INTERPLAY BETWEEN SPACETIME CURVATURE, SPEED OF LIGHT AND QUANTUM DEFORMATIONS OF RELATIVISTIC SYMMETRIES

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Noncommutative and generalized geometry in string theory, gauge theory and related physical models

Corfu, September 2022

References

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- Ballesteros, GG, Gutierrez-Sagredo, Herranz, *The κ-Newtonian and κ-Carrollian algebras and their noncommutative spacetimes* , PLB 2020
- Ballesteros, GG, Herranz, *Lorentzian Snyder spacetimes and their Galilei and Carroll limits from projective geometry,* CQG 2020
- Ballesteros, GG, Gutierrez-Sagredo, Herranz, *Curved momentum spaces from quantum (anti–)de Sitter groups in (3+1) dimensions* , PRD 2018
- Ballesteros, GG, Gutierrez-Sagredo, Herranz, *Curved momentum spaces from quantum groups with cosmological constant* , PLB 2017

The Poincaré algebra of special relativity

- Special relativistic symmetries are described by the Poincaré Lie algebra $\mathfrak{p}(3+1)$ generated by time translations P_0, spatial translation P_a, boosts K_a, and rotations J_a

$$
\begin{aligned}
&[J_a, J_b] = \epsilon_{abc} J_c \qquad && [K_a, P_0] = P_a \qquad && [P_0, P_a] = 0 \\
&[J_a, P_b] = \epsilon_{abc} P_c \qquad && [K_a, P_b] = \delta_{ab} P_0 \qquad && [P_a, P_b] = 0 \\
&[J_a, K_b] = \epsilon_{abc} K_c \qquad && [K_a, K_b] = -\epsilon_{abc} J_c \qquad && [P_0, J_a] = 0
\end{aligned}
$$

Casimir: $\mathscr{C} = P_0^2 - \mathbf{P}^2$

- Minkowski spacetime $\mathbf{M}^{3+1}$ is obtained as the homogeneous space of the Poincaré Lie group with isotropy subgroup given by Lorentz

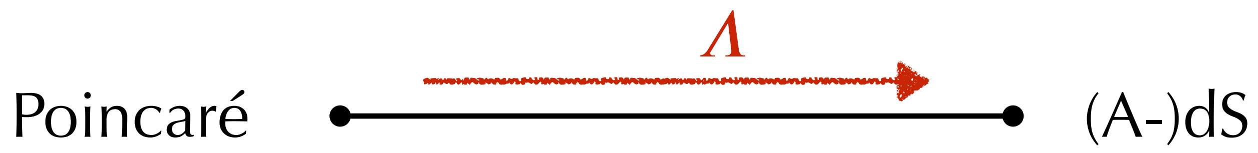
$$
\mathbf{M}^{3+1} \equiv \mathrm{ISO}(3,1)/\mathrm{SO}(3,1)
$$

Classical deformation of the Poincaré algebra: (A)dS symmetries

✦ **Classical** deformations of the Poincaré algebra

Governed by the curvature parameter Λ: (Anti-)de Sitter symmetries

Squared-length scale: the induced modifications are relevant at length scales comparable to $\sqrt{\Lambda}$



Classical deformation of the Poincaré algebra: (A)dS symmetries

✦ **Classical** deformations of the Poincaré algebra

Linearly deformed commutation rules:

$$[J_a, J_b] = \epsilon_{abc} J_c \qquad [K_a, P_0] = P_a \qquad [P_0, P_a] = 0$$
$$[J_a, P_b] = \epsilon_{abc} P_c \qquad [K_a, P_b] = \delta_{ab} P_0 \qquad [P_a, P_b] = 0$$
$$[J_a, K_b] = \epsilon_{abc} K_c \qquad [K_a, K_b] = -\epsilon_{abc} J_c \qquad [P_0, J_a] = 0$$

$$\mathscr{C} = P_0^2 - \mathbf{P}^2$$

$\downarrow \Lambda$

$$[J_a, J_b] = \epsilon_{abc} J_c \qquad [K_a, P_0] = P_a \qquad [P_0, P_a] = -\Lambda K_a$$
$$[J_a, P_b] = \epsilon_{abc} P_c \qquad [K_a, P_b] = \delta_{ab} P_0 \qquad [P_a, P_b] = \Lambda \epsilon_{abc} J_c$$
$$[J_a, K_b] = \epsilon_{abc} K_c \qquad [K_a, K_b] = -\epsilon_{abc} J_c \qquad [P_0, J_a] = 0$$

$$\mathscr{C} = P_0^2 - \mathbf{P}^2 - \Lambda \left(\mathbf{J}^2 - \mathbf{K}^2 \right)$$

Classical deformation of the Poincaré algebra: (A)dS symmetries

- (Anti-)de Sitter algebra in the kinematical basis

$$[J_a, J_b] = \epsilon_{abc} J_c \qquad [K_a, P_0] = P_a \qquad [P_0, P_a] = -\Lambda\, K_a$$
$$[J_a, P_b] = \epsilon_{abc} P_c \qquad [K_a, P_b] = \delta_{ab} P_0 \qquad [P_a, P_b] = \Lambda\, \epsilon_{abc}\, J_c$$
$$[J_a, K_b] = \epsilon_{abc} K_c \qquad [K_a, K_b] = -\epsilon_{abc} J_c \qquad [P_0, J_a] = 0$$

Casimir:
$$\mathscr{C} = P_0^2 - \mathbf{P}^2 - \Lambda \left(\mathbf{J}^2 - \mathbf{K}^2 \right)$$

- (anti-)de Sitter spacetime $(\mathbf{A})\mathbf{dS}^{3+1}$ is obtained as a homogeneous space:

$$\Lambda < 0: \mathbf{AdS}^{3+1} \equiv SO(3,2)/SO(3,1)$$

$$\Lambda > 0: \mathbf{dS}^{3+1} \equiv SO(4,1)/SO(3,1)$$

$$\Lambda = 0: \mathbf{M}^{3+1} \equiv ISO(3,1)/SO(3,1)$$

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de Sitter spacetime, phase space and particle kinematics

$$\left[H = \sqrt{\Lambda}\right]$$

- line element in comoving coordinates (1+1 dimensions)

$$ds^2 = (dx^0)^2 - e^{2Hx^0}(dx^1)^2$$

- algebra of symmetries in the comoving basis (Poisson brackets!)

$$\begin{aligned}\{\mathcal{P}_0, \mathcal{P}_1\} &= H\,\mathcal{P}_1\\ \{\mathcal{P}_0, \mathcal{N}\} &= \mathcal{P}_1 - H\,\mathcal{N}\\ \{\mathcal{P}_1, \mathcal{N}\} &= \mathcal{P}_0\end{aligned}$$

- mass Casimir $\mathcal{C}_{dS} = \mathcal{P}_0^2 - \mathcal{P}_1^2 + 2H\mathcal{N}\mathcal{P}_1$

- representation of symmetry generators:

$$\begin{aligned}\{x^\mu, x^\nu\} &= 0\,,\\ \{x^\mu, p_\nu\} &= -\delta^\mu_\nu\,,\\ \{p_\mu, p_\nu\} &= 0\,.\end{aligned} \quad\longrightarrow\quad \begin{aligned}\mathcal{P}_0 &= p_0 - Hx^1p_1\\ \mathcal{P}_1 &= p_1\\ \mathcal{N} &= x^1p_0 + p_1\left(\tfrac{1-e^{-2Hx^0}}{2H} - \tfrac{H}{2}(x^1)^2\right)\end{aligned}$$

- the massless condition $\mathcal{C}_{dS} = 0$ relates energy and spatial momentum, encoding energy redshift

$$p_0 = |p_1|e^{-Hx^0}$$

- particles worldline (using Hamiltonian formalism)

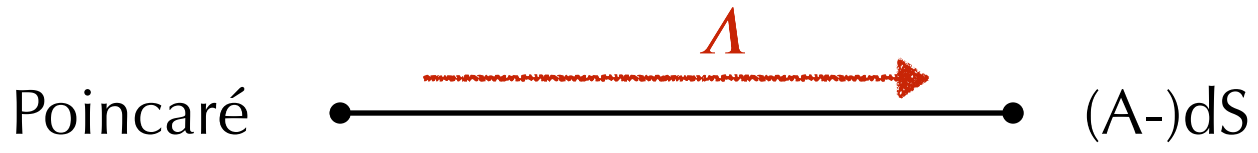
$$\begin{aligned}\dot{x}^1 &= \{\mathcal{C}_{dS}, x^1\} = -2e^{-2Hx^0}p_1\\ \dot{x}^0 &= \{\mathcal{C}_{dS}, x^0\} = 2p_0\end{aligned} \quad\longrightarrow\quad x^1(x^0) - \bar{x}^1 \equiv \int_{\bar{x}^0}^{x^0} \frac{\dot{x}^1}{\dot{x}^0}dx^0 = \left(\frac{e^{-H\bar{x}^0} - e^{-Hx^0}}{H}\right)$$

Deforming the Poincaré algebra of special relativity

- **Classical** deformations of the Poincaré algebra

  Governed by the curvature parameter Λ: (Anti-)de Sitter symmetries

  Squared-length scale: the induced modifications are relevant at length scales comparable to $\sqrt{\Lambda}$



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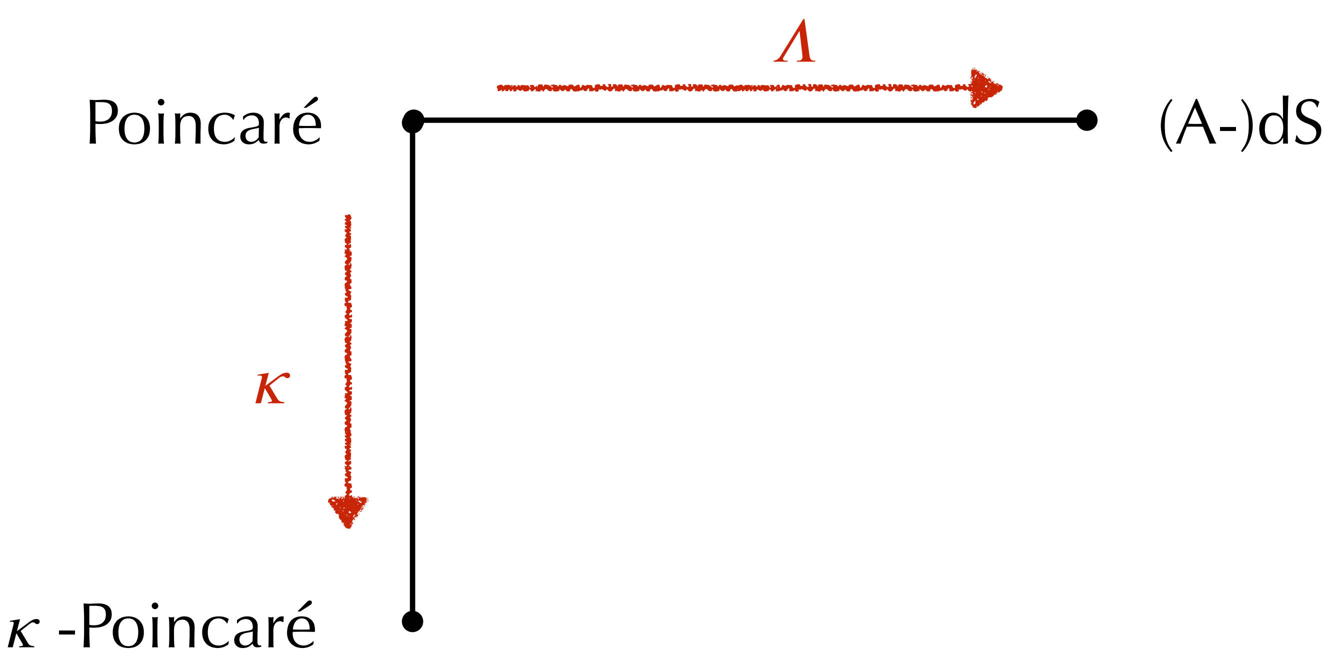
✦ **Quantum** deformations of the Poincaré algebra

Governed by the quantum deformation parameter κ

Energy scale: the induced modifications are relevant at energies comparable to κ



Quantum deformation of the Poincaré algebra: κ -Poincaré

✦ **Quantum** deformations of the Poincaré algebra ⇒ Hopf algebra
(we show the κ -Poincaré algebra in the bicrossproduct basis)

Nonlinearly deformed commutation rules:

$$[J_a, J_b] = \epsilon_{abc} J_c \qquad [K_a, P_0] = P_a \qquad [P_0, P_a] = 0$$

$$[J_a, P_b] = \epsilon_{abc} P_c \qquad [K_a, P_b] = \delta_{ab} P_0 \qquad [P_a, P_b] = 0$$

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$$\mathscr{C} = P_0^2 - \mathbf{P}^2$$

$$\Big\downarrow \kappa$$

$$[K_a, P_b] = \delta_{ab} \left(\frac{\kappa}{2} \left(1 - \mathrm{e}^{-2P_0/\kappa}\right) + \frac{1}{2\kappa} \mathbf{P}^2 \right) - \frac{1}{\kappa} P_a P_b$$

