Physical Tuning

Vincenzo Branchina

Department of Physics and Astronomy “E. Majorana”
University of Catania and INFN, Italy

September 6, 2022

C. Branchina, VB, F. Contino, ArXiv: 2208.05431
see also:C. Branchina, VB, F. Contino, N. Darvishi, ArXiv:2204.10582, accepted PRD

Workshop on the Standard Model and Beyond, Corfu, Greece,
August 28 - September 8, 2022

Keywords

Effective Field Theory

Wilsonian Renormalization Group

RENORMALIZATION AND EFFECTIVE LAGRANGIANS

Joseph POLCHINSKI*

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Received 27 April 1983

There is a strong intuitive understanding of renormalization, due to Wilson, in terms of the scaling of effective lagrangians. We show that this can be made the basis for a proof of perturbative renormalization. We first study renormalizability in the language of renormalization group flows for a toy renormalization group equation. We then derive an exact renormalization group equation for a four-dimensional $\lambda\phi^4$ theory with a momentum cutoff. We organize the cutoff dependence of the effective lagrangian into relevant and irrelevant parts, and derive a linear equation for the irrelevant part. A lengthy but straightforward argument establishes that the piece identified as irrelevant actually is so in perturbation theory. This implies renormalizability. The method extends immediately to any system in which a momentum-space cutoff can be used, but the principle is more general and should apply for any physical cutoff. Neither Weinberg's theorem nor arguments based on the topology of graphs are needed.

QFT contains an **ultimate scale** $\Lambda_{phys} \;\Rightarrow\; \mathcal{L}_{\Lambda_{phys}}$

(For notational convenience : $\Lambda_{phys} \to \Lambda$)

Below Λ : Effective Field Theory (EFT): ok $\mathcal{L}_\Lambda$

Above Λ : UV completion needed : $\not{\mathcal{L}}_\Lambda$

Standard Model

EFT **valid up to** a certain scale Λ $(M_P,\ M_{GUT},\ \ldots) =$ **physical cut-off**

Effective Lagrangian $\mathcal{L}_{SM}^{(\Lambda)}$ describes processes for momenta $p \lesssim \Lambda$

Un-suppressed quantum fluctuations $\Rightarrow$ $m_H^2 \sim \Lambda^2$

"**Quadratic sensitivity**" to the ultimate scale of the theory

Note: $m_H^2 \sim \Lambda^2$ is $m_H^2(\mu)$ at $\mu = \Lambda$

If Λ too large $\Rightarrow$ $m_H^2(\Lambda)$ "**unnaturally**" large

$\Rightarrow$ problem of "**hierarchy**" with Fermi scale μ_F

where $m_H^2(\mu_F) \sim (125\ \text{GeV})^2$

Several attempts to "solve" this naturalness/hierarchy (NH) problem.

Let's focus on some of them ...

1. Quantum Gravity Miracle

see for instance Giudice, PoS EPS-HEP2013, 163 (2013)

Assumption - The UV completion of the SM provides the condition

$$m_H^2(\Lambda) \ll \Lambda^2$$

Conspiracy among the SM couplings at Λ (example: Veltman condition)

In this scenario

(i) **Naturalness** “solved” from physics “**outside**” the SM realm: the condition is a **left-over** of its UV completion

(ii) **Hierarchy** solved “**inside**” the SM: the **perturbative RG equation** for $m_H^2(\mu)$ is considered ($\gamma \ll 1$ is the perturbative anomalous mass dimension)

$$\mu \frac{d}{d\mu} m_H^2(\mu) = \gamma \, m_H^2(\mu)$$

$\Rightarrow$ $m_H^2(\mu_F)$ and $m_H^2(\Lambda)$ of the **same order** $\Rightarrow$ **no problem of hierarchy**

2. Self-organized criticality

see for instance Pawlowski, Reichert, Wetterich, M. Yamada, Phys. Rev. D99, 086010 (2019)

The **key equation** is **again** the RG equation for the running Higgs mass $m_H^2(\mu)$

$$\mu \frac{d}{d\mu} m_H^2(\mu) = \gamma \, m_H^2(\mu)$$

but now assumed that **gravity** provides a **non-perturbative** value for γ (~ 2)

In this case, the **large hierarchy** between the Fermi scale μ_F and the UV scale Λ can be **accommodated** $\Rightarrow$ **no Naturalness / Hierarchy NH problem**

3. Dimensional Regularization

see for instance Salvio, Strumia, JHEP 06, 080 (2014)

Some authors suggest **DR endowed with special physical properties** that make it the **correct “physical” way** to calculate the radiative corrections in QFT. If no new heavy particles are coupled to the Higgs boson, the NH problem would seems to be **absent from the beginning**

These approaches cannot solve the NH problem

Why?

