Planck Formula for the Gluon Parton Distribution in the Proton.

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Summary

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2) PARTON MODEL TO DESCRIBE SCALE INVARIANCE IN DEEP INELASTIC SCATTERING .

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Summary

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7) CONSEQUENCES OF THE EQUILIBRE CONDITIONS WITH RESPECT TO THE ELEMENTAR QCD PROCESSES : THE ISOSPIN AND SPIN ASYMMETRIES IN THE PROTON SEA ARE DERIVED BY THE VALENCE PARTON DISTRIBUTIONS .

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9) COMPARISON OF THE GLUON DISTRIBUTIONS AT $Q^2 = 4$ FOUND BY THE ATLAS COLLABORATION FROM THE ANALYSIS OF THE EXPERIMENTS AT HERA AND LHC WITH THE PLANCK FORMULA IMPLIED BY THE STATISTICAL APPROACH .

10) PION PARTON DISTRIBUTIONS FROM DRELL-YAN PAIRS PRODUCTION IN PION NUCLEON SCATTERING .

11) CONCLUSIONS

GENERAL QCD PROPERTIES

The sign of the β function for the renormalization group imples that QCD accounts for the CONFINEMENT of the quarks (''INFRARED SLAVERY'') and for the scale invariance of the structure functions, which describe DEEP INELASTIC SCATTERING (''ASYMPTOTIC FREEDHOM''). The proton and the other baryons, which at small Q^2 behave as states with three quarks combined into a color singlet, at high Q^2 appear as an incoherent set of quarks, gluons and antiquarks with distributions, which obey the sum rules of the parton model as the condition that at high P_z :

$$\int_0^1 \Sigma_i x p_i(x) dx = 1$$

where x is the fraction of the proton momentum carried by the parton i and the quark number sum rules, which imply for the proton :

$$\int_0^1 [u(x) - \bar{u}(x)] dx = 2$$

$$\int_0^1 [d(x) - \bar{d}(x)] dx = 1$$

DGLAP EQUAtIONS

QCD implies logarithmic violations of scale invariance described by DGLAP equations, which allow to deduce the parton distributions at a Q^2 larger than a sufficiently high Q_0^2 to allow the perturbative expansion from the ones at Q_0^2, for which the standard form:

$$Ax^B(1-x)^C P(x)$$

is assumed with the parameter A, B e C and the polynome P(x) depending on the parton and such a form holds for the non polarized and for the polarized distributions . This form allows to solve the DGLAP equations by computing the momenta :

$$\int_0^1 x^n p_i(x) dx$$

FORM OF THE DISTRIBUTIONS AT Q_0^2

Parton model and the consequent scale invariance hold for large values of Q^2 and $(p+q)^2 = M^2 + Q^2(\frac{1}{x} - 1)$ larger than M^2 and therefore the values $x = 0$ e $x = 1$ are exscluded as well as their neighboroods with amplitudes decreasing with Q^2 .

Therefore to fix the power behaviour around these points has not a strong motivation .

To fix the distributions at Q_0^2 one may be inspired by experiment, which suggests a role of quantum statistical mechanics .

PAULI PRINCIPLE TO ACCOUNT FOR THE ISOSPIN ASYMMETRY OF THE PROTON SEA

Several years ago Niegawa and Sisiki and Feynman and Field stated that in the proton $\bar{d}(x)$ is larger than $\bar{u}(x)$ as a consequence of Pauli principle, since "one produces less $u\bar{u}$ pairs than $d\bar{d}$ pairs in the proton, since there are two valence u quarks in the proton and only one valence d quark" .

EXPERIMENTAL CONFIRMS OF THE ROLE OF PAULI PRINCIPLE

The statement by Niegawa and Sisiki and Feynman and Field has been confirmed by the defect in the Gottfried sum rule :

$$\int_0^1 \frac{F_2^p(x) - F_2^n(x)}{x} dx = \frac{u + \bar{u} - d - \bar{d}}{3} = 0.24$$

instead of

$$\frac{1}{3}$$

and by the asymmetry in the Drell-Yan pairs production in pd or pp scattering, which shows a continuous increasing of the ratio

$$\frac{\bar{d}(x)}{\bar{u}(x)}$$

always larger than 1 .

The role of Pauli principle implies a role of quantum statistical mechanics .