Casimir: $$\mathscr{C}_\kappa = 4\kappa^2 \sinh^2(P_0/2\kappa) - e^{P_0/\kappa}\, \mathbf{P}^2$$

•*Lukierski, Nowicki, Ruegg, PLB 1992*
•*Lukierski, Ruegg, Nowicki, Tolstoi, PLB 1991*
•*Majid, Ruegg, PLB 1994*

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$$\mathscr{C} = P_0^2 - \mathbf{P}^2$$

κ

$$[K_a, P_b] = \delta_{ab} \left(\frac{\kappa}{2} \left(1 - \mathrm{e}^{-2P_0/\kappa}\right) + \frac{1}{2\kappa} \mathbf{P}^2 \right) - \frac{1}{\kappa} P_a P_b$$

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Quantum deformation of the Poincaré algebra: κ -Poincaré

- **Quantum** deformations of the Poincaré algebra ⇒ Hopf algebra
  (we show the κ -Poincaré algebra in the bicrossproduct basis)

  Deformed action on multi-particle states (coproducts)
  (and conservation rules of energy-momentum in interactions)

$$\Delta(X) = X \otimes 1 + 1 \otimes X$$

$$\Big\downarrow \kappa$$

$$\begin{aligned}
\Delta(P_0) &= P_0 \otimes 1 + 1 \otimes P_0, \\
\Delta(J_a) &= J_a \otimes 1 + 1 \otimes J_a, \\
\Delta(P_a) &= P_a \otimes 1 + \mathrm{e}^{-P_0/\kappa} \otimes P_a, \\
\Delta(K_a) &= K_a \otimes 1 + \mathrm{e}^{-P_0/\kappa} \otimes K_a + \frac{1}{\kappa} \epsilon_{abc} P_b \otimes J_c \,.
\end{aligned}$$

•*Lukierski, Nowicki, Ruegg, PLB 1992*
•*Lukierski, Ruegg, Nowicki, Tolstoi, PLB 1991*
•*Majid, Ruegg, PLB 1994*

Quantum deformation of the Poincaré algebra: κ -Poincaré

✦ **Quantum** deformations of the Poincaré algebra

The κ-Poincaré algebra (as a Lie bialgebra) is generated by the r-matrix

$$r_\Lambda = \frac{1}{\kappa}(K_1 \wedge P_1 + K_2 \wedge P_2 + K_3 \wedge P_3)$$

Which produces the cocommutators via $\delta(X) = [X \otimes 1 + 1 \otimes X, r_\Lambda]$, $X \in \{P_0, P_a, J_a, K_a\}$

$$\delta(P_0) = \delta(J_a) = 0$$

$$\delta(P_a) = \frac{1}{\kappa} P_a \wedge P_0$$

$$\delta(K_1) = \frac{1}{\kappa}(K_1 \wedge P_0 + P_2 \wedge J_3 - P_3 \wedge J_2),$$

$$\delta(K_2) = \frac{1}{\kappa}(K_2 \wedge P_0 - P_1 \wedge J_3 + P_3 \wedge J_1),$$

$$\delta(K_3) = \frac{1}{\kappa}(K_3 \wedge P_0 + P_1 \wedge J_2 - P_2 \wedge J_1)\,.$$

• *Maslanka, JPA 1993*
•*Zakrzewski, JPA 1994*

κ -Poincaré phase space: free particle worldlines

$[\ell \equiv 1/\kappa]$

✦ representation of the algebra of symmetries on phase space (Poisson brackets)

$$\{p_1, p_0\} = 0$$
$$\{x^1, x^0\} = 0$$
$$\{x^\mu, p_\nu\} = \delta^\mu_\nu$$

→

$$\mathcal{P}_0 = p_0$$
$$\mathcal{P}_1 = p_1$$
$$\mathcal{N} = p_1 x^0 + \left(\frac{1 - e^{-2\ell p_0}}{2\ell} - \frac{\ell}{2} p_1^2\right) x^1$$

✦ evolution of phase space coordinates is given by the Hamilton equations with the κ-Poincaré Casimir as Hamiltonian

$$\dot{x}^0 = \{\mathcal{C}_\ell, x^0\} = \frac{1}{\ell}\left(e^{\ell p_0} - e^{-\ell p_0}\right) - \ell p_1^2 e^{\ell p_0}$$
$$\dot{x}^1 = \{\mathcal{C}_\ell, x^1\} = 2\, p_1\, e^{\ell p0}.$$

✦ Then the free particle worldline reads $x^1(x^0) = x^1(0) + v(p) x^0$

→ $$v(p) = \frac{e^{p_0/\kappa}\sqrt{e^{2p_0/\kappa} + 1 - 2\, e^{p_0/\kappa}\cosh(m/\kappa)}}{1 - e^{p_0/\kappa}\cosh(m/\kappa)} \rightarrow e^{p_0/\kappa}$$ for m=0

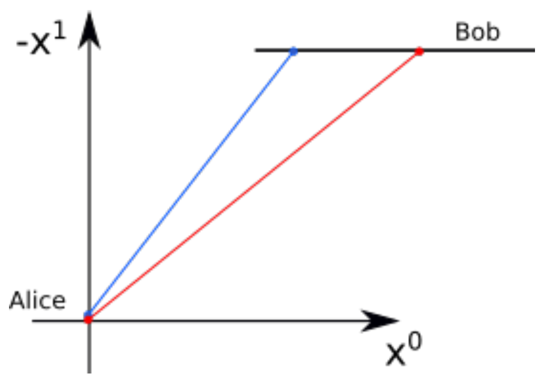
•*Amelino-Camelia, Barcaroli, GG, Loret, CQG 2013*
•*GG, Barcaroli, PRD 2016*

Particle worldlines and relativity of locality

$[\ell \equiv 1/\kappa]$

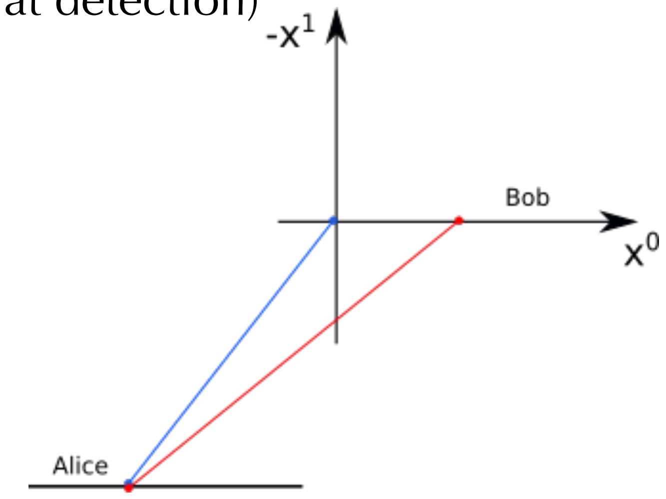
worldlines of two massless particles emitted simultaneously with different energies

worldlines seen by Alice
(local at emission)



$$x_A^1 = -e^{\ell p_0} x_A^0$$
$$x_A^1 = -e^{\ell \tilde{p}_0} x_A^0$$

worldlines seen by Bob
(local at detection)



$$x_B^0 = \mathcal{T}_a \triangleright x_A^0 = x_A^0 - a^0$$
$$x_B^1 = \mathcal{T}_a \triangleright x_A^1 = x_A^1 - a^1$$

✦ using coordinates dual to momenta:
(energy dependent worldlines)

•Amelino-Camelia, Loret, Rosati, PLB 2011
•Amelino-Camelia, Barcaroli, GG, Loret, CQG 2013

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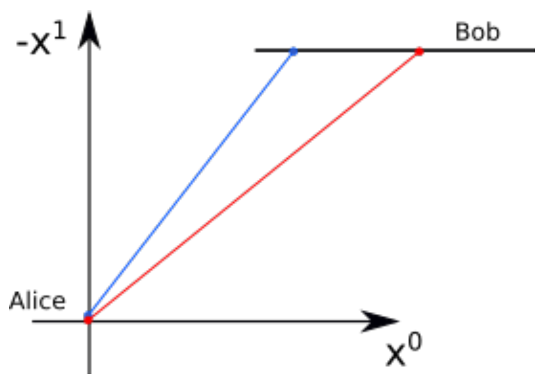

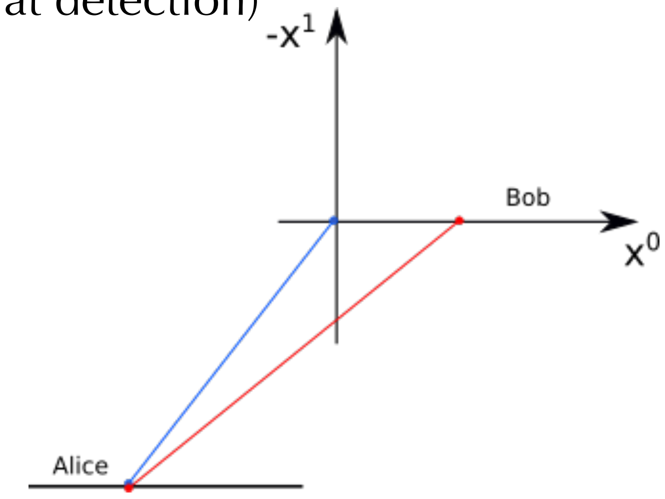

$$x^1_A = -e^{\ell p_0} x^0_A$$
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$$x^0_B = \mathcal{T}_a \triangleright x^0_A = x^0_A - a^0$$
$$x^1_B = \mathcal{T}_a \triangleright x^1_A = x^1_A - a^1$$

✦ using 'κ-Minkowski coordinates':
(energy independent worldlines
but deformed action of translations)

$$\chi^1 = x^1$$
$$\chi^0 = x^0 - \ell x^1 p_1$$

$$\{\chi^0, \chi^1\} = \ell \chi^1$$
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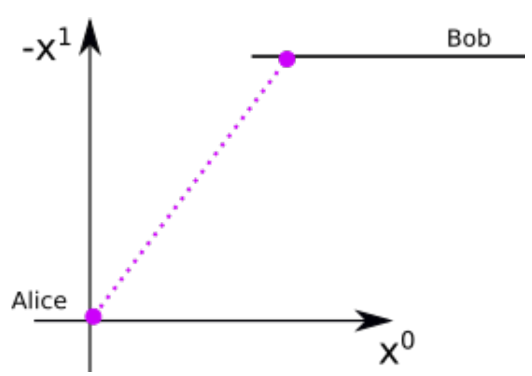

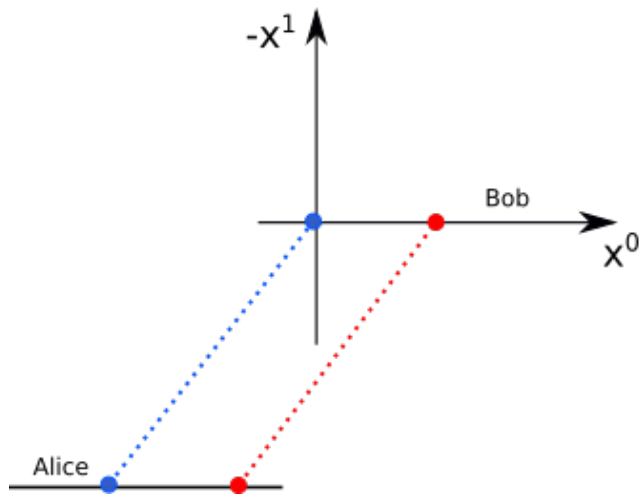

$$\chi^1 - \bar{\chi}^1 = \chi^0 - \bar{\chi}^0$$

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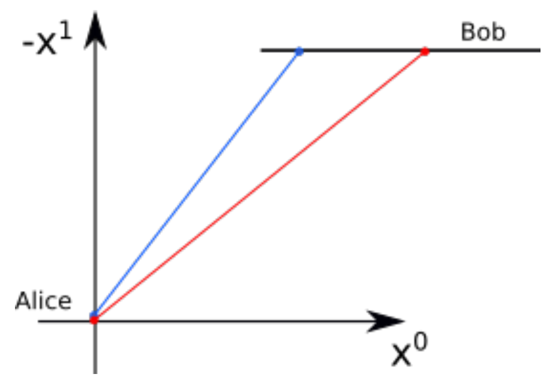

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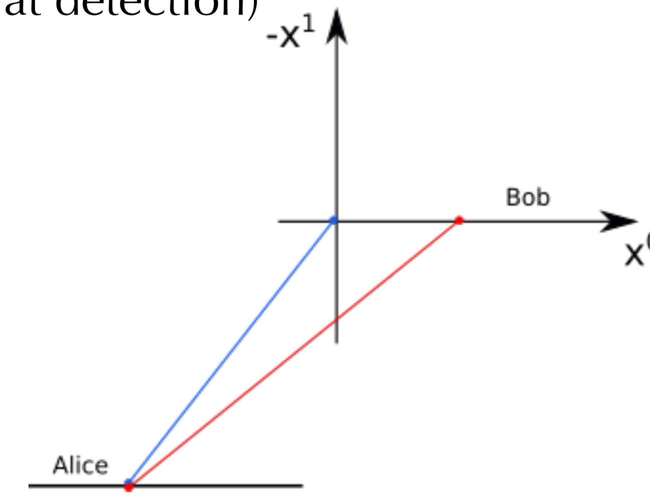

