Holography in anomaly flow in orbifold gauge theory

Yutaka Hosotani, Osaka Univ

Universality in anomaly flow
by Y. Hosotani
PTEP 2022, 073B01 (2205.00154)

Anomaly flow by an Aharonov-Bohm phase
by Funatsu, Hatanaka, Hosotani, Orikasa, Yamatsu
PTEP 2022, 043B04 (2202.01393)

Corfu Summer Institute 2022
Workshop on Standard Model and Beyond
Corfu, Greece, 7 September 2022

Introduction

Chiral fermion ⟺ Chiral anomaly

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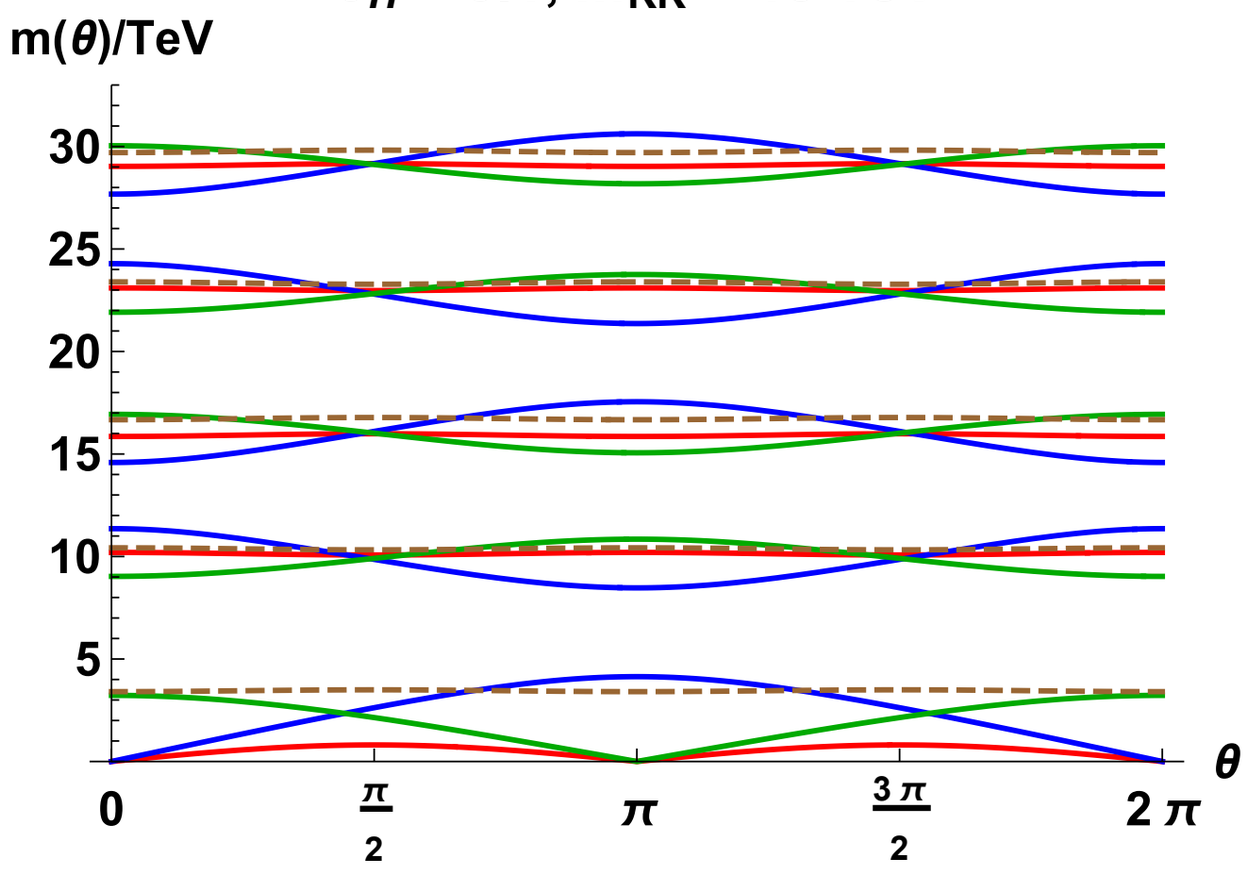

GUT inspired
$SU(3) \times SO(5) \times U(1)$
GHU in RS

PRD.104.115018 (2021)

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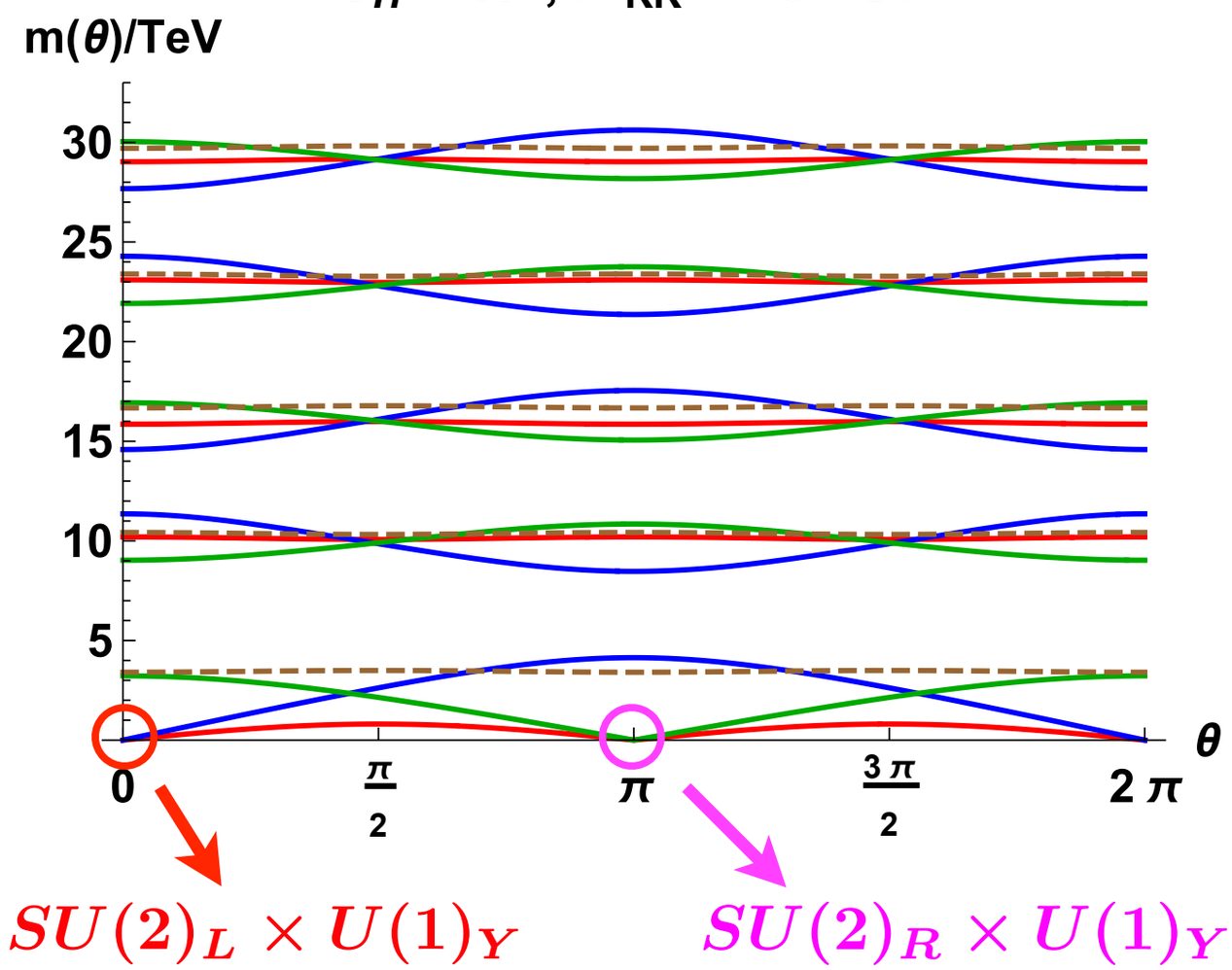

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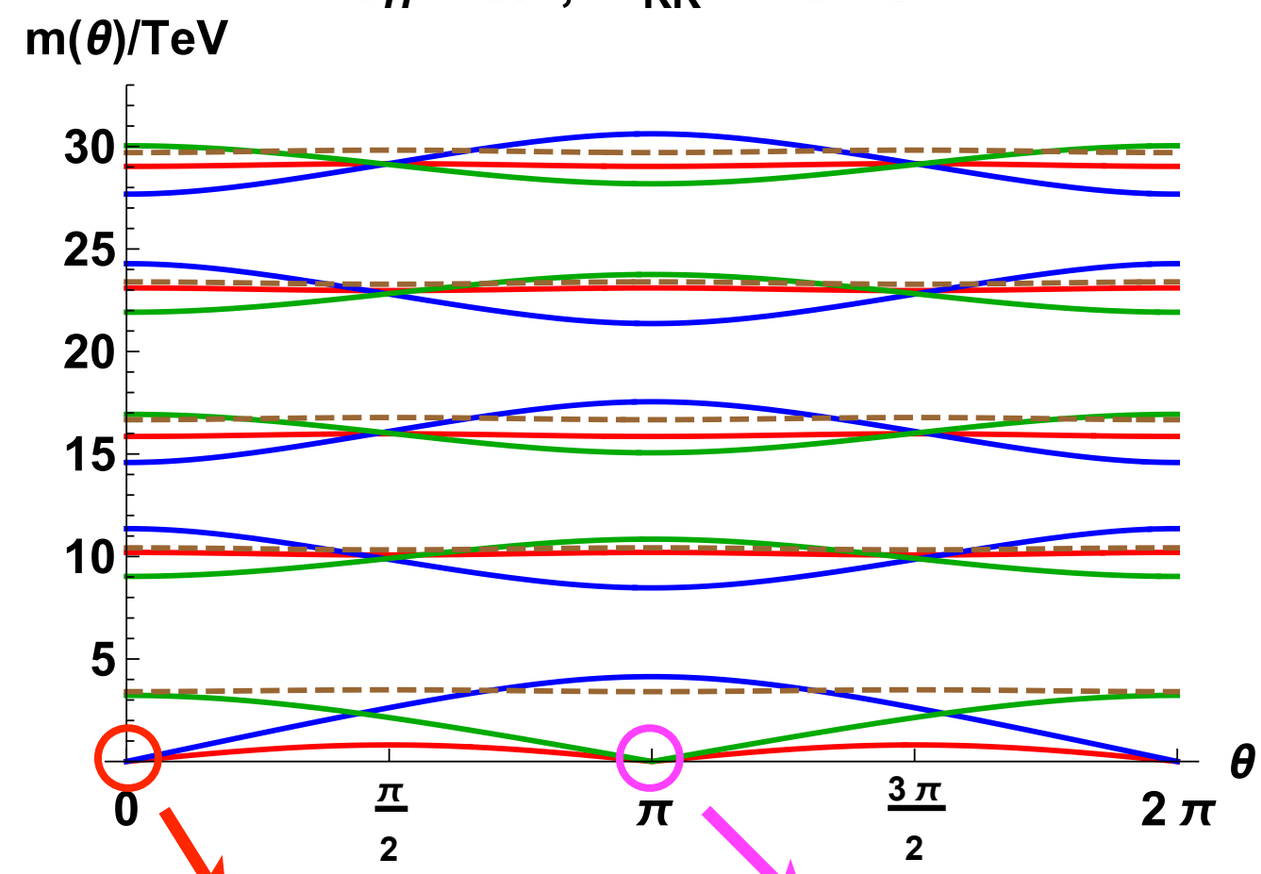

top: chiral ⟶ vectorlike

GUT inspired $SU(3) \times SO(5) \times U(1)$ GHU in RS

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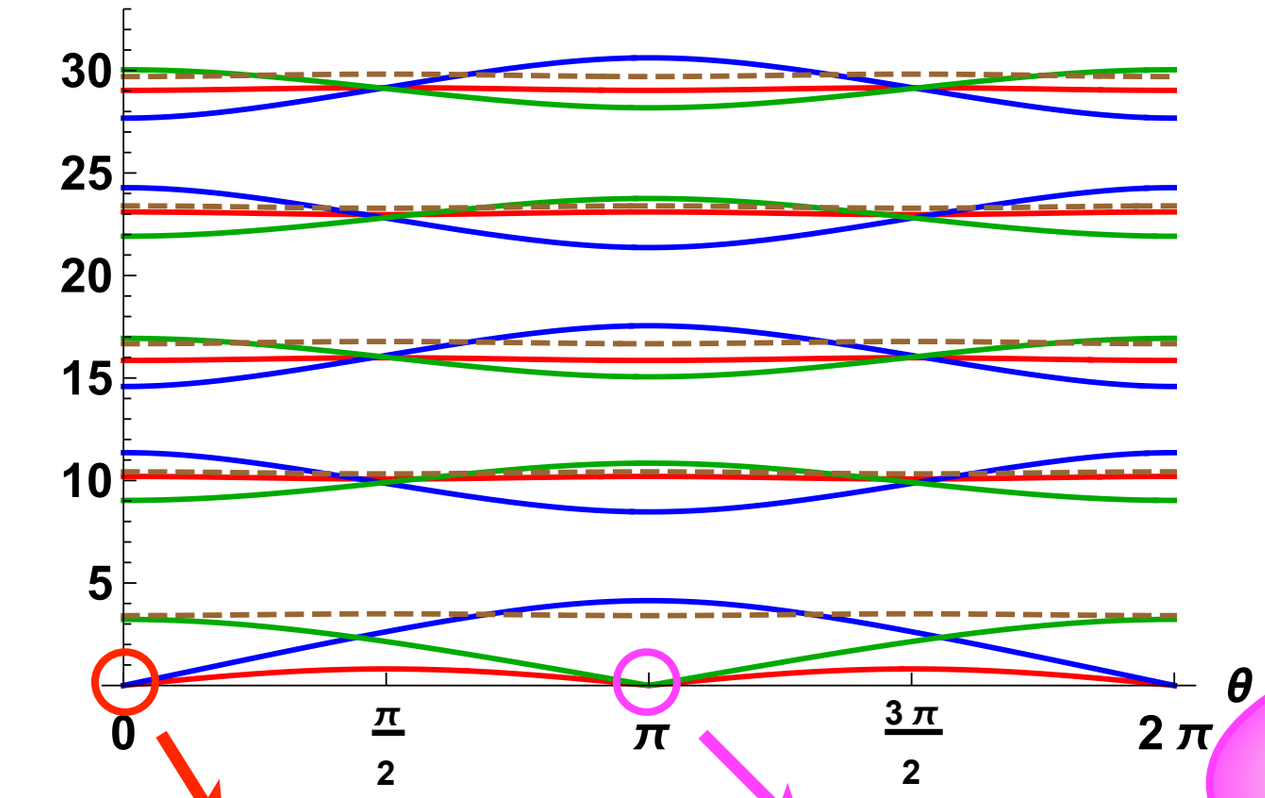

GUT inspired $SU(3) \times SO(5) \times U(1)$ GHU in RS

PRD.104.115018 (2021)

top: chiral ⟶ vectorlike

What is the fate of chiral anomaly?