OTHER EXPERIMENTAL FACTS CONFIRM THE ROLE OF QUANTUM STATISTICAL MECHANICS FOR PARTON DISTRIBUTIONS

Other phenomenological facts suggest the role of quantum statistical mechanics for parton distributions beyond the role of Pauli principle to explain the isospin asymmetry in the proton sea, $\bar{d}(x)$ larger $\bar{u}(x)$:

1) The correlation between the first moments of the valence partons and the shape of their distributions, which are broader in x for the partons with higher first moment (as for the Fermi sphere, which implies an increasing mean energy with the number of the fermions) .

2) The common Boltzmann behaviour

$$\exp\left(-\frac{x}{\bar{x}}\right)$$

for x larger of the highest "potential" , $X(u^{\uparrow}) = 0.461$.

These facts have given rise to the conclusion that the functions, which give the probability that a parton, defined by its "flavor" and helicity, carries the percentage x of the hadron momentum in deep inelastic scattering, is fixed by quantum statistical mechanics .

THE ROLE OF QUANTUM STATISTICAL MECHANICS

The role of Pauli principle implies that the gas of fermions is degenerate with occupation nunmbers not small, implying that the phase space is limited . In statistical mechanics the variable is the energy, because of the constraint :

$\Sigma_i n_i \epsilon_i = E$

Since the variable appearing in the parton model sum rules is x, the "temperature" and the "potentials" of the valence quarks should also be numbers, respectively $\bar{x}$ and X_q^h, depending on the flavor, q, and the helicity, h, of the quark .

FORM OF THE DISTRIBUTIONS AT Q_0^2

The role of Pauli principle leads to the proposal of quanrum statistical parton distributions for the partons as boundary condition for the DGLAP equation at a Q_0^2, which separates the non perturbative and the perturbative regimes of the evolution . Fermi-Dirac functions for the fermions and Planck (a Bose-Einstein with vanishing "potential") for the gluons .

S. Sohaily, F. Tramontano and F. B.

QUANTUM STATISTICAL PARTON DISTRIBUTIONS

Twenty years ago Claude Bourrely, Jacques Soffer and F. B. proposed the following expression for the parton distributions at $Q^2 = 4(GeV)^2$:

$$xq(x) = \frac{Ax^b X_q}{(\exp \frac{x - X_q}{\bar{x}} + 1)} + \frac{\tilde{A}x^{\tilde{b}}}{(\exp \frac{x}{\bar{\bar{x}}} + 1)}$$

$$x\bar{q}(x) = \frac{\bar{A}x^{2b}}{X_q(\exp \frac{x + X_q}{\bar{x}} + 1)} + \frac{\tilde{A}x^{\tilde{b}}}{(\exp \frac{x}{\bar{\bar{x}}} + 1)}$$

for the valence partons and for their antiparticles with opposite helicities, where the first term depend on the "potential" depending on the flavor (u or d) and the helicity of the parton, while the second term, the diffractive one, is the same for the valence quarks and their antiparticles ; $\bar{x}$ plays the role of the "temperature" and X_q play the role of "potential" .

QUANTUM STATISTICAL PARTON DISTRIBUTIONS

For the gluons we have a Planck formula , a Bose-Einstein formula with vanishing potential :

$$xG(x) = \frac{A_G x^{b_G}}{(\exp \frac{x}{\bar{x}} - 1)}$$

The "ad hoc" factors , X_q for the valence partons and

$$\frac{1}{X_q}$$

for their antiparticles, introduced to comply with the data, have been accounted by the extension to the transverse momenta .

THE EQUILIBRIUM CONDITIONS FOR THE QCD PROCESSES

The equilibrium with respect to the two processes elementary QCD, the emission of a gluon by a fermion and the conversion of a gluon into a $q\bar{q}$ pair with opposite helicities, has the important conseguences to predict a vanishing potential for the gluons and opposite values for the potentials of the quarks and of their antiparticles with opposite helicity . The Bose-Einstein formula for the gluons $xG(x)$ becomes a Planck formula :

$$\frac{1}{(\exp \frac{x}{\bar{x}} - 1)}$$

and $\Delta G(x) = 0$

DETERMINATION OF THE PARAMETERS FROM DATA

By comparing with the data on unpolarized and polarized deep inelastic scattering from incident leptons and on the production of Drell-Yan pairs in pp and pd scattering we got the following values for the parameters, which descrive the parton distributions of the light fermions and of the gluons at $Q^2 = 4$,