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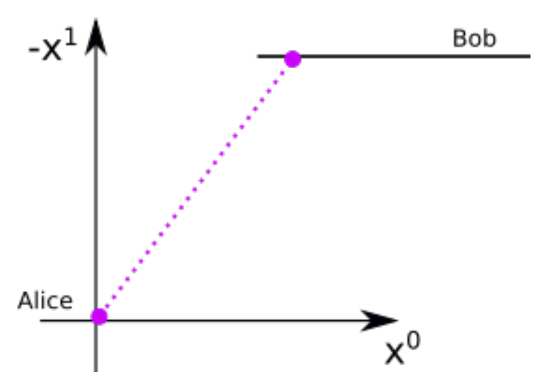
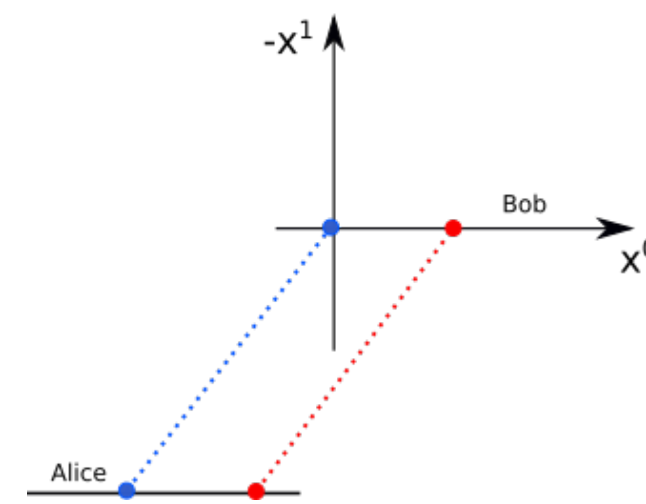


$$\chi^1 - \bar{\chi}^1 = \chi^0 - \bar{\chi}^0$$



$$\chi^0_B = \mathcal{T}_a \triangleright \chi^0_A = \chi^0_A - a^0 + a^1 \ell p_1$$
$$\chi^1_B = \mathcal{T}_a \triangleright \chi^1_A = \chi^1_A - a^1$$

✦ measurements done <u>locally</u> (i.e. at spatial origin of each observer) do not depend on choice of coordinates: Alice emits the particles at the same time and Bob detects then with a time delay

$$\Delta x^0 = a^0 \left(e^{-\ell \Delta p_0} - 1\right)$$

•*Amelino-Camelia, Loret, Rosati, PLB 2011*
•*Amelino-Camelia, Barcaroli, GG, Loret, CQG 2013*

Duality between classical and quantum deformation

$H \equiv \sqrt{\Lambda}$

de Sitter phase space

$$\{x^\mu, p_\nu\} = -\delta^\mu_\nu$$

$$\begin{aligned}\{x^1, x^0\} &= 0\\ \{\mathcal{N}, x^0\} &= x^1\\ \{\mathcal{N}, x^1\} &= \frac{1-e^{-2Hx^0}}{2H} - \frac{H}{2}(x^1)^2\end{aligned}$$

$$\begin{aligned}\{p_0, p_1\} &= 0\\ \{p_0, \mathcal{N}\} &= p_1 e^{-2Hx^0}\\ \{p_1, \mathcal{N}\} &= p_0 - Hp_1x^1\end{aligned}$$

$$\begin{aligned}\{\mathcal{P}_0, \mathcal{P}_1\} &= H\,\mathcal{P}_1\\ \{\mathcal{P}_0, \mathcal{N}\} &= \mathcal{P}_1 - H\,\mathcal{N}\\ \{\mathcal{P}_1, \mathcal{N}\} &= \mathcal{P}_0\end{aligned}$$

the Casimir of the algebra $\{\mathcal{N}, \mathcal{P}_0, \mathcal{P}_1\}$ determines the dispersion relation

the Casimir of the algebra $\{\mathcal{N}, x^0, x^1\}$ determines the worldline (and is the Newton-Wigner operator)

κ-Poincaré phase space

$\ell \equiv 1/\kappa$

$$\{p_\nu, x^\mu\} = -\delta^\mu_\nu$$

$$\begin{aligned}\{\mathcal{P}_1, \mathcal{P}_0\} &= 0\\ \{\mathcal{N}, \mathcal{P}_0\} &= \mathcal{P}_1\\ \{\mathcal{N}, \mathcal{P}_1\} &= \frac{1-e^{-2\ell\mathcal{P}_0}}{2\ell} - \frac{\ell}{2}\mathcal{P}_1^2\end{aligned}$$

$$\begin{aligned}\{x^0, x^1\} &= 0\\ \{x^0, \mathcal{N}\} &= x^1 e^{-2\ell p_0}\\ \{x^1, \mathcal{N}\} &= x^0 - \ell x^1 p_1\end{aligned}$$

$$\begin{aligned}\{\chi^0, \chi^1\} &= \ell\chi^1\\ \{\chi^0, \mathcal{N}\} &= \chi^1 - \ell\mathcal{N}\\ \{\chi^1, \mathcal{N}\} &= \chi^0\end{aligned}$$

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Duality between classical and quantum deformation

de Sitter spacetime	de Sitter momentum space
$H \equiv \sqrt{\Lambda}$	$\ell \equiv 1/\kappa$
spacetime metric $$ds^2 = (dx^0)^2 - e^{2Hx^0}(dx^1)^2$$	momentum space metric $$ds_p^2 = dp_0^2 - e^{2\ell p_0} dp_1^2$$
worldline $$x^1 = \frac{1 - e^{-Hx^0}}{H}$$	dispersion relation $$p_1 = \frac{1 - e^{-\ell p_0}}{\ell}$$
dispersion relation $$p_1 = -e^{Hx^0} p_0$$	worldline $$x^1 = -e^{\ell p_0} x^0$$
generators of translations $$\mathcal{P}_1 = p_1$$ $$\mathcal{P}_0 = p_0 - Hx^1 p_1$$	'k-Minkowski coordinates' $$\chi^1 = x^1$$ $$\chi^0 = x^0 - \ell x^1 p_1$$

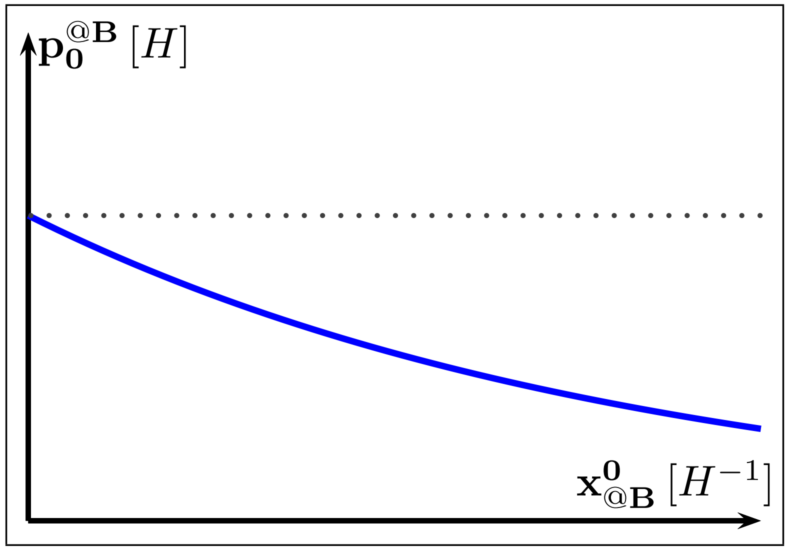
•*Amelino-Camelia, Barcaroli, Gubitosi, Loret, CQG 2013*

✦ related to the fact that in Hopf algebras noncommutativity induces curvature in the dual space, and viceversa

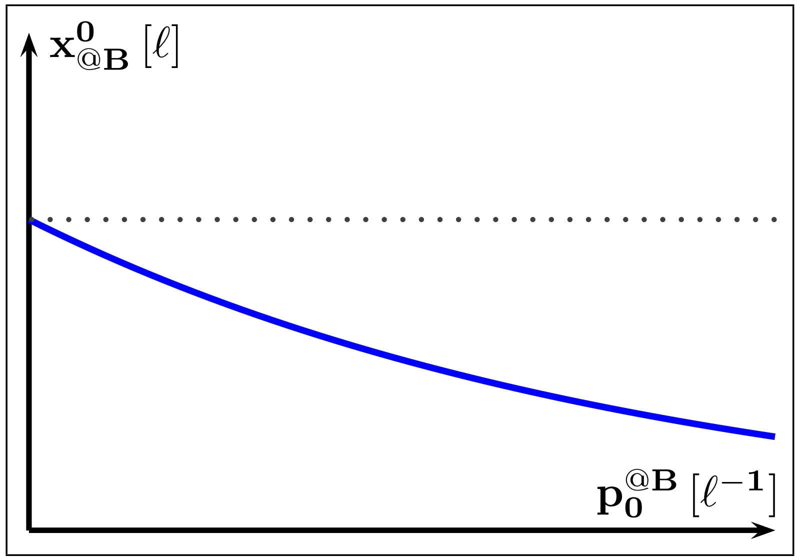
•*Majid ArXiv: hep-th/0604130*

Duality between de Sitter spacetime and de Sitter momentum space

- correlation between time of detection and energy, for fixed energy of emission (de Sitter spacetime) - *redshift*



- correlation between time of detection and energy, for fixed time of emission (de Sitter momentum space) - *time shift*



•*Amelino-Camelia, Barcaroli, Gubitosi, Loret, CQG 2013*

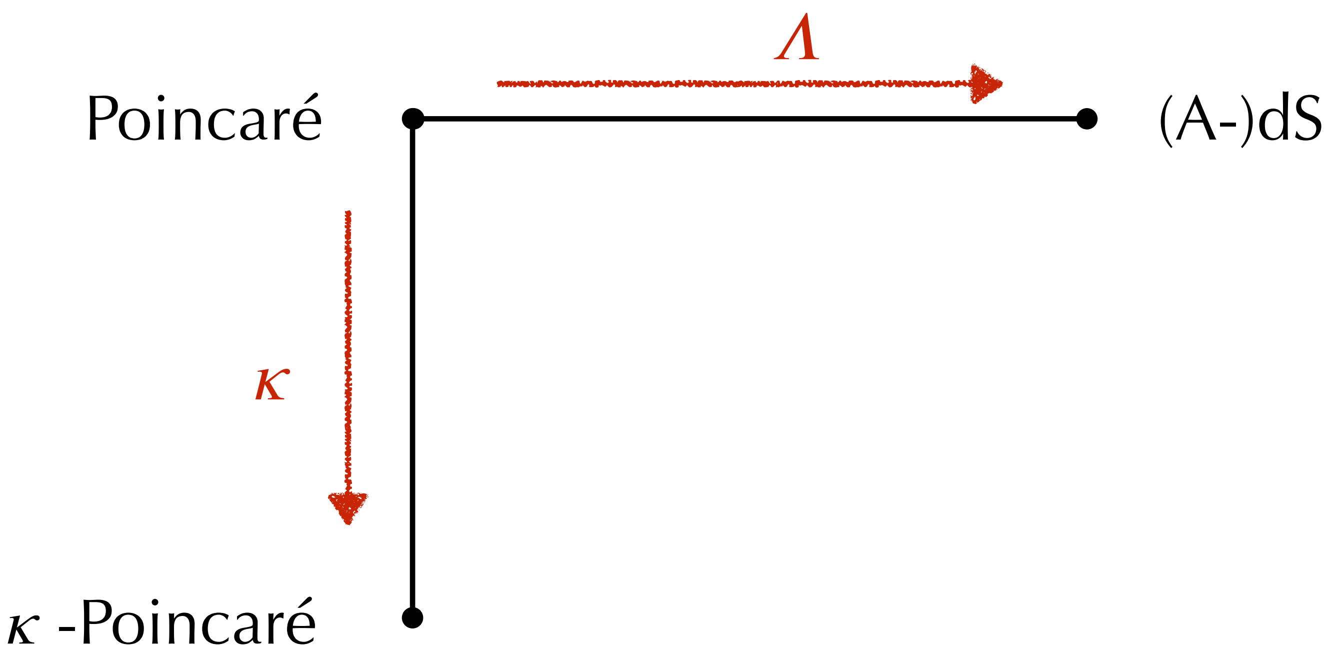
Deforming the Poincaré algebra of special relativity

