Dimensional regularization and γ_5 — no-compromise* approach to the BMHV scheme

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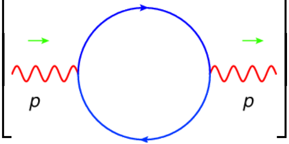
Collaborators: Bélusca-Maïto, Ilakovac, Kühler, Mađor-Božinović

* or: traditional/old-fashioned/stubborn. . .

Example: QED Ward identity

Check QED transversality of photon self energy

p_μ []

$$= p_\mu \int d^D k \frac{\mathrm{Tr}(\not{k}\gamma^\mu(\not{k}+\not{p})\gamma^\nu)}{k^2(k+p)^2}$$

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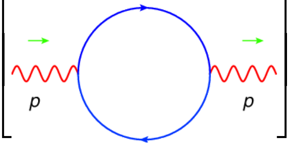
$p_\mu \Big[$  $\Big]$

$$= p_\mu \int d^D k \frac{\mathrm{Tr}(\not{k}\gamma^\mu(\not{k}+\not{p})\gamma^\nu)}{k^2(k+p)^2}$$

using $\not{p} = (\not{k}+\not{p}) - \not{k}$ gives zero:

$$= \int d^D k \frac{(k+p)^2}{(k+p)^2}\frac{\mathrm{Tr}(\not{k}\gamma^\nu)}{k^2} - \int d^D k \frac{k^2}{k^2}\frac{\mathrm{Tr}((\not{k}+\not{p})\gamma^\nu)}{(k+p)^2} = 0$$

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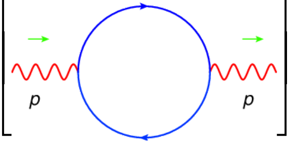
p_μ [

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$$= p_\mu \int d^D k \frac{\mathrm{Tr}(\not{k}\gamma^\mu(\not{k}+\not{p})\gamma^\nu)}{k^2(k+p)^2}$$

What happens if we do the numerator algebra in purely 4-dimensions?

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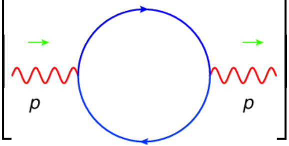
p_μ []

$$= p_\mu \int d^D k \, \frac{\mathrm{Tr}(\not{k}\gamma^\mu(\not{k}+\not{p})\gamma^\nu)}{k^2(k+p)^2}$$

What happens if we do the numerator algebra in purely 4-dimensions?

$$= \int d^D k \, \frac{(\bar{k}+\bar{p})^2}{(k+p)^2} \frac{\mathrm{Tr}(\bar{\not{k}}\bar{\gamma}^\nu)}{k^2} - \int d^D k \, \frac{\bar{k}^2}{k^2} \frac{\mathrm{Tr}((\bar{\not{k}}+\bar{\not{p}})\bar{\gamma}^\nu)}{(k+p)^2} \neq 0$$

GAUGE INVARIANCE BROKEN!

The problem: γ_5 and DReg

Three properties in 4-dimensions:

$$\{\gamma_5, \gamma^\mu\} = 0, \quad (1)$$

$$\mathrm{Tr}(\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4i\epsilon^{\mu\nu\rho\sigma}, \quad (2)$$

$$\mathrm{Tr}(\Gamma_1 \Gamma_2) = \mathrm{Tr}(\Gamma_2 \Gamma_1)\,. \quad (3)$$

Inconsistent in $D \neq 4$ (can prove that trace=0).

Give up at least one $\Rightarrow$ many proposals!

BMHV scheme — non-anticommuting γ_5

QFT consistent, unitary; breaks symmetries, complicated

- "*D*-dim space" split into pure 4-dim space $\oplus$ (-2ϵ)-dim space

$$X^\mu = \bar{X}^\mu + \hat{X}^\mu$$

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$$

$$\{\gamma_5, \bar{\gamma}^\mu\} = 0$$

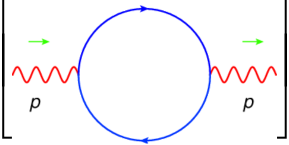
$$[\gamma_5, \hat{\gamma}^\mu] = 0$$

Goals:

- Take seriously, apply to 1-loop, 2-loop ... EW calculations
- Here — Technical task: restore gauge invariance
- Progress will feed back to other schemes

Example: 2-loop result in our model as a result of γ_5

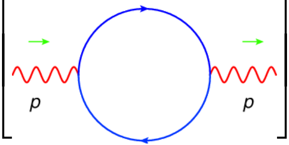
$$\Bigg[$$



$$\Bigg]$$

$$\propto \frac{ie^4}{3 \cdot 256 \pi^4} \Bigg[\left(\frac{673}{23} - 6 \log(-\overline{p}^2) - 24\zeta(3) \right) (\overline{p}^\mu \overline{p}^\nu - \overline{p}^2 \overline{g}^{\mu\nu}) + \frac{11}{8} \overline{p}^\mu \overline{p}^\nu \Bigg],$$

Example: 2-loop result in our model as a result of γ_5

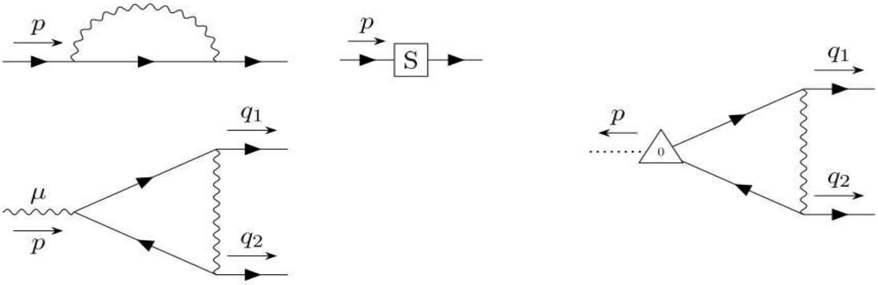


$$
\left[\text{diagram}\right] \propto \frac{ie^4}{3\cdot 256\pi^4}\left[\left(\frac{673}{23} - 6\log(-\overline{p}^2) - 24\zeta(3)\right)(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{g}^{\mu\nu}) + \frac{11}{8}(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{g}^{\mu\nu})\right],
$$

Breaking can be compensated by additional counterterm:

$$
\mathcal{L}_{\text{fin-ct}} \propto -\frac{e^4}{3\cdot 256\pi^4}\frac{11}{16}\bar{A}_\mu\overline{\partial}^2\bar{A}^\mu
$$

Preview: Problem in a nutshell



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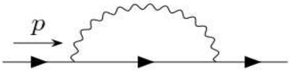

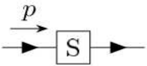

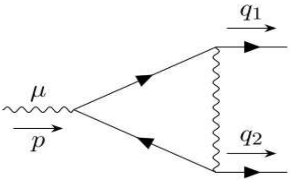

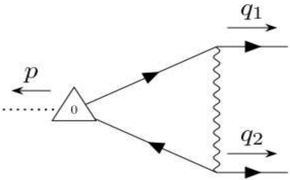

Ward identity
violated

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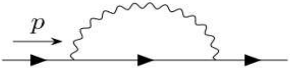

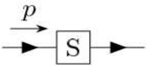

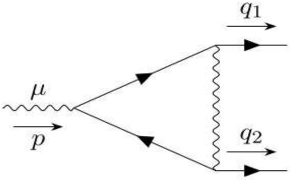

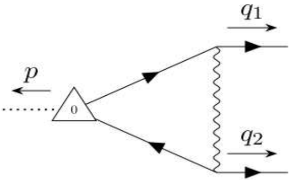

Ward identity violated

compensated by special c.t.

(our main task)

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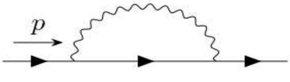

