Moduli stabilization at $h^{2,1}=50$

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Workshop on Holography and the Swampland

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This talk is based on ::

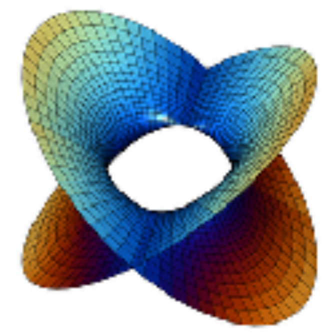
- *The tadpole conjecture at large complex-structure*
  E. Plauschinn
  arXiv:2109.00029
- *Moduli Stabilization in Asymptotic Flux Compactifications*
  T. Grimm, D. van de Heisteeg, E. Plauschinn
  arXiv:2110.05511
- *The tadpole conjecture in asymptotic limits*
  M. Graña, T. Grimm, D. van de Heisteeg, A. Herraez, E. Plauschinn
  arXiv:2204.05331
- *Moduli stabilization in type IIB orientifolds at* $h^{2,1}=50$
  E. Plauschinn, K. Tsagkaris
  arXiv:2207.13721

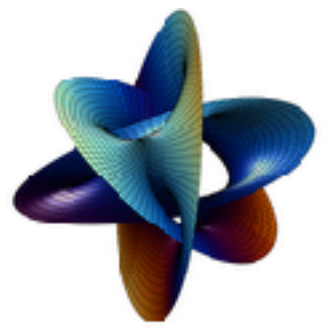
motivation

Some motivation.

motivation — the landscape

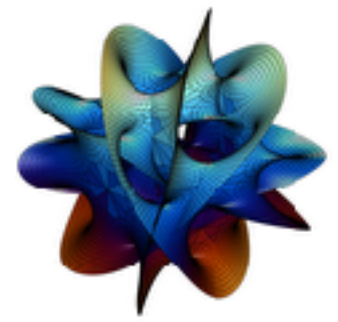
Compactifications of string theory give rise to an abundance of lower-dimensional theories — the string theory landscape.



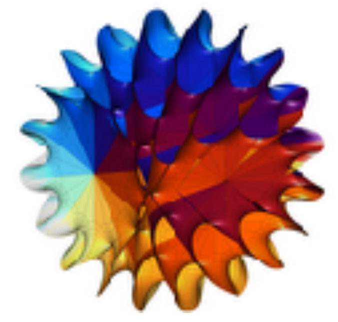


Famous estimates for its size are 10^{500}, 10^{930}, 10^{1500}, 10^{272000}.

Bousso, Polchinski — 2000
Schellekens — 2016
Lerche, Lüst, Schellekens — 1987
Taylor, Wang — 2015



But often one is only interested in four-dimensional theories with no or few massless scalar fields.