✦ **Classical** deformations of the Poincaré algebra

Governed by the curvature parameter Λ: (Anti-)de Sitter symmetries

✦ **Quantum** deformations of the Poincaré algebra

Governed by the quantum deformation parameter κ



Deforming the Poincaré algebra of special relativity

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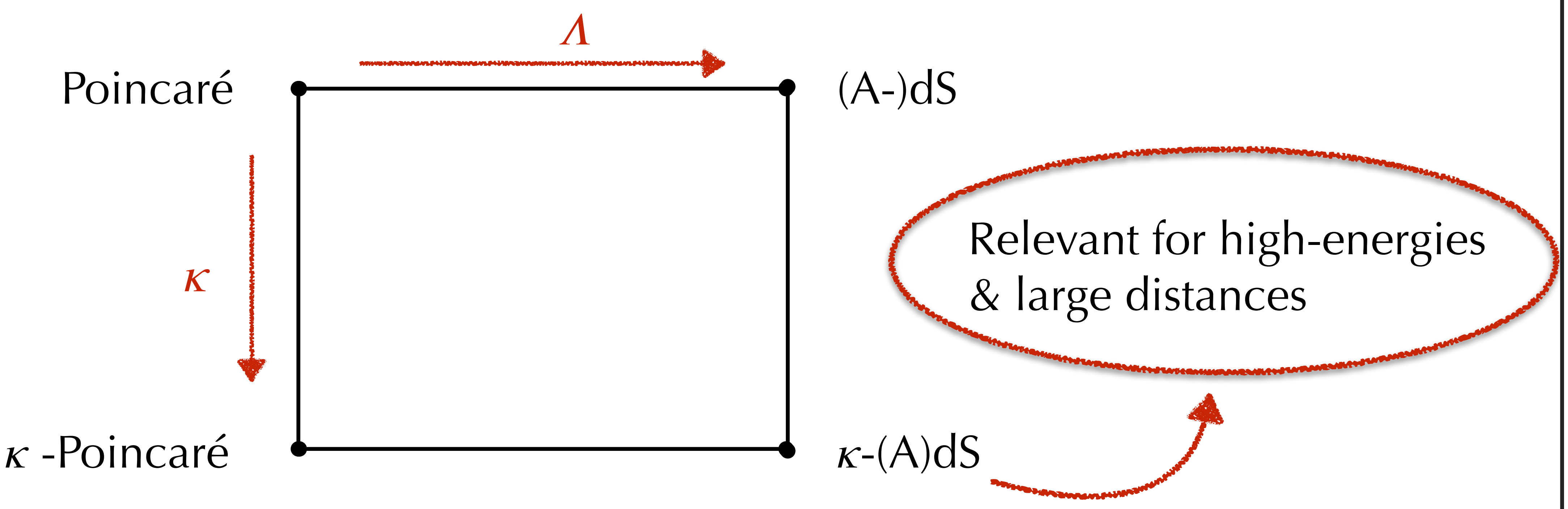

Classical+Quantum deformation of the Poincaré algebra: κ -(A)dS

✦ **Classical+Quantum** deformations of the Poincaré algebra

$$\left[\eta \equiv -\Lambda^2\right]$$

The 3+1 κ-(A)dS (Poisson) algebra is generated by the r-matrix

$$r_\Lambda = \frac{1}{\kappa}(K_1 \wedge P_1 + K_2 \wedge P_2 + K_3 \wedge P_3 + \eta J_1 \wedge J_2)$$

Which produces the cocommutators via $\delta(X) = [X \otimes 1 + 1 \otimes X, r_\Lambda]$, $X \in$ (A)dS

$$\delta(P_0) = \delta(J_3) = 0, \qquad \delta(J_1) = \frac{\eta}{\kappa} J_1 \wedge J_3, \qquad \delta(J_2) = \frac{\eta}{\kappa} J_2 \wedge J_3,$$

$$\delta(P_1) = \frac{1}{\kappa}(P_1 \wedge P_0 - \eta P_3 \wedge J_1 - \eta^2 K_2 \wedge J_3 + \eta^2 K_3 \wedge J_2),$$

$$\delta(P_2) = \frac{1}{\kappa}(P_2 \wedge P_0 - \eta P_3 \wedge J_2 + \eta^2 K_1 \wedge J_3 - \eta^2 K_3 \wedge J_1),$$

$$\delta(P_3) = \frac{1}{\kappa}(P_3 \wedge P_0 + \eta P_1 \wedge J_1 + \eta P_2 \wedge J_2 - \eta^2 K_1 \wedge J_2 + \eta^2 K_2 \wedge J_1),$$

$$\delta(K_1) = \frac{1}{\kappa}(K_1 \wedge P_0 + P_2 \wedge J_3 - P_3 \wedge J_2 - \eta K_3 \wedge J_1),$$

$$\delta(K_2) = \frac{1}{\kappa}(K_2 \wedge P_0 - P_1 \wedge J_3 + P_3 \wedge J_1 - \eta K_3 \wedge J_2),$$

$$\delta(K_3) = \frac{1}{\kappa}(K_3 \wedge P_0 + P_1 \wedge J_2 - P_2 \wedge J_1 + \eta K_1 \wedge J_1 + \eta K_2 \wedge J_2).$$

The r-matrix is unique if one asks that P_0 has trivial coproduct

•Ballesteros, Herranz, Musso Naranjo, PLB 2017

Classical+Quantum deformation of the Poincaré algebra: κ -(A)dS

- **Classical+Quantum** deformations of the Poincaré algebra

$$[J_a, J_b] = \epsilon_{abc} J_c \qquad [K_a, P_0] = P_a \qquad [P_0, P_a] = 0$$
$$[J_a, P_b] = \epsilon_{abc} P_c \qquad [K_a, P_b] = \delta_{ab} P_0 \qquad [P_a, P_b] = 0$$
$$[J_a, K_b] = \epsilon_{abc} K_c \qquad [K_a, K_b] = -\epsilon_{abc} J_c \qquad [P_0, J_a] = 0$$

$$\mathscr{C} = P_0^2 - \mathbf{P}^2$$

$\downarrow$ $\Lambda + \kappa$

Translations sector:
noncommutativity is induced by curvature, nonlinearity by quantum deformation

$$\{P_1, P_2\} = -\eta^2 \frac{\sinh(2\frac{\eta}{\kappa} J_3)}{2\eta/\kappa} - \frac{\eta}{2\kappa}\left(2P_3^2 + \eta^2(J_1^2 + J_2^2)\right) - \frac{\eta^5}{4\kappa^3} e^{-2\frac{\eta}{\kappa}J_3}\left(J_1^2 + J_2^2\right)^2$$

$$\{P_1, P_3\} = \frac{1}{2}\eta^2 J_2 \left(1 + e^{-2\frac{\eta}{\kappa}J_3}\left[1 + \frac{\eta^2}{\kappa^2}\left(J_1^2 + J_2^2\right)\right]\right) + \frac{\eta}{\kappa} P_2 P_3$$

$$\{P_2, P_3\} = -\frac{1}{2}\eta^2 J_1 \left(1 + e^{-2\frac{\eta}{\kappa}J_3}\left[1 + \frac{\eta^2}{\kappa^2}\left(J_1^2 + J_2^2\right)\right]\right) - \frac{\eta}{\kappa} P_1 P_3$$

$$\left[\eta \equiv -\Lambda^2\right]$$

Classical+Quantum deformation of the Poincaré algebra: κ -(A)dS

✦ **Classical+Quantum** deformations of the Poincaré algebra

$$[J_a, J_b] = \epsilon_{abc} J_c \qquad [K_a, P_0] = P_a \qquad [P_0, P_a] = 0$$
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$$\mathscr{C} = P_0^2 - \mathbf{P}^2$$

↓ $\Lambda + \kappa$

$$\left[\eta \equiv -\Lambda^2\right]$$

Rotations sector:

$$\{J_1, J_2\} = \frac{e^{2\frac{\eta}{\kappa}J_3} - 1}{2\eta/\kappa} - \frac{\eta}{2\kappa}\left(J_1^2 + J_2^2\right), \qquad \{J_1, J_3\} = -J_2, \qquad \{J_2, J_3\} = J_1$$

$$\Delta(J_3) = J_3 \otimes 1 + 1 \otimes J_3 \qquad \Delta(J_1) = J_1 \otimes e^{\frac{\eta}{\kappa}J_3} + 1 \otimes J_1 \qquad \Delta(J_2) = J_2 \otimes e^{\frac{\eta}{\kappa}J_3} + 1 \otimes J_2$$

Deformed rotation sector! This is a pure "$\Lambda + \kappa$" effect and only present in 3+1D

Classical+Quantum effects on particle propagation

✦ The interplay of curvature and quantum deformation can be seen at the level of redshift and worldlines in the 1+1 case (3+1 case is more complicated because of the nontrivial rotation sector)

the dispersion relation and the worldlines show explicitly the interplay, already at first order in the deformation parameters:

$\left[H = \sqrt{\Lambda}\right]$

$[\ell \equiv 1/\kappa]$

$$p_0 = -p_1\left(1 - Hx^0 - \ell p_1\left(\frac{1}{2} - Hx^0\right)\right)$$

$$x^1 - \bar{x}^1 = (x^0 - \bar{x}^0)(1 - \ell p_1) - \frac{1}{2}H\left((x^0)^2 - (\bar{x}^0)^2\right)(1 - 2\ell p_1)$$

these features are inherited by observable properties, such as the time delay in the travel time of photons with different energies and the energy redshift of a photon traveling between far away observers:

$$\Delta x^0 = \ell a^0 \Delta p_0(1 + Ha^0)$$

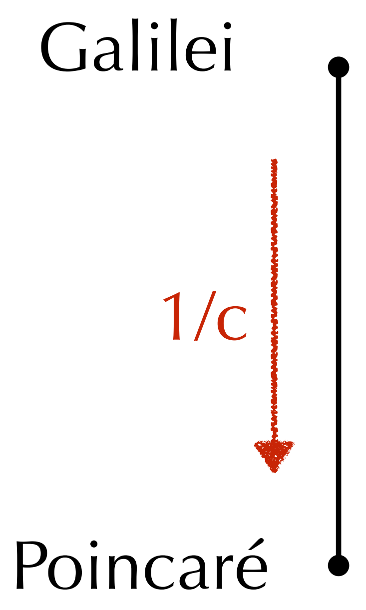
$$\Delta p_0 = -Hp_0x^0(1 + \frac{\ell}{2}p_0)$$

• *Barcaroli, Gubitosi, PRD 2016*

Speed-of-light contractions of the Poincaré algebra

✦ **Galilean** contraction (speed/space contraction) $1/c \to 0$

Small-velocities/large time intervals
' Absolute time': the light cone opens along t=0, spatial leaves at constant time

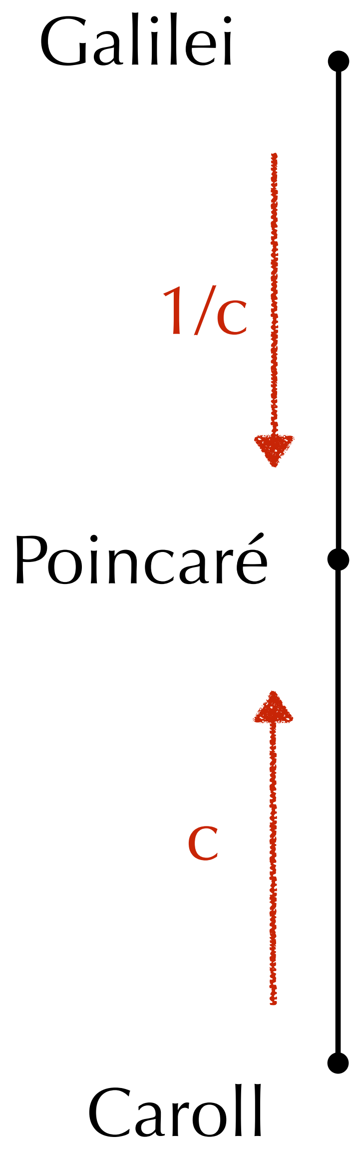


•*Bacry, Lévy-Leblond, J. Math. Phys. 1968*
•*Lévy-Leblond, Ann. Inst. Henry Poincaré, 1965*

Speed-of-light contractions of the Poincaré algebra

✦ **Carroll** contraction (speed/time contraction) $c \to 0$

Large space intervals
' Absolute space': the light cone closes along t, time leaves at constant space



