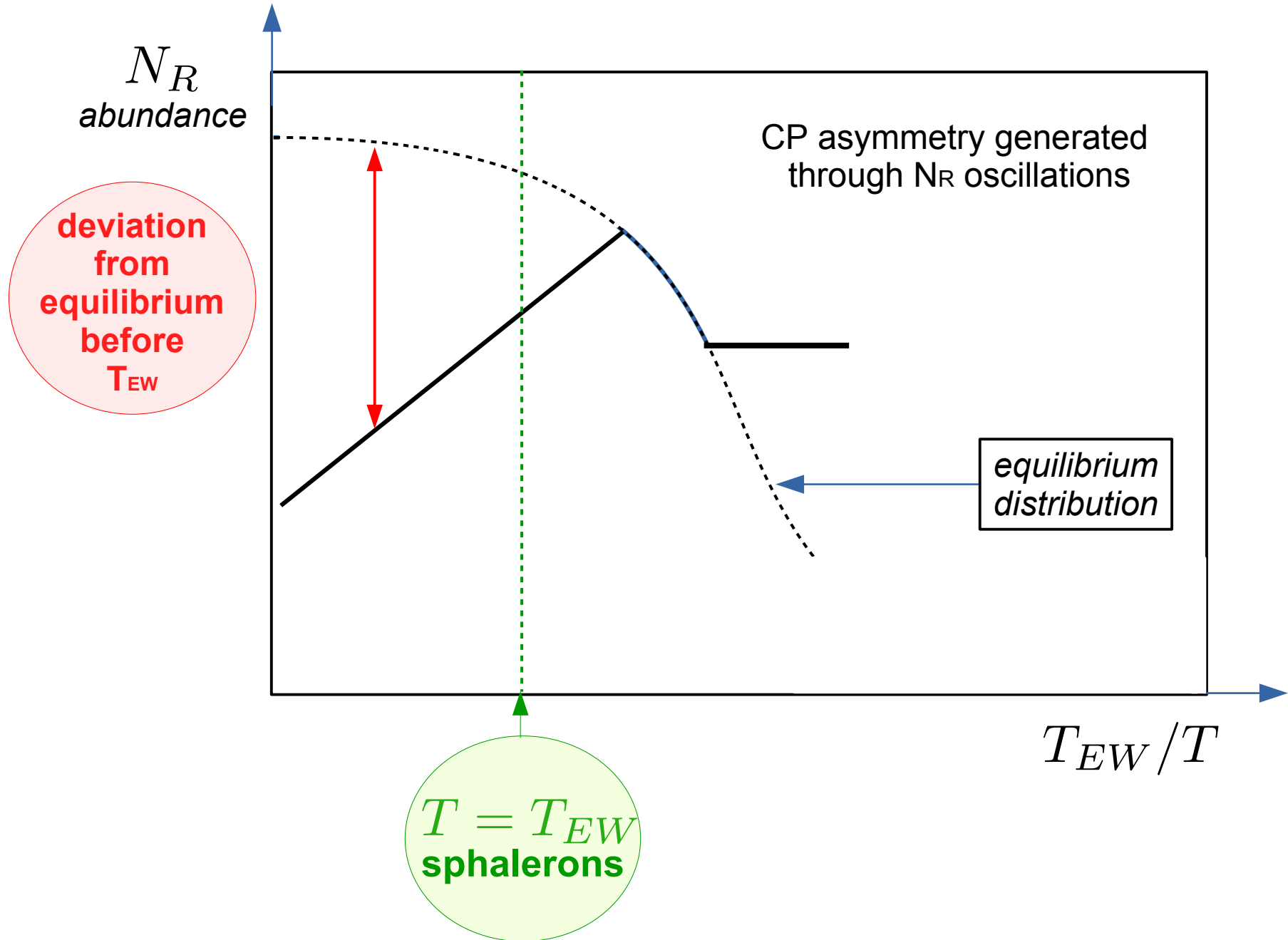


Role of shape uncertainty



- Sensitivity driven by spectral information.
- Marginal impact of global normalization error.

Low Scale Leptogenesis (ARS)