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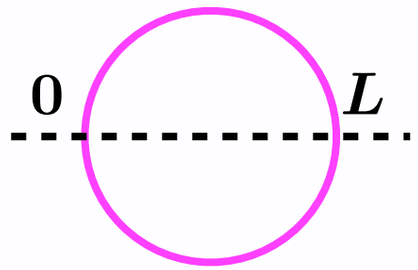
$SU(2)$ GHU in $M^4 \times (S^1/Z_2)$

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix}(x, y_j - y) = P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}(x, y_j + y) P_j^{-1}$$

$$\Psi(x, y_j - y) = P_j \gamma^5 \Psi(x, y_j + y)$$

$$P_0 = P_1 = \tau^3 \qquad SU(2) \Rightarrow U(1)$$



$$(y_0, y_1) = (0, L)$$

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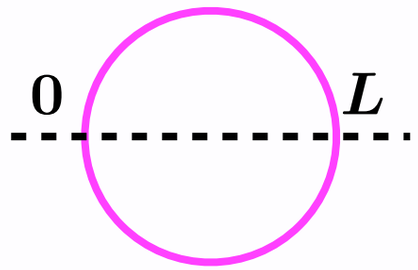

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AB phase θ_H

$$P e^{i g_A \int_0^{2L} dy \langle A_y \rangle} = e^{i \theta_H \tau^2}$$

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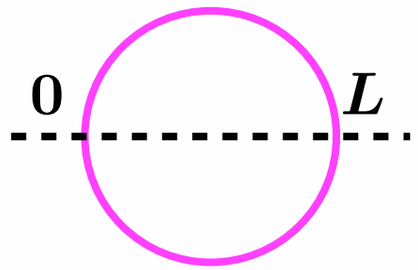

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$$\begin{pmatrix} A_\mu^1 \\ A_\mu^3 \end{pmatrix} = \sum_{n=-\infty}^{\infty} B_\mu^{(n)}(x) \frac{1}{\sqrt{\pi R}} \begin{pmatrix} \sin \frac{ny}{R} \\ \cos \frac{ny}{R} \end{pmatrix}$$

$$m_{B^{(n)}} = \frac{1}{R} \left| n + \frac{\theta_H}{\pi} \right|$$

Massless mode

$\theta_H = 0$	$B_\mu^{(0)}$
π	$B_\mu^{(-1)}$
2π	$B_\mu^{(-2)}$

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Spectrum

$$\begin{pmatrix} u_R \\ d_R \end{pmatrix} = \sum_n \psi_R^{(n)}(x) \frac{1}{\sqrt{\pi R}} \begin{pmatrix} \cos \frac{ny}{R} \\ \sin \frac{ny}{R} \end{pmatrix}$$

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$$m_{\psi^{(n)}} = \frac{1}{R} \left| n + \frac{\theta_H}{2\pi} \right|$$

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$$\theta_H = 0 : \quad u_R \, , \; d_L$$

chiral

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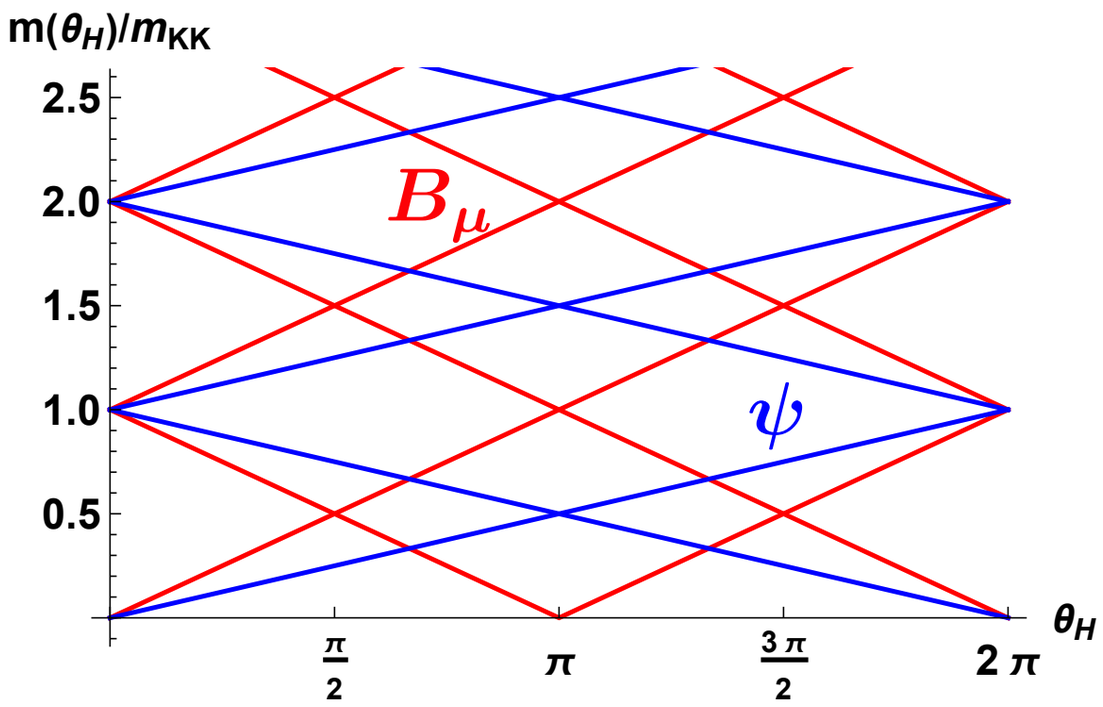
chiral

Spectrum in flat space



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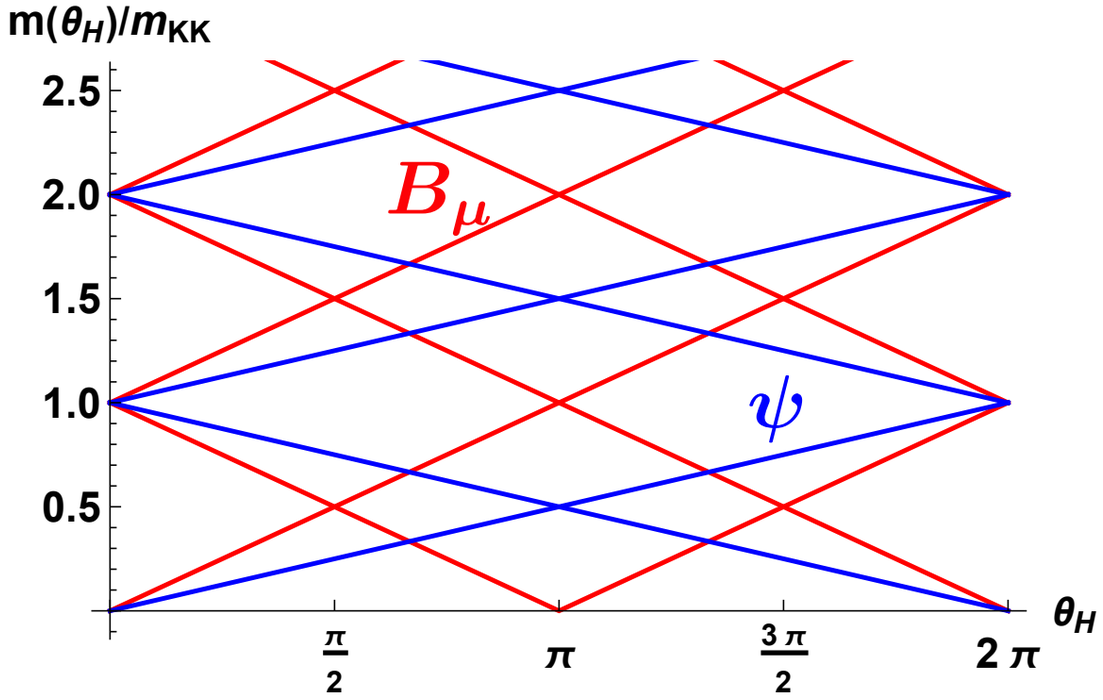
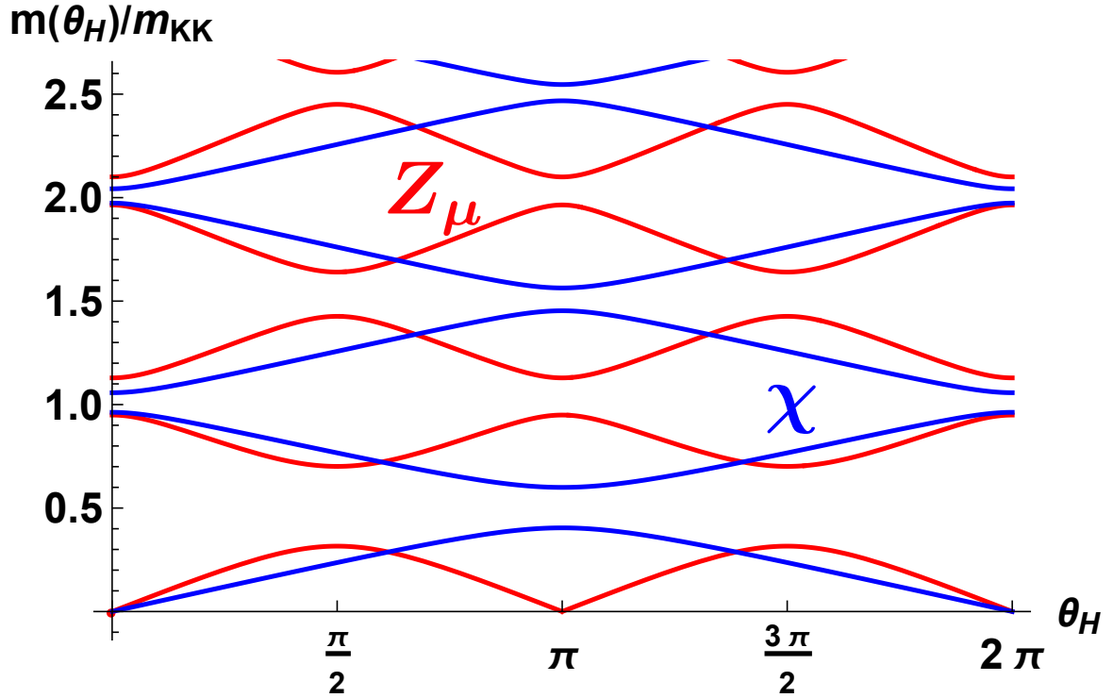


Spectrum in RS, z_L=10



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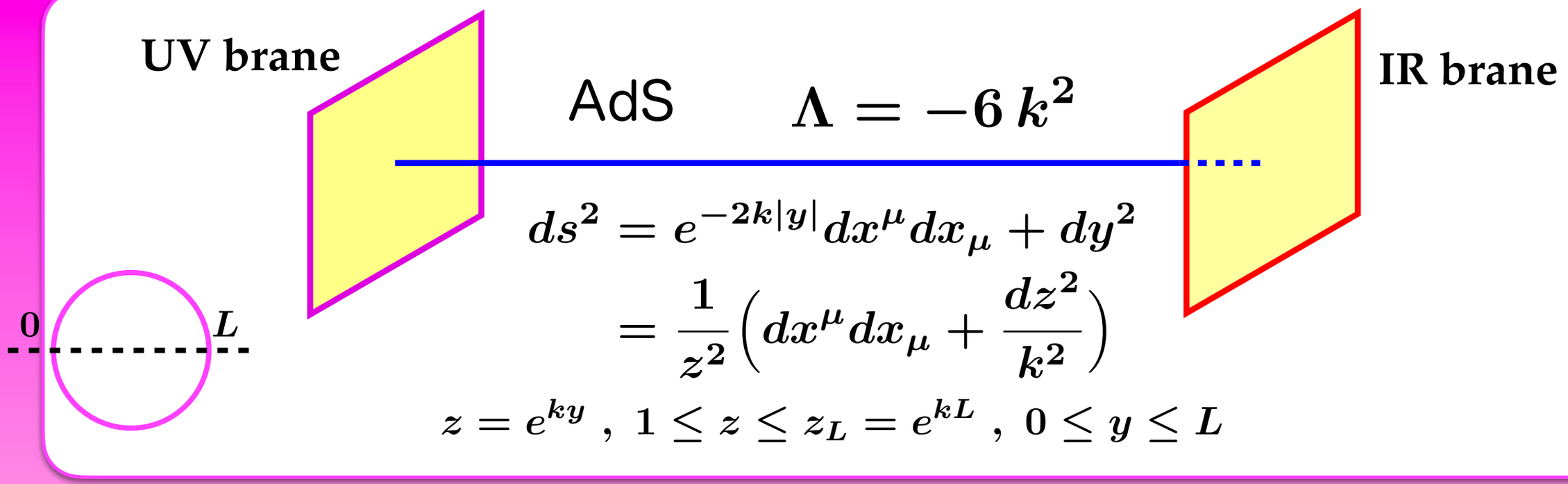
$SU(2)$ GHU in Randall-Sundrum



$$ds^2 = e^{-2k|y|} dx^\mu dx_\mu + dy^2$$

$$= \frac{1}{z^2}\left(dx^\mu dx_\mu + \frac{dz^2}{k^2}\right)$$

$$z = e^{ky}\ ,\ 1 \le z \le z_L = e^{kL}\ ,\ 0 \le y \le L$$

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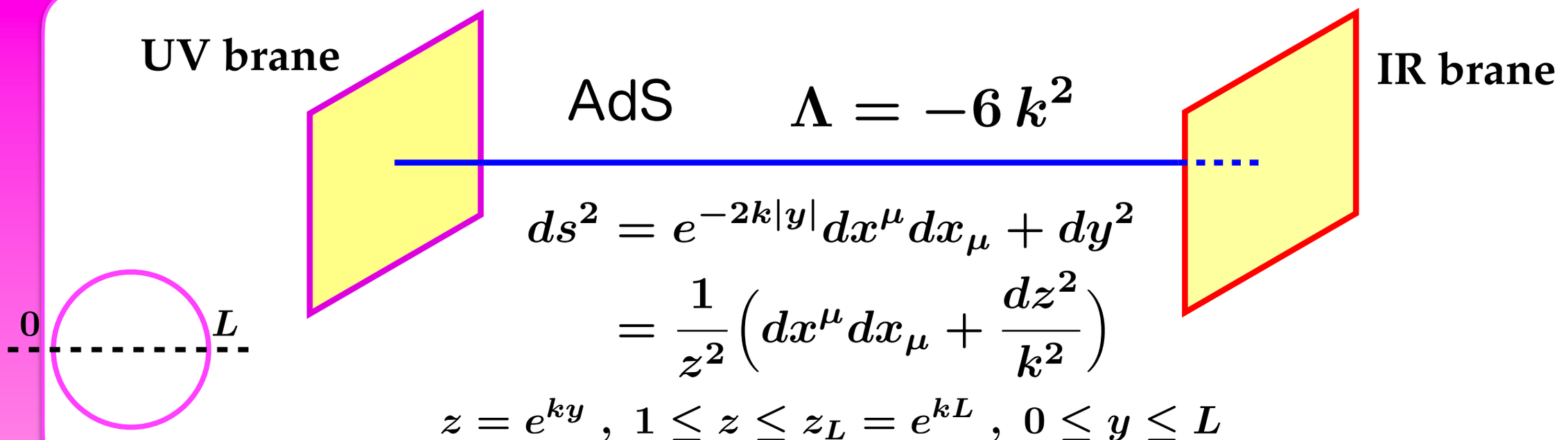

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$$\begin{pmatrix} A_\mu^1 \\ A_\mu^3 \end{pmatrix} = \frac{1}{\sqrt{L}} \sum_{n=0}^{\infty} Z_\mu^{(n)}(x) \begin{pmatrix} h_n(y) \\ k_n(y) \end{pmatrix}$$

$$\check{\Psi} = \frac{1}{z^4}\Psi$$

$$\begin{pmatrix} \check{u}_R \\ \check{d}_R \end{pmatrix} = \sqrt{k} \sum_{n=0}^{\infty} \chi_R^{(n)}(x) \begin{pmatrix} f_{Rn}(y) \\ g_{Rn}(y) \end{pmatrix}$$

$$\begin{pmatrix} \check{u}_L \\ \check{d}_L \end{pmatrix} = \sqrt{k} \sum_{n=0}^{\infty} \chi_L^{(n)}(x) \begin{pmatrix} f_{Ln}(y) \\ g_{Ln}(y) \end{pmatrix}$$

