Graham Ross, Semi-Classical v Loops and Quantum Gravity

Martin Einhorn[1,2] Ian Jack[3] Tim Jones[3] Graham Ross[4]

[1]KITP
University of California, Santa Barbara

[2]University of Michigan

[3]Dept. of Mathematical Sciences
University of Liverpool

[4]Dept. of Theoretical Physics
University of Oxford

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Outline

First "Encounters"

In 1973 I was a postgraduate in Oxford. I was fortunate in that my allocated supervisor (who had not undergone “conversion to QFT and gauge theories”) went away on sabbatical and I persuaded John C. Taylor to take me on.

My first paper

"Gauge Theories and the Pion Mass Difference" ,

an attempt to generalise the successful calculation of this mass difference to inclusion of weak interactions in the newly popular gauge models was submitted in Nov. 1973. I then noticed this paper

“Neutral weak currents and the proton-neutron mass difference, A. Love, G.G. Ross, Nucl.Phys. B56 (1973) 440. ”

which I had intended to have a go at next. From then on I watched for papers by Graham in every preprint list!

Meanwhile asymptotic freedom had been discovered, and I embarked on my thesis calculation, which I then extended to the recently invented supersymmetric gauge theory, while a postdoc at Sussex. At that time considerable effort went into constructing models with Han-Nambu quarks, and ones that were asymptotically free but with the gauge symmetry spontaneously broken by the Higgs mechanism, in order to give the vector bosons masses. I recall for instance

“Scaling Behavior in a Class of Massive Nonabelian Gauge Theories", H.D. Politzer Graham G. Ross(Rutherford), Nucl.Phys.B 75 (1974) 269-284.

As the confinement paradigm took hold, I worked on lattice gauge theories while Graham remained faithful to the continuum and wrote a lot of excellent papers with a variety of collaborators.

Low Energy Supersymmetry

It was the development of low energy supersymmetry in 1981 which brought me back to mainstream gauge theory models, with the appearance of the paper

"Supersymmetry and the Scale of Unification", S. Dimopoulos, S. Raby, F. Wilczek, Phys.Rev.D 24 (1981) 1681-1683.

I was working with Marty Einhorn in 1981. We were surprised by the omission in this paper of contributions to the β-functions from the Higgs multiplet, thereby leaving the prediction of $\sin^2\theta_W$ unchanged from the non-susy case. We set out to rectify this, extend the RG analysis to two loops and calculate the $\frac{m_b}{m_\tau}$ mass ratio.

"The Weak Mixing Angle and Unification Mass in Supersymmetric SU(5)", M.B. Einhorn, D.R.T. Jones, Nucl.Phys.B 196 (1982) 475

Regarding the one-loop analysis, there appeared just before us:

"$SU(2)_L \otimes U(1)$ Breaking as a Radiative Effect of Supersymmetry Breaking in Guts" L. Ibanez, G.G. Ross, Phys. Lett. B110 (1982) 215,

part of a long series of significant papers by Graham on supersymmetric gauge theories.

In 1984 I was in Aspen with Green and Schwarz when they made the crucial discovery which led to string theory taking centre stage.

My contribution to string theory was made in 1982 and relates to the equation

$$196884 = 196883 + 1. \quad \text{(Robert Griess)}$$

Graham embraced the strings revolution and I studied his papers in particular hoping for insight. We sometimes worked on similar things, such as models of flavour but it was interest in Anomaly Mediation that brought us together.

Anomaly Mediation

I am not sure when we first met. I spent a year at CERN 2003-4, and he was there in a neighbouring office. It was then that we began to discuss joint interests. At the time Ian Jack and I had been working for some years on RG aspects of supersymmetric theories, including some exact results for β-functions in the case of Anomaly Mediated Supersymmetry Breaking (AMSB). With Graham we developed and pursued the phenomenology of an extension of the MSSM wherein the tachyonic slepton problem was resolved by the introduction of a Fayet-Iliopoulos (FI) term.