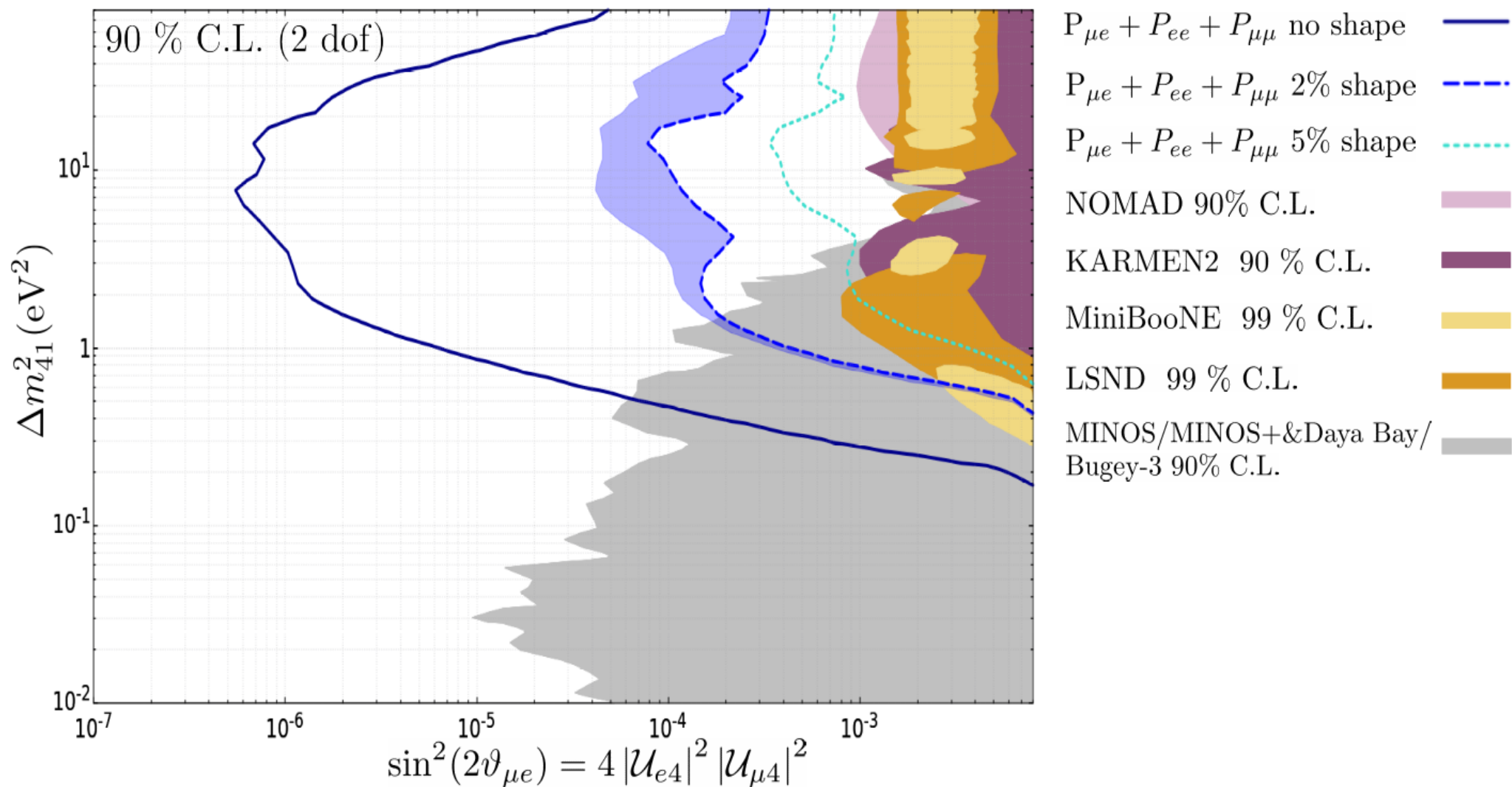
sterile Neutrinos: 3+1

$$\mathcal{P}_{\mu e} = 4|U_{\mu 4}|^2|U_{e 4}|^2 \sin^2 \frac{\Delta m_{41}^2 L}{4E}$$

$$\mathcal{P}_{\mu\mu} = 1 - 4|U_{\mu 4}|^2 \sin^2 \frac{\Delta m_{41}^2 L}{4E}$$

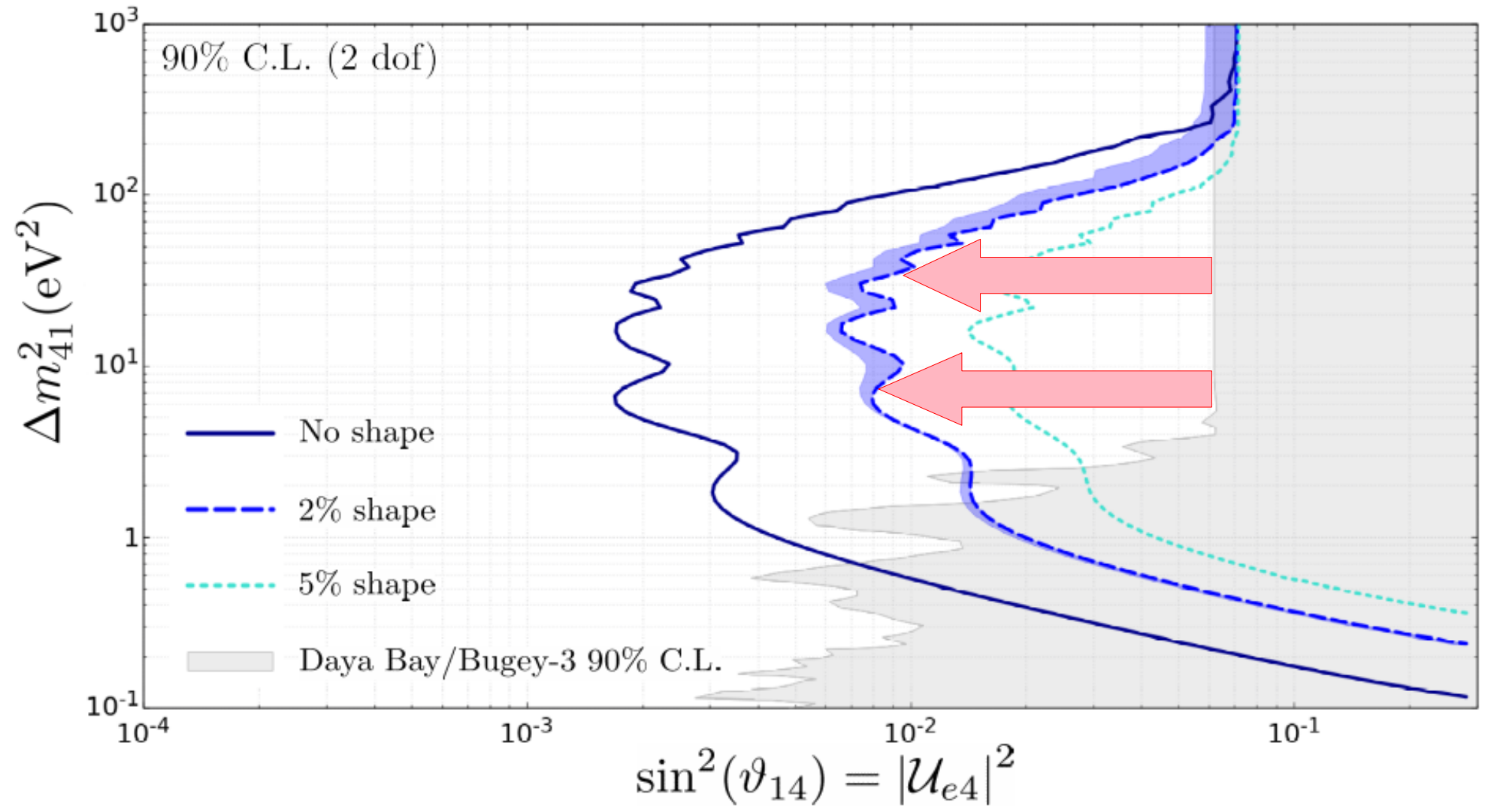
$$\mathcal{P}_{ee} = 1 - 4|U_{e 4}|^2 \sin^2 \frac{\Delta m_{41}^2 L}{4E}$$