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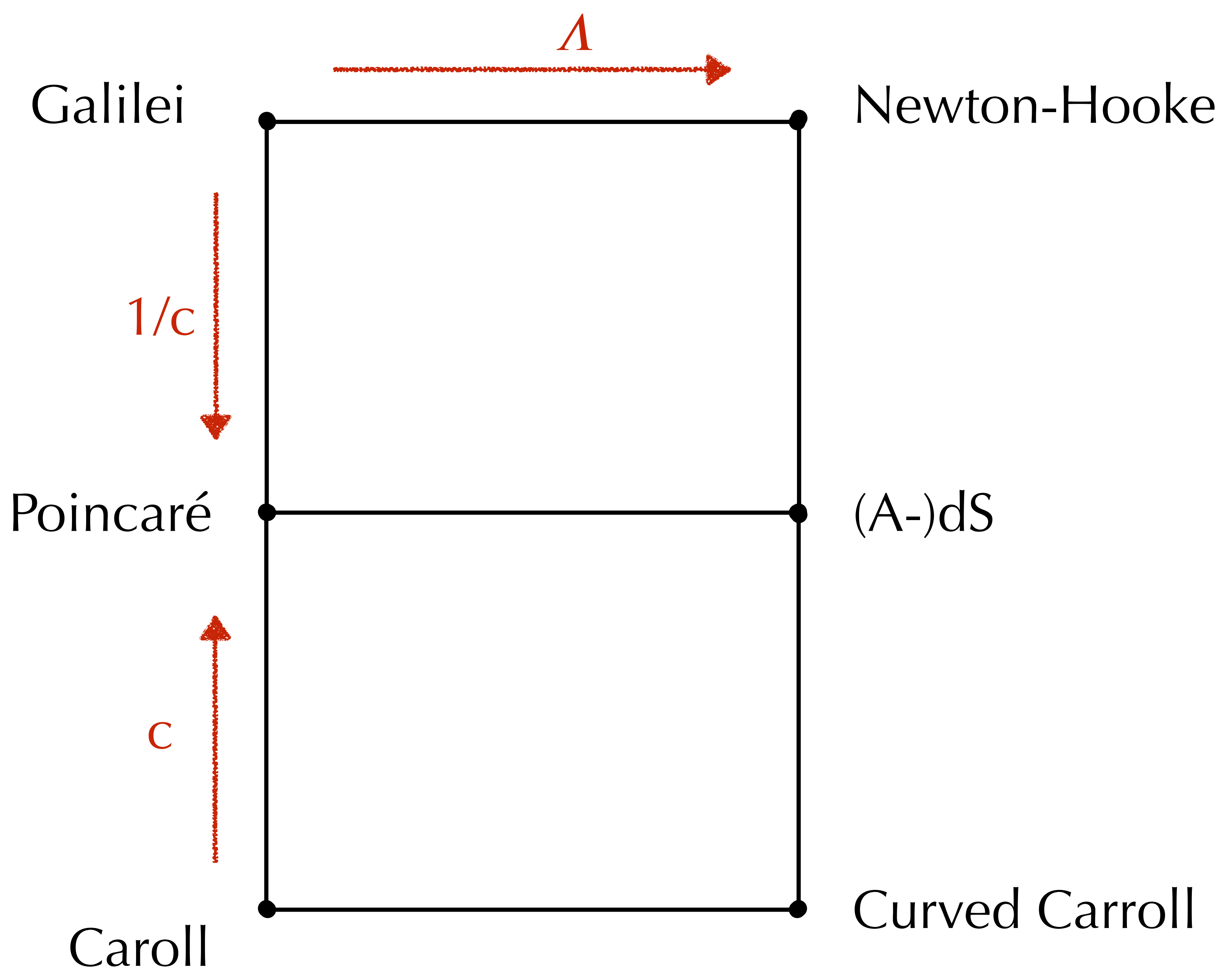
Speed-of-light contractions of the (A)dS algebra

✦ **Galilei and Carroll** contractions can be performed also in the presence of curvature



Speed-of-light contractions of the (A)dS algebra

- **Galilei and Carroll** contractions can be performed also in the presence of curvature

  - Galilei: Inönü–Wigner contraction, induced by the algebra automorphism $\mathscr{P}(P_0, P_a, K_a, J_a) = (P_0, -P_a, -K_a, J_a)$ (speed-space contraction)

    This allows us to perform the rescaling $P_a = \frac{\bar{P}_a}{c}, K_a = \frac{\bar{K}_a}{c}$

$$[K_a, P_b] = \delta_{ab}\frac{1}{c^2}P_0$$
$$[K_a, K_b] = -\epsilon_{abc}\frac{1}{c^2}J_c$$
$$[P_a, P_b] = \Lambda\epsilon_{abc}\frac{1}{c^2}J_c$$

$\xrightarrow{1/c \to 0}$

$$[K_a, P_b] = 0$$
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$$[P_a, P_b] = 0$$

$$\mathscr{C} = P_0^2 - c^2\mathbf{P}^2 - \Lambda\left(\mathbf{J}^2 - c^2\mathbf{K}^2\right) \xrightarrow{1/c \to 0} \mathscr{C} = \mathbf{P}^2 - \Lambda\,\mathbf{K}^2$$

  - Newtonian spacetimes $\mathbf{N}_\Lambda^{3+1}$ are obtained as the coset space

$$\mathbf{N}_\Lambda^{3+1} = \mathrm{N}_\Lambda/\mathrm{H}, \qquad \mathrm{H} = \mathrm{ISO}(3) = \{\mathbf{K}, \mathbf{J}\}$$

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$$
\begin{aligned}
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\left[P_a, P_b\right] &= \Lambda\epsilon_{abc}\frac{1}{c^2}J_c
\end{aligned}
\quad \xrightarrow{1/c \to 0} \quad
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$\xrightarrow{c \to 0}$

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This allows us to perform the rescaling $P_0 = c\bar{P}_0\,, K_a = c\bar{K}_a$

$$\begin{aligned}[K_a, K_b] &= -\epsilon_{abc}c^2 J_c \\ [K_a, P_0] &= c^2 P_a \\ [P_a, P_b] &= \Lambda \epsilon_{abc} J_c\end{aligned} \quad \xrightarrow{c \to 0} \quad \begin{aligned}[K_a, K_b] &= 0 \\ [K_a, P_0] &= 0 \\ [P_a, P_b] &= \Lambda \epsilon_{abc} J_c\end{aligned}$$

$$\mathcal{C} = \frac{1}{c^2}P_0^2 - \mathbf{P}^2 - \Lambda\left(\mathbf{J}^2 - \frac{1}{c^2}\mathbf{K}^2\right) \quad \xrightarrow{c \to 0} \quad \mathcal{C} = {P_0}^2 + \Lambda\,\mathbf{K}^2$$

✦ Carrollian spacetimes $\mathbf{C}_\Lambda^{3+1}$ are obtained as the homogeneous space

$$\mathbf{C}_\Lambda^{3+1} = \mathrm{C}_\Lambda/\mathrm{H}, \qquad \mathrm{H} = \mathrm{ISO}(3) = \{\mathbf{K}, \mathbf{J}\}$$

Speed-of-light contractions of the (A)dS algebra

✦ **Galilei and Carroll** contractions can be performed also in the presence of curvature

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$$\xrightarrow{c \to 0}$$

$$[K_a, K_b] = 0 \qquad [K_a, P_0] = 0 \qquad [P_a, P_b] = \Lambda \epsilon_{abc} J_c$$

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$$

Speed-of-light contractions of the (A)dS algebra

	Galilean limit	(A)dS	Carrollian limit
$[J_a, J_b]$		$\epsilon_{abc}J_c$	
$[J_a, P_b]$		$\epsilon_{abc}P_c$	
$[J_a, K_b]$		$\epsilon_{abc}K_c$	
$[J_a, P_0]$		0	
$[K_a, K_b]$	0	$-\epsilon_{abc}J_c$	0
$[K_a, P_b]$	0	$\delta_{ab}P_0$	$\delta_{ab}P_0$
$[K_a, P_0]$	P_a	P_a	0
$[P_a, P_b]$	0	$\Lambda\epsilon_{abc}J_c$	$\Lambda\epsilon_{abc}J_c$
$[P_a, P_0]$	ΛK_a	ΛK_a	ΛK_a

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$[J_a, K_b]$		$\epsilon_{abc} K_c$	
$[J_a, P_0]$		0	
$[K_a, K_b]$	0	$-\epsilon_{abc} J_c$	0
$[K_a, P_b]$	0	$\delta_{ab} P_0$	$\delta_{ab} P_0$
$[K_a, P_0]$	P_a	P_a	0
$[P_a, P_b]$	0	$\Lambda \epsilon_{abc} J_c$	$\Lambda \epsilon_{abc} J_c$
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Nonrelativistic boost

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$[K_a, K_b]$	0	$-\epsilon_{abc}J_c$	0
$[K_a, P_b]$	0	$\delta_{ab}P_0$	$\delta_{ab}P_0$
$[K_a, P_0]$	P_a	P_a	0
$[P_a, P_b]$	0	$\Lambda\epsilon_{abc}J_c$	$\Lambda\epsilon_{abc}J_c$
$[P_a, P_0]$	ΛK_a	ΛK_a	ΛK_a

Absolute time

Nonrelativistic boost

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$[J_a, P_0]$		0	
$[K_a, K_b]$	0	$-\epsilon_{abc}J_c$	0
$[K_a, P_b]$	0	$\delta_{ab}P_0$	$\delta_{ab}P_0$
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$[K_a, K_b]$	0	$-\epsilon_{abc}J_c$	0
$[K_a, P_b]$	0	$\delta_{ab}P_0$	$\delta_{ab}P_0$
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Nonrelativistic boost

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Absolute space

Flat spatial slices even when curvature is nonzero

Speed-of-light contractions of the (A)dS algebra

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$[J_a, P_b]$	—	$\epsilon_{abc}P_c$	—
$[J_a, K_b]$	—	$\epsilon_{abc}K_c$	—
$[J_a, P_0]$	—	0	—
$[K_a, K_b]$	0	$-\epsilon_{abc}J_c$	0
$[K_a, P_b]$	0	$\delta_{ab}P_0$	$\delta_{ab}P_0$
$[K_a, P_0]$	P_a	P_a	0
$[P_a, P_b]$	0	$\Lambda\epsilon_{abc}J_c$	$\Lambda\epsilon_{abc}J_c$
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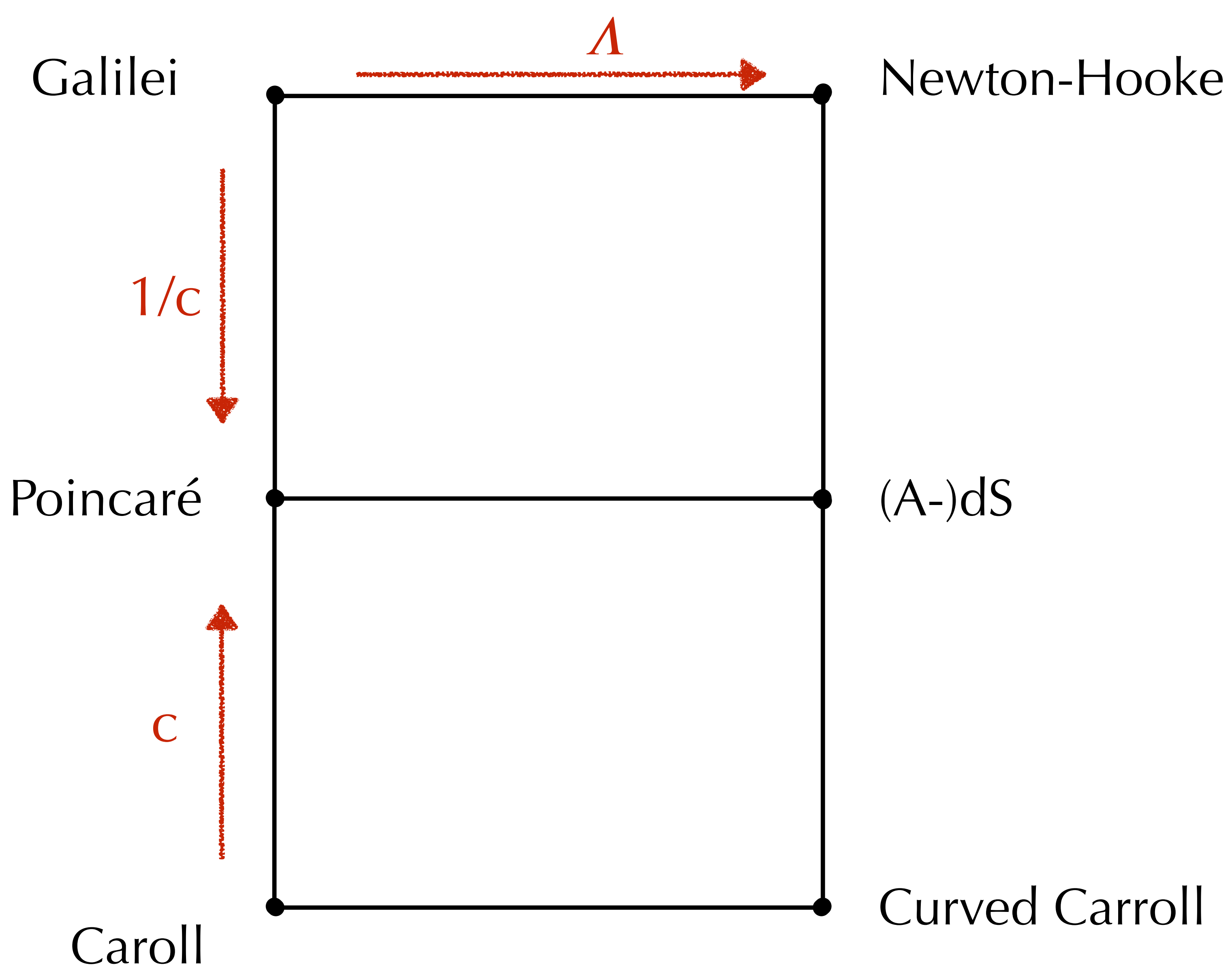
✦ Can we import this classical intuition into the realm of quantum symmetries?