The EFT nature of the SM **not properly** and **fully** taken into account

Statement: The SM in an Effective Theory

Meaning:

(A) the parameters (masses, couplings) $g_i(\Lambda)$ in $\mathcal{L}_{SM}^{(\Lambda)}$ result from **integrating out** the **higher energy dof** related to the **UV completion** of the SM

(B) the same parameters $g_i(\mu)$ at scales $\mu < \Lambda$ result from **integrating out** the modes of the **fields that appear in** $\mathcal{L}_{SM}^{(\Lambda)}$ in the range $[\mu, \Lambda]$.

Wilson Lesson

Effective Field Theory (Wilson)

Action $\mathcal{S}_\Lambda[\Phi] = \int d^d x \, \mathcal{L}_\Lambda$ with $\Phi(x) = \sum_{0<|p|<\Lambda} \varphi_p e^{ipx}$

$$\Phi(x) = \varphi(x) + \varphi'(x) \; ; \; \varphi(x) = \sum_{0<|p|<k} \varphi_p e^{ipx} \quad \varphi'(x) = \sum_{k<|p|<\Lambda} \varphi_p e^{ipx}$$

Wilsonian Action at $k < \Lambda$ $\quad S_k[\varphi] \quad \Leftarrow \quad e^{-S_k[\varphi]} \equiv \int D[\varphi'] e^{-S_\Lambda[\varphi+\varphi']}$

Wilsonian Action at $k - \delta k$ $\quad S_{k-\delta k}[\varphi] \quad \Leftarrow \quad e^{-S_{k-\delta k}[\varphi]} = \int D[\varphi'] e^{-S_k[\varphi+\varphi']}$

$$\varphi(x) = \sum_{0<|p|<k-\delta k} \varphi_p e^{ipx} \qquad \varphi'(x) = \sum_{k-\delta k<|p|<k} \varphi_p e^{ipx}$$

Legendre Effective Action $\Gamma[\varphi] = S_{k=0}[\varphi]$; Action $\mathcal{S}_\Lambda[\varphi] = S_{k=\Lambda}[\varphi]$

$$S_{k-\delta k}[\varphi] = -\ln\left(\int D[\varphi'] e^{-S_k[\varphi+\varphi']}\right)$$

$$\varphi(x) = \sum_{0<|p|<k-\delta k} \varphi_p e^{ipx} \qquad \varphi'(x) = \sum_{k-\delta k<|p|<k} \varphi_p e^{ipx}$$

$$S_k[\varphi] = \int d^d x \left(U_k(\varphi) + \frac{Z_k(\varphi)}{2} \partial_\mu\varphi\,\partial_\mu\varphi + Y_k(\varphi)\,(\partial_\mu\varphi\,\partial_\mu\phi)^2 + W_k(\varphi)\,(\varphi\,\partial_\mu\partial_\mu\varphi)^2 + \cdots \right)$$

Local Potential Approximation $Z_k(\varphi) = 1 \;,\; Y_k(\varphi) = W_k(\varphi) = \cdots = 0$

Homogeneous background $\varphi(x) = \varphi_0$ $\left(U_k''(\varphi) \equiv \frac{\partial^2 U_k(\varphi)}{\partial\varphi^2} \right)$

$$U_{k-\delta k}(\varphi_0) = U_k(\varphi_0) + \frac{1}{2}\int_{[k-\delta k, k]} \frac{d^d p}{(2\pi)^d} \ln\left(\frac{p^2 + U_k''(\varphi_0)}{p^2 + U_k''(0)}\right)$$

$$k\frac{\partial}{\partial k} U_k(\varphi_0) = -\frac{k^d}{(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \ln\left(\frac{k^2 + U_k''(\varphi_0)}{k^2 + U_k''(0)}\right)$$