3+1 Sterile Neutrinos: $P_{\mu\mu} + P_{\mu e} + P_{ee}$

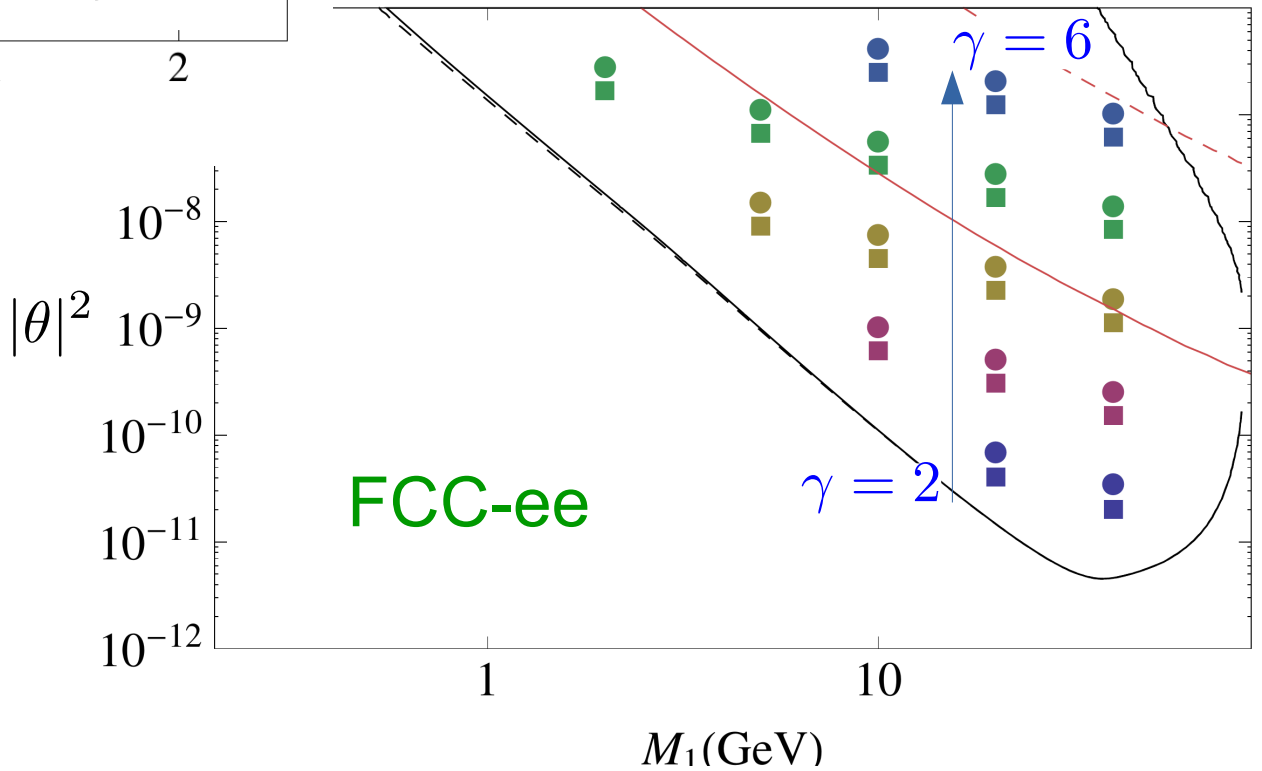
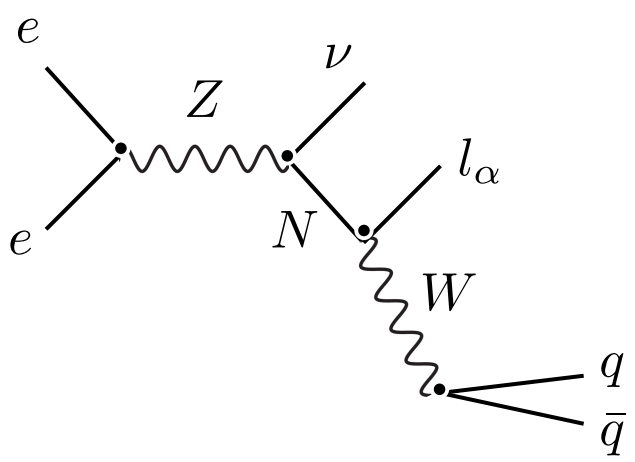
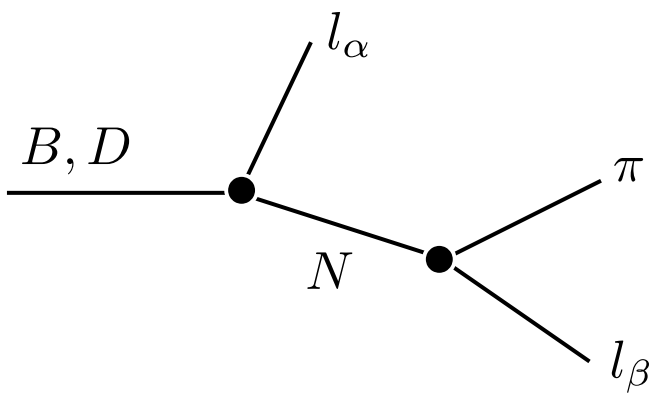
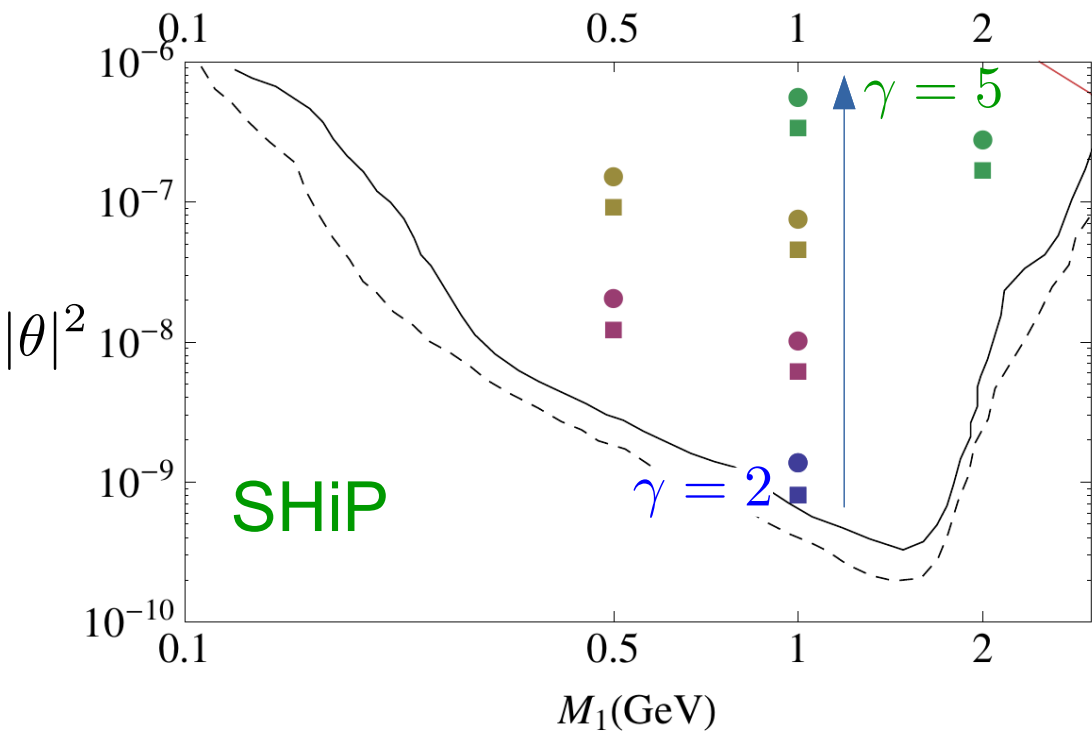