Spectrum

$$Z_\mu^{(n)}: \ SC'(1;\lambda_n) + \lambda_n \sin^2\theta_H = 0$$

$$C(z;\lambda) = \frac{\pi}{2}\lambda z z_L F_{1,0}(\lambda z, \lambda z_L)$$
$$S(z;\lambda) = -\frac{\pi}{2}\lambda z F_{1,1}(\lambda z, \lambda z_L)$$

$$F_{\alpha,\beta}(u,v) \equiv J_\alpha(u)Y_\beta(v) - Y_\alpha(u)J_\beta(v)$$

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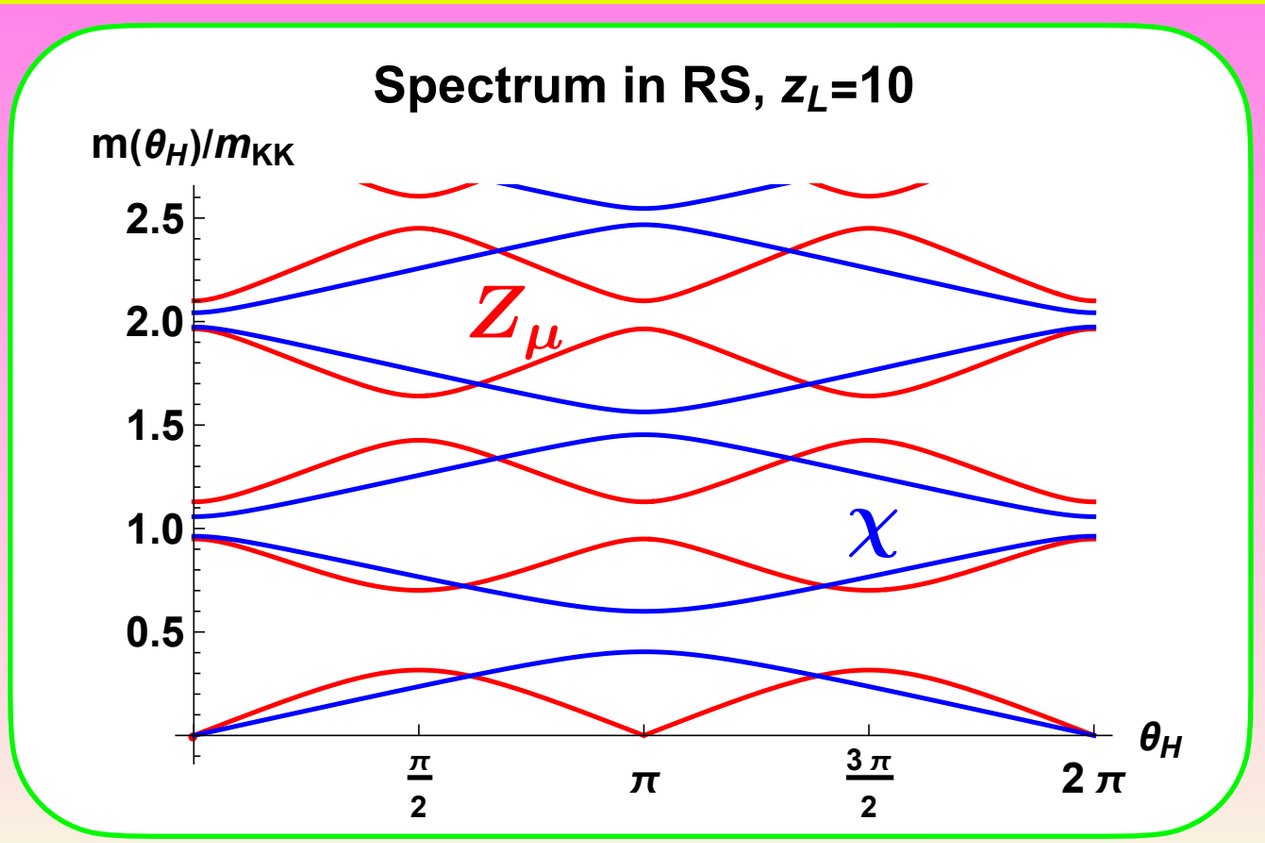

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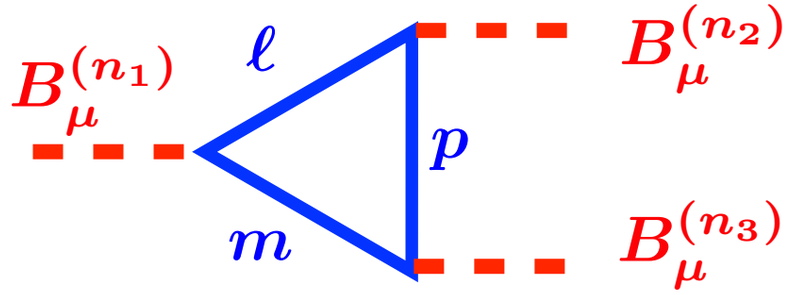
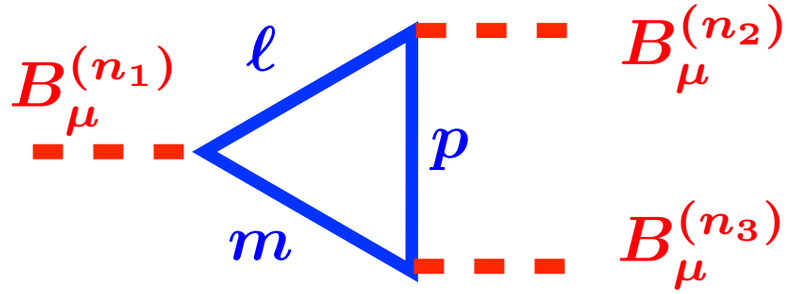
Gauge couplings and anomaly in flat space



$$\frac{g_4}{2}\sum_{n,m,\ell=-\infty}^{\infty} B_\mu^{(n)}\left\{ s^R_{nm\ell}\,\psi_R^{(m)\dagger}\bar{\sigma}^\mu\psi_R^{(\ell)} + s^L_{nm\ell}\,\psi_L^{(m)\dagger}\sigma^\mu\psi_L^{(\ell)} \right\}$$

$\delta_{n,m+\ell}$ $\qquad$ $\delta_{n,m+\ell}$

Gauge couplings and anomaly in flat space



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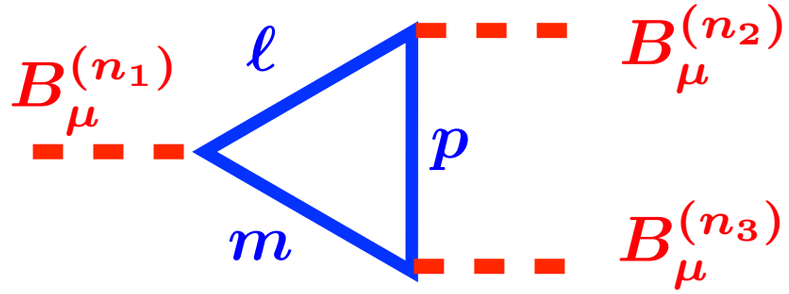
Anomaly coefficient

$$\partial_\mu j^\mu_{(n)} \Rightarrow -\left(\frac{g_4}{2}\right)^3 \sum_\ell \sum_m \frac{b_{n\ell m}}{16\pi^2}\, B^{(\ell)}_{\mu\nu}\widetilde{B}^{(m)\mu\nu}$$

$$b_{n_1 n_2 n_3} = \sum_{m,\ell,p}\left\{ s^R_{n_1 m\ell} s^R_{n_2 \ell p} s^R_{n_3 p m} + s^L_{n_1 m\ell} s^L_{n_2 \ell p} s^L_{n_3 p m}\right\}$$

Gauge couplings and anomaly in flat space



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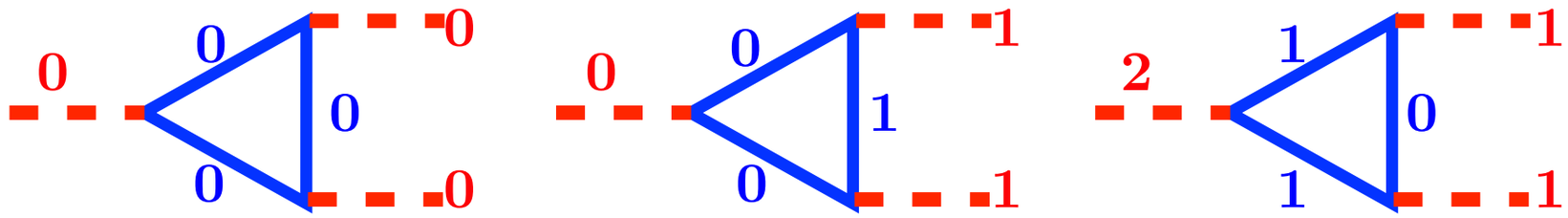
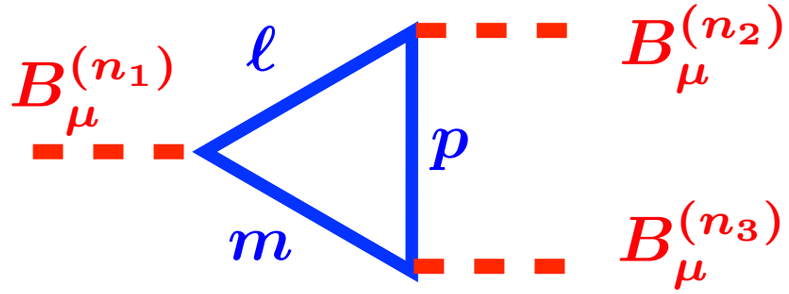
$$= \begin{cases} 2 \\ 0 \end{cases} \text{ for } n_1+n_2+n_3 = \begin{cases} \text{even} \\ \text{odd} \end{cases}$$



Gauge couplings and anomaly in flat space



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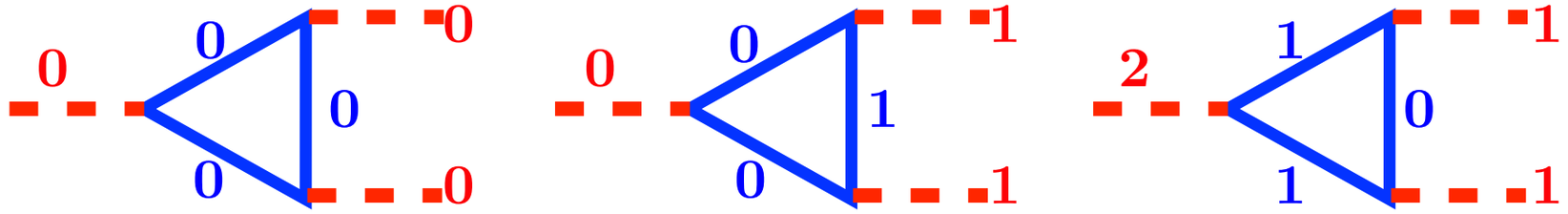
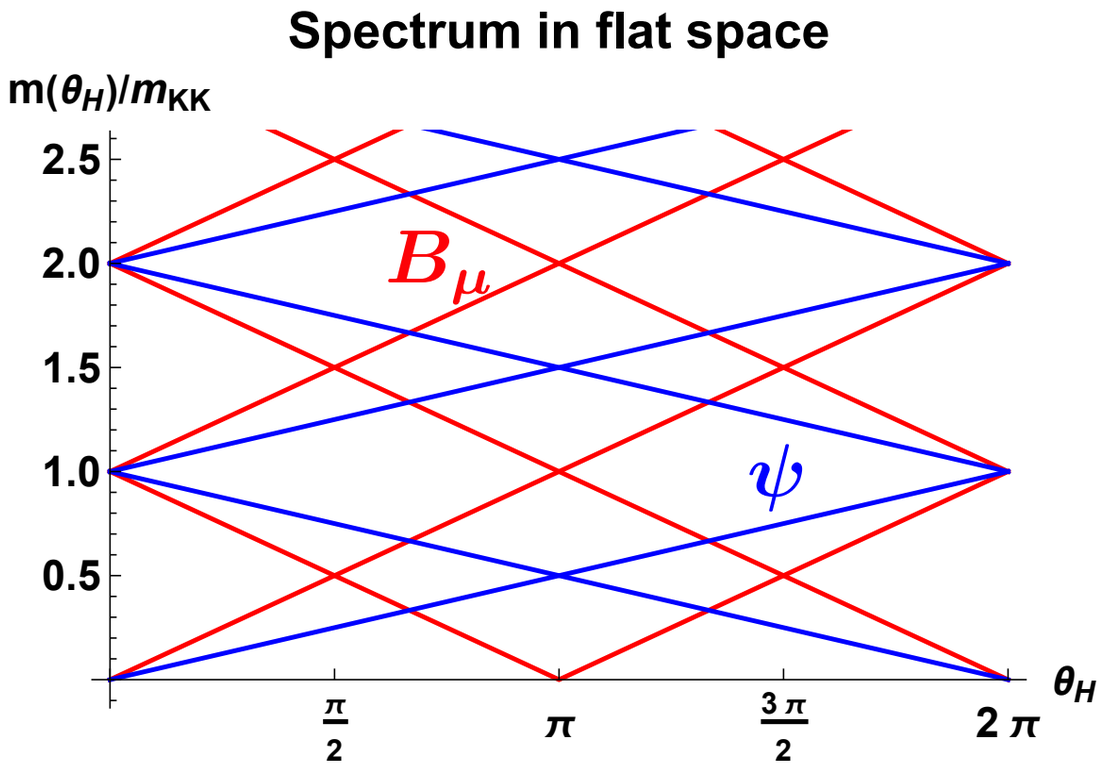
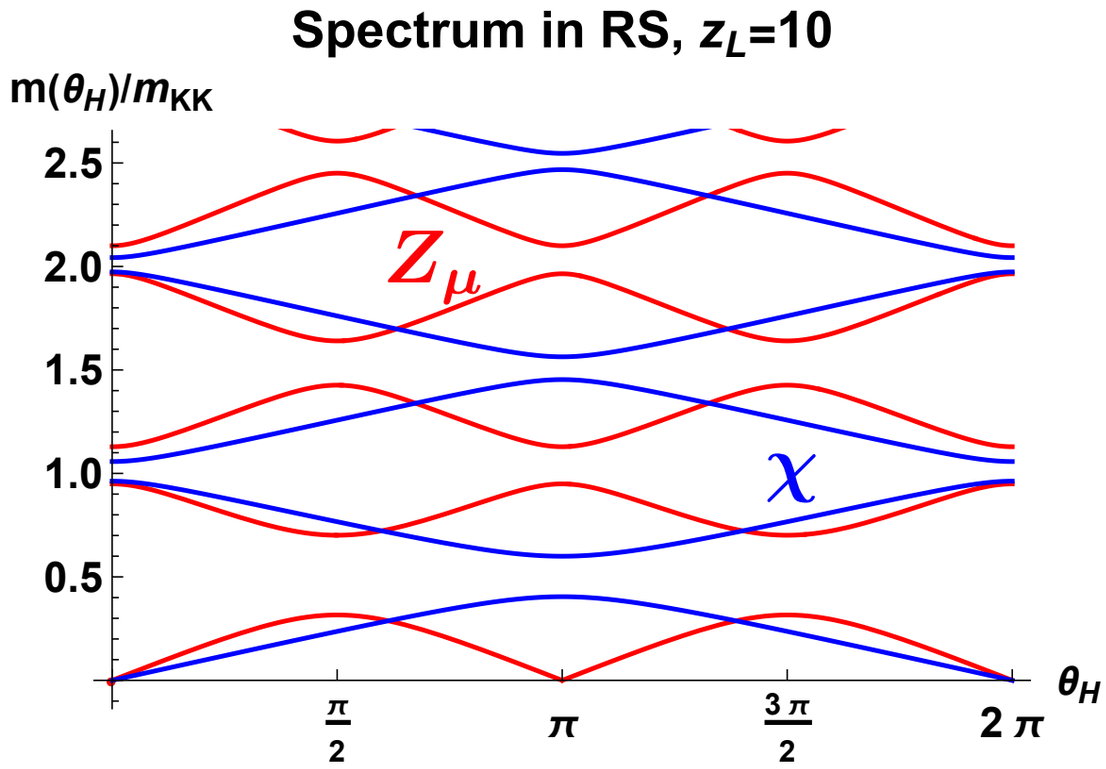
θ_H-independent ?