Non-perturbative RG equation for $U_k(\varphi_0)$. Inserting in this equation the polynomial expansion ($Z(2)$ symmetry $\varphi_0 \to -\varphi_0$ assumed)

$$U_k(\varphi_0) = \frac{1}{2} m_k^2 \varphi_0^2 + \frac{\lambda_k}{4!} \varphi_0^4 + \frac{\lambda_k^{(6)}}{6!} \varphi_0^6 + \frac{\lambda_k^{(8)}}{8!} \varphi_0^8 + \cdots$$

$\Rightarrow$ **RG Equations for the couplings** $\left(N_D = 2/(4\pi)^{D/2} \Gamma(D/2)\right)$

$$k \frac{dm^2(k)}{dk} = -\frac{k^D N_D}{2} \frac{\lambda(k)}{k^2 + m^2(k)}$$

$$k \frac{d\lambda(k)}{dk} = -\frac{k^D N_D}{2} \left[\frac{\lambda_6(k)}{k^2 + m^2(k)} - 3 \frac{\lambda^2(k)}{(k^2 + m^2(k))^2} \right]$$

$$k \frac{d\lambda_6(k)}{dk} = -\frac{k^D N_D}{2} \left[\frac{\lambda_8(k)}{k^2 + m^2(k)} - 15 \frac{\lambda(k)\lambda_6(k)}{(k^2 + m^2(k))^2} + 30 \frac{\lambda^3(k)}{(k^2 + m^2(k))^3} \right]$$

$\ldots$

Scalar Theory in $d = 4$ dimensions

Wilsonian action $S_k[\phi] = \int d^4 x \left(\frac{1}{2} \partial_\mu \phi \partial_\mu \phi + U_k(\phi) \right)$

Truncating the potential $U_k(\phi) = \frac{1}{2} m_k^2 \phi^2 + \frac{1}{4!} \lambda_k \phi^4 \quad \Rightarrow$

$$k \frac{dm_k^2}{dk} = -\frac{k^4}{16\pi^2} \frac{\lambda_k}{k^2 + m_k^2}$$
$$k \frac{d\lambda_k}{dk} = \frac{k^4}{16\pi^2} \frac{3\lambda_k^2}{(k^2 + m_k^2)^2}$$

When $m_k^2 \ll k^2$ in the whole range of integration, well approximated by

$$k \frac{dm_k^2}{dk} = -\frac{\lambda_k}{16\pi^2} k^2 + \frac{\lambda_k}{16\pi^2} m_k^2$$
$$k \frac{d\lambda_k}{dk} = \frac{3\lambda_k^2}{16\pi^2}$$

Taking "SM-like" boundaries, $m(\mu_F) = 125.7$ GeV and $\lambda(\mu_F) = 0.1272$, numerical solutions to the two systems coincide with **great accuracy** (!)

$$k\frac{dm_k^2}{dk} = -\frac{\lambda_k}{16\pi^2}k^2 + \frac{\lambda_k}{16\pi^2}m_k^2$$

$$k\frac{d\lambda_k}{dk} = \frac{3\lambda_k^2}{16\pi^2}$$

Can be **solved analytically** with no further approximations. Second Equation:

$$\lambda(\mu) = \frac{\lambda_\Lambda}{1 - \frac{3}{16\pi^2}\lambda_\Lambda \log\left(\frac{\mu}{\Lambda}\right)}$$

Inserting in the First Equation ⇒ **Non-perturbative** RG equation for $m^2(\mu)$

($E_{\frac{2}{3}}(x)$ is the generalized exponential integral function $E_p(x)$ with $p = \frac{2}{3}$)

$$m^2(\mu) = \frac{1}{3 \cdot 2^{2/3}\left(3\lambda_\Lambda \log\left(\frac{\mu}{\Lambda}\right) - 16\pi^2\right)}$$

$$\times \left[2^{2/3}\Lambda^2 e^{\frac{32\pi^2}{3\lambda_\Lambda}} \times \left(16\pi^2 - 3\lambda_\Lambda \log\left(\frac{\mu}{\Lambda}\right)\right) E_{\frac{2}{3}}\left(\frac{32\pi^2}{3\lambda_\Lambda} - 2\log\left(\frac{\mu}{\Lambda}\right)\right)\right.$$