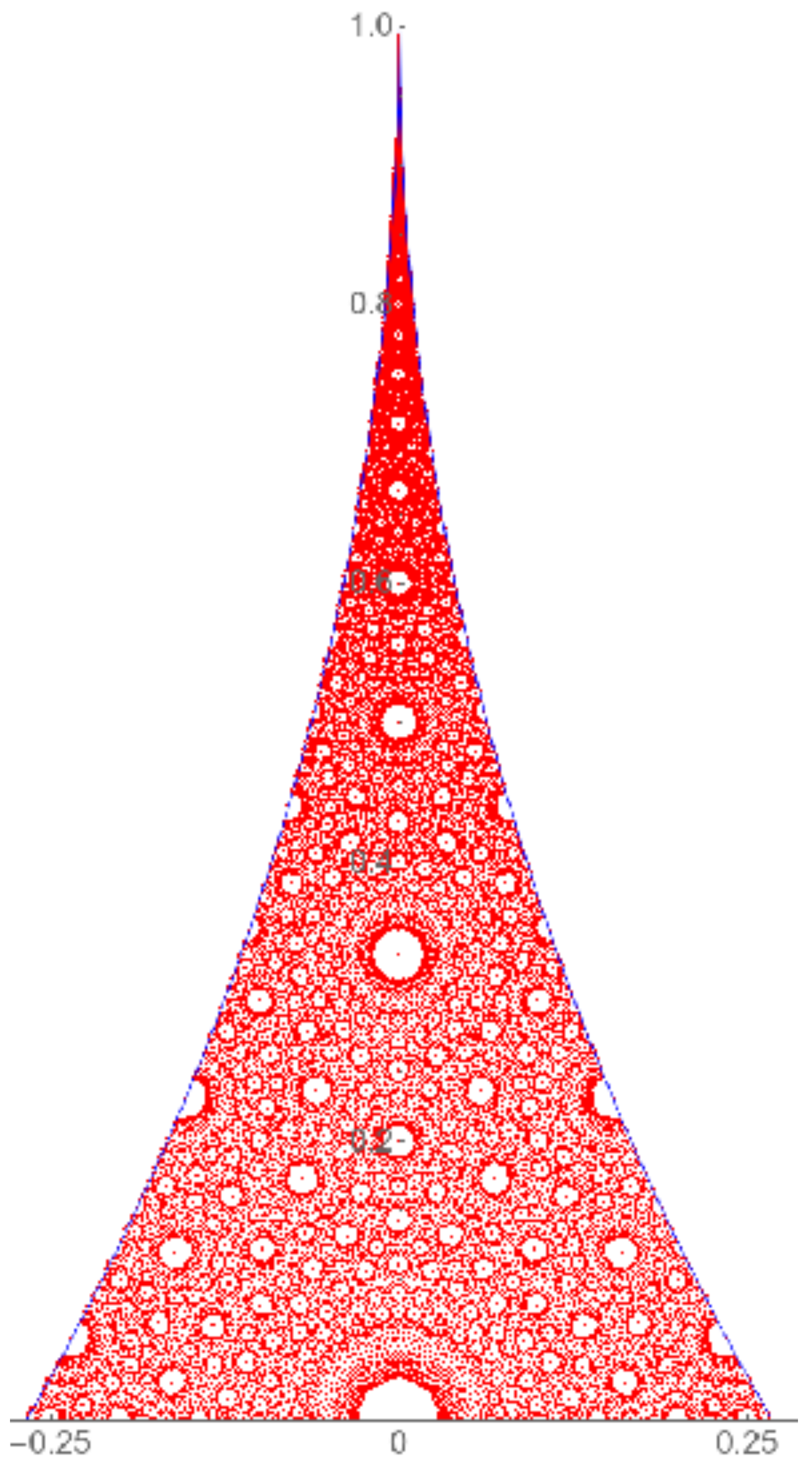


In type IIB orientifold compactifications on Calabi-Yau three-folds, fluxes generate a mass-term for the complex-structure and axio-dilaton moduli.

An underlying assumption of the KKLT and Large-Volume scenarios is that all of these moduli can be stabilized in a suitable regime.

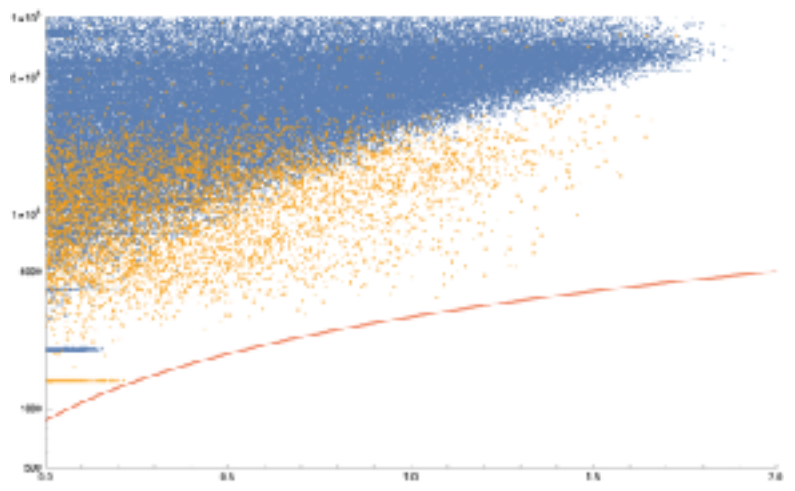
It became clear recently that this assumption can fail.