3+1 Sterile Neutrinos:

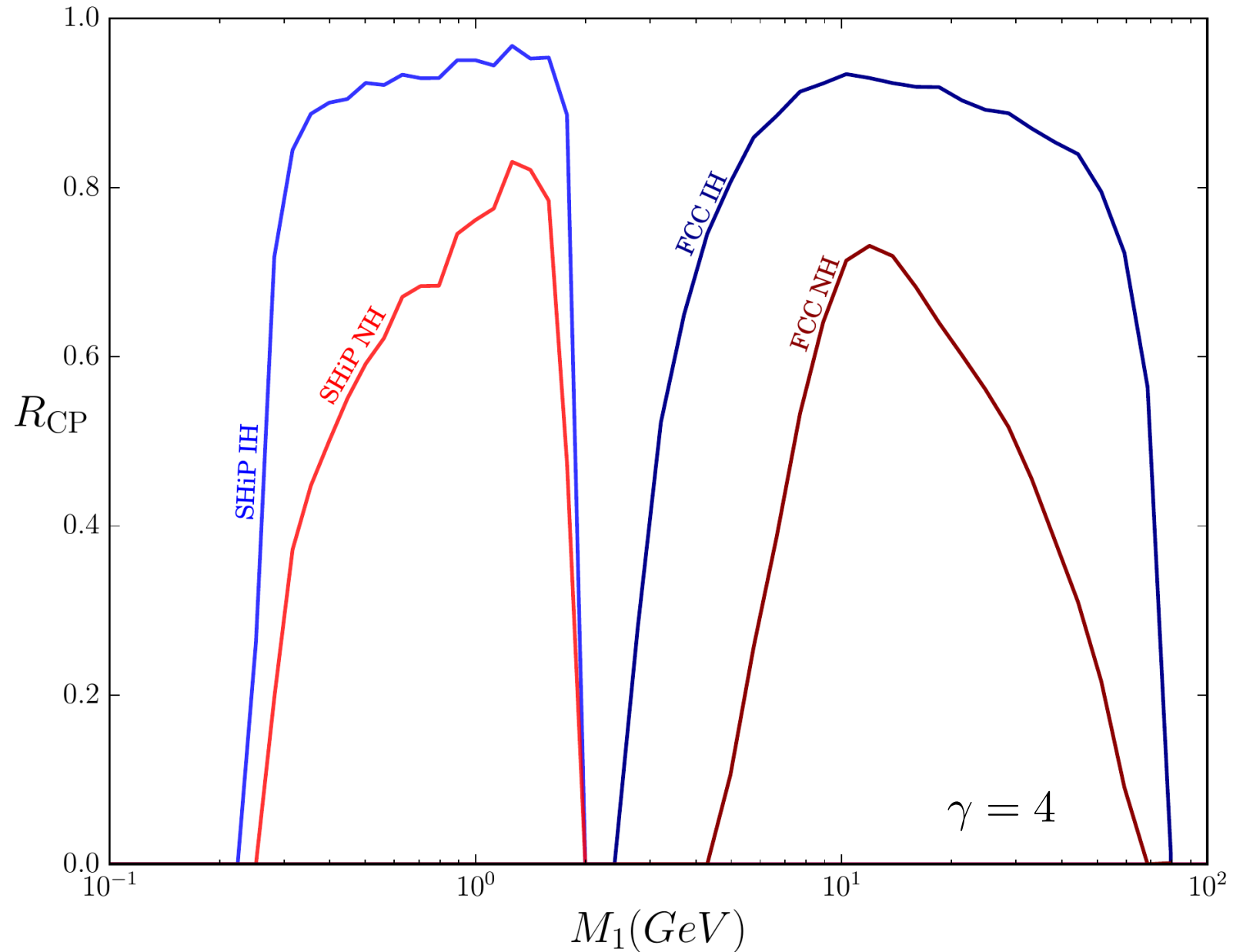
P_{ee}



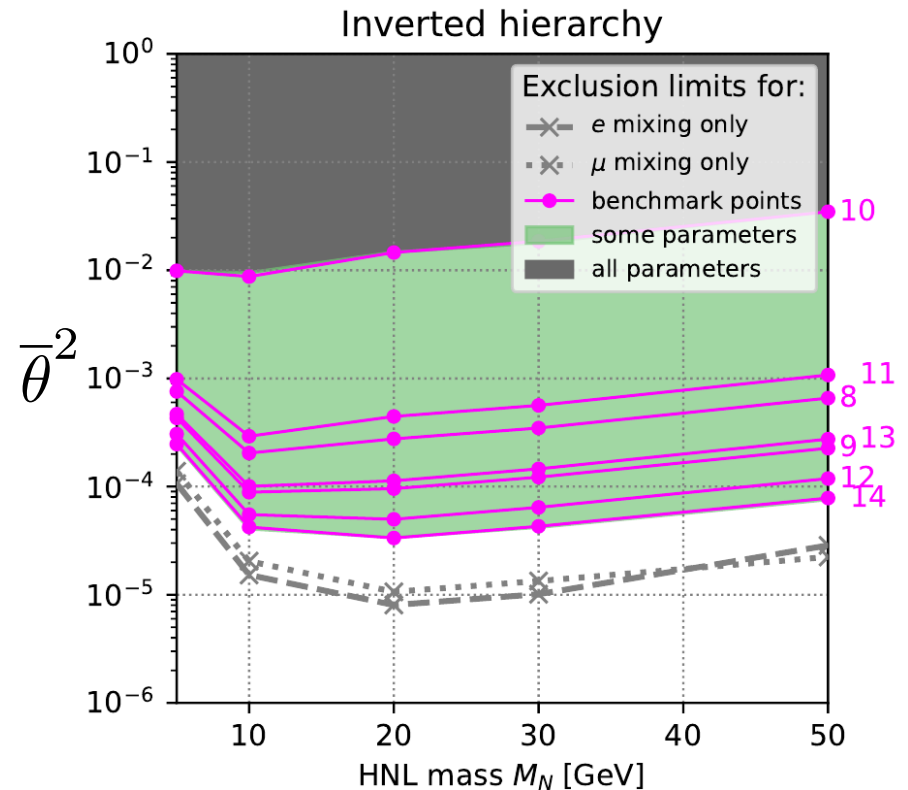
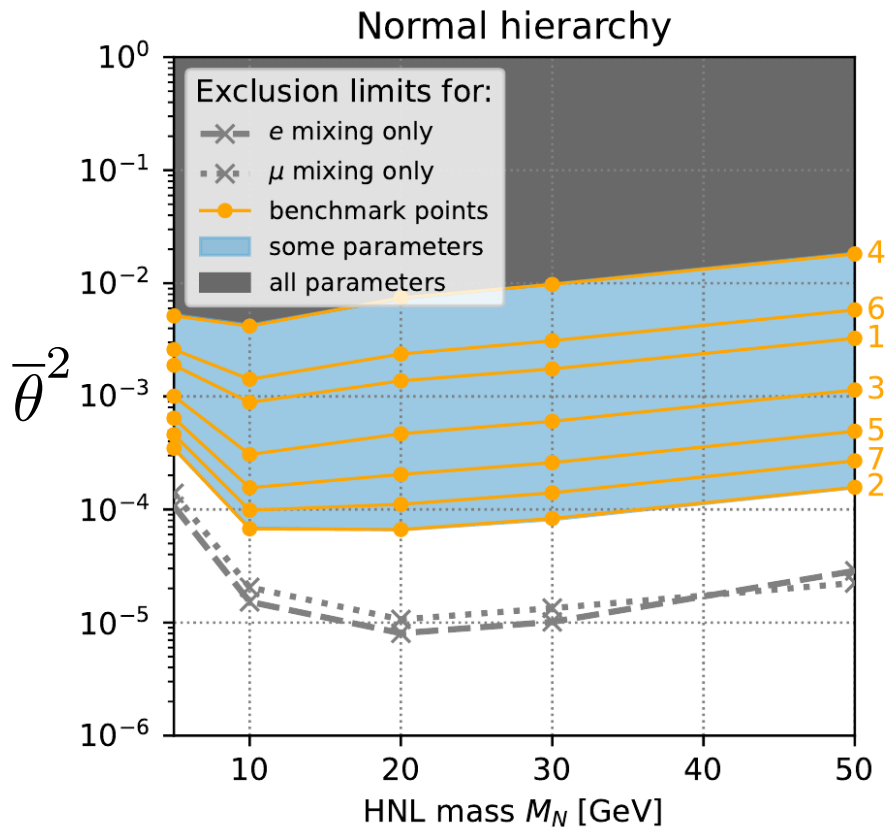
For instance...



5 σ discovery PMNS CP-violation



Flavor pattern vs sensitivity



- Interpretation of ATLAS data depends on assumptions about “flavor mixing pattern”

Tastet, Ruchayskiya, Timiryasov 2107.12980

See talks by **Tastet and Xabier Marciano**

- Same conclusion applies to other experimental searches.