Possible relativistic models

- One can consider quantum deformations of each of these classical relativistic models



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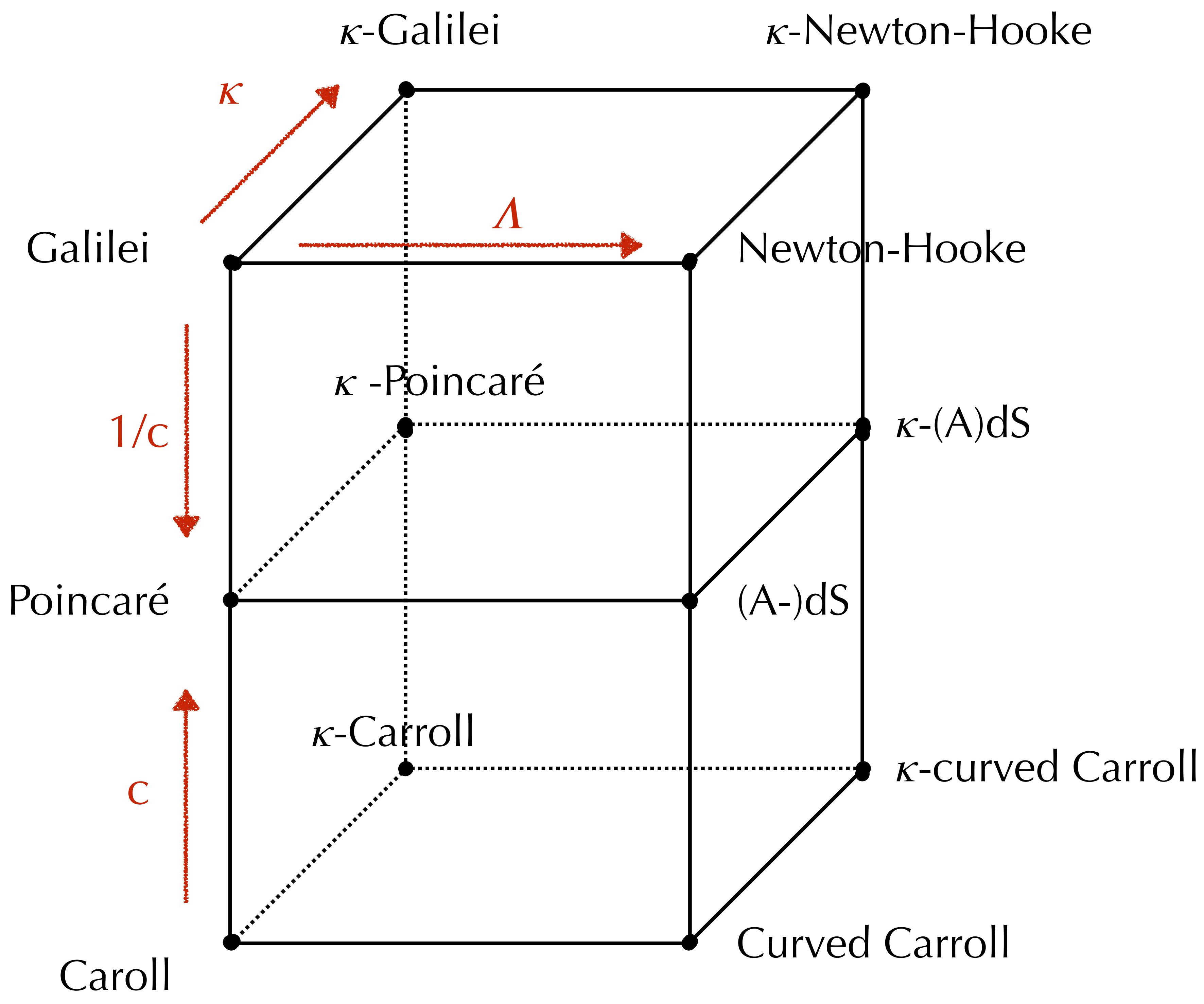

Speed-of-light contraction of quantum-deformed algebras

- The easiest way to work out the Galilean and Carrollian quantum algebras is via contraction of the κ-(A)dS algebra

- This contraction might require a rescaling of the quantum deformation parameter, along with the generators, in order to obtain meaningful structures, either at the level of the r-matrix or at the level of the cocommutators

Galilei contraction (flat case):

Contraction at the level of the r-matrix $r = \frac{c^2}{\kappa}\left(K_1 \wedge P_1 + K_2 \wedge P_2 + K_3 \wedge P_3\right)$ requires a rescaling of κ, however this produces trivial cocommutators

Contraction can be performed directly at the level of the cocommutators and does not require to rescale κ

$$[K_a, P_b] = \frac{\delta_{ab}}{c^2}\left[\frac{\kappa}{2}\left(1 - e^{-2P_0/\kappa}\right) + c^2\frac{\mathbf{P}^2}{2\kappa}\right] - \frac{P_a P_b}{\kappa}$$

$$[K_a, K_b] = -\frac{\epsilon_{abc}}{c^2}J_c$$

$1/c \to 0$

$$[K_a, P_b] = \delta_{ab}\frac{\mathbf{P}^2}{2\kappa} - \frac{P_a P_b}{\kappa}$$

$$[K_a, K_b] = 0$$

$$\mathscr{C}_\kappa = \mathrm{e}^{P_0/\kappa}\,\mathbf{P}^2$$

The coproducts are the same as in κ-Poincaré

•Ballesteros, GG, Gutierrez-Sagredo, Herranz, PLB 2020
•Giller, Kosinski, Majewski, Maslanka, Kunz, PLB 1992

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$$[K_a, P_0] = P_a c^2$$
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$$[K_a, K_b] = -c^2\epsilon_{abc}J_c$$

$c \to 0$ ⟶

$$[K_a, P_0] = 0$$
$$[K_a, P_b] = \delta_{ab}\frac{\kappa}{2}\left(1 - e^{-2P_0/\kappa}\right)$$
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Speed-of-light contraction of quantum-deformed algebras

$$[\eta \equiv -\Lambda^2]$$

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$$\{J_1,J_2\} = \frac{e^{2\eta J_3/\kappa}-1}{2\eta/\kappa} - \frac{\eta}{2\kappa}\left(J_1^2+J_2^2\right), \qquad \{J_1,J_3\} = -J_2, \qquad \{J_2,J_3\} = J_1,$$

$$\begin{array}{lll}
\{J_1,P_1\} = \frac{\eta}{\kappa}J_1P_2, & \{J_1,P_2\} = P_3 - \frac{\eta}{\kappa}J_1P_1, & \{J_1,P_3\} = -P_2, \\
\{J_2,P_1\} = -P_3 + \frac{\eta}{\kappa}J_2P_2, & \{J_2,P_2\} = -\frac{\eta}{\kappa}J_2P_1, & \{J_2,P_3\} = P_1, \\
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\{K_a,P_0\} = P_a, & \{P_0,P_a\} = \eta^2K_a, & \{P_0,J_a\} = 0,
\end{array}$$

$$\begin{array}{ll}
\{K_1,P_1\} = \frac{1}{2\kappa}\left(P_2^2+P_3^2-P_1^2\right) + \frac{\eta^2}{2\kappa}\left(K_2^2+K_3^2-K_1^2\right), & \{P_1,K_3\} = \frac{1}{\kappa}\left(P_1P_3+\eta^2K_1K_3+\eta K_2P_3\right), \\
\{K_2,P_2\} = \frac{1}{2\kappa}\left(P_1^2+P_3^2-P_2^2\right) + \frac{\eta^2}{2\kappa}\left(K_1^2+K_3^2-K_2^2\right), & \{P_3,K_1\} = \frac{1}{\kappa}\left(P_1P_3+\eta^2K_1K_3-\eta P_2K_3\right), \\
\{K_3,P_3\} = \frac{1}{2\kappa}\left[(P_1+\eta K_2)^2+(P_2-\eta K_1)^2-P_3^2-\eta^2K_3^2\right], & \{P_2,K_3\} = \frac{1}{\kappa}\left(P_2P_3+\eta^2K_2K_3-\eta K_1P_3\right), \\
\{P_1,K_2\} = \frac{1}{\kappa}\left(P_1P_2+\eta^2K_1K_2-\eta P_3K_3\right), & \{P_3,K_2\} = \frac{1}{\kappa}\left(P_2P_3+\eta^2K_2K_3+\eta P_1K_3\right), \\
\{P_2,K_1\} = \frac{1}{\kappa}\left(P_1P_2+\eta^2K_1K_2+\eta P_3K_3\right), & \{K_a,K_b\} = -\frac{\eta}{\kappa}\epsilon_{abc}K_cK_3, \qquad \{P_a,P_b\} = -\frac{\eta}{\kappa}\epsilon_{abc}P_cP_3.
\end{array}$$

Casimir:
$$\mathcal{C}_\kappa = e^{P_0/\kappa}\left(\mathbf{P}^2+\eta^2\mathbf{K}^2\right)\left[\cosh(\eta J_3/\kappa) + \frac{\eta^2}{2\kappa^2}(J_1^2+J_2^2)e^{-\eta J_3/\kappa}\right]$$
$$-2\eta^2 e^{P_0/\kappa}\left[\frac{\sinh(\eta J_3/\kappa)}{\eta}R_3 + \frac{1}{\kappa}\left(J_1R_1+J_2R_2+\frac{\eta}{2\kappa}(J_1^2+J_2^2)R_3\right)e^{-\eta J_3/\kappa}\right]$$

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$$\{J_2,P_1\}=-P_3+\tfrac{\eta}{\kappa}J_2P_2, \qquad \{J_2,P_2\}=-\tfrac{\eta}{\kappa}J_2P_1, \qquad \{J_2,P_3\}=P_1,$$

$$\{J_3,P_1\}=P_2, \qquad \{J_3,P_2\}=-P_1, \qquad \{J_3,P_3\}=0,$$

$$\{J_1,K_1\}=\tfrac{\eta}{\kappa}J_1K_2, \qquad \{J_1,K_2\}=K_3-\tfrac{\eta}{\kappa}J_1K_1, \qquad \{J_1,K_3\}=-K_2,$$

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$$\{J_3,K_1\}=K_2, \qquad \{J_3,K_2\}=-K_1, \qquad \{J_3,K_3\}=0,$$

$$\{K_a,P_0\}=P_a, \qquad \{P_0,P_a\}=\eta^2K_a, \qquad \{P_0,J_a\}=0,$$

$$\{K_1,P_1\} = \frac{1}{2\kappa}\left(P_2^2+P_3^2-P_1^2\right)+\frac{\eta^2}{2\kappa}\left(K_2^2+K_3^2-K_1^2\right), \qquad \{P_1,K_3\} = \frac{1}{\kappa}\left(P_1P_3+\eta^2K_1K_3+\eta K_2P_3\right),$$

$$\{K_2,P_2\} = \frac{1}{2\kappa}\left(P_1^2+P_3^2-P_2^2\right)+\frac{\eta^2}{2\kappa}\left(K_1^2+K_3^2-K_2^2\right), \qquad \{P_3,K_1\} = \frac{1}{\kappa}\left(P_1P_3+\eta^2K_1K_3-\eta P_2K_3\right),$$

$$\{K_3,P_3\} = \frac{1}{2\kappa}\left[(P_1+\eta K_2)^2+(P_2-\eta K_1)^2-P_3^2-\eta^2K_3^2\right], \qquad \{P_2,K_3\} = \frac{1}{\kappa}\left(P_2P_3+\eta^2K_2K_3-\eta K_1P_3\right),$$

$$\{P_1,K_2\} = \frac{1}{\kappa}\left(P_1P_2+\eta^2K_1K_2-\eta P_3K_3\right), \qquad \{P_3,K_2\} = \frac{1}{\kappa}\left(P_2P_3+\eta^2K_2K_3+\eta P_1K_3\right),$$

$$\{P_2,K_1\} = \frac{1}{\kappa}\left(P_1P_2+\eta^2K_1K_2+\eta P_3K_3\right), \qquad \{K_a,K_b\} = -\frac{\eta}{\kappa}\epsilon_{abc}K_cK_3, \qquad \{P_a,P_b\} = -\frac{\eta}{\kappa}\epsilon_{abc}P_cP_3.$$

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$$\mathcal{C}_\kappa = e^{P_0/\kappa}\left(\mathbf{P}^2+\eta^2\mathbf{K}^2\right)\left[\cosh(\eta J_3/\kappa)+\frac{\eta^2}{2\kappa^2}(J_1^2+J_2^2)e^{-\eta J_3/\kappa}\right]$$
$$-2\eta^2 e^{P_0/\kappa}\left[\frac{\sinh(\eta J_3/\kappa)}{\eta}R_3+\frac{1}{\kappa}\left(J_1R_1+J_2R_2+\frac{\eta}{2\kappa}(J_1^2+J_2^2)R_3\right)e^{-\eta J_3/\kappa}\right]$$