Bena, Dudas, Graña, Lüst — 2018
Betzler, EP — 2019
Braun, Valandro — 2020
Bena, Blabäck, Graña, Lüst - 2020
Junghans — 2022
Gao, Hebecker, Schreyer, Venken — 2022



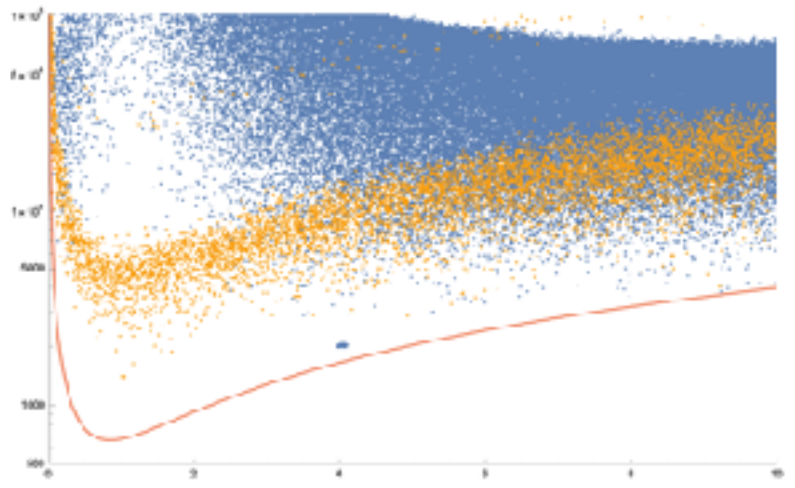
The tadpole conjecture states that for a large number of complex-structure moduli, not all of them can be consistently stabilized by fluxes.

Bena, Blåbäck, Graña, Lüst — 2020



Goal for this talk ::

- Review the status of the tadpole conjecture, and
- study it for a concrete example.



outline

1. motivation
2. tadpole conjecture
3. moduli stabilization
4. $h^{2,1}$=50
5. summary

Explanation of the tadpole conjecture.

D-branes and O-planes are charged under Ramond-Ramond gauge potentials and therefore contribute to Bianchi identities as sources. The integrated expressions are the tadpole cancellation conditions.

In F-theory, the D3-brane tadpole equation reads (with χ the Euler number of the four-fold)

$$N_{\mathrm{D3}} + \frac{N_{\mathrm{flux}}}{2} = \frac{\chi}{24} \qquad \xrightarrow{h^{3,1}\gg 1} \qquad \frac{N_{\mathrm{flux}}}{2} \leq \frac{h^{3,1}}{4}\,.$$

The tadpole conjecture states that in the large $h^{3,1}$-limit, the flux number satisfies

$$\frac{N_{\mathrm{flux}}}{2} > \frac{n_{\mathrm{stab}}}{3} \qquad \longrightarrow \qquad n_{\mathrm{stab}} < \frac{3}{4}\,h^{3,1}\,.$$

Bena, Blåbäck, Graña, Lüst — 2020

Comments ::

- The tadpole conjecture also applies to type IIB orientifold compactifications.
- If true, the conjecture implies that the landscape of theories with no massless scalar fields is smaller than expected.
- The KKLT and Large Volume scenarios propose mechanisms to obtain de-Sitter vacua in string theory. A crucial step is moduli stabilization by fluxes — the tadpole conjecture scrutinizes this step.

Status of the conjecture.

Status ::

- The conjecture has been verified for smooth F-theory compactifications on $K3 \times K3$.

  Bena, Blåbäck, Graña, Lüst — 2020 & 2021

- It has been verified for F-theory compactifications on CY_4 with a weak Fano base.

  Bena, Brodie, Graña — 2021

- It has been argued that for type IIB compactifications in the large-complex-structure limit the conjecture is satisfied.

  EP — 2021

- It has been argued that for F-theory compactifications in any asymptotic regime the conjecture is satisfied.