Anomaly flow





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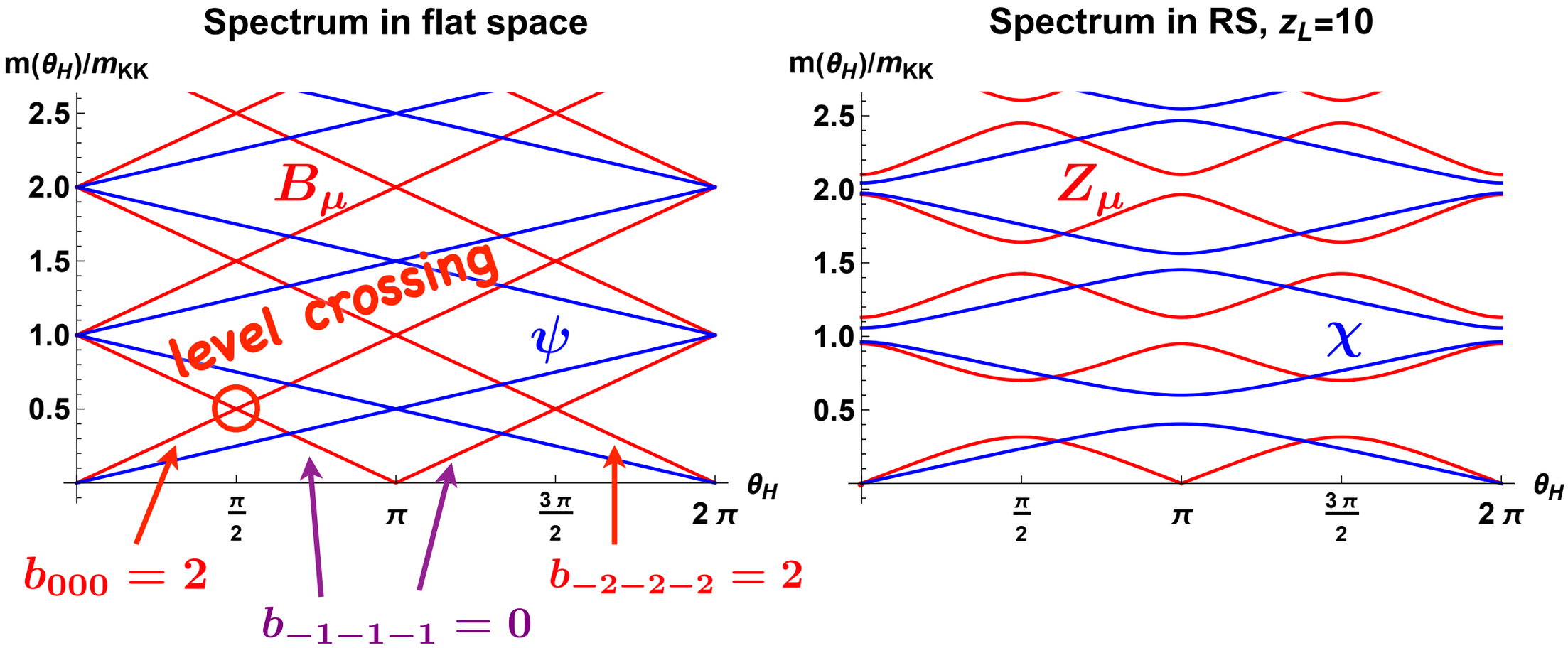
Anomaly flow



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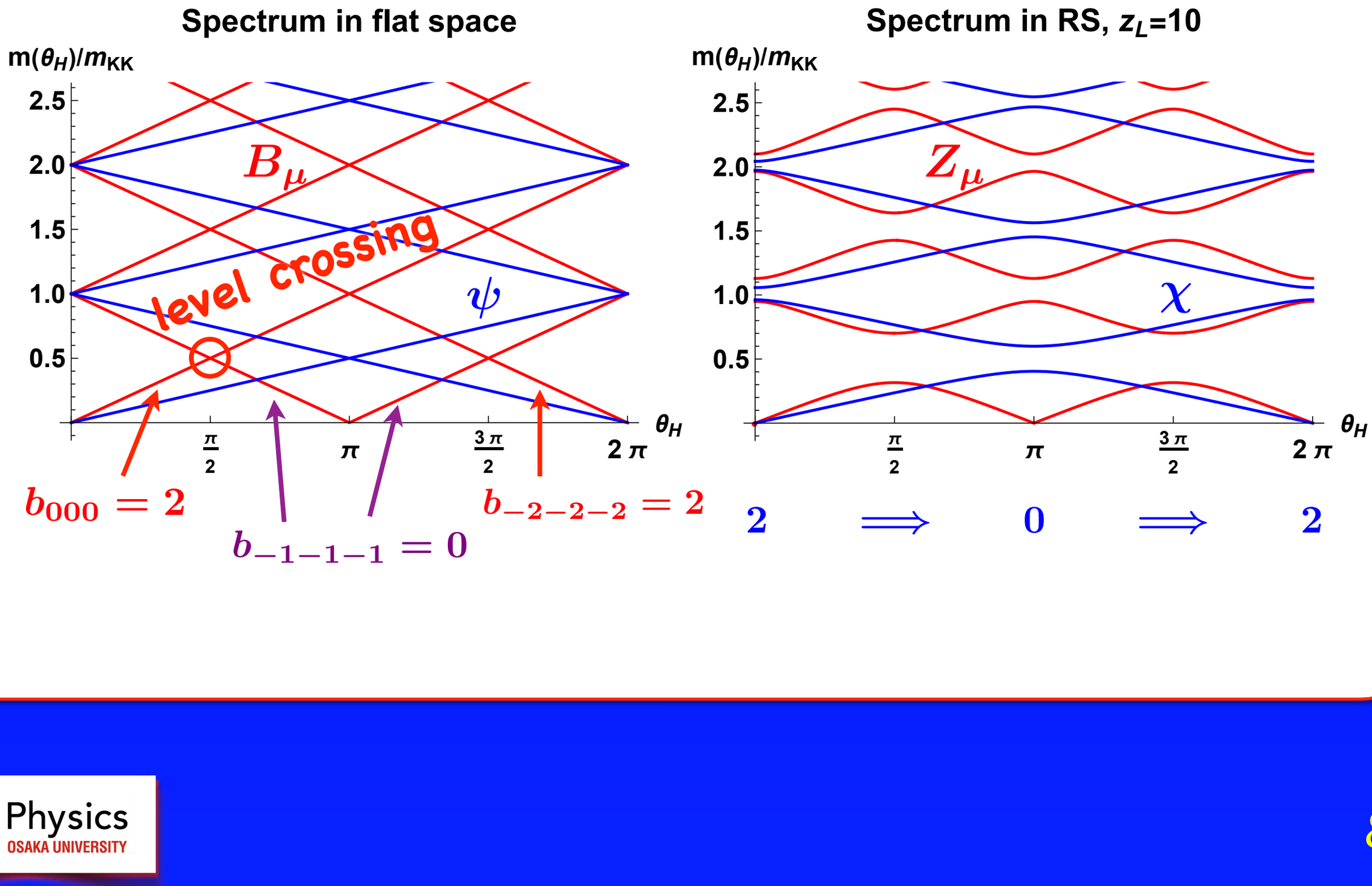

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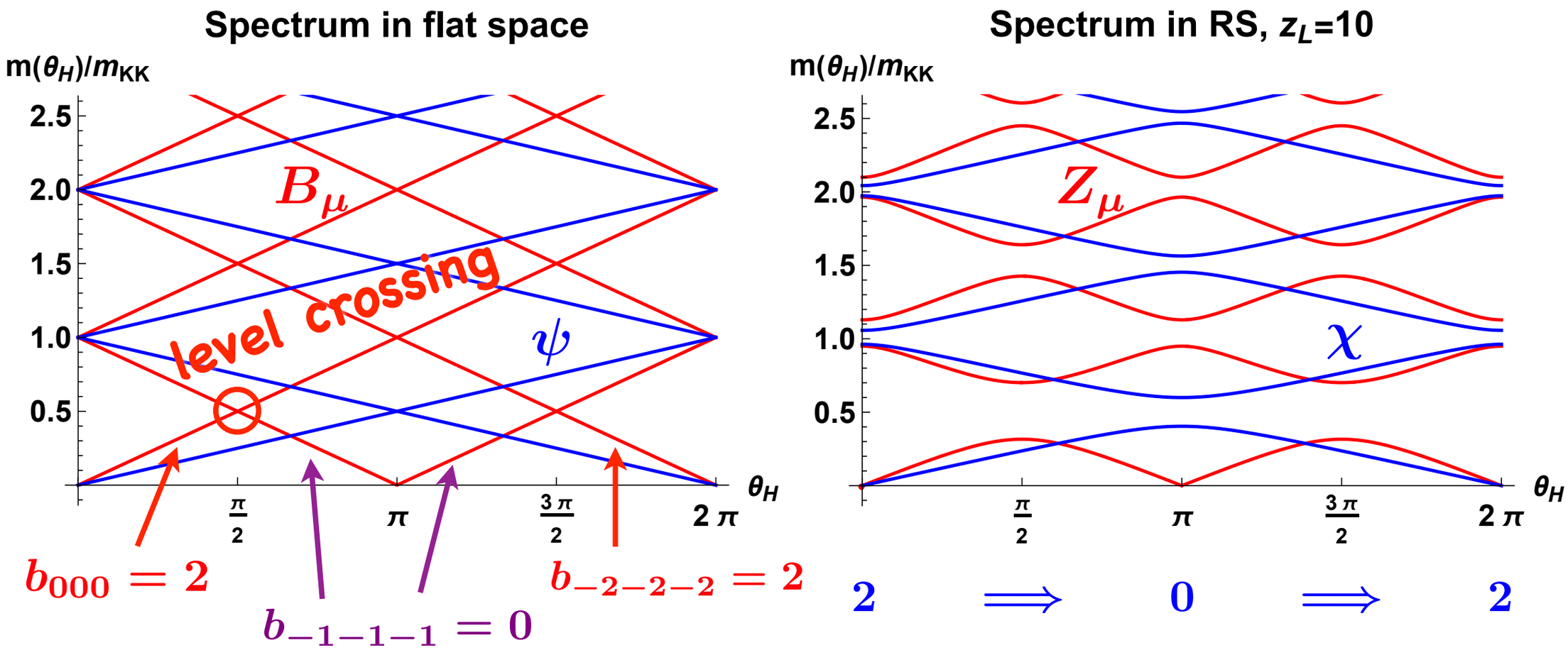

Anomaly must flow with θ_H !

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Couplings in RS

$$\frac{g_4}{2}\sum_{n,\ell,m=0}^{\infty} Z_\mu^{(n)}\left\{t^R_{n\ell m}\,\chi_R^{(\ell)\dagger}\bar\sigma^\mu\chi_R^{(m)} + t^L_{n\ell m}\,\chi_L^{(\ell)\dagger}\sigma^\mu\chi_L^{(m)}\right\}$$

$$t^R_{n\ell m} = \frac{k}{2}\int_{-L}^{L} dy\, e^{k|y|}\left\{h_n\left(f^*_{R\ell}g_{Rm} + g^*_{R\ell}f_{Rm}\right) + k_n\left(f^*_{R\ell}f_{Rm} - g^*_{R\ell}g_{Rm}\right)\right\}$$

$$t^L_{n\ell m} = -\frac{k}{2}\int_{-L}^{L} dy\, e^{k|y|}\left\{\ f_{Rm}\to f_{Lm}\ ,\ g_{Rm}\to g_{Lm}\ \right\}$$

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$$\frac{g_4}{2}\sum_{n,\ell,m=0}^{\infty} Z_\mu^{(n)}\left\{t^R_{n\ell m}\,\chi_R^{(\ell)\dagger}\bar\sigma^\mu\chi_R^{(m)} + t^L_{n\ell m}\,\chi_L^{(\ell)\dagger}\sigma^\mu\chi_L^{(m)}\right\}$$

$$t^R_{n\ell m} = \frac{k}{2}\int_{-L}^{L} dy\, e^{k|y|}\left\{h_n\left(f^*_{R\ell}g_{Rm} + g^*_{R\ell}f_{Rm}\right) + k_n\left(f^*_{R\ell}f_{Rm} - g^*_{R\ell}g_{Rm}\right)\right\}$$

$$t^L_{n\ell m} = -\frac{k}{2}\int_{-L}^{L} dy\, e^{k|y|}\left\{\ f_{Rm}\to f_{Lm}\ ,\ g_{Rm}\to g_{Lm}\ \right\}$$

Anomaly coefficient

$$a_{n_1n_2n_3} = \sum_{m,\ell,p}\left\{t^R_{n_1m\ell}t^R_{n_2\ell p}t^R_{n_3pm} + t^L_{n_1m\ell}t^L_{n_2\ell p}t^L_{n_3pm}\right\}$$