$\bar{x} = 0.099$, $X_{u^\uparrow} = 0.461$, $X_{d^\downarrow} = 0.302$, $X_{u^\downarrow} = 0.298$, $X_{d^\uparrow} = 0.228$
$b_U = 0.41$, $b_G = 0.747$, $\tilde{b} = -0.253$, $A = 1.75$, $\bar{A} = 1.91$, $\tilde{A} = 0.083$,

EXPERIMENTAL CONFIRMS OF THE PARTON DISTRIBUTIONS PROPOSED IN THE 2002 PAPER

The parton distributions proposed in the 2002 paper have received many confirms by experiments performed after . It happened for the measurements of the polarized structure functions of the nucleons, $g_1^p(x)$ and $g_1^n(x)$, for the spin asymmetries of the proton sea measured at RHIC in the production of the charged weak bosons with polarized beams, which are consistent with the predictions $\Delta\bar{u}(x)$ positive and $\Delta\bar{d}(x)$ negative . More precisely one predicts :

$\Delta\bar{d}(x)$ negative, $\Delta\bar{u}(x)$ positive and smaller than $\bar{d}(x) - \bar{u}(x)$ smaller than $\Delta\bar{u}(x) - \Delta\bar{d}(x)$

While the first three inequalities are confirmed experimentally, the measurement of the isovector spin asymmetry :

$\Delta\bar{u}(x) - \Delta\bar{d}(x)$

is difficult, while its important contribution to the Bjorken sum rule is welcome .

PLANCK FORMULA FOR THE GLUON PARTON DISTRIBUTION IN THE PROTON

Recently a further confirm of the role of Quantum Mechanical Statistics for parton distributions has been obtained by comparing the gluon parton distribution found by the ATLAS collaboration by analyzing of the experiments performed at HERA and LHC with the Planck formula proposed in the 2002 work:

$$xG(x) = \frac{A_G x^{b_G}}{(\exp \frac{x}{\bar{x}} - 1)}$$

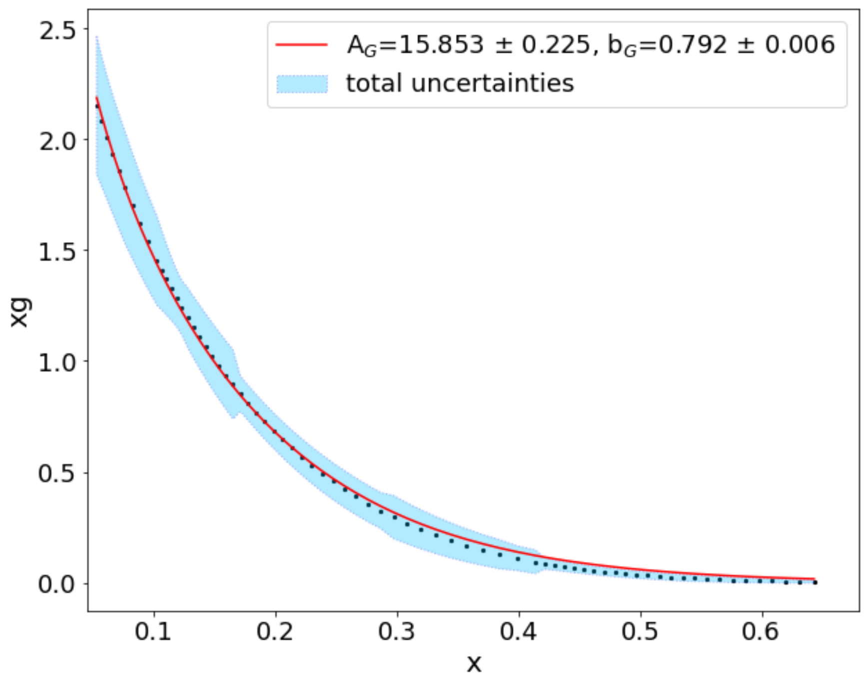
One obtains a very good description of the experimental form with the same value of $\bar{x}$, which describes the behaviour of the valence partons around the values of their potential and the Boltzmann behaviour proportional to

$$\exp\left(-\frac{x}{\bar{x}}\right)$$

above them and with values for A_G and b_G very similar to the ones found in the 2002 work,

	2002	**2022**
A_G	14.3	15.85
b_G	0.75	0.79

PLANCK FORMULA FOR THE GLUON PARTON DISTRIBUTION IN THE PROTON



The red curve represents the best fit of the gluon momentum distribution $xg(x)$ obtained in the ATLAS experiment, performed using Planck formula, with A_G and b_G as free parameters, and $\bar{x} = 0.099$. The dots correspond to the experimental points, and the (cyan) shaded area to their uncertainty.