  Graña, Grimm, v.d. Heisteeg, Herraez, EP — 2022

- The conjecture has been verified for a type IIB compactification with $h^{2,1}_{-} = 50$.

  EP, Tsagkaris — 2022

Criticism ::

- A linear scenario that can violate the tadpole conjecture has been proposed — and opposed. Currently no corresponding explicit model is known.

Marchesano, Prieto, Wiesner — 2021
Lüst - 2021
Graña, Grimm, v.d. Heisteeg, Herraez, EP — 2021
EP, Tsagkaris — 2022

- The tadpole conjecture applies to smooth compactifications, but could be violated for spaces with singularities.

Bena, Blåbäck, Graña, Lüst — 2020
Gao, Hebecker, Schreyer, Venken — 2022

- A database of orientifold compactifications with large negative tadpole charge has been published. Though, currently no explicit model violating the conjecture is known.

Crinò, Quevedo, Schachner, Valandro — 2022
see also Junghans — 2022

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3. **moduli stabilization**
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Moduli stabilization in type IIB orientifolds.

Consider type IIB Calabi-Yau orientifold compactifications on $\mathcal{X}$ with O3-/O7-planes. The effective four-dimensional theory contains

$h^{2,1}_{-}$	complex-structure moduli	$z^i = u^i + i\,v^i\,,$
1	axio-dilaton	$\tau = c + i\,s\,,$
$h^{1,1}$	Kähler-sector moduli	$T_a\,,\; G_{\hat{a}}\,.$

The dynamics of the moduli fields (at weak string coupling) is described by the Kähler potential

$$\mathcal{K} = -\log\bigl[-i(\tau-\bar{\tau})\bigr] - \log\left[+i\int_{\mathcal{X}}\Omega\wedge\bar{\Omega}\right] - 2\log\mathcal{V}\,.$$

The holomorphic three-form depends on the complex-structure moduli z^i and can be expanded in an integral symplectic basis as $(I = 0, \ldots, h^{2,1}_-)$

$$\Omega = X^I \alpha_I - \mathcal{F}_I \beta^I \,, \qquad\qquad z^i = X^i / X^0 \,.$$

In the large-complex-structure limit the periods $\mathcal{F}_I = \partial_I \mathcal{F}$ of Ω can be determined from the prepotential

$$\mathcal{F} = \mathcal{F}_{\rm pert} + \mathcal{F}_{\rm inst} \,, \qquad\qquad \mathcal{F}_{\rm pert} = -\frac{1}{3!} \frac{\tilde{\kappa}_{ijk} X^i X^j X^k}{X^0} + \frac{1}{2!} a_{ij} X^i X^j + b_i X^i X^0 + \frac{1}{2!} c (X^0)^2 \,,$$

$$\mathcal{F}_{\rm inst} = -\frac{1}{(2\pi i)^3} (X^0)^2 \sum_{\vec{q}} N_{\vec{q}} \, \mathrm{Li}_3 \left(e^{2\pi i \, q_i X^i / X^0} \right) .$$

Three-form fluxes F_3 and H_3 generate a potential for the moduli z^i and τ . It is encoded in the superpotential

$$W = \int_{\mathcal{X}} \Omega \wedge (F_3 - \tau H_3)\,.$$

The scalar potential in the four-dimensional theory (for a no-scale Kähler potential) reads (with $\alpha = (\tau, z^i)$)

$$V = e^{\mathcal{K}} F_\alpha G^{\alpha\overline{\beta}}\,\overline{F}_{\overline{\beta}}\,,$$

$$F_\alpha = \partial_\alpha W + (\partial_\alpha \mathcal{K}) W\,,$$

$$G_{\alpha\overline{\beta}} = \partial_\alpha \partial_{\overline{\beta}} \mathcal{K}\,.$$

Global minima of this potential are given by $F_\alpha = 0$. These conditions can fix moduli.

Tadpole-cancellation condition.

D-branes and O-planes are charged under Ramond-Ramond gauge potentials and therefore contribute to Bianchi identities as sources. The integrated expressions are the tadpole cancellation conditions.