Note: $\quad t^{R/L}_{n\ell m} = t^{R/L}_{n\ell m}(\theta_H, z_L, c)$

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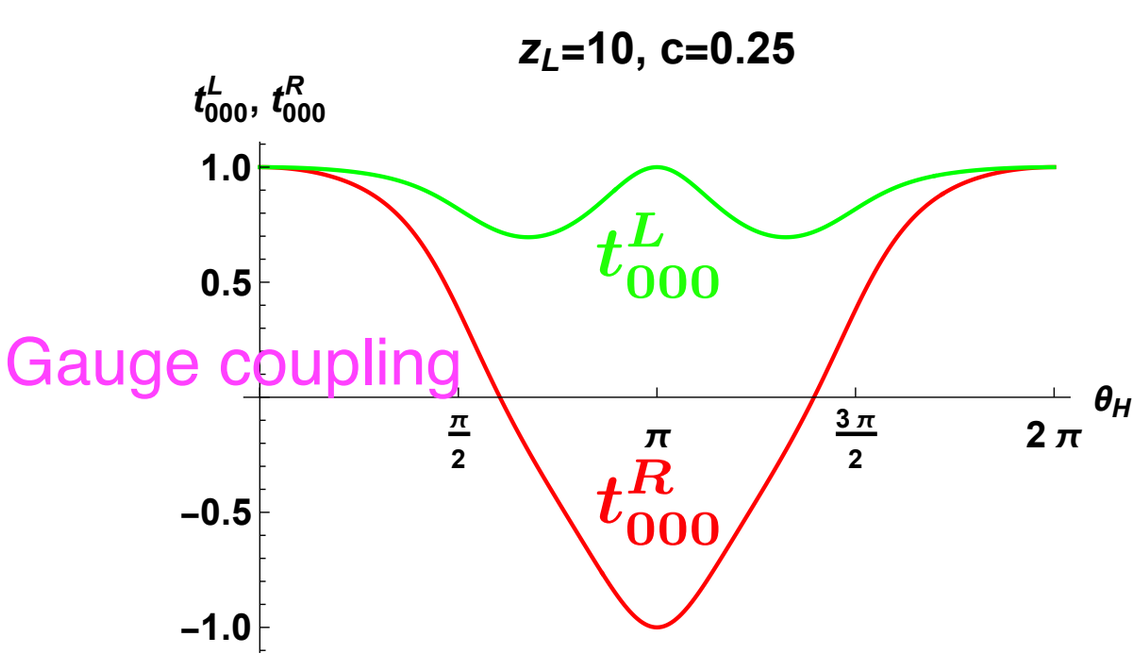
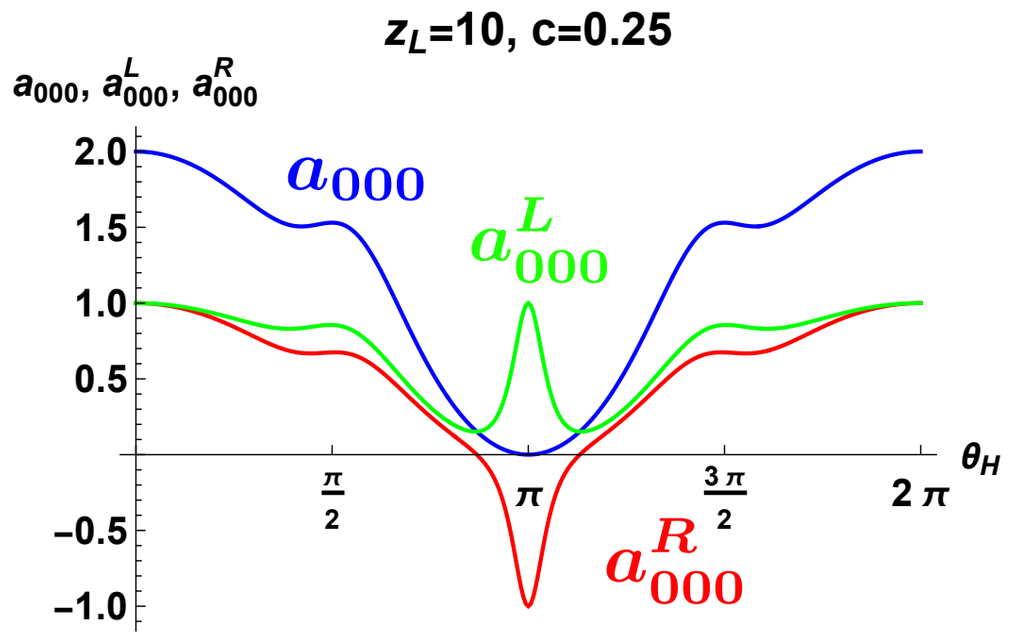
Anomaly flows !



Gauge coupling



Anomaly coefficient

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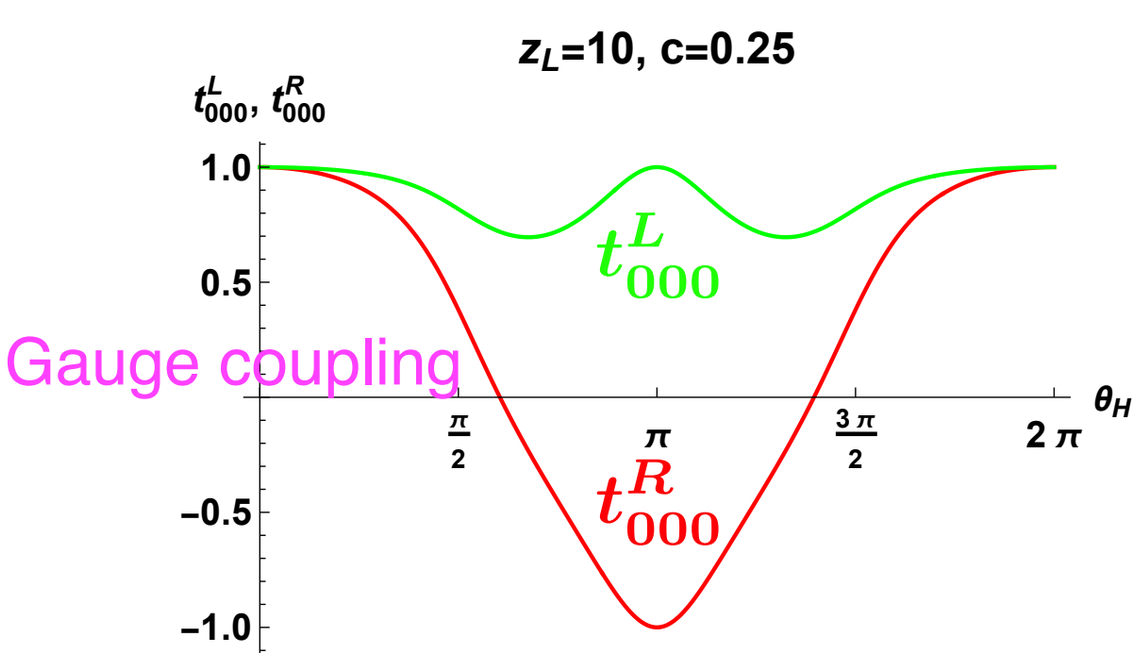

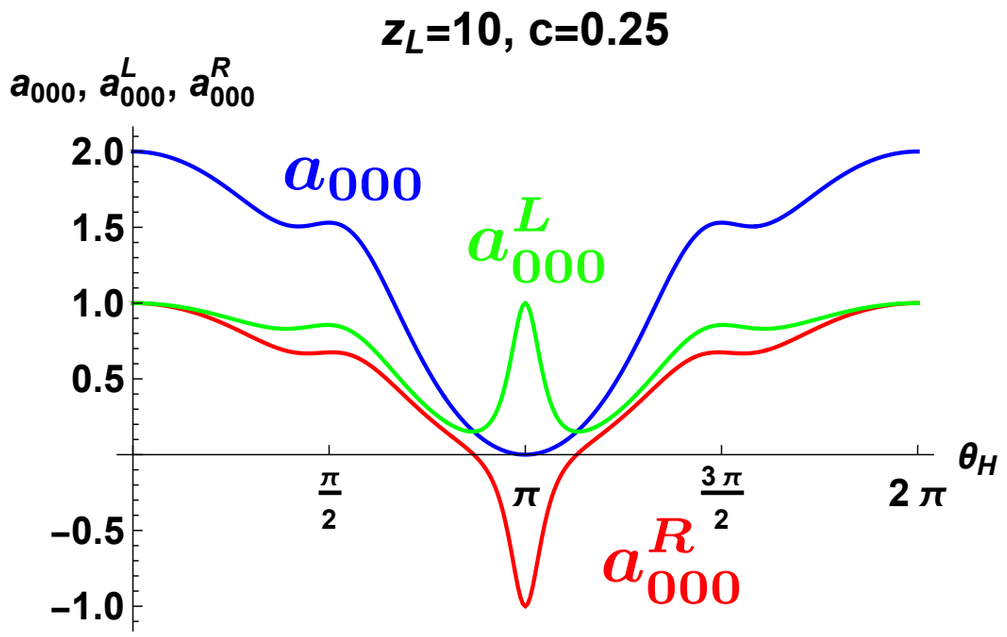

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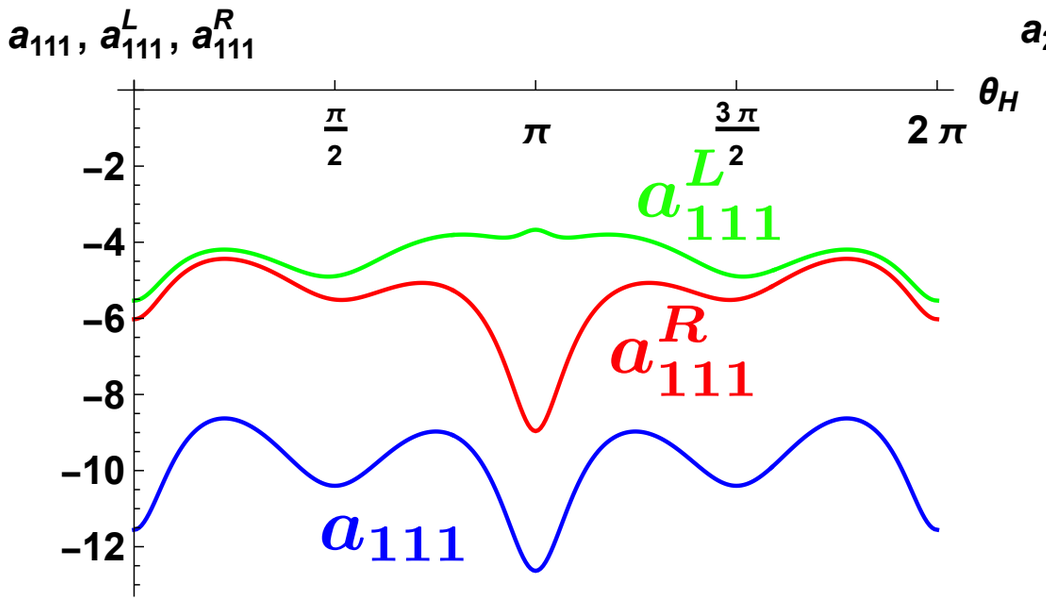

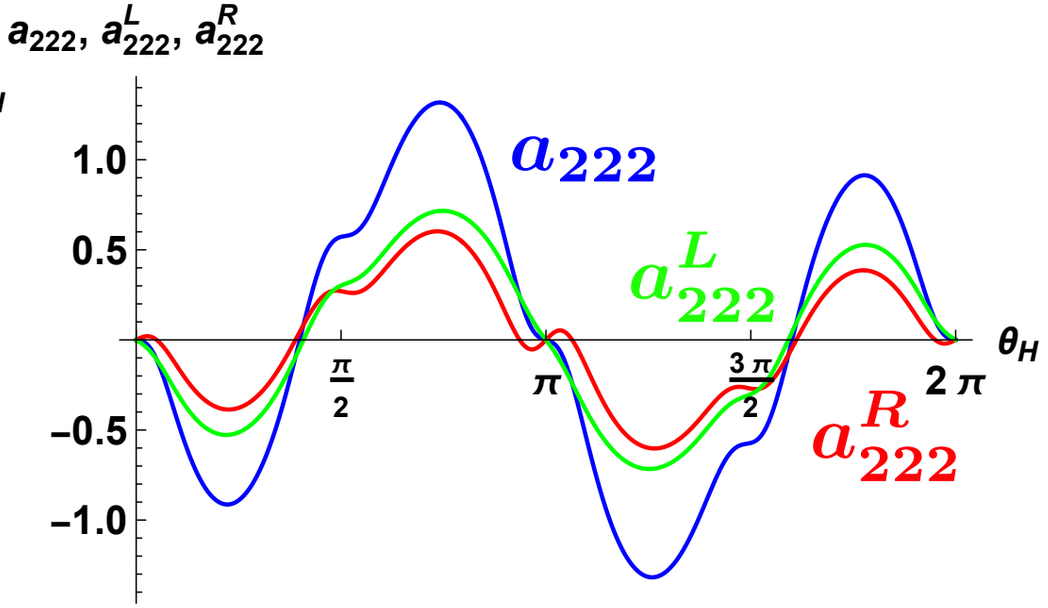

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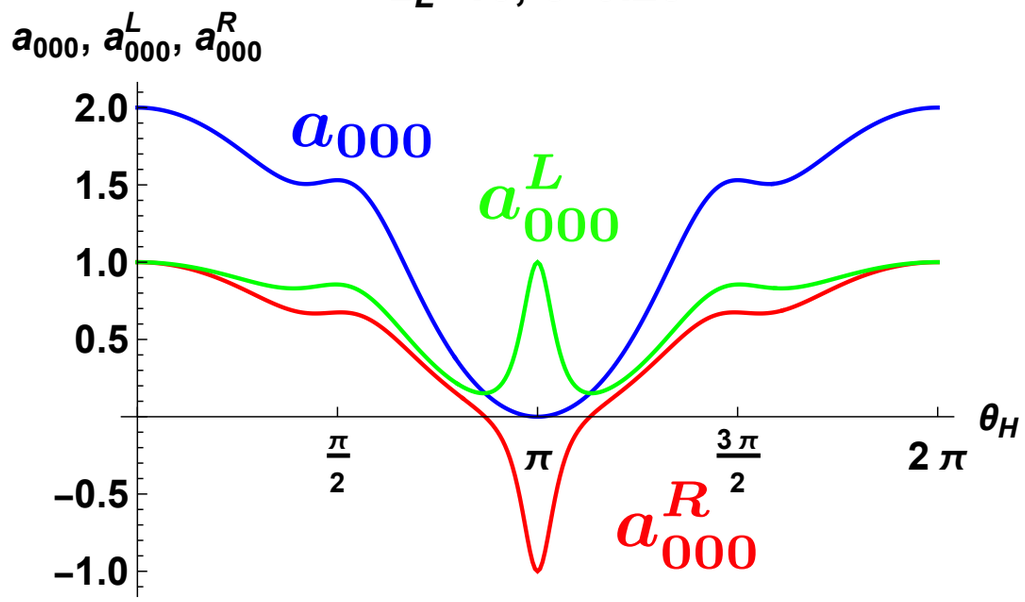
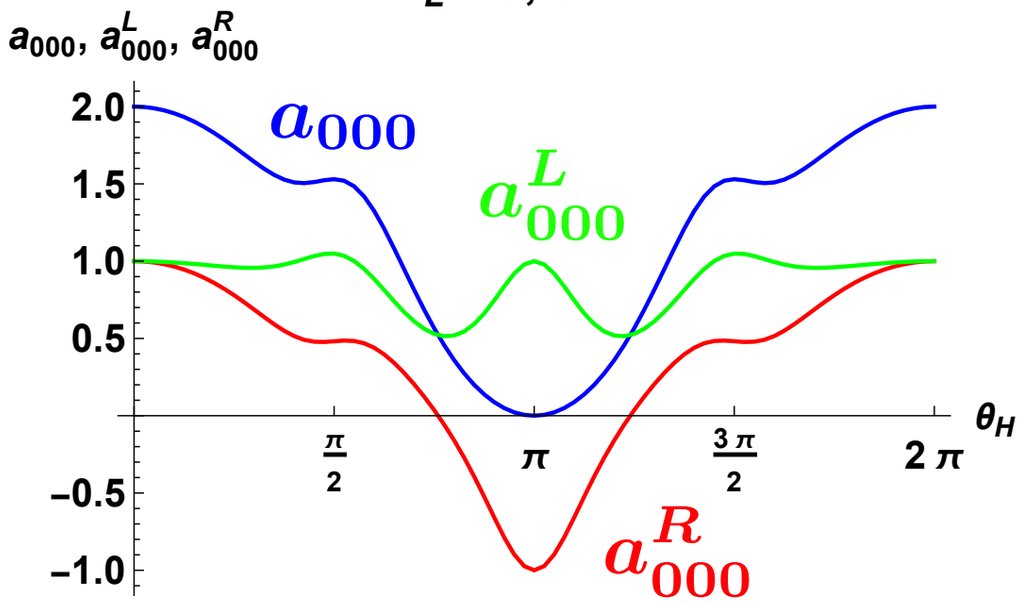
c-independence