$$\left. + 4\lambda_\Lambda \sqrt[3]{-\frac{1}{\lambda_\Lambda}}\left(\Lambda^2 e^{\frac{32\pi^2}{3\lambda_\Lambda}} E_{\frac{2}{3}}\left(\frac{32\pi^2}{3\lambda_\Lambda}\right) + 3m_\Lambda^2\right) \times \left(3\pi \log\left(\frac{\mu}{\Lambda}\right) - \frac{16\pi^3}{\lambda_\Lambda}\right)^{2/3}\right]$$

Nice features of this **Non-perturbative** evolution equation for $m^2(\mu)$
(replace $\lambda_\Lambda \to \lambda$)

1) Expanding for $\lambda \ll 1$ and $\mu^2 \ll \Lambda^2 \Rightarrow$ **well-known perturbative result**

$$m_\mu^2 = m_\Lambda^2 + \frac{\lambda}{32\pi^2}\left(\Lambda^2 - m_\Lambda^2 \log\frac{\Lambda^2}{\mu^2}\right)$$

2) Also: very interesting **non-perturbative** approximation, obtained by replacing λ_k with λ in the rhs of the RG equation for $m^2(\mu)$

$$m_\mu^2 = \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}}\left(m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2 - \lambda}\right) - \frac{\lambda\mu^2}{32\pi^2 - \lambda}$$

Excellent approximation for $m^2(\mu)$ (see **ugly equation** previous page)
Important result

$$m^2(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}} \left(m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2 - \lambda}\right) - \frac{\lambda\mu^2}{32\pi^2 - \lambda}$$

Contains several lessons

1) How the **fine-tuning** operates in the **Wilsonian framework**
Boundary at the UV scale $\mu = \Lambda$ for $m^2(\mu)$: m_Λ^2 and $\frac{\lambda\Lambda^2}{32\pi^2-\lambda}$ need to be fine-tuned if at the IR scale μ_{low} we want $m\left(\mu_{low}\right) \sim \mathcal{O}(100)$ GeV.

2) For most of the running towards IR, **flow dominated by the** μ^2 **term**
When $\left(m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2-\lambda}\right) \sim \frac{\lambda\mu^2}{32\pi^2-\lambda}$, **first term takes over**

3) Define

$$m_r^2(\mu) \equiv m^2(\mu) + \frac{\lambda\,\mu^2}{32\pi^2 - \lambda}$$

$$m^2(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}} \left(m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2 - \lambda}\right) - \frac{\lambda\mu^2}{32\pi^2 - \lambda}$$

3) Defining the combination

$$m_r^2(\mu) \equiv m^2(\mu) + \frac{\lambda\,\mu^2}{32\pi^2 - \lambda}$$

from the above equation we see that $m_r^2(\mu)$ obeys the RG equation

($\gamma = \frac{\lambda}{16\pi^2}$ = mass anomalous dimension at one-loop)

$$\mu\frac{d}{d\mu} m_r^2(\mu) = \gamma\, m_r^2(\mu)$$

This coincides with the one-loop improved RG equation for the renormalized running mass $\Rightarrow$ $m_r^2(\mu)$ **is the renormalized running mass**

4) The quantity

$$m_{cr}^2(\mu) \equiv -\frac{\lambda\,\mu^2}{32\pi^2 - \lambda}$$

subtracted to $m^2(\mu)$ in the definition of $m_r^2(\mu)$

$$m_r^2(\mu) \equiv m^2(\mu) + \frac{\lambda\,\mu^2}{32\pi^2 - \lambda}$$

is the **critical mass**

Integrating the Eq. for $m_r^2(\mu)$ (previous page)

$$m_r^2(\mu) = \left(\frac{\mu}{\mu_0}\right)^{\frac{\lambda}{16\pi^2}} m_r^2(\mu_0)$$

Comments

We derived equation

$$\mu \frac{d}{d\mu} m_r^2(\mu) = \frac{\lambda}{16\pi^2}\, m_r^2(\mu)$$

in the **Wilsonian framework**, namely from the **Wilsonian RG flow equation**

$$\mu \frac{d}{d\mu} m^2(\mu) = -\frac{\lambda}{16\pi^2}\, \mu^2 + \frac{\lambda}{16\pi^2} m^2(\mu)$$

whereas it is usually derived in the context of "**technical schemes**" as **dimensional**, **heat kernel**, or **zeta function** regularization.