Anomaly mediation, Fayet-Iliopoulos D-terms and precision sparticle spectra, R. Hodgson, I. Jack, D.R.T. Jones, G.G. Ross, Nucl.Phys.B 728 (2005) 192

The soft supersymmetry breakings take the following characteristic form:

$$\begin{aligned} M_i &= m_0\beta_{g_i}/g_i \ \text{gaugino masses} \\ h_{t,b,\tau} &= -m_0\beta_{Y_{t,b,\tau}} \ \ \phi^3 \text{ couplings} \\ (m^2)^i{}_j &= \tfrac{1}{2}m_0^2\mu\tfrac{d}{d\mu}\gamma^i{}_j + kY_i\delta^i{}_j \ \ \phi^*\phi \text{ masses} \\ m_3^2 &= \kappa m_0\mu - m_0\beta_\mu \ \ H_1H_2 \text{ mass} \end{aligned} \tag{1}$$

where the FI-term contribution is indicated in red.
One might think that if the FI term is associated with an additional U_1' broken at some high scale M, then by the decoupling theorem, all effects of the U_1' would be suppressed at energies $E << M$ by powers of $1/M$. We showed that with a FI term this is not the case and it is quite natural for there to be $O(M_{\text{susy}})$ scalar mass contributions arising from the presence of the FI term, with $M >> M_{\text{susy}}$.

The sparticle mass scale is determined by the mass parameters m_0 and k. Requiring the U_1' to be anomaly free results in various interesting sum rules.

Anomaly Mediation and Dimensional Transmutation

Graham and I both felt that the introduction of a second scale (the FI term) to the anomaly mediated MSSM was an unattractive feature, though it works! It dawned on us that we could start with a *scale invariant MSSM* $\otimes U_1'$ model, with the scale of U_1' breaking arising by dimensional transmutation, and the only explicit terms of dimension two and three in the Lagrangian being those associated with AMSB. The U_1' breaking scale also determines the right-handed neutrino masses, which in turn determine the observable neutrino masses via the usual see-saw mechanism.

A brief account of this appears in

"Anomaly mediation and dimensional transmutation", D.R.T. Jones, G.G. Ross, Phys.Lett.B 642 (2006) 540.

We planned to follow up but but somehow we drifted off in different directions. Time to revisit this model perhaps?

I will return to Graham at the end of my talk.

Semi-Classical v Loops

In autumn 2019 we noticed the paper

Feynman diagrams and the large charge expansion in $3-\varepsilon$ dimensions, G. Badel, G. Cuomo, A. Monin, R. Rattazzi, Phys.Lett.B 802 (2020) 135202

and were impressed by the success of the comparison of semi-classical calculations (with which I was unfamiliar) with perturbative ones (with which I was). This paper carried out, in particular, a calculation in simple $\lambda\phi^4$ of the anomalous dimension of the operator ϕ^n to two loops. This struck us as fertile ground for Ian Jack and me to plough, though we did not begin until well into 2020.

Motivation: Many particle amplitudes are of obvious relevance to the LHC. Of course we would like to do quark and gluon amplitudes, but a lot can be learned from scalar theories, specifically scale invariant ones. The study of this in scale invariant scalar theories is also of interest in the Conformal Field theory context. There are three such theories with a ϕ^M interaction:

$\phi^4(d = 4), \phi^3(d = 6), \phi^6(d = 3)$

Approaches that extend the reach of (or even transcend the need for) perturbation theory have always been challenging, and are all the more interesting now because of the increased importance attached to multi-leg amplitudes, which can present formidable calculational obstacles at higher loop orders.
Another motivation for studying this class of theories is their (classical) scale invariance (CSI). As remarked in

O. Antipin, J. Bersini, F. Sannino, Z. Wang and C. Zhang,"Charging the $O(N)$ model," Phys. Rev. D102 (2020) 4, 045011,

the Standard Model (SM) is "almost" CSI. Indeed, in 1973, Coleman and Weinberg (CW) had hoped to argue that the SM might indeed be viable with the omission of the Higgs (wrong-sign) (mass)^2 term, with *dimensional transmutation* generating the physical mass scale in the CSI theory. This attractive idea failed. Neglecting Yukawa couplings led to a Higgs mass prediction which was too small; and including the top quark Yukawa coupling destabilised the Higgs vacuum altogether