Relevant here is the D3-brane tadpole condition

$$0 = N_{\text{flux}} + 2\,N_{\text{D3}} + Q_{\text{D3}}\,.$$

$$Q_{\mathbf{D3}} < 0$$

$$Q_{\text{D3}} = -\frac{N_{\text{O3}}}{2} - \sum_{\text{D7}_\text{i}} \left[\int_{\Gamma_{\text{D7}_\text{i}}} \text{tr}\left[\mathsf{F}^2_{\text{D7}_\text{i}}\right] + N_{\text{D7}_\text{i}} \frac{\chi(\Gamma_{\text{D7}_\text{i}})}{12} \right] - \sum_{\text{O7}_\text{j}} \frac{\chi(\Gamma_{\text{O7}_\text{j}})}{6}$$

$$N_{\text{D3}} \geq 0$$

$$N_{\text{flux}} = \int_{\mathcal{X}} F_3 \wedge H_3 \geq 0$$

The tadpole problem.

Moduli stabilization ::

- Fluxes stabilize the axio-dilaton and complex-structure moduli.
- Fluxes are restricted by the tadpole cancellation condition.

Tadpole problem ::

- Can all moduli be stabilized by fluxes within the tadpole bound?

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$h^{2,1}=50$

The setting.

$h^{2,1}$=50 — setting & technical challenges

Consider an example with $h^{2,1}_{-} = 50$ complex-structure moduli (to which the tadpole conjecture applies).

The data of the (perturbative) prepotential in the large-complex-structure-limit is determined from an example in the Kreuzer-Skarke list using CYTools.

Demirtas, McAllister, Rios-Tascon

Numerical solutions to the minimum conditions are extremely difficult to find. A new algorithm has been developed and 10^5 solutions we constructed.

EP, Tsagkaris — 2022

Solutions to the minimum conditions are valid in the weak-string-coupling and large-complex-structure limit

$$\mathrm{Im}\,\tau = s \gg 1\,,$$

$$\frac{\mathcal{F}_{\mathrm{inst}\,I}}{\mathcal{F}_{\mathrm{pert}\,I}} \ll 1 \qquad \longrightarrow \qquad \mathrm{c} \gg 10^{-2}\,.$$