When quantum fluctuations are calculated within a "technical scheme" we only have **access to first equation**: blind to the fact that $m_r^2(\mu)$ is **physically** obtained only after the **subtraction** $m^2(\mu) \;\to\; m_r^2(\mu) \equiv m^2(\mu) - m_{cr}^2(\mu)$

When quantum fluctuations are calculated within the Wilsonian "physical scheme" $\;\Rightarrow\;$ **we see how the renormalized mass emerges**

Question 1

Should we identify the **physical** running mass $m^2_{phys}(\mu)$ with

$$m^2(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}} \left(m^2_\Lambda + \frac{\lambda\Lambda^2}{32\pi^2 - \lambda}\right) - \frac{\lambda\mu^2}{32\pi^2 - \lambda}$$

or with

$$m^2_r(\mu) = \left(\frac{\mu}{\mu_0}\right)^{\frac{\lambda}{16\pi^2}} m^2_r(\mu_0)$$

Original mass $m^2(\mu)$ or subtracted mass $m^2_r(\mu)$?

In QFT $m^2_{phys}(\mu)$ is usually identified with $m^2_r(\mu)$

Running couplings $g_i(\mu) \Leftarrow$ integrating out quantum fluctuations in $[\mu, \Lambda]$
$g_i(\mu)$: **effective couplings** at the scale μ. True, in particular, for the mass.

$\Rightarrow$ Identify $m^2_{phys}(\mu)$ with $m^2(\mu)$ **not with the subtracted** $m^2_r(\mu)$

Question 2

But ... in QFT textbooks $m^2_{phys}(\mu)$ **usually identified** with $m^2_r(\mu)$

What did we do **wrong**?

Let us **compare** how $m^2(\mu)$ and $m^2_r(\mu)$ **depend** on μ

For **sufficiently low values** of μ (IR regime) $m^2(\mu)$ and $m^2_r(\mu)$ **coincide**

Overlap μ **region** given by

$$\frac{\lambda\mu^2}{32\pi^2-\lambda} \ll \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}} \left(m^2_\Lambda + \frac{\lambda\Lambda^2}{32\pi^2-\lambda}\right)$$

Therefore: the above equation shows the **limitations** of

$$\mu\frac{d}{d\mu}m^2_r(\mu) = \gamma\, m^2_r(\mu)$$

If we are interested in energy scales μ **above this region** we must go back to the original flow equation, that has a **much wider range of validity**

Standard Model - RG Equation for the Higgs mass

$$\mu \frac{d}{d\mu} m_H^2 = \frac{\alpha(\mu)}{16\pi^2}\mu^2 + \gamma(\mu)\, m_H^2$$

$\alpha(\mu)$: combination of SM couplings (gauge, Yukawa, scalar). At one-loop:

$$16\pi^2 \alpha(\mu) = 12 y_t^2 - 12\lambda - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2$$

$\gamma(\mu)$: mass anomalous dimension

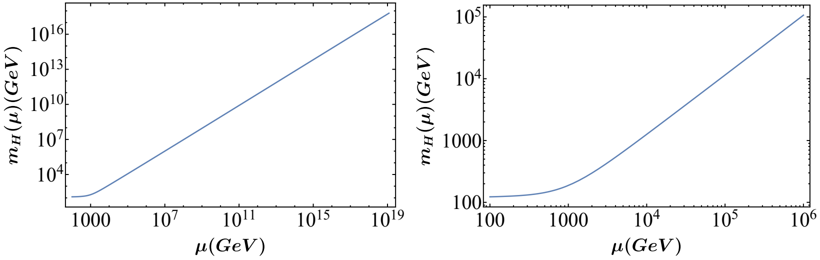
$$16\pi^2 \gamma(\mu) = 6 y_t^2 + 12\lambda - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2$$

Integrating the RG equation for $m_H^2(\mu)$

$$m_H^2(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\gamma} \left(m_H^2(\Lambda) - \frac{\alpha\,\Lambda^2}{2-\gamma}\right) + \frac{\alpha\,\mu^2}{2-\gamma}$$