There followed a series of papers

"Anomalous dimensions for ϕ^n in scale invariant $d = 3$ theory.", Phys. Rev. D **102**, 085012 (2020)

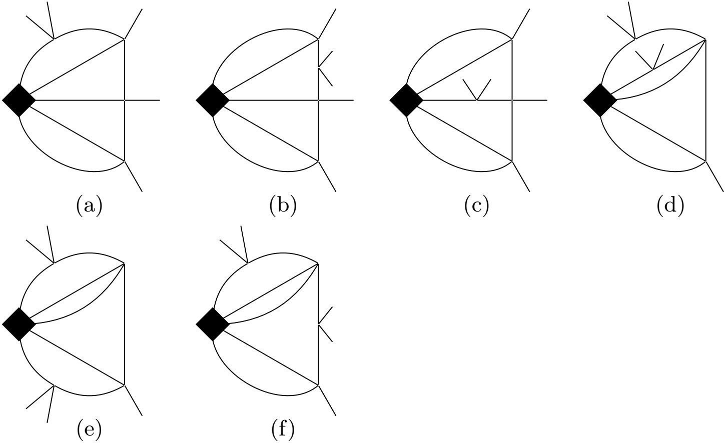
"Anomalous dimensions at large charge in $d = 4$ O(N) theory," Phys. Rev. D **103**, 085013 (2021)

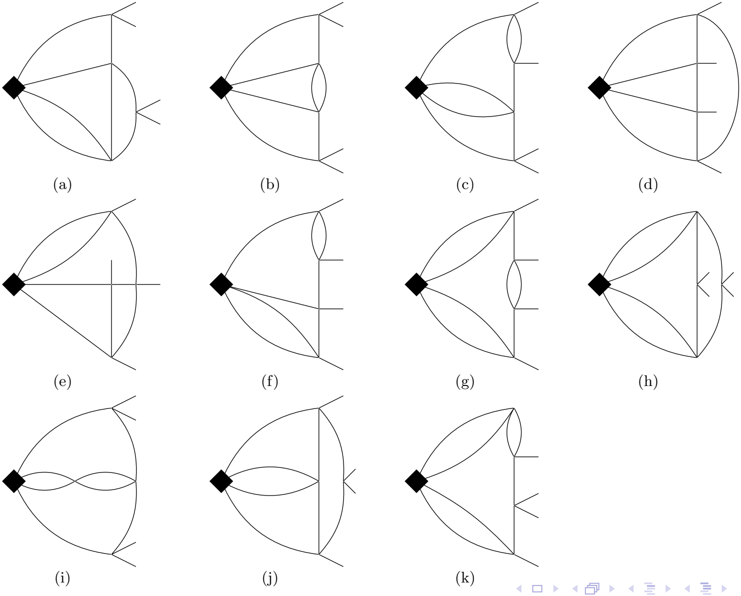
"Anomalous dimensions at large charge for $U(N) \otimes U(N)$ theory in three and four dimensions", Phys.Rev.D **104** (2021) 10, 105017

"Scaling dimensions at large charge for ϕ^3 theory in six dimensions", Phys.Rev.D **105** (2022) 4, 045021

We pushed some anomalous dimension calculations to higher loops and continued the generally successful comparison of these results with the semi-classical calculations.

As an example, I show the graphs responsible for the leading *n* and next to leading contributions to the anomalous dimension of the ϕ^n operator at four loops:

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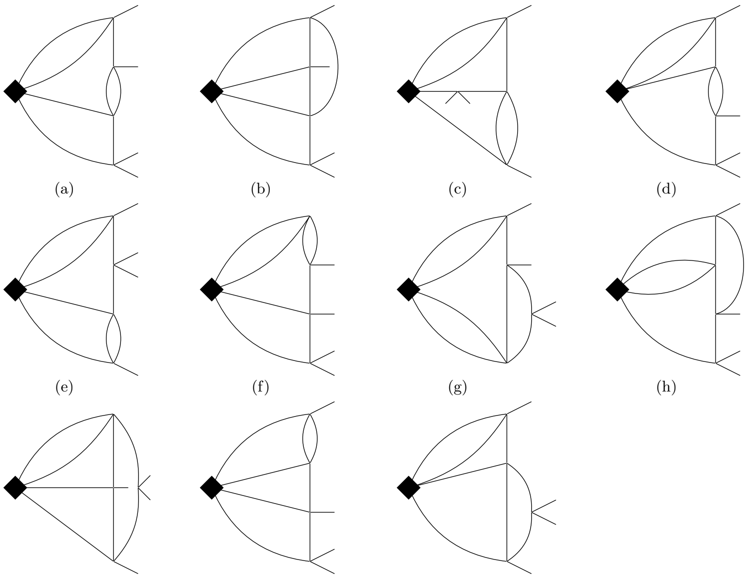

(a) (b) (c) (d) (e) (f) (g) (h) (i) (j) (k)

I will avoid a detailed description of our work. Suffice to say that, generally speaking our new results reinforced the agreement with the semi-classical calculations, where applicable.

It would be interesting to purse semi-classics and the comparison with perturbation theory for other scale invariant cases such as the Thirring model, the Wess-Zumino model or even QCD.

Scale Invariant Gravity

Marty Einhorn and I were very impressed by the paper

F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B 659 (2008) 703

with its use of the scale invariant $\xi R\phi^2$ term to generate inflation via a large value for ξ. We pursued the implementation of this idea in the context of supergravity,

"Inflation with Non-minimal Gravitational Couplings in Supergravity", M.B. Einhorn, D.R.T. Jones, JHEP 03 (2010) 026

Interest in the $\xi R\phi^2$ term got us interested in *scale invariant* quantum gravity, with the Einstein term $M_P^2 R$ is generated by a vev for a scalar field via dimensional transmutation (DT), much in the manner of the classic Coleman and Weinberg paper.

Scale Invariant Gravity

Scale Invariant Gravity takes the form

$$S = \int d^4x \sqrt{g} \left(M_P^2 R + \alpha R_{\mu\nu\rho\sigma}^2 + \beta R_{\mu\nu}^2 + \gamma R^2 \right) + S_{\text{matter}}$$

We explored coupling to scale invariant $\lambda\phi^4$ when the gravitational couplings can induce a vev for the scalar field, first for a single scalar field and then for scalar representations in gauge theories.

This led to a series of papers:

“Naturalness and Dimensional Transmutation in Classically Scale-Invariant Gravity,” JHEP **03** (2015) 047.

“Gauss-Bonnet coupling constant in classically scale-invariant gravity,” Phys. Rev. D 91 (2015) 8, 084039.
“Induced Gravity I: Real Scalar Field,” JHEP **01** (2016) 019.

“Induced Gravity II: Grand Unification,” JHEP **05** (2016) 185,

“Zero modes in de Sitter background,” JHEP **03** (2017) 144,

“Renormalizable, asymptotically free gravity without ghosts or tachyons", Phys.Rev. **D96** (2017) 12, 124025
“Grand Unified Theories in Renormalisable, Classically Scale Invariant Gravity", JHEP **10** (2019) 012

Outcomes

- Dimensionless transmutation can give a non zero $\langle\phi\rangle$ in a theory with scalar fields coupled to R^2 gravity, and hence generate an Einstein term in the “low energy” theory.
- In the simplest model, the attraction basin of the only UV stable FP does not include the region in which DT minima occur, so in this region the theory becomes strongly coupled or must be modified at high scales.
- More complicated models can remedy this, and also the nonzero $\langle\phi\rangle$ can break a Grand Unified symmetry. For $SO(12)$, in a region of parameter space, Dimensional Transmutation occurs, with the adjoint vacuum expectation value breaking $SO(12) \to SU(6) \otimes U(1)$, and producing a Low Energy Effective Theory having Einstein-Hilbert gravity. Certain minima are locally stable and lie within the catchment basin of the ultraviolet fixed points. The scenario may be compatible with a form of Higgs inflation.
- Problems: Unitarity, the electroweak scale, naturalness, ...