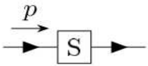

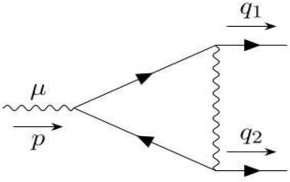

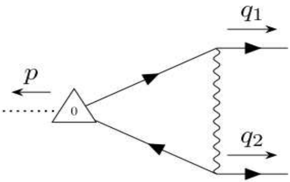

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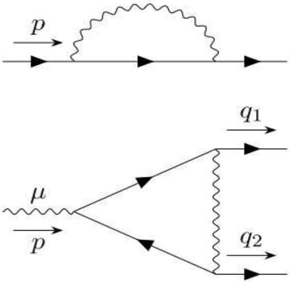
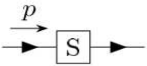
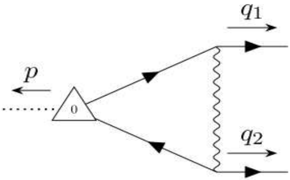
$$S^1_{\text{fct}} = \frac{e^2}{16\pi^2} \int d^4 x \left\{ \ldots + \left(\frac{5+\xi}{6} \right) (\mathcal{Y}^j_R)^2 \left(\bar{\psi}_j i \bar{\partial\!\!\!/} \, P_{\text{R}} \, \psi_j \right) \right\}.$$

Preview: Problem in a nutshell







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Alternative: breaking via $\frac{\epsilon}{\epsilon}$ term

(tool)

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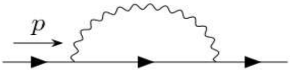

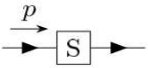

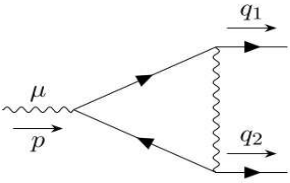

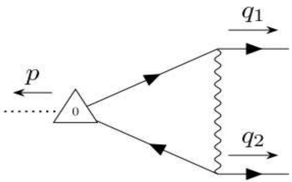

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see also: SUSY in DRed (3-loop) [DS'05, Hollik,DS'05, DS,Unger'18]

Plan here: chiral "QED" (only $P_R\psi$) at 1-/2-loop

[Bélusca-Maïto, Ilakovac,Kühler Mađor-Božinović, DS, 2021]

1. Define D-dimensional Lagrangian compute symmetry breaking
2. Determine 1-loop UV divs $\rightsquigarrow \mathcal{L}_{\text{sct}}$
3. Determine 1-loop violation of Slavnov-Taylor identity
4. Determine 1-loop symmetry-restoring counterterms $\rightsquigarrow \mathcal{L}_{\text{fct}}$
5. Repeat at 2-loop new features?

1. Define D-dimensional Lagrangian

Abelian theory like $\mathrm{U}(1)_Y$-part of SM, only ψ_{Ri} interact

Description of symmetry: gauge invariance $\rightarrow$ BRST invariance $\rightarrow$ Slavnov-Taylor identity is required for renormalized theory: $S(\Gamma_{\mathrm{ren}}) = 0$

1. Define D-dimensional Lagrangian

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Description of symmetry: gauge invariance $\rightarrow$ BRST invariance $\rightarrow$ Slavnov-Taylor identity is required for renormalized theory: $S(\Gamma_{\text{ren}}) = 0$

$\mathcal{L}$ in D-dim? Our choice has D-dim kinetic, 4-dim interaction term:

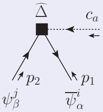
$$\mathcal{L}_{\text{fermions}} = i\overline{\psi}_i \partial\!\!\!/ \psi_i + e\mathcal{Y}_{Ri}\overline{\psi_R}_i A\!\!\!/ \psi_{Ri} \,.$$

1. Define D-dimensional Lagrangian

Abelian theory like $U(1)_Y$-part of SM, only ψ_{Ri} interact

$\mathcal{L}$ in D-dim breaks D-dim gauge/BRST invariance
$\Rightarrow$ and leads to breaking of tree-level Slavnov-Taylor identity

$$\mathcal{S}_d(S_0) = \widehat{\Delta} \equiv \int \mathrm{d}^d x \, (e\mathcal{Y}_{Ri}) c \left\{ \overline{\psi}_i \left(\overleftarrow{\widehat{\partial\!\!\!/}} P_\mathrm{R} + \overrightarrow{\widehat{\partial\!\!\!/}} P_\mathrm{L} \right) \psi_i \right\} .$$



$$= (e\mathcal{Y}_{Ri}) \left(\widehat{p\!\!\!/}_1 P_\mathrm{R} + \widehat{p\!\!\!/}_2 P_\mathrm{L} \right)_{\alpha\beta}$$

This is the core of the difficulties.
Can be written as a
local Feynman rule

2. Compute Green functions to determine UV divs

Many 1-loop diagrams (not shown) $\rightsquigarrow$ divergent counterterms:

$$S^1_{\text{sct}} = S^1_{\text{sct,inv}} + S^1_{\text{sct,break}}\,,$$

...skip details...