ν_τ appearance channel

ν_τ detection:

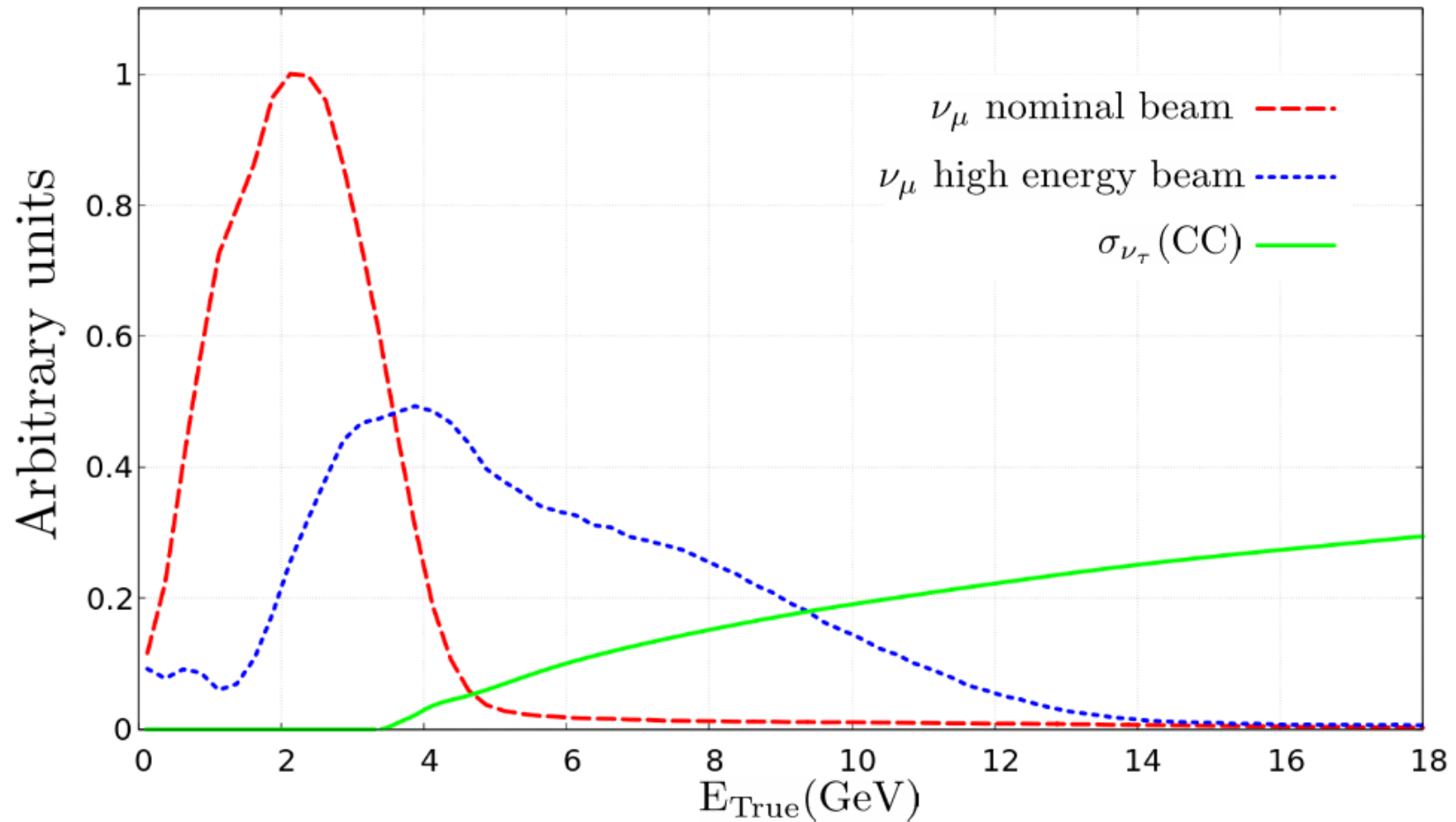
- Energy threshold of τ production 3.2 GeV.
- Short lifetime of τ , indirect measurement via hadronic decays ($\sim 65\%$ branching ratio).
- NC background. We have considered a sample in which 30% of the hadronic events are identified keeping 0.5% of NC background.

de Gouvêa, Kelly, Stenico, Pasquini 1904.07265

See talks by Pedro Machado & Adam Aurisano

ν_τ appearance channel

- Energy threshold of τ production 3.2 GeV.