Unitarity and First Order Formalism

In the hope of gaining insight into the Unitarity issue we have been working on the first order (Palantini) formalism as appled to R^2 gravity. First order formalism entails treating the metric and connection as independent fields.
In Einstein gravity, this was introduced by Einstein himself to simplify the derivation of the field equations. Thus from

$$S = M_P^2 \sqrt{(}g)R$$

$g^{\mu\nu}$ becomes effectively an auxiliary field, and its equation of motion gives the Einstein equation:

$$R_{(\mu\nu)} - \frac{1}{2} g_{\mu\nu} R = 0$$

The equation of motion for Γ then restores the usual definition of Γ in terms of g.

First Order and R^2 Gravity

First Order formalism makes a big difference in R^2 gravity; for example the number of independent R^2 invariants is different:

$$
\begin{aligned}
S_{RQG} &= \int d^4x\sqrt{g}\,[\alpha R^2 + \beta_1 R_{(\mu\nu)}R_{(\kappa\lambda)}g^{\mu\kappa}g^{\nu\lambda} - 2\beta_2 R_{(\mu\nu)}\tilde{R}_{\lambda}{}^{\mu}g^{\nu\lambda} \\
&+ \beta_3\tilde{R}_{\nu}{}^{\mu}\tilde{R}_{\mu}{}^{\nu} + \beta_6\tilde{R}_{\kappa}{}^{\mu}\tilde{R}_{\lambda}{}^{\nu}g_{\mu\nu}g^{\kappa\lambda} + \beta_8\tilde{R}_{\kappa}{}^{\mu}\overline{R}_{\mu\nu}g^{\kappa\nu} \\
&+ \beta_9\overline{R}_{\mu\nu}\overline{R}_{\kappa\lambda}g^{\mu\kappa}g^{\nu\lambda} + R_{\mu\nu\rho}{}^{\sigma}(\gamma_1 g^{\rho\kappa}g^{\mu\tau}R_{\kappa\sigma\tau}{}^{\nu} + \gamma_3 g^{\nu\lambda}g^{\mu\kappa}R_{\kappa\lambda\sigma}{}^{\rho} \\
&+ \gamma_4 g^{\rho\lambda}g^{\mu\kappa}R_{\kappa\lambda\sigma}{}^{\nu} \\
&+ \gamma_5 g^{\nu\tau}g^{\mu\kappa}R_{\kappa\sigma\tau}{}^{\rho} + \gamma_6 g_{\sigma\upsilon}g^{\rho\lambda}g^{\nu\tau}g^{\mu\kappa}R_{\kappa\lambda\tau}{}^{\upsilon} + \gamma_7 g^{\rho\tau}g^{\mu\kappa}R_{\kappa\sigma\tau}{}^{\nu})]
\end{aligned}
$$

Here $\tilde{R}_{\mu}{}^{\sigma} \equiv g^{\nu\rho}R_{\mu\nu\rho}{}^{\sigma}(\Gamma)$, and $R \equiv g^{\rho\mu}R_{(\mu\rho)}(\Gamma)$, where

$$R_{\mu\rho} \equiv R_{\mu\sigma\rho}{}^{\sigma}, \qquad \overline{R}_{\mu\nu} \equiv R_{\mu\nu\rho}{}^{\rho}.$$

The Riemann tensor does not respect all the familiar identities. There have been previous studies, notably

M.Borunda et al, JCAP 11 (2008) 008.

The result is a vastly more complicated analysis of the RG evolution of the dimensionless couplings, and of the issue of DT. Our main motive is to get insight into the issue of Unitarity. Work in progress........

Graham again

To conclude:

I enjoyed my only too brief period of collaborating with Graham, and wish we had worked together more. Also that I had been capable of giving him more opposition on the squash court! He was much too good for me.

He was a great scientist and a good friend. He and Ruth were wonderful dinner companions. I only have two pictures, where I suspect Graham is describing our work to a sceptic:

Graham and Stuart

Graham and Stuart