3. Determine 1-loop violation of Slavnov-Taylor id.

Ultimate structure at 1-loop (finite ct to be determined)

$$\Gamma^{(1)}_{\text{DReg}} = \Gamma^{(1)} + S^1_{\text{sct}} + S^1_{\text{fct}}\,,$$

Evaluate STI at 1-loop order, div-parts cancel, fin-parts t.b.d.

$$\mathcal{S}_d(\Gamma^{(1)}_{\text{DReg}}) = \underbrace{\mathcal{S}_d(\Gamma^{(1)})|_{\text{finite}}} + \mathcal{S}_d S^1_{\text{fct}}$$

Left term means: breaking of regularized STI; must be computed.

In principle this corresponds to checking all STIs/WIs, e.g. Fermion 2-point/3-point function etc.

3. Determine 1-loop violation of Slavnov-Taylor id.

Ultimate structure at 1-loop (finite ct to be determined)

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Left term means: breaking of regularized STI; must be computed.

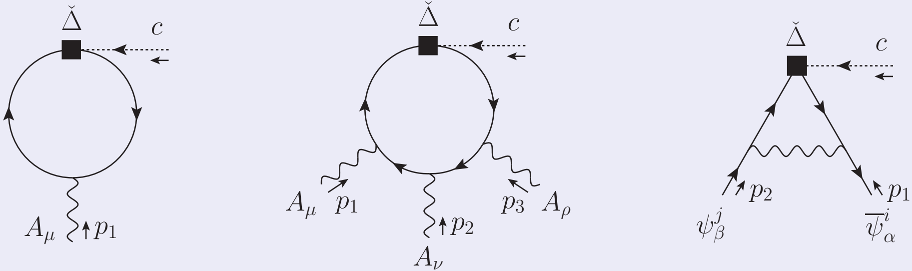
In principle this corresponds to checking all STIs/WIs, e.g. Fermion 2-point/3-point function etc.

But can be simplified by using quantum action principle (BM)

$$\mathcal{S}_d(\Gamma^{(1)}) = \widehat{\Delta}\cdot\Gamma^{(1)}\,,$$

Bonneau (1980): only power-counting divergent diagrams matter!

The complete set of power-counting divergent 1-loop diagrams with insertion of $\widehat{\Delta}$:



Results mean: breaking of three concrete WI/STIs.

They have the form $\frac{\epsilon/\text{evanescent}}{\epsilon} \times$ (local)

$\rightsquigarrow$ local counterterms can repair the symmetry!

(There is an additional diagram corresponding to the fermion triangle loop and the true anomaly (assumed absent))

4. Determine symmetry-restoring counterterms

$$\mathcal{S}_d(\Gamma^{(1)})|_{\text{finite}} + \mathcal{S}_d S^1_{\text{fct}} \stackrel{!}{=} 0$$

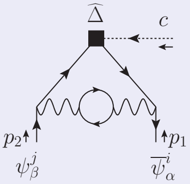
Requiring this renormalized STI to hold leads to the result

$$S^1_{\text{fct}} = \frac{e^2}{16\pi^2} \int d^4x \left\{ \frac{-\operatorname{Tr}(\mathcal{Y}_R^2)}{6} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + \frac{e^2 \operatorname{Tr}(\mathcal{Y}_R^4)}{12} (\bar{A}^2)^2 \right. \\ \left. + \left(\frac{5+\xi}{6}\right) (\mathcal{Y}_R^j)^2 \left(\bar{\psi}_j i \bar{\not{\partial}} P_R \psi_j\right) \right\}.$$

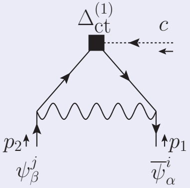
This is the full 1-loop result of symmetry-restoring counterterms for this chiral QED in BMHV scheme for our Lagrangian.
Finite, NON-evanescent counterterms. Not gauge invariant!
Modify both self-energies and A^4 interaction