The coproducts are the same as in κ-(A)dS

•*Ballesteros, GG, Gutierrez-Sagredo, Herranz, PLB 2020*

Speed-of-light contraction of quantum-deformed algebras

$[\eta \equiv -\Lambda^2]$

✦ Galilei contraction of the κ-(A)dS algebra

$$
\begin{array}{lll}
\{J_1,J_2\} = \dfrac{e^{2\eta J_3/\kappa}-1}{2\eta/\kappa} - \dfrac{\eta}{2\kappa}\left(J_1^2+J_2^2\right), & \{J_1,J_3\} = -J_2, & \{J_2,J_3\} = J_1, \\
\{J_1,P_1\} = \frac{\eta}{\kappa}J_1P_2, & \{J_1,P_2\} = P_3 - \frac{\eta}{\kappa}J_1P_1, & \{J_1,P_3\} = -P_2, \\
\{J_2,P_1\} = -P_3 + \frac{\eta}{\kappa}J_2P_2, & \{J_2,P_2\} = -\frac{\eta}{\kappa}J_2P_1, & \{J_2,P_3\} = P_1, \\
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\{K_a,P_0\} = P_a, & \{P_0,P_a\} = \eta^2 K_a, & \{P_0,J_a\} = 0,
\end{array}
$$

$$
\begin{array}{ll}
\{K_1,P_1\} = \dfrac{1}{2\kappa}\left(P_2^2+P_3^2-P_1^2\right) + \dfrac{\eta^2}{2\kappa}\left(K_2^2+K_3^2-K_1^2\right), & \{P_1,K_3\} = \dfrac{1}{\kappa}\left(P_1P_3+\eta^2K_1K_3+\eta K_2P_3\right), \\
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$$-2\eta^2 e^{P_0/\kappa}\left[\frac{\sinh(\eta J_3/\kappa)}{\eta}R_3 + \frac{1}{\kappa}\left(J_1R_1+J_2R_2+\frac{\eta}{2\kappa}(J_1^2+J_2^2)R_3\right)e^{-\eta J_3/\kappa}\right]$$

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$$
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$$

$$
\begin{aligned}
&\{K_1,P_1\} = \frac{1}{2\kappa}\left(P_2^2+P_3^2-P_1^2\right) + \frac{\eta^2}{2\kappa}\left(K_2^2+K_3^2-K_1^2\right), \qquad \{P_1,K_3\} = \frac{1}{\kappa}\left(P_1P_3+\eta^2K_1K_3+\eta K_2P_3\right),\\
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$$
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\mathcal{C}_\kappa = {} & e^{P_0/\kappa}\left(\mathbf{P}^2+\eta^2\mathbf{K}^2\right)\left[\cosh(\eta J_3/\kappa) + \frac{\eta^2}{2\kappa^2}(J_1^2+J_2^2)e^{-\eta J_3/\kappa}\right]\\
&-2\eta^2 e^{P_0/\kappa}\left[\frac{\sinh(\eta J_3/\kappa)}{\eta}R_3 + \frac{1}{\kappa}\left(J_1R_1+J_2R_2+\frac{\eta}{2\kappa}(J_1^2+J_2^2)R_3\right)e^{-\eta J_3/\kappa}\right]
\end{aligned}
$$

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•*Ballesteros, GG, Gutierrez-Sagredo, Herranz, PLB 2020*

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$$
\begin{aligned}
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&-2\eta^2 e^{P_0/\kappa}\left[\frac{\sinh(\eta J_3/\kappa)}{\eta}R_3 + \frac{1}{\kappa}\left(J_1R_1+J_2R_2+\frac{\eta}{2\kappa}(J_1^2+J_2^2)R_3\right)e^{-\eta J_3/\kappa}\right]
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✦ Galilei contraction of the κ-(A)dS algebra

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$$\{K_2,P_2\} = \frac{1}{2\kappa}\left(P_1^2+P_3^2-P_2^2\right) + \frac{\eta^2}{2\kappa}\left(K_1^2+K_3^2-K_2^2\right),$$

$$\{K_3,P_3\} = \frac{1}{2\kappa}\left[(P_1+\eta K_2)^2 + (P_2-\eta K_1)^2 - P_3^2 - \eta^2 K_3^2\right],$$

$$\{P_1,K_2\} = \frac{1}{\kappa}\left(P_1P_2 + \eta^2 K_1K_2 - \eta P_3K_3\right),$$

$$\{P_2,K_1\} = \frac{1}{\kappa}\left(P_1P_2 + \eta^2 K_1K_2 + \eta P_3K_3\right),$$

$$\{P_1,K_3\} = \frac{1}{\kappa}\left(P_1P_3 + \eta^2 K_1K_3 + \eta K_2P_3\right),$$

$$\{P_3,K_1\} = \frac{1}{\kappa}\left(P_1P_3 + \eta^2 K_1K_3 - \eta P_2K_3\right),$$

$$\{P_2,K_3\} = \frac{1}{\kappa}\left(P_2P_3 + \eta^2 K_2K_3 - \eta K_1P_3\right),$$

$$\{P_3,K_2\} = \frac{1}{\kappa}\left(P_2P_3 + \eta^2 K_2K_3 + \eta P_1K_3\right),$$

$$\{K_a,K_b\} = -\frac{\eta}{\kappa}\epsilon_{abc}K_cK_3, \qquad \{P_a,P_b\} = -\frac{\eta}{\kappa}\epsilon_{abc}P_cP_3.$$

Casimir: $$\mathcal{C}_\kappa = e^{P_0/\kappa}\left(\mathbf{P}^2+\eta^2\mathbf{K}^2\right)\left[\cosh(\eta J_3/\kappa) + \frac{\eta^2}{2\kappa^2}(J_1^2+J_2^2)e^{-\eta J_3/\kappa}\right]$$
$$-2\eta^2 e^{P_0/\kappa}\left[\frac{\sinh(\eta J_3/\kappa)}{\eta}R_3 + \frac{1}{\kappa}\left(J_1R_1 + J_2R_2 + \frac{\eta}{2\kappa}(J_1^2+J_2^2)R_3\right)e^{-\eta J_3/\kappa}\right]$$

The coproducts are the same as in κ-(A)dS

•*Ballesteros, GG, Gutierrez-Sagredo, Herranz, PLB 2020*

Speed-of-light contraction of quantum-deformed algebras

$[\eta \equiv -\Lambda^2]$

- Galilei contraction of the κ-(A)dS algebra

$$\{J_1,J_2\} = \frac{e^{2\eta J_3/\kappa}-1}{2\eta/\kappa} - \frac{\eta}{2\kappa}\left(J_1^2+J_2^2\right), \qquad \{J_1,J_3\} = -J_2, \qquad \{J_2,J_3\} = J_1,$$

$$\begin{array}{lll}
\{J_1,P_1\} = \frac{\eta}{\kappa}J_1P_2, & \{J_1,P_2\} = P_3 - \frac{\eta}{\kappa}J_1P_1, & \{J_1,P_3\} = -P_2, \\
\{J_2,P_1\} = -P_3 + \frac{\eta}{\kappa}J_2P_2, & \{J_2,P_2\} = -\frac{\eta}{\kappa}J_2P_1, & \{J_2,P_3\} = P_1, \\
\{J_3,P_1\} = P_2, & \{J_3,P_2\} = -P_1, & \{J_3,P_3\} = 0, \\
\{J_1,K_1\} = \frac{\eta}{\kappa}J_1K_2, & \{J_1,K_2\} = K_3 - \frac{\eta}{\kappa}J_1K_1, & \{J_1,K_3\} = -K_2, \\
\{J_2,K_1\} = -K_3 + \frac{\eta}{\kappa}J_2K_2, & \{J_2,K_2\} = -\frac{\eta}{\kappa}J_2K_1, & \{J_2,K_3\} = K_1, \\
\{J_3,K_1\} = K_2, & \{J_3,K_2\} = -K_1, & \{J_3,K_3\} = 0, \\
\{K_a,P_0\} = P_a, & \{P_0,P_a\} = \eta^2K_a, & \{P_0,J_a\} = 0,
\end{array}$$

$$\begin{array}{ll}
\{K_1,P_1\} = \frac{1}{2\kappa}\left(P_2^2+P_3^2-P_1^2\right) + \frac{\eta^2}{2\kappa}\left(K_2^2+K_3^2-K_1^2\right), & \{P_1,K_3\} = \frac{1}{\kappa}\left(P_1P_3+\eta^2K_1K_3+\eta K_2P_3\right), \\
\{K_2,P_2\} = \frac{1}{2\kappa}\left(P_1^2+P_3^2-P_2^2\right) + \frac{\eta^2}{2\kappa}\left(K_1^2+K_3^2-K_2^2\right), & \{P_3,K_1\} = \frac{1}{\kappa}\left(P_1P_3+\eta^2K_1K_3-\eta P_2K_3\right), \\
\{K_3,P_3\} = \frac{1}{2\kappa}\left[(P_1+\eta K_2)^2+(P_2-\eta K_1)^2-P_3^2-\eta^2K_3^2\right], & \{P_2,K_3\} = \frac{1}{\kappa}\left(P_2P_3+\eta^2K_2K_3-\eta K_1P_3\right), \\
\{P_1,K_2\} = \frac{1}{\kappa}\left(P_1P_2+\eta^2K_1K_2-\eta P_3K_3\right), & \{P_3,K_2\} = \frac{1}{\kappa}\left(P_2P_3+\eta^2K_2K_3+\eta P_1K_3\right), \\
\{P_2,K_1\} = \frac{1}{\kappa}\left(P_1P_2+\eta^2K_1K_2+\eta P_3K_3\right), & \{K_a,K_b\} = -\frac{\eta}{\kappa}\epsilon_{abc}K_cK_3, \qquad \{P_a,P_b\} = -\frac{\eta}{\kappa}\epsilon_{abc}P_cP_3.
\end{array}$$

Casimir:
$$\mathcal{C}_\kappa = e^{P_0/\kappa}\left(\mathbf{P}^2+\eta^2\mathbf{K}^2\right)\left[\cosh(\eta J_3/\kappa) + \frac{\eta^2}{2\kappa^2}(J_1^2+J_2^2)e^{-\eta J_3/\kappa}\right]$$
$$-2\eta^2 e^{P_0/\kappa}\left[\frac{\sinh(\eta J_3/\kappa)}{\eta}R_3 + \frac{1}{\kappa}\left(J_1R_1+J_2R_2+\frac{\eta}{2\kappa}(J_1^2+J_2^2)R_3\right)e^{-\eta J_3/\kappa}\right]$$

The coproducts are the same as in κ-(A)dS

•*Ballesteros, GG, Gutierrez-Sagredo, Herranz, PLB 2020*

Speed-of-light contraction of quantum-deformed algebras

✦ Carroll contraction of the κ-(A)dS algebra

$$[\eta \equiv -\Lambda^2]$$

$$\begin{aligned}
&\{J_a, J_b\} = \epsilon_{abc} J_c, && \{J_a, P_b\} = \epsilon_{abc} P_c, && \{J_a, K_b\} = \epsilon_{abc} K_c, \\
&\{K_a, P_0\} = 0, && \{K_a, K_b\} = 0, && \{P_0, J_a\} = 0, \\
&\{P_0, P_a\} = \eta^2 K_a, && \{P_a, P_b\} = -\eta^2 \epsilon_{abc} J_c, &&
\end{aligned}$$

$$\{K_a, P_b\} = \delta_{ab} \left(\frac{1 - \mathrm{e}^{-2P_0/\kappa}}{2/\kappa} + \frac{\eta^2}{2\kappa} \mathbf{K}^2 \right) - \frac{\eta^2}{\kappa} K_a K_b.$$

Casimir: $$\mathcal{C}_\kappa = 2\kappa^2 \left(\cosh(P_0/\kappa) - 1\right) - \eta^2 \mathrm{e}^{P_0/\kappa} \mathbf{K}^2.$$

Coproducts:

$$\Delta(P_0) = P_0 \otimes 1 + 1 \otimes P_0, \qquad \Delta(J_a) = J_a \otimes 1 + 1 \otimes J_a,$$

$$\Delta(P_a) = P_a \otimes 1 + \mathrm{e}^{-P_0/\kappa} \otimes P_a - \frac{\eta^2}{\kappa} \epsilon_{abc} K_b \otimes J_c,$$

$$\Delta(K_a) = K_a \otimes 1 + \mathrm{e}^{-P_0/\kappa} \otimes K_a,$$

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Speed-of-light contraction of quantum-deformed algebras

✦ Carroll contraction of the κ-(A)dS algebra

$$[\eta \equiv -\Lambda^2]$$

$$\{J_a, J_b\} = \epsilon_{abc} J_c, \qquad \{J_a, P_b\} = \epsilon_{abc} P_c, \qquad \{J_a, K_b\} = \epsilon_{abc} K_c,$$
$$\{K_a, P_0\} = 0, \qquad \{K_a, K_b\} = 0, \qquad \{P_0, J_a\} = 0,$$
$$\{P_0, P_a\} = \eta^2 K_a, \qquad \{P_a, P_b\} = -\eta^2 \epsilon_{abc} J_c,$$
$$\{K_a, P_b\} = \delta_{ab} \left(\frac{1 - \mathrm{e}^{-2P_0/\kappa}}{2/\kappa} + \frac{\eta^2}{2\kappa} \mathbf{K}^2 \right) - \frac{\eta^2}{\kappa} K_a K_b.$$

Casimir: $$\mathcal{C}_\kappa = 2\kappa^2 \left(\cosh(P_0/\kappa) - 1\right) - \eta^2 \mathrm{e}^{P_0/\kappa} \mathbf{K}^2.$$