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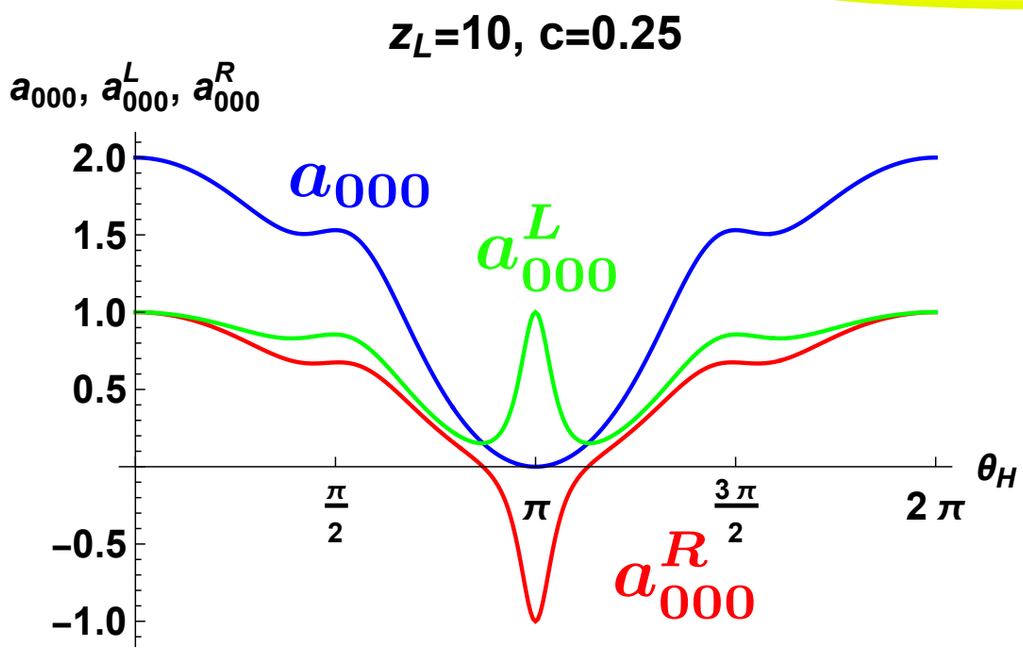

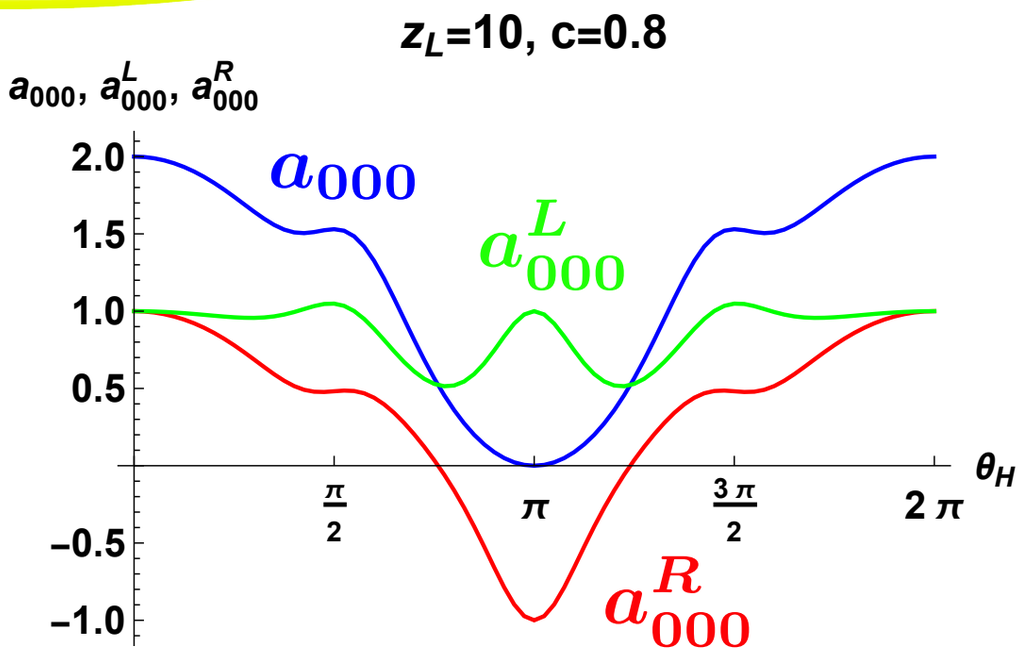

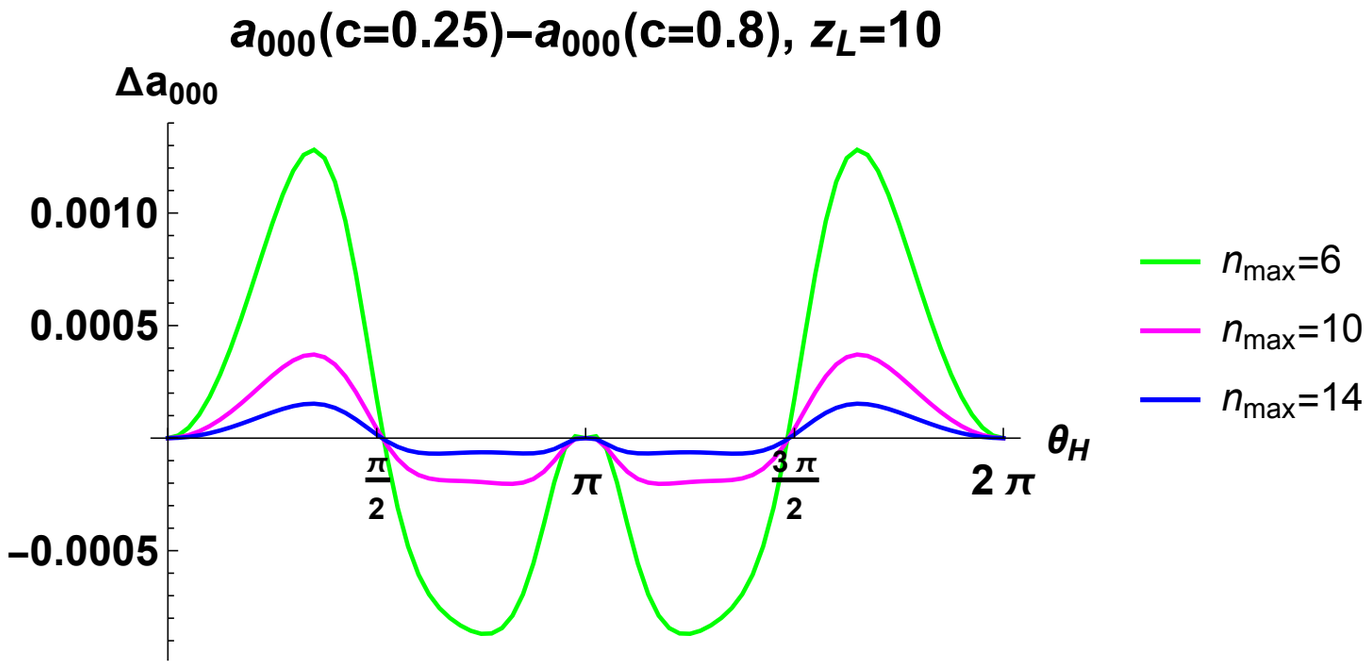

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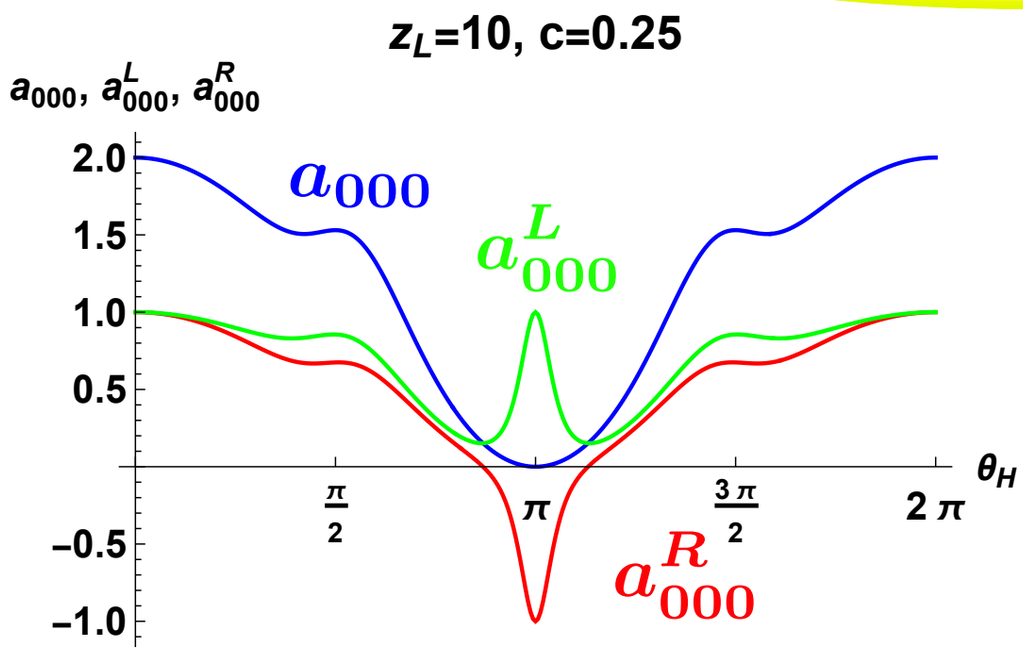
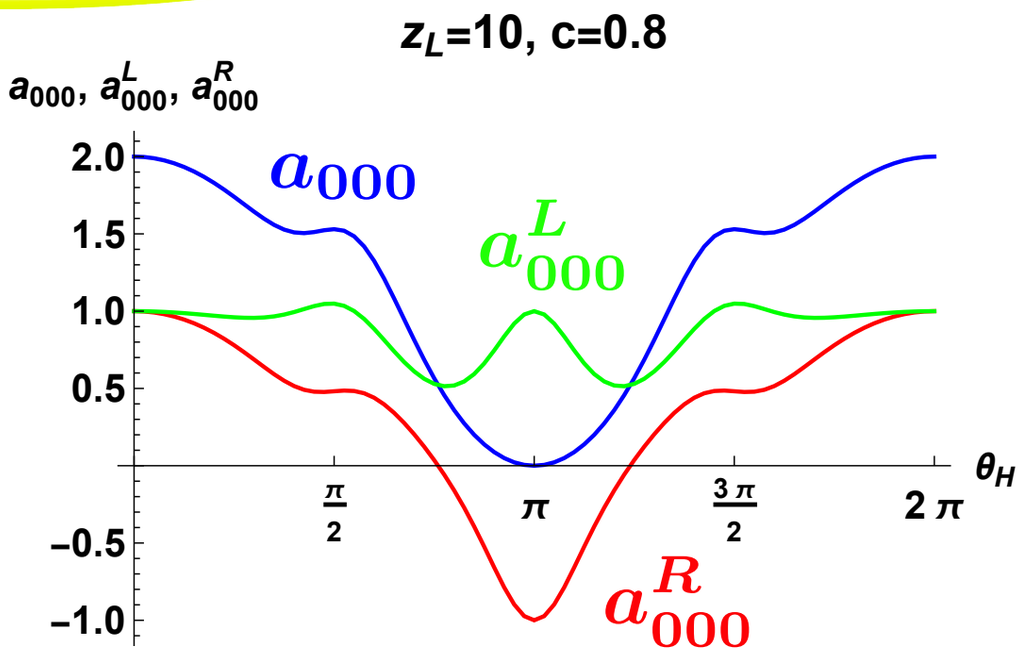
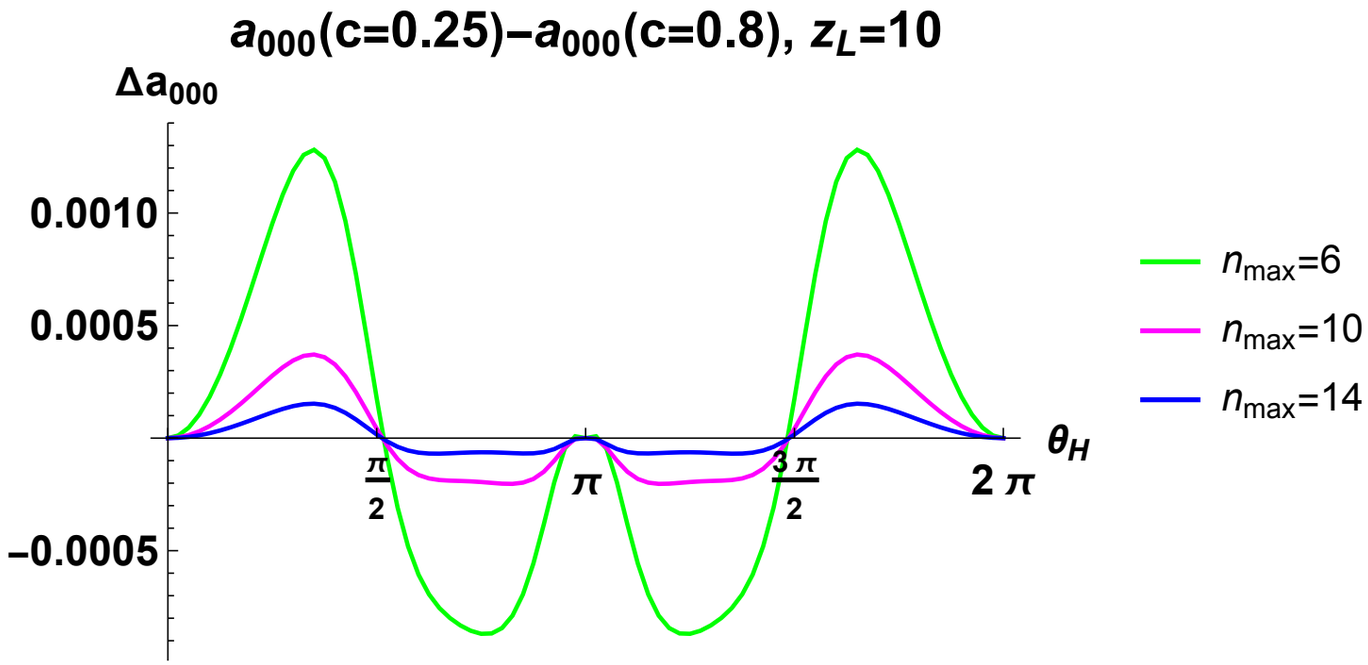
c-independence



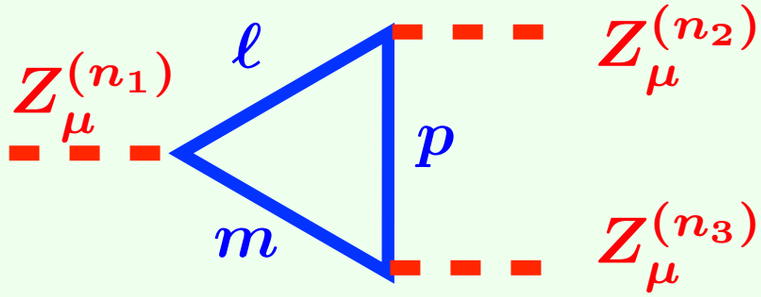




Why ?