thanks to J. Moritz

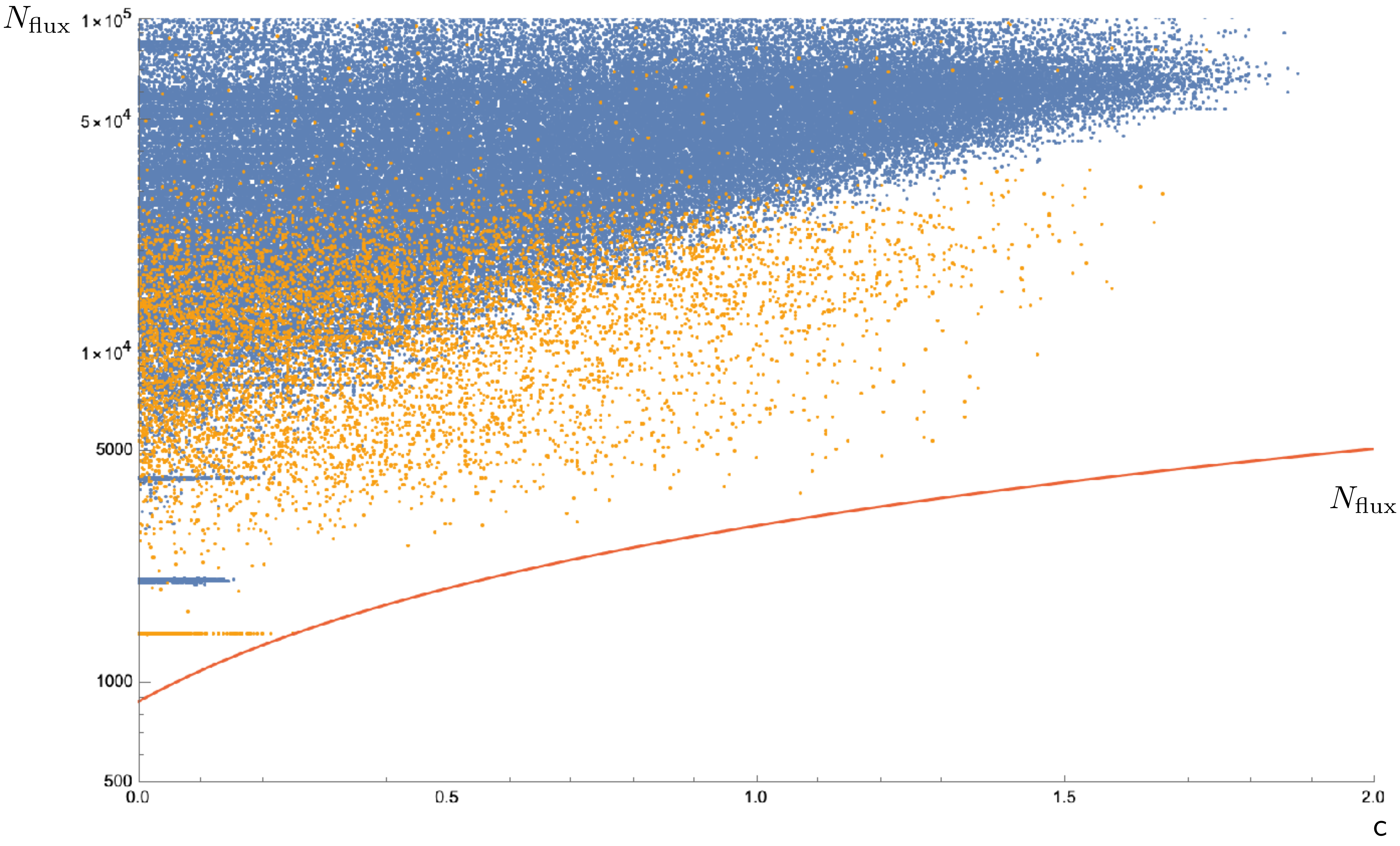
For $h^{2,1}_{-} = 50$ a bound on the orientifold charge Q_{D3} can be estimated as

$$Q_{\mathrm{D3}} \gtrsim -1000 \qquad \longrightarrow \qquad N_{\mathrm{flux}} \lesssim 1000\,.$$