- ν_τ detection: we follow de Gouvêa, Kelly, Stenico, Pasquini 1904.07265

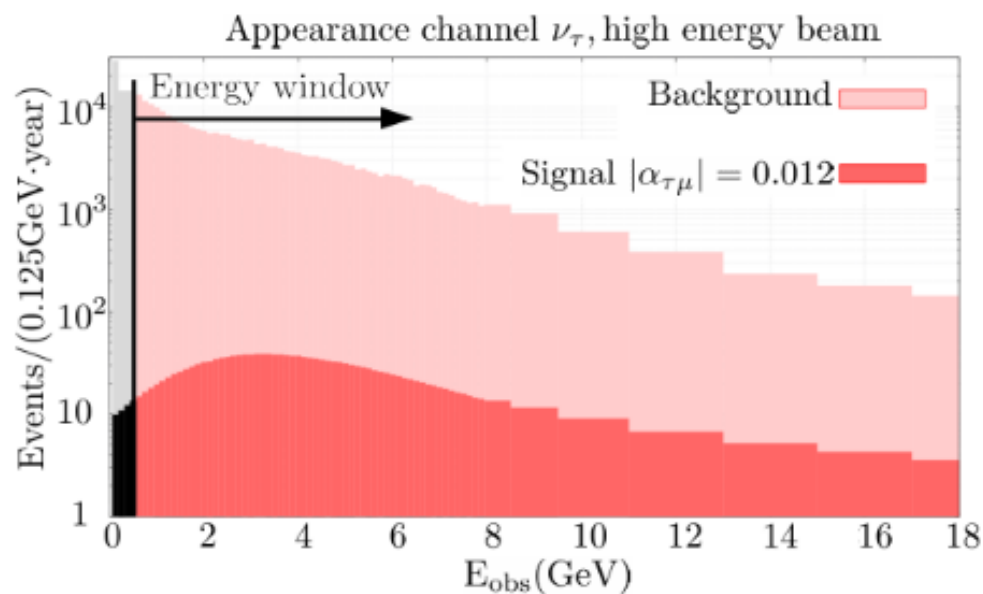
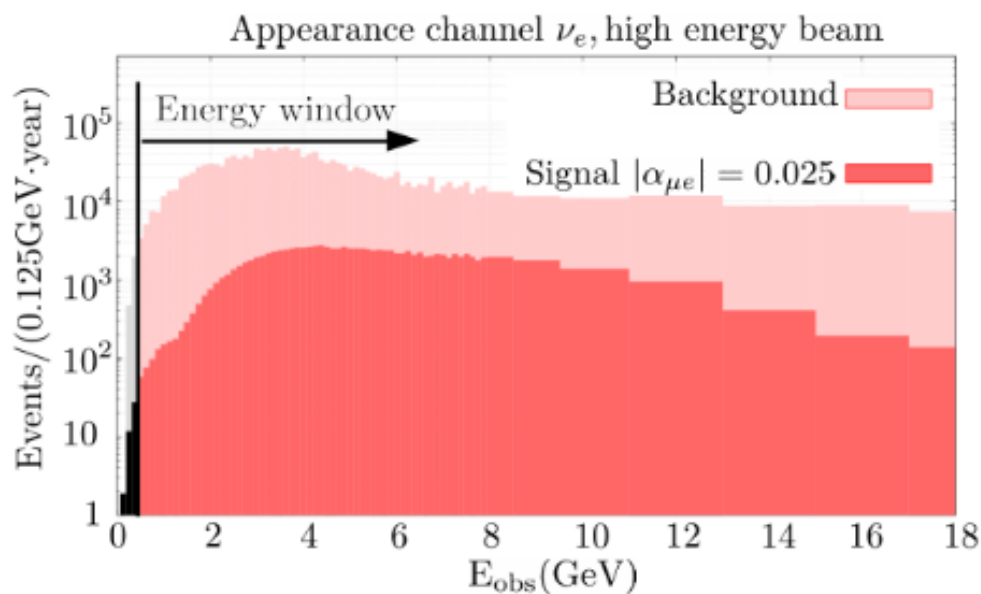
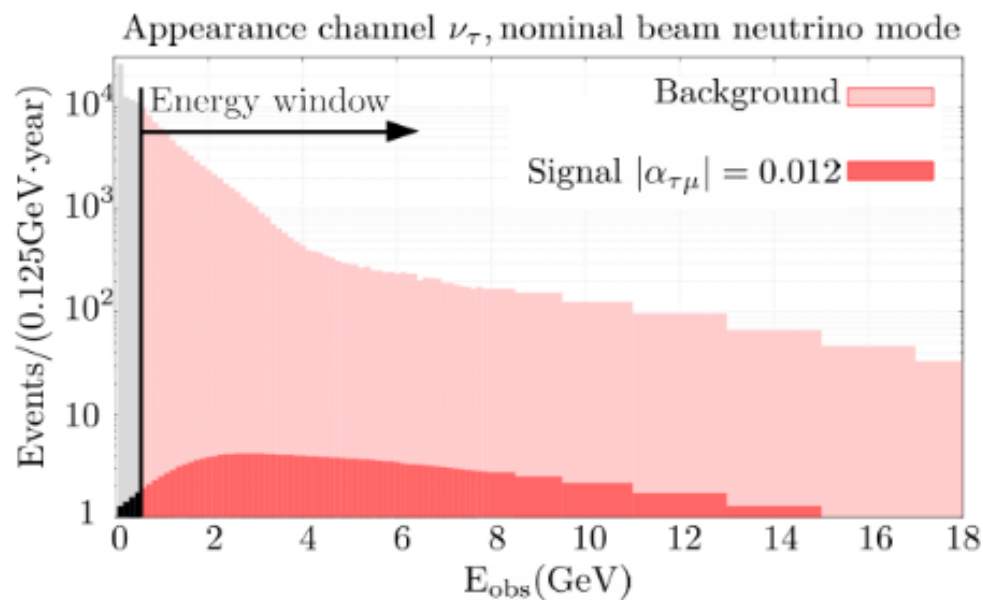
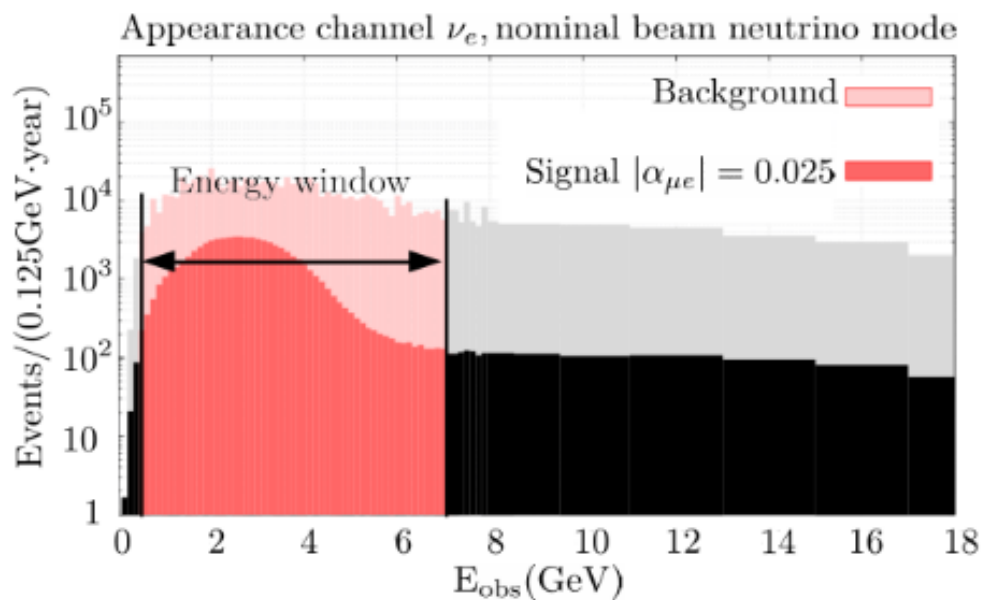
DUNE set up

Globes files

DUNE Collaboration, arXiv:2103.04797 [hep] 8 Mar 2021.