Coproducts:
$$\Delta(P_0) = P_0 \otimes 1 + 1 \otimes P_0, \qquad \Delta(J_a) = J_a \otimes 1 + 1 \otimes J_a,$$
$$\Delta(P_a) = P_a \otimes 1 + \mathrm{e}^{-P_0/\kappa} \otimes P_a - \frac{\eta^2}{\kappa} \epsilon_{abc} K_b \otimes J_c,$$
$$\Delta(K_a) = K_a \otimes 1 + \mathrm{e}^{-P_0/\kappa} \otimes K_a,$$

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Speed-of-light contraction of quantum-deformed algebras

✦ Carroll contraction of the κ-(A)dS algebra $\left[\eta \equiv -\Lambda^2\right]$

$$
\begin{aligned}
&\{J_a, J_b\} = \epsilon_{abc} J_c, \qquad \{J_a, P_b\} = \epsilon_{abc} P_c, \qquad \{J_a, K_b\} = \epsilon_{abc} K_c, \\
&\{K_a, P_0\} = 0, \qquad \{K_a, K_b\} = 0, \qquad \{P_0, J_a\} = 0, \\
&\{P_0, P_a\} = \eta^2 K_a, \qquad \{P_a, P_b\} = -\eta^2 \epsilon_{abc} J_c, \\
&\{K_a, P_b\} = \delta_{ab} \left(\frac{1 - e^{-2P_0/\kappa}}{2/\kappa} + \frac{\eta^2}{2\kappa} \mathbf{K}^2 \right) - \frac{\eta^2}{\kappa} K_a K_b.
\end{aligned}
$$

Casimir: $$\mathcal{C}_\kappa = 2\kappa^2 \left(\cosh(P_0/\kappa) - 1\right) - \eta^2 e^{P_0/\kappa} \mathbf{K}^2.$$

Coproducts:

$$
\begin{aligned}
&\Delta(P_0) = P_0 \otimes 1 + 1 \otimes P_0, \qquad \Delta(J_a) = J_a \otimes 1 + 1 \otimes J_a, \\
&\Delta(P_a) = P_a \otimes 1 + e^{-P_0/\kappa} \otimes P_a - \frac{\eta^2}{\kappa} \epsilon_{abc} K_b \otimes J_c, \\
&\Delta(K_a) = K_a \otimes 1 + e^{-P_0/\kappa} \otimes K_a,
\end{aligned}
$$

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Speed-of-light contraction of quantum-deformed algebras

✦ Carroll contraction of the κ-(A)dS algebra

$$[\eta \equiv -\Lambda^2]$$

$$\{J_a, J_b\} = \epsilon_{abc}J_c, \qquad \{J_a, P_b\} = \epsilon_{abc}P_c, \qquad \{J_a, K_b\} = \epsilon_{abc}K_c,$$
$$\{K_a, P_0\} = 0, \qquad \{K_a, K_b\} = 0, \qquad \{P_0, J_a\} = 0,$$
$$\{P_0, P_a\} = \eta^2 K_a, \qquad \{P_a, P_b\} = -\eta^2 \epsilon_{abc} J_c,$$
$$\{K_a, P_b\} = \delta_{ab}\left(\frac{1-\mathrm{e}^{-2P_0/\kappa}}{2/\kappa} + \frac{\eta^2}{2\kappa}\mathbf{K}^2\right) - \frac{\eta^2}{\kappa}K_a K_b.$$

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$$\mathcal{C}_\kappa = 2\kappa^2\left(\cosh(P_0/\kappa) - 1\right) - \eta^2 \mathrm{e}^{P_0/\kappa}\,\mathbf{K}^2.$$

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$$\Delta(P_0) = P_0 \otimes 1 + 1 \otimes P_0, \qquad \Delta(J_a) = J_a \otimes 1 + 1 \otimes J_a,$$
$$\Delta(P_a) = P_a \otimes 1 + \mathrm{e}^{-P_0/\kappa} \otimes P_a - \frac{\eta^2}{\kappa}\epsilon_{abc} K_b \otimes J_c,$$
$$\Delta(K_a) = K_a \otimes 1 + \mathrm{e}^{-P_0/\kappa} \otimes K_a,$$

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Speed-of-light contraction of quantum-deformed algebras

✦ Carroll contraction of the κ-(A)dS algebra $\quad [\eta \equiv -\Lambda^2]$

$$\{J_a, J_b\} = \epsilon_{abc} J_c, \qquad \{J_a, P_b\} = \epsilon_{abc} P_c, \qquad \{J_a, K_b\} = \epsilon_{abc} K_c,$$
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$$\{K_a, P_b\} = \delta_{ab} \left(\frac{1 - \mathrm{e}^{-2P_0/\kappa}}{2/\kappa} + \frac{\eta^2}{2\kappa} \mathbf{K}^2 \right) - \frac{\eta^2}{\kappa} K_a K_b.$$

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$$\mathcal{C}_\kappa = 2\kappa^2 \left(\cosh(P_0/\kappa) - 1\right) - \eta^2 \mathrm{e}^{P_0/\kappa} \mathbf{K}^2.$$

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Speed-of-light contraction of quantum-deformed algebras

✦ Carroll contraction of the κ-(A)dS algebra $\left[\eta \equiv -\Lambda^2\right]$

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$$\{K_a, P_0\} = 0, \qquad \{K_a, K_b\} = 0, \qquad \{P_0, J_a\} = 0,$$
$$\{P_0, P_a\} = \eta^2 K_a, \qquad \{P_a, P_b\} = -\eta^2 \epsilon_{abc} J_c,$$
$$\{K_a, P_b\} = \delta_{ab} \left(\frac{1 - \mathrm{e}^{-2P_0/\kappa}}{2/\kappa} + \frac{\eta^2}{2\kappa} \mathbf{K}^2 \right) - \frac{\eta^2}{\kappa} K_a K_b.$$

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Speed-of-light contraction of quantum-deformed algebras

✦ Carroll contraction of the κ-(A)dS algebra

$$[\eta \equiv -\Lambda^2]$$

$$
\begin{array}{lll}
\{J_a, J_b\} = \epsilon_{abc} J_c, & \{J_a, P_b\} = \epsilon_{abc} P_c, & \{J_a, K_b\} = \epsilon_{abc} K_c, \\
\{K_a, P_0\} = 0, & \{K_a, K_b\} = 0, & \{P_0, J_a\} = 0, \\
\{P_0, P_a\} = \eta^2 K_a, & \{P_a, P_b\} = -\eta^2 \epsilon_{abc} J_c, &
\end{array}
$$

$$\{K_a, P_b\} = \delta_{ab} \left(\frac{1 - \mathrm{e}^{-2P_0/\kappa}}{2/\kappa} + \frac{\eta^2}{2\kappa} \mathbf{K}^2 \right) - \frac{\eta^2}{\kappa} K_a K_b.$$

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$$\Delta(K_a) = K_a \otimes 1 + \mathrm{e}^{-P_0/\kappa} \otimes K_a,$$

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Possible relativistic models

	(curved) κ-Galilei	κ-(A)dS	(curved) κ-Carroll
$[J_a, J_b]$	|	|	|
$[J_a, P_b]$	anisotropy $\sim \frac{\Lambda}{\kappa}$	anisotropy $\sim \frac{\Lambda}{\kappa}$	isotropy
$[J_a, K_b]$	|	|	|
$[J_a, P_0]$	—	0	—
$[K_a, K_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa})$	$-\epsilon_{abc}J_c + O(\frac{\sqrt{\Lambda}}{\kappa})$	0
$[K_a, P_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa}, \frac{1}{\kappa})$	$\delta_{ab}P_0 + O(\frac{\Lambda}{\kappa}, \frac{1}{\kappa})$	$\delta_{ab}P_0 + O(\frac{\Lambda}{\kappa}, \frac{1}{\kappa})$
$[K_a, P_0]$	P_a	P_a	0
$[P_a, P_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa})$	$\Lambda\epsilon_{abc}J_c + O(\frac{\sqrt{\Lambda}}{\kappa})$	$\Lambda\epsilon_{abc}J_c$
$[P_a, P_0]$	ΛK_a	ΛK_a	ΛK_a

The "$\Lambda + \kappa$" effects are milder in the Carroll models

Possible relativistic models

	(curved) κ-Galilei	κ-(A)dS	(curved) κ-Carroll
$[J_a, J_b]$	anisotropy $\sim \frac{\Lambda}{\kappa}$	anisotropy $\sim \frac{\Lambda}{\kappa}$	isotropy
$[J_a, P_b]$			
$[J_a, K_b]$			
$[J_a, P_0]$	0	0	0
$[K_a, K_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa})$	$-\epsilon_{abc}J_c + O(\frac{\sqrt{\Lambda}}{\kappa})$	0
$[K_a, P_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa}, \frac{1}{\kappa})$	$\delta_{ab}P_0 + O(\frac{\Lambda}{\kappa}, \frac{1}{\kappa})$	$\delta_{ab}P_0 + O(\frac{\Lambda}{\kappa}, \frac{1}{\kappa})$
$[K_a, P_0]$	P_a	P_a	0
$[P_a, P_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa})$	$\Lambda\epsilon_{abc}J_c + O(\frac{\sqrt{\Lambda}}{\kappa})$	$\Lambda\epsilon_{abc}J_c$
$[P_a, P_0]$	ΛK_a	ΛK_a	ΛK_a

Carroll contraction restores isotropy

The "$\Lambda + \kappa$" effects are milder in the Carroll models

Possible relativistic models

	(curved) κ-Galilei	κ-(A)dS	(curved) κ-Carroll
$[J_a, J_b]$			
$[J_a, P_b]$	anisotropy $\sim \frac{\Lambda}{\kappa}$	anisotropy $\sim \frac{\Lambda}{\kappa}$	isotropy
$[J_a, K_b]$			
$[J_a, P_0]$	0	0	0
$[K_a, K_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa})$	$-\epsilon_{abc}J_c + O(\frac{\sqrt{\Lambda}}{\kappa})$	0
$[K_a, P_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa}, \frac{1}{\kappa})$	$\delta_{ab}P_0 + O(\frac{\Lambda}{\kappa}, \frac{1}{\kappa})$	$\delta_{ab}P_0 + O(\frac{\Lambda}{\kappa}, \frac{1}{\kappa})$
$[K_a, P_0]$	P_a	P_a	0
$[P_a, P_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa})$	$\Lambda\epsilon_{abc}J_c + O(\frac{\sqrt{\Lambda}}{\kappa})$	$\Lambda\epsilon_{abc}J_c$
$[P_a, P_0]$	ΛK_a	ΛK_a	ΛK_a

Carroll contraction restores isotropy

Non-commuting boosts

The "$\Lambda + \kappa$" effects are milder in the Carroll models

Possible relativistic models

	(curved) κ-Galilei	κ-(A)dS	(curved) κ-Carroll
$[J_a, J_b]$			
$[J_a, P_b]$	anisotropy $\sim \frac{\Lambda}{\kappa}$	anisotropy $\sim \frac{\Lambda}{\kappa}$	isotropy
$[J_a, K_b]$			
$[J_a, P_0]$	0	0	0
$[K_a, K_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa})$	$-\epsilon_{abc}J_c + O(\frac{\sqrt{\Lambda}}{\kappa})$	0
$[K_a, P_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa}, \frac{1}{\kappa})$	$\delta_{ab}P_0 + O(\frac{\Lambda}{\kappa}, \frac{1}{\kappa})$	$\delta_{ab}P_0 + O(\frac{\Lambda}{\kappa}, \frac{1}{\kappa})$
$[K_a, P_0]$	P_a	P_a	0
$[P_a, P_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa})$	$\Lambda\epsilon_{abc}J_c + O(\frac{\sqrt{\Lambda}}{\kappa})$	$\Lambda\epsilon_{abc}J_c$
$[P_a, P_0]$	ΛK_a	ΛK_a	ΛK_a

Carroll contraction restores isotropy

Non-commuting boosts

No absolute time

The "$\Lambda + \kappa$" effects are milder in the Carroll models

Possible relativistic models

	(curved) κ-Galilei	κ-(A)dS	(curved) κ-Carroll
$[J_a, J_b]$			
$[J_a, P_b]$	anisotropy $\sim \frac{\Lambda}{\kappa}$	anisotropy $\sim \frac{\Lambda}{\kappa}$	isotropy
$[J_a, K_b]$			
$[J_a, P_0]$		0	
$[K_a, K_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa})$	$-\epsilon_{abc}J_c + O(\frac{\sqrt{\Lambda}}{\kappa})$	0
$[K_a, P_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa}, \frac{1}{\kappa})$	$\delta_{ab}P_0 + O(\frac{\Lambda}{\kappa}, \frac{1}{\kappa})$	$\delta_{ab}P_0 + O(\frac{\Lambda}{\kappa}, \frac{1}{\kappa})$
$[K_a, P_0]$	P_a	P_a	0
$[P_a, P_b]$	$O(\frac{\sqrt{\Lambda}}{\kappa})$	$\Lambda\epsilon_{abc}J_c + O(\frac{\sqrt{\Lambda}}{\kappa})$	$\Lambda\epsilon_{abc}J_c$
$[P_a, P_0]$	ΛK_a	ΛK_a	ΛK_a

Carroll contraction restores isotropy

Non-commuting boosts

No absolute time

Absolute space

The "$\Lambda + \kappa$" effects are milder in the Carroll models

Thanks!