Holography in anomaly flow

Anomaly coefficient



$$a_{n_1 n_2 n_3} = \sum_{m,\ell,p} \left\{ t^R_{n_1 m \ell} t^R_{n_2 \ell p} t^R_{n_3 p m} + t^L_{n_1 m \ell} t^L_{n_2 \ell p} t^L_{n_3 p m} \right\}$$

Holography in anomaly flow

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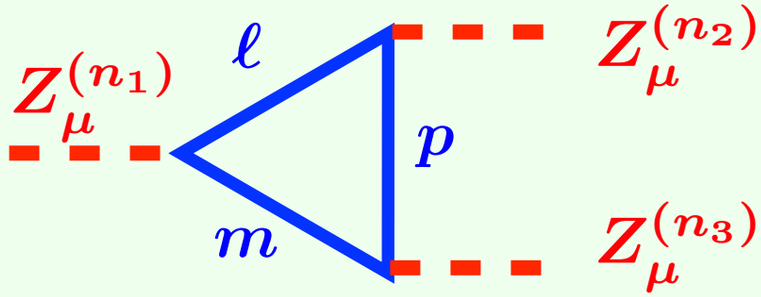

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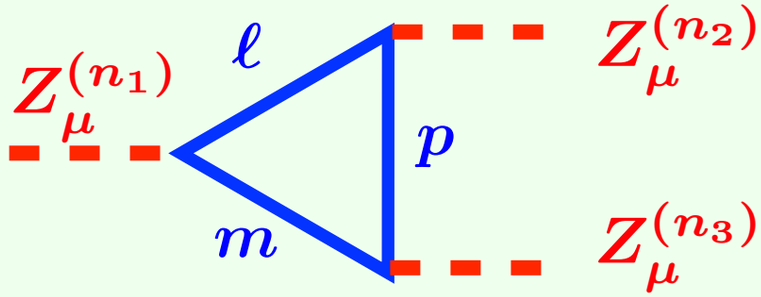
Method 1 (i) Evaluate $t^{R/L}_{nm\ell}$. (ii) Do $\sum_{m,\ell,p}$.

Holography in anomaly flow

Anomaly coefficient



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Method 1 (i) Evaluate $t^{R/L}_{nm\ell}$. (ii) Do $\sum_{m,\ell,p}$.

Method 2 (i) Do $\sum_{m,\ell,p}$. (ii) Do $\int dy_1 dy_2 dy_3$.

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$$a_{n_1 n_2 n_3} = \left(\frac{k}{2}\right)^3 \int\int\int_{-a}^{2L-a} dy_1 dy_2 dy_3 \, e^{\sigma(y_1)+\sigma(y_2)+\sigma(y_3)}$$

$$\times \Big[\, k_1 k_2 k_3 \big\{ A_R(1,2) A_R(2,3) A_R(3,1) - B_R(1,2) B_R(2,3) B_R(3,1)$$

$$+ B_L(1,2) B_L(2,3) B_L(3,1) - A_L(1,2) A_L(2,3) A_L(3,1) \big\}$$

$$+ k_1 h_2 h_3 \{ \cdots \} + h_1 k_2 h_3 \{ \cdots \} + h_1 h_2 k_3 \{ \cdots \} \Big]$$

$$k_j = k_{n_j}(y_j) \ , \quad h_j = h_{n_j}(y_j)$$

$$\begin{pmatrix} A_R(j,k) \\ B_R(j,k) \end{pmatrix} = \sum_{n=0}^{\infty} \begin{pmatrix} f_{Rn}(y_j) f^*_{Rn}(y_k) \\ g_{Rn}(y_j) g^*_{Rn}(y_k) \end{pmatrix}$$

Anomaly formula

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$$a_{n\ell m}(\theta_H, z_L) = Q_0 k_n(0) k_\ell(0) k_m(0) + Q_1 k_n(L) k_\ell(L) k_m(L)$$

$$(Q_0, Q_1) = \begin{cases} (+1,+1) & \text{for type 1A} \\ (-1,-1) & \text{for type 1B} \\ (+1,-1) & \text{for type 2A} \\ (-1,+1) & \text{for type 2B} \end{cases}$$

(P_0, P_1) for Ψ_R

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(P_0, P_1) for Ψ_R

Determined by
wave functions of gauge fields at y=0, L,
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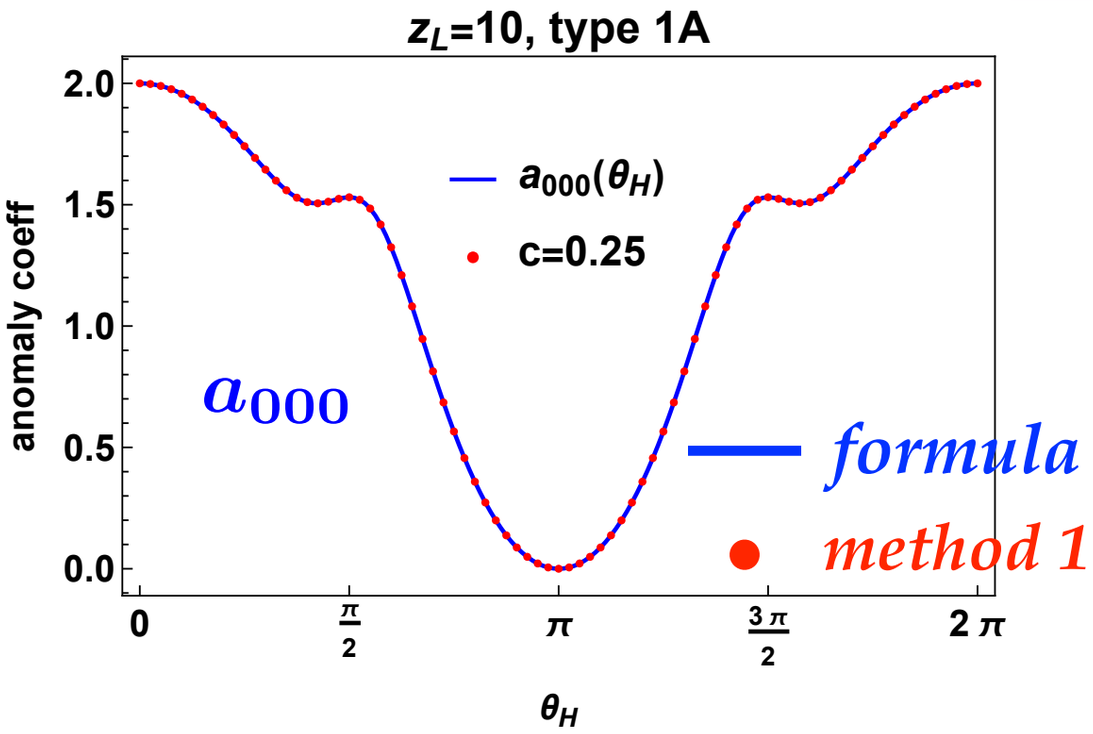
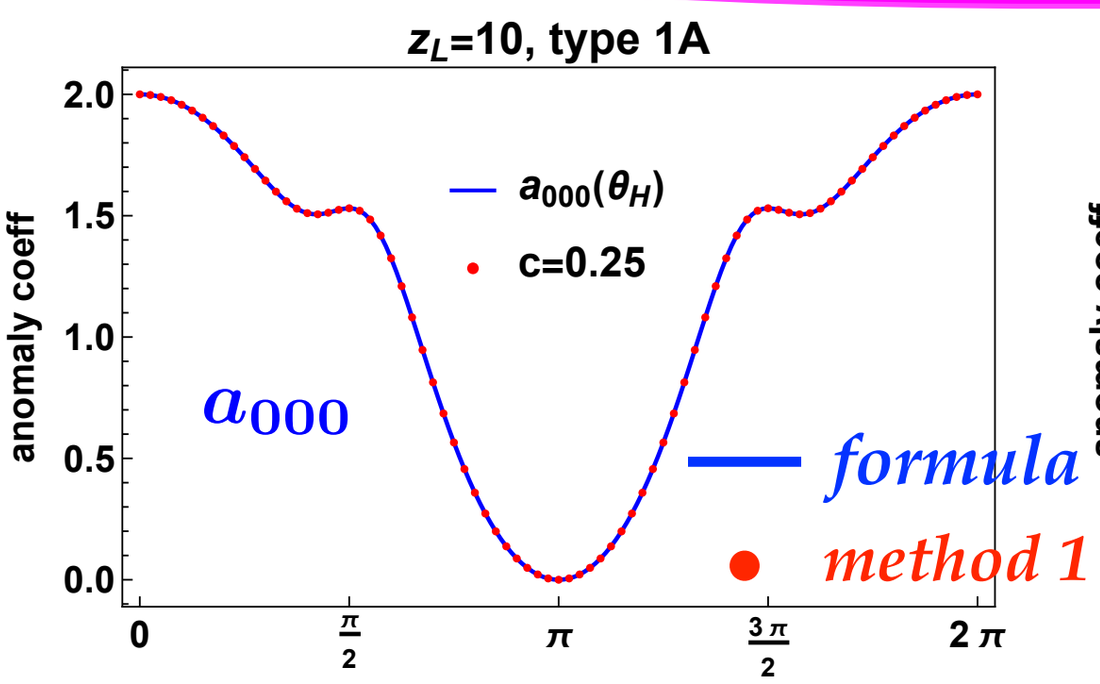
Holography