Flux configuration

Beam configuration	Power	E_p	PoT/yr	t_ν (yr)	$t_{\bar{\nu}}$ (yr)	M_{det}
Nominal	1.2 MW	120 GeV	1.1×10^{21}	3.5	3.5	67.2 tons
High-Energy	1.2 MW	120 GeV	1.1×10^{21}	3.5	–	67.2 tons



Running mode	Sample	Contribution	Event rates ($\times 10^5$)	$E_{\text{obs}}^{\text{max}}$ (GeV)
ν mode (nominal)	ν_e -like	Intrinsic cont.	20.18	7.125
		Flavor mis-ID	4.61	
		NC	6.77	
	ν_μ -like	$\nu_\mu, \bar{\nu}_\mu$ CC ($P_{\mu\mu} = 1$)	2,235.72	7.125
		NC	17.35	
	ν_τ -like	$\nu_\tau, \bar{\nu}_\tau$ CC ($P_{\mu\tau} = 1$)	39.33	18
NC		3.23		
$\bar{\nu}$ mode (nominal)	$\bar{\nu}_e$ -like	Intrinsic cont.	11.18	7.125
		Flavor mis-ID	1.07	
		NC	3.89	
	$\bar{\nu}_\mu$ -like	$\nu_\mu, \bar{\nu}_\mu$ CC ($P_{\mu\mu} = 1$)	1,013.42	7.125
		NC	9.76	
	$\bar{\nu}_\tau$ -like	$\nu_\tau, \bar{\nu}_\tau$ CC ($P_{\mu\tau} = 1$)	27.75	18
NC		1.80		
ν mode (HE)	ν_e -like	Intrinsic cont.	38.10	18
		Flavor mis-ID	12.98	
		NC	30.51	
	ν_μ -like	$\nu_\mu, \bar{\nu}_\mu$ CC ($P_{\mu\mu} = 1$)	5,784.30	18
		NC	72.15	
	ν_τ -like	$\nu_\tau, \bar{\nu}_\tau$ CC ($P_{\mu\tau} = 1$)	259.67	18
NC		9.42		

Event sample	Contribution	Benchmark 1		Benchmark 2		Benchmark 3	
		σ_{norm}	σ_{shape}	σ_{norm}	σ_{shape}	σ_{norm}	σ_{shape}
ν_e -like	Signal	5%	—	5%	—	5%	—
	Intrinsic cont.	10%	—	10%	2%	10%	5%
	Flavor mis-ID	5%	—	5%	2%	5%	5%
	NC	10%	—	10%	2%	10%	5%
ν_μ -like	$\nu_\mu, \bar{\nu}_\mu$ CC (signal)	10%	—	10%	2%	10%	5%
	NC	10%	—	10%	2%	10%	5%
ν_τ -like	Signal	20%	—	20%	—	20%	—
	NC	10%	—	10%	2%	10%	5%

$$\chi_{\min}^2(\{\Theta\}) = \min_{\{\xi, \zeta\}} \left[\chi_{\text{stat}}^2(\{\Theta, \xi, \zeta\}) + \sum_s \left(\frac{\zeta_s}{\sigma_{\text{norm},s}} \right)^2 + \sum_b \left(\frac{\zeta_b}{\sigma_{\text{norm},b}} \right)^2 + \sum_i \left(\frac{\xi_i^{\text{sig}}}{\sigma_{\text{shape},\text{sig}}} \right)^2 + \sum_i \left(\frac{\xi_i^{\text{bg}}}{\sigma_{\text{shape},\text{bg}}} \right)^2 \right],$$

$$\chi_{\text{stat}}^2(\{\Theta, \xi, \zeta\}) = \sum_i 2 \left(N_i(\{\Theta, \xi, \zeta\}) - O_i + O_i \ln \frac{O_i}{N_i(\{\Theta, \xi, \zeta\})} \right)$$

$$N_i(\{\Theta, \xi, \zeta\}) = \sum_s (1 + \xi_i^{\text{sig}} + \zeta_s) s_i(\{\Theta\}) + \sum_b (1 + \xi_i^{\text{bg}} + \zeta_b) b_i(\{\Theta\})$$

Present Bounds (update)

2σ

90% C.L.

	Non-Unitarity ($M > \text{EW}$)	Non-Unitarity ($M \gtrsim 1 \text{ eV}$)
α_{ee}	$1.3 \cdot 10^{-3}$	$8.4 \cdot 10^{-3}$ (SOLAR)
$\alpha_{\mu\mu}$	$2.2 \cdot 10^{-4}$	$5.0 \cdot 10^{-3}$ (MINOS)
$\alpha_{\tau\tau}$	$2.8 \cdot 10^{-3}$	$6.5 \cdot 10^{-2}$ (ATM)
$ \alpha_{\mu e} $	$6.8 \cdot 10^{-4}$ ($2.4 \cdot 10^{-5}$)	$9.2 \cdot 10^{-3}$ $1.4 \cdot 10^{-2}$ $1.1 \cdot 10^{-2}$
$ \alpha_{\tau e} $	$2.7 \cdot 10^{-3}$	
$ \alpha_{\tau\mu} $	$1.2 \cdot 10^{-3}$	

Fernandez-Martinez, Hernandez-Garcia, JLP
 1605.08774
 Blennow, Coloma, Fernandez-Martinez,
 Hernandez-Garcia, JLP 1609.08637
 Argüelles et al, 2203.10811

$$\alpha_{\alpha\beta} \leq 2\sqrt{\alpha_{\alpha\alpha}\alpha_{\beta\beta}}$$