Repeat at 2-loop order

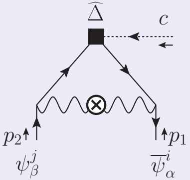
2-loop Slavnov-Taylor breaking — many diagrams of four types:

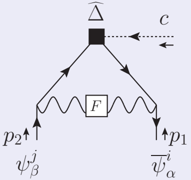


2-loop insertion of $\widehat{\Delta}$



1-loop insertion of Δ^1_{ct}





insertion of $\widehat{\Delta}$ into 1-loop diagram with 1-loop ct insertion

Sum gives $\mathcal{S}_d(\Gamma^{(2)})|_{\mathrm{finite}}$ =local. Can cancel by local counterterms

4. Determine sym-restoring counterterms at 2-loop

$$\mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}} + \mathcal{S}_d S^2_{\text{fct}} \stackrel{!}{=} 0$$

Requiring this renormalized STI to hold leads to the result

$$S^2_{\text{fct}} = \frac{e^4}{(16\pi^2)^2} \int \mathrm{d}^4 x \left\{ \mathrm{Tr}(\mathcal{Y}_R^4) \frac{11}{48} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + e^2 \frac{\mathrm{Tr}(\mathcal{Y}_R^6)}{8} (\bar{A}^2)^2 \right.$$
$$\left. -(\mathcal{Y}_R^j)^2 \left(\frac{127}{36} (\mathcal{Y}_R^j)^2 - \frac{1}{27} \mathrm{Tr}(\mathcal{Y}_R^2) \right) \left(\bar{\psi}_j i \bar{\partial\!\!\!/} P_{\mathrm{R}} \psi_j \right) \right\}$$

This is the full 2-loop result of symmetry-restoring counterterms for this chiral QED in BMHV scheme for our Lagrangian.
Finite, NON-evanescent counterterms. Not gauge invariant!
Same structure as at 1-loop

Generalization to YM

[Bélusca-Maïto, Ilakovac, Mađor-Božinović, DS, 2020]

symmetry-restoring counterterm for YM+fermions+scalars (1-loop)

$$S^1_{\text{fct,restore}} =$$

$$\frac{\hbar}{16\pi^2}\Bigg\{ g^2\frac{S_2(R)}{6}\left(5S_{GG} + S_{GGG} - \int d^4x\, G^{a\,\mu}\partial^2 G^a_\mu\right) + \frac{Y_2(S)}{3}\overline{S_{\Phi\Phi}}$$

$$+g^2\frac{(T_R)^{abcd}}{3}\int d^4x\,\frac{g^2}{4}G^a_\mu G^{b\,\mu}G^c_\nu G^{d\,\nu} - \frac{(\mathcal{C}_R)^{ab}_{mn}}{3}\int d^4x\,\frac{g^2}{2}G^a_\mu G^{b\,\mu}\Phi^m\Phi^n$$

$$+g^2\left(1+\frac{\xi-1}{6}\right)C_2(R)S_{\overline{\psi}\psi} - \frac{\left((Y^m_R)^* T_{\overline{R}}{}^a Y^m_R\right)_{ij}}{2}\int d^4x\, g\overline{\psi}_i \not{G}^a P_{\mathrm{R}}\psi_j$$

$$-g^2\frac{\xi C_2(G)}{4}\left(S_{\bar{R}c\psi_R} + S_{Rc\overline{\psi_R}}\right)\Bigg\}\,,$$

Finite, NON-evanescent counterterms. Not gauge invariant!
Modify all self-energies and some interactions!
But rather compact, universal, could be implemented e.g. in FeynArts

Summary and outlook

- γ_5 is problematic in DReg, BMHV scheme is rigorous:
- γ_5 non-anticommuting, distinguish 4-dim and ϵ-dim quantities
- gauge invariance broken already in $\mathcal{L}_D$ and at loop level

Results:

- Symmetry-restoring counterterms: 1-loop YM, 2-loop abelian
- Method established, result has compact simple structure

Outlook:

- 2-loop YM, 2-loop EWSM, 3-loop
- automatize, implement in FeynArts, FeynRules etc
- alternative $\mathcal{L}_D$, schemes (Larin, FDH, DRed, etc)