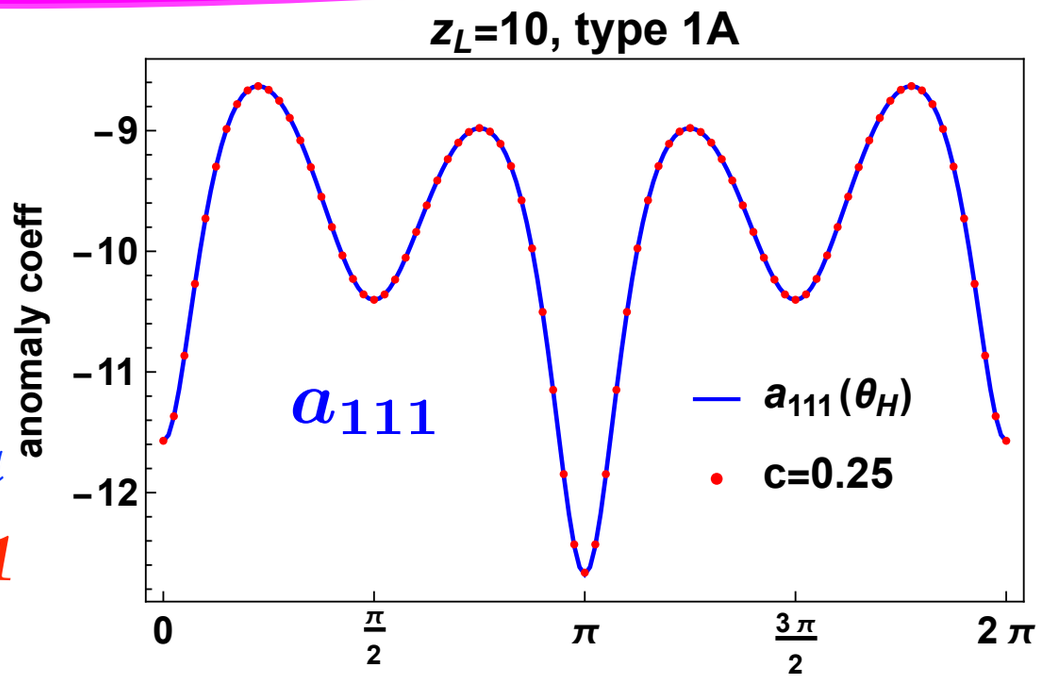
Anomaly flow and holography

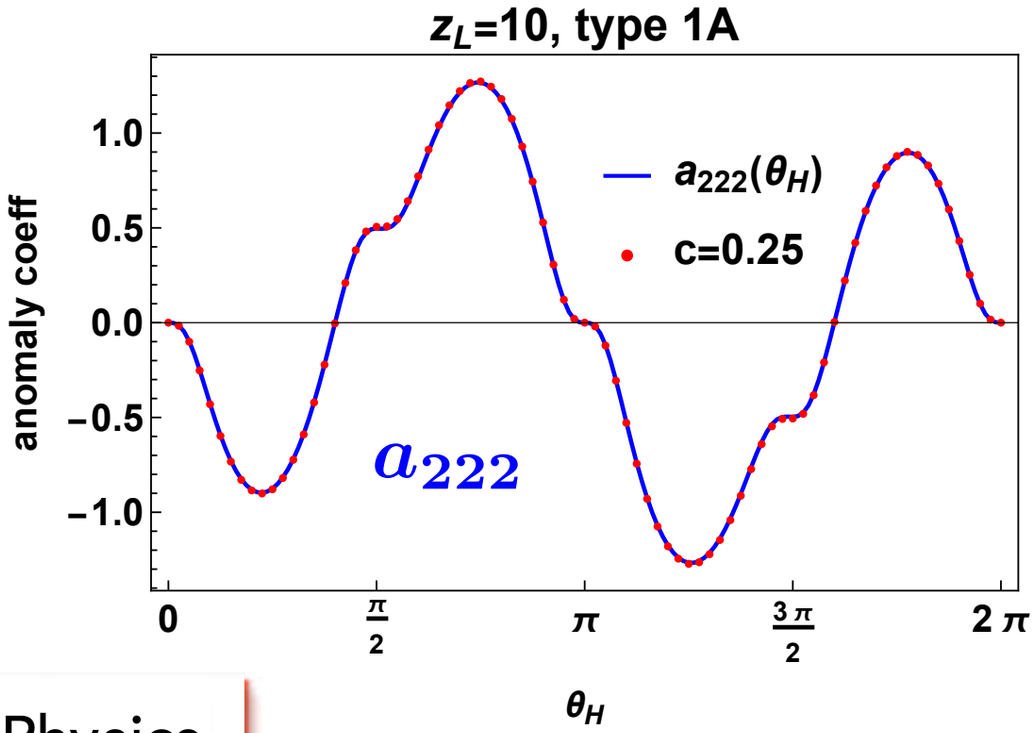
Anomaly flow and holography

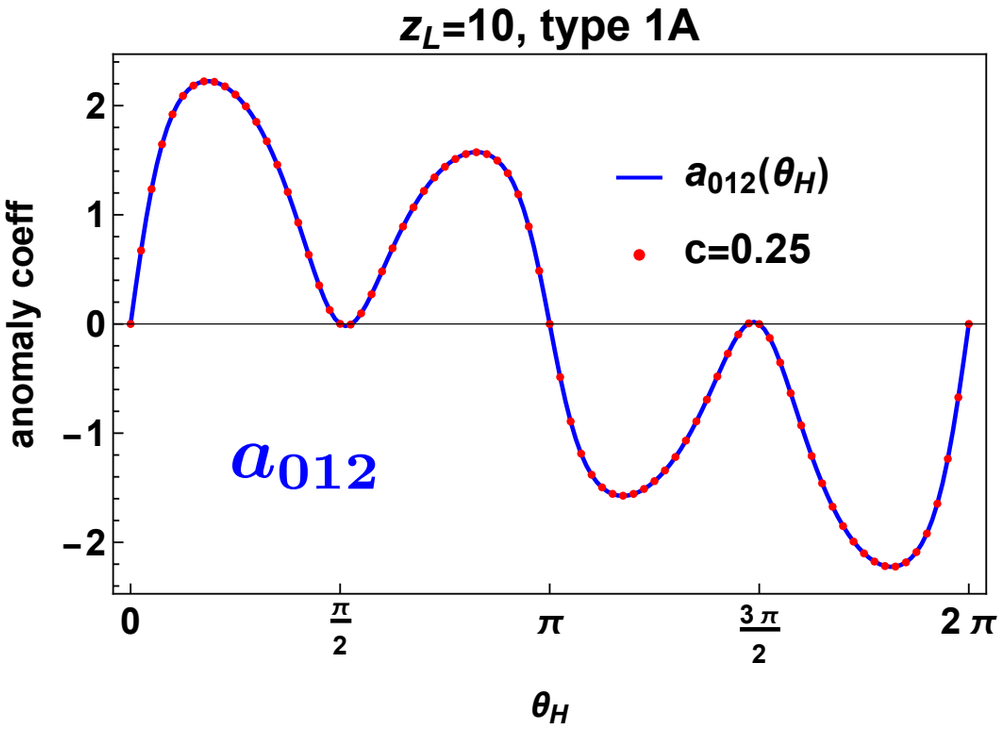


Anomaly flow and holography









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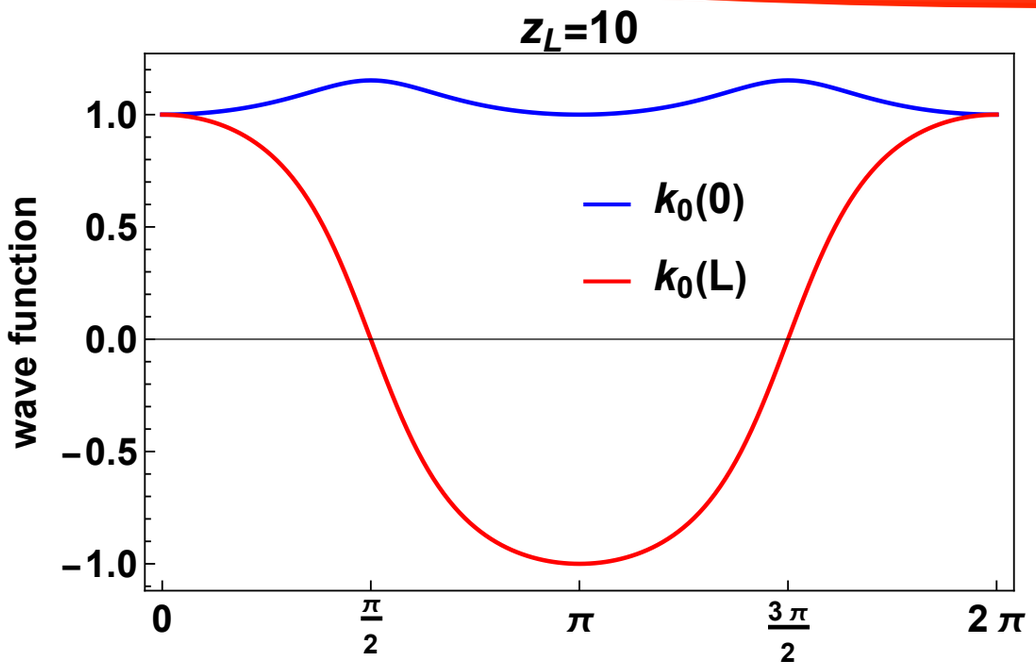
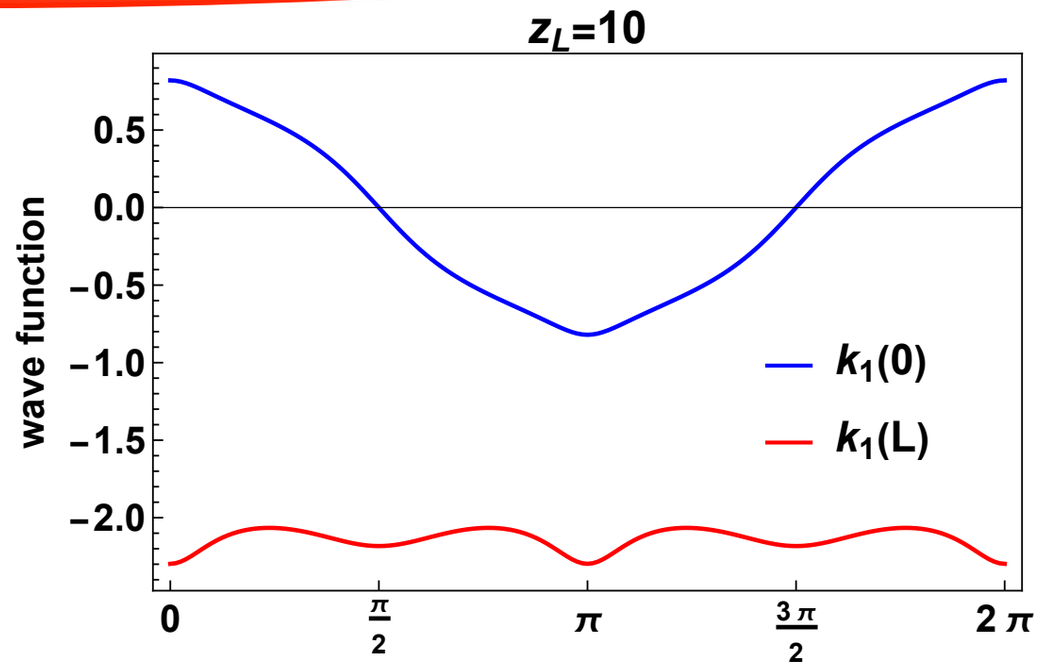
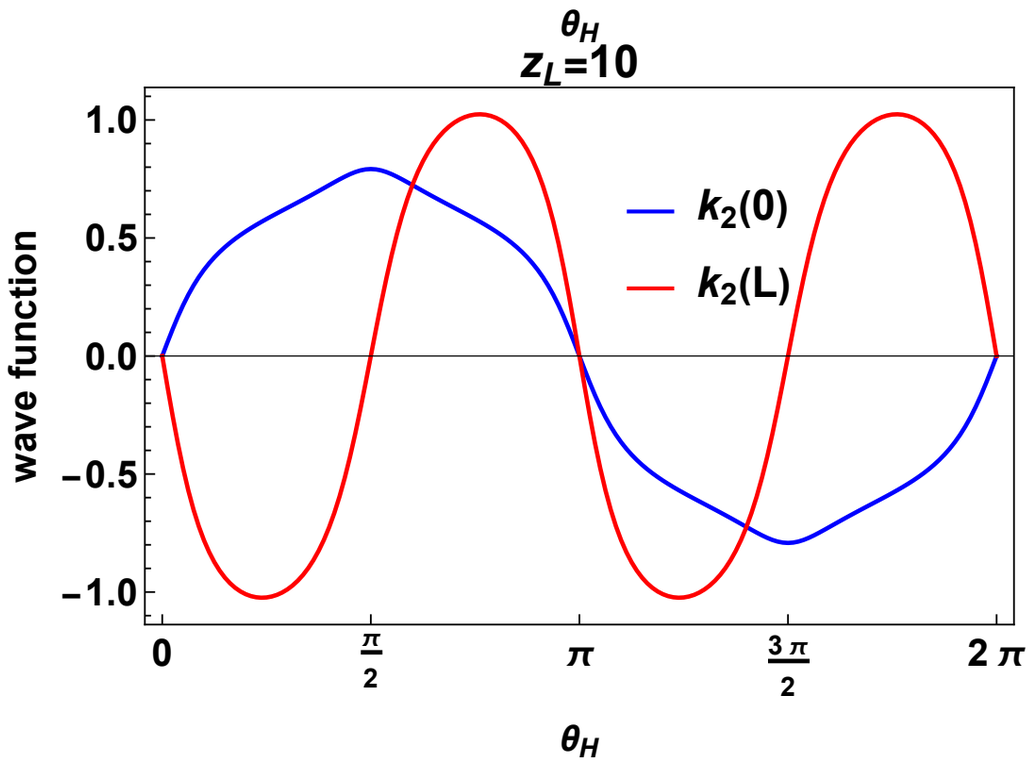
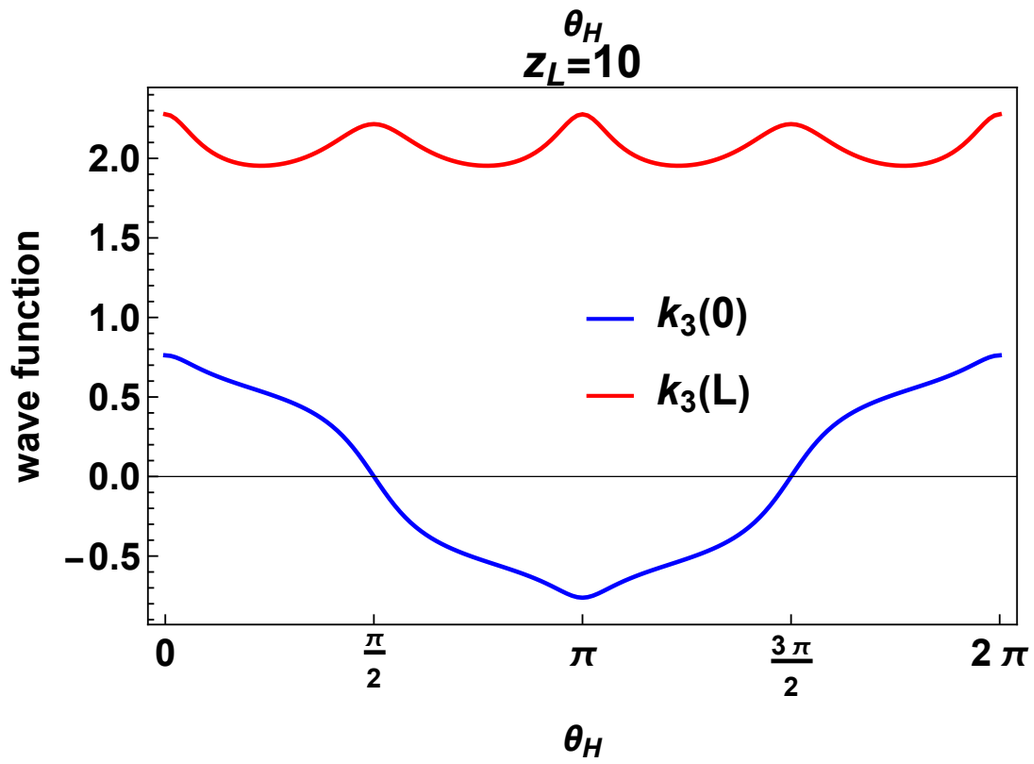
Gauge wave functions $k_n(0), k_n(L)$









Anomaly cancellation

Anomaly cancellation

$$\partial_\mu j^\mu_{(n)} \Rightarrow -\left(\frac{g_4}{2}\right)^3 \sum_\ell \sum_m \frac{a_{n\ell m}}{16\pi^2} Z^{(\ell)}_{\mu\nu} \widetilde{Z}^{(m)\mu\nu}$$

$$a_{n\ell m}(\theta_H, z_L) = Q_0 k_n(0) k_\ell(0) k_m(0) + Q_1 k_n(L) k_\ell(L) k_m(L)$$

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All anomalies are cancelled at once if

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All anomalies are cancelled at once if

$$n_{1A} = n_{1B}\ ,\ n_{2A} = n_{2B}$$

More generally if

$$(n_{1A} - n_{1B}) + (n_{2A} - n_{2B}) + (\hat{n}_R - \hat{n}_L) = 0\ ,$$
$$(n_{1A} - n_{1B}) - (n_{2A} - n_{2B}) = 0$$

↑ brane fermions at $y = 0$

Summary

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Chiral anomaly flows by θ_H

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In RS, no level crossing,

smooth variation of $a_{n\ell m}(\theta_H)$

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Anomaly cancellation in GHU

Flat space limit

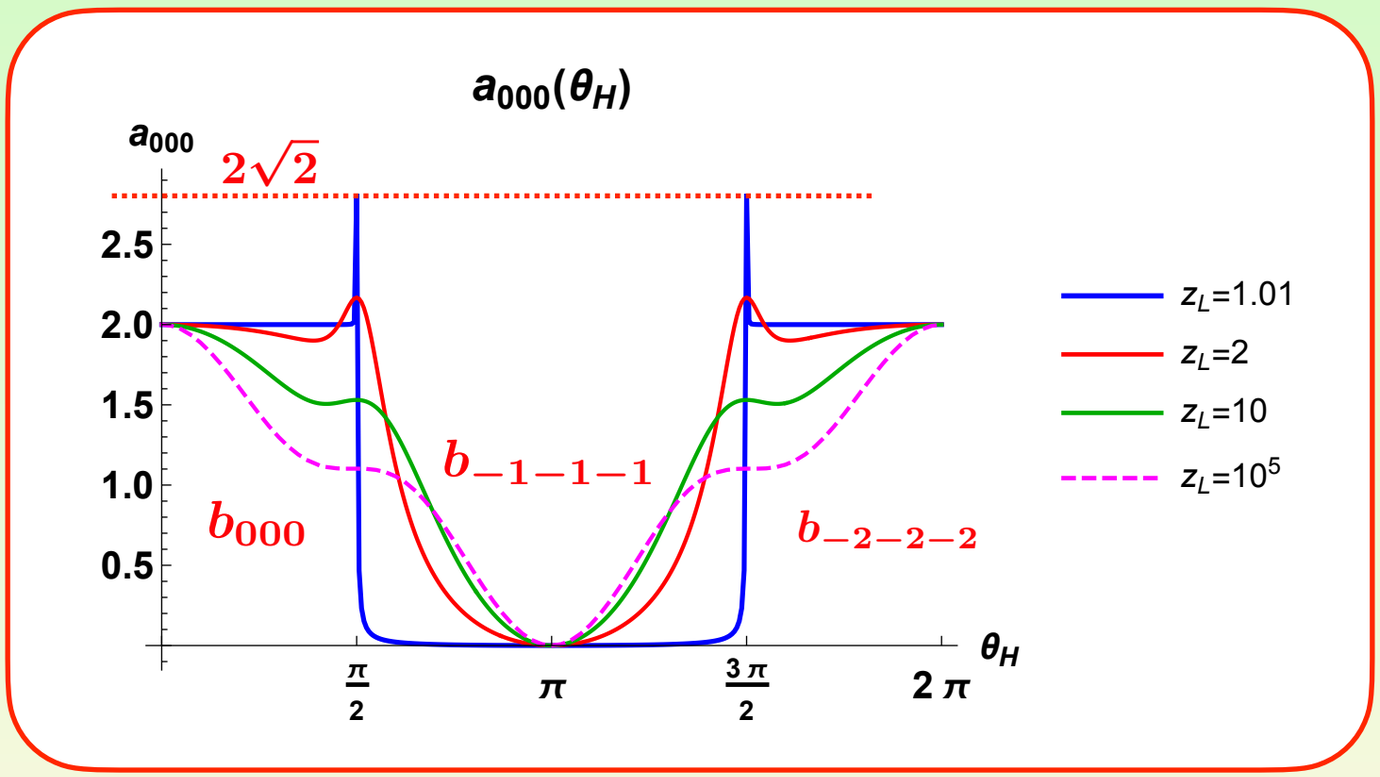
$$\text{flat } M^4 \times (S^1/Z_2) \qquad 0 \le y \le L = \pi R$$

$$\text{RS: } ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + \frac{dz^2}{k^2}\right) \quad (1 \le z \le z_L = e^{kL})$$

$$z = e^{ky}, \quad \text{AdS curvature} = -6k^2 \qquad m_{\rm KK} = \frac{\pi k}{z_L - 1} \implies \frac{1}{R}$$

$$b_{n\ell m} = Q_0 k_n^{\rm flat}(0) k_\ell^{\rm flat}(0) k_m^{\rm flat}(0) + Q_1 k_n^{\rm flat}(L) k_\ell^{\rm flat}(L) k_m^{\rm flat}(L)$$

$$k_n^{\rm flat}(y) = \cos\frac{n\pi y}{L} \quad \Rightarrow \quad b_{n\ell m} = Q_0 + (-1)^{n+\ell+m} Q_1$$



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