DRELL-YAN PAIR PRODUCTION IN PION NUCLEON SCATTERING

Experimentally at high x_F the production of Drell-Yan pairs on nuclear targets with incident negative pions dominates with respect to incident antiprotons : this phenomenon is accounted in the framework of the statistical approach by assuming a high value for the "potential" of the valence fermions, $\bar{u}$ and d, 0.75 . With this high value of the "potential" the first moment of the diffractive term of the antiparticles of the valence fermions is negligible with the consequence that the quark number sum rule requires that the first moment of the non diffractive term of the valence fermion is very near to 1 .

ADVANTAGE OF THE PLANCK FORMULA

The main motivation for the polynomial parametrization is the semplicity of the Mellin transform, which allows to solve the DGLAP equations, exploiting the Q^2 dependance of the momenta of the parton distributions, but this is instead the reason to adopt a parametrization, which better describes the high x values, which have an increasing relevance with the high n momenta . In fact, while the C parameters depend on the behaviour near $x = 1$, where the experimental information is scarce and at the initial Q_0^2 that region does not belong to the domain of validity of the parton model, all the parameters, which fix the high x behaviour in the statistical approach are fixed by experiments at values of x, where the experimental information is very good and in the domain of validity of the parton model .

THE METHOD OF MELLIN TRANSFORM WOULD BE MORE PRECISE STARTING WITH A BETTER PARAMETRIZATION AR HIGH X

Adopting the parametrization suggested by the role of Quantum Statistical Mechanics, which has received many experimental confirms and allows to describe at the same time unpolarized and polarized parton distribution, will supply a more precise description at high x, a region where the direct experimental information is not the desired one for the research of physics beyond the standard model .

CONCLUSIONS

1) The agreement of the fermion distributions found in 2002 inspired by quantum statistical mechanics with the ones found by Hera is an important confirm of the validity of the statistical approach, which has been motivated by the idea of Niegawa, Sisiki, Feynman and Fields that the Pauli principle implies the isospin asymmetry in the proton sea .

2) The theory has been improved with the extension to the transverse components of the momentum and with the hypothesis that the statistical distributions are the boundary condition at low Q^2 of the Altarelli e Parisi equations (DGLAP).

CONCLUSIONS

3) The decreasing at high x and the ratios between the distributions of the valence partons are better described by the statistical distributions than by the che standard ones:

$Ax^B(1-x)^C P(x)$.

In fact the ratios change faster in the range :

$X_{d^\uparrow}, X_{u^\uparrow}$ (0.22, 0.46)

than for values larger than $X_{u^\uparrow}$.

This allows a more precise determination of the distributions at high x, where the experimental information is scarce .

CONCLUSIONS

4) An interesting property of the statistical approach is the fact that the high x behaviour depends on free parameters, which are fixed by measurements in a region of x, where both the statistical and the systematic errors are small, the region (0.22, 0.46), where the valence partons dominate, in such a way to provide the factor for the Boltzmann behaviour

$$\exp\left(\frac{-x}{\bar{x}}\right)$$

.

Also it predicts the isospin and spin asymmetries of the sea .

CONCLUSIONS

5) The agreement of the gluon distribution found by the ATLAS Collaboration from the analysis of the experiments at HERA and LHC with the Plabck formula predicted by the statistical approach with the SAME value of the "temperature", $\bar{x} = 0.099$, which describes the behaviour of the valence quark distributions around their "potentials" and above them, and similar values of the other two parameters to the one found in the 2002 work is an important confirm that the boundary conditions to the DGLAP equationss is fixed by Quantum Mechanical Statistics, confirming the role of Pauli principle advocated by Niegawa, Sisiki, Feynman and Fields .

CONCLUSIONS

6) The statistical approach is sucessfully applied to the production of Drell-Yan pairs in pion nucleon scattering with a large ""poten-tial" found for the valence partons, which explains the dominance at high x_F with incident negative pion with respect to incident antiproton .

7) HELP IS NEEDED FROM SCIENTISTS, WHICH THINK THAT THE CONTENT OF THIS SEMINAR IS RIGHT, BECAUSE UN-FORTUNATELY THERE IS A TENDENCY TO IGNORE THE CONSEQUENCES OF ITS CONTENT