Crinò, Quevedo, Schachner, Valandro — 2022

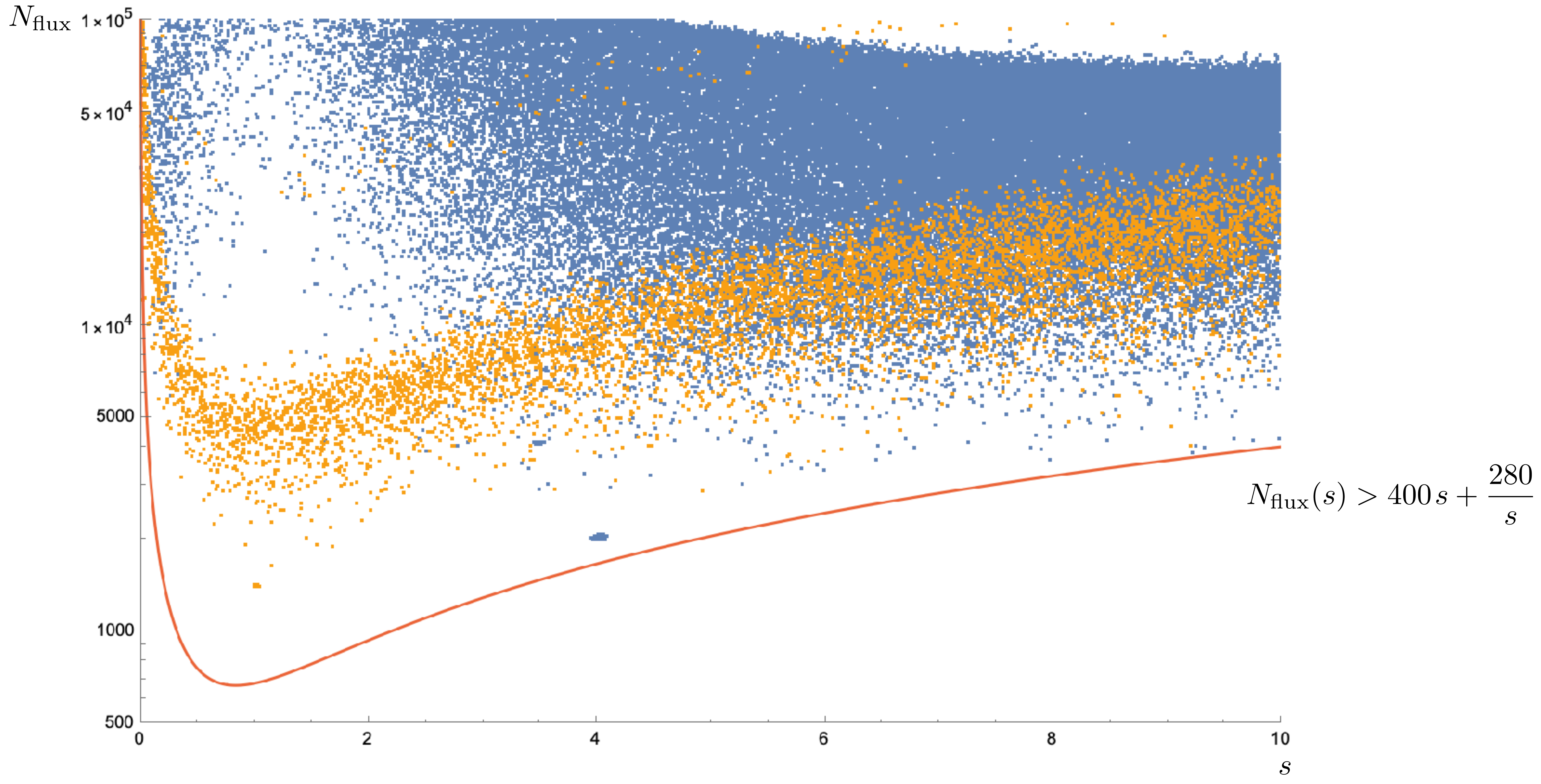
Results.

$h^{2,1}$=50 — results I



$N_{\rm flux}(\mathsf{c}) > 2100\,\mathsf{c} + 880$

$h^{2,1}$=50 — results II



$h^{2,1}$=50 — results III

	$s \geq 1$	$s \geq 2$	$s \geq 5$	$s \geq 10$
c $\geq 10^{-3}$	1400	1991	3023	6157
c $\geq 10^{-2}$	1400	1991	3023	6157
c $\geq 10^{-1}$	1405	1992	3304	6157
c ≥ 0.5	2993	3912	4886	13218
c ≥ 1	3717	5379	9345	21384

Smallest values for N_{flux} for given bounds on c and s.
Tadpole cancellation requires $N_{\text{flux}} \lesssim 1000$.

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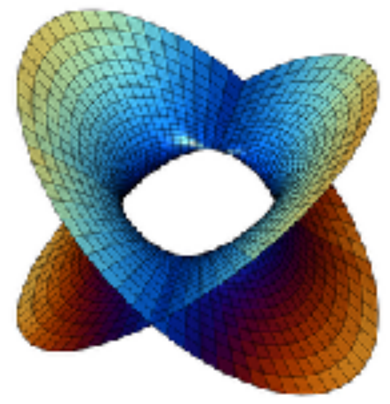
summary

Summary ::

- String-theory compactifications provide a large number effective four-dimensional theories — the string-theory landscape.
- The tadpole conjecture suggests that the landscape of theories with no/few massless scalar fields is smaller than expected.
- The tadpole conjecture has been verified for a concrete example with $h^{2,1}=50$. Technical challenges were addressed.



$$\frac{N_{\text{flux}}}{2} > \frac{n_{\text{stab}}}{3}$$