Very good analytical approximation to the flow

Figure : **Numerical solution** to RG equation and its **Analytical approximation** plotted: indistinguishable



$$\mu \frac{d}{d\mu} m_H^2 = \frac{\alpha(\mu)}{16\pi^2}\mu^2 + \gamma(\mu)\, m_H^2$$

$$m_H^2(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\gamma} \left(m_H^2(\Lambda) - \frac{\alpha\,\Lambda^2}{2-\gamma}\right) + \frac{\alpha\,\mu^2}{2-\gamma}$$

As for Scalar Theory: define **critical mass** and **subtracted mass**

$$m^2_{H,\mathrm{cr}}(\mu) \equiv \frac{\alpha\,\mu^2}{2-\gamma} \qquad \text{and} \qquad m^2_{H,r}(\mu) \equiv m^2_H(\mu) - m^2_{H,\mathrm{cr}}(\mu)$$

From which we immediately see that

$$\mu \frac{d}{d\mu} m^2_{H,r}(\mu) = \gamma\; m^2_{H,r}(\mu) \qquad \Rightarrow \qquad m^2_{H,r}(\mu) = \left(\frac{\mu}{\mu_0}\right)^{\gamma} m^2_{H,r}(\mu_0)$$

This equation **coincides** with the one-loop improved RG equation for the renormalized running mass $\Rightarrow$ $m^2_{H,r}(\mu)$ = **renormalized running Higgs mass**

However the original equation is

$$\mu \frac{d}{d\mu} m^2_H = \frac{\alpha(\mu)}{16\pi^2}\mu^2 + \gamma(\mu)\, m^2_H \;\Rightarrow\; m^2_H(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\gamma}\left(m^2_H(\Lambda) - \frac{\alpha\,\Lambda^2}{2-\gamma}\right) + \frac{\alpha\,\mu^2}{2-\gamma}$$

$$\mu \frac{d}{d\mu} m^2_{H,r}(\mu) = \gamma \; m^2_{H,r}(\mu) \qquad ; \qquad \mu \frac{d}{d\mu} m^2_H \quad = \frac{\alpha(\mu)}{16\pi^2}\mu^2 + \gamma(\mu)\, m^2_H$$

The two flows **coincide** for values of μ such that

$$\frac{\alpha\, \mu^2}{2-\gamma} \ll \left(\frac{\mu}{\Lambda}\right)^{\gamma} \left(m^2_H(\Lambda) - \frac{\alpha\, \Lambda^2}{2-\gamma} \right)$$

Physical Lessons

Fine-tuning of $m^2_H(\Lambda)$ has a profound **physical meaning**: provides the boundary at the UV scale Λ for the RG flow of $m^2_H(\mu)$

Large value of $m^2_H(\Lambda)$ is **physically necessary** and welcome

Quadratic running lasts for most of the $m^2_H(\mu)$ flow towards the IR

Multiplicative renormalization **emerges** while flowing towards the **IR**

In schemes as DR we only have access to $m^2_{H,r}(\mu)$ flow (Why is this the case? ... Backup slides). **Physical perspective** the latter is an **emergent property** of the running, that rises when the flow approaches the IR

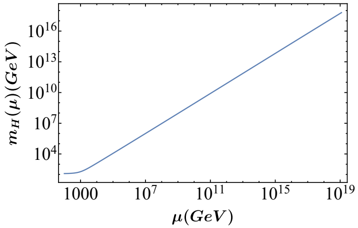
Change in the usual paradigm

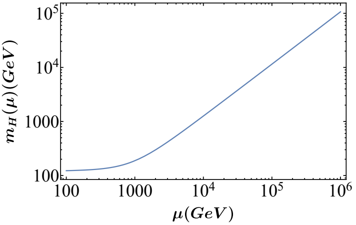
Wilsonian framework **physically required** $\Rightarrow$ **large hierarchy** between UV and IR values of m_H^2 & **fine-tuning** of $m_H^2(\Lambda)$ are **physically necessary**

Multiplicative renormalization valid only at **sufficiently low energies**

The "**elbow**" near $\mu \sim 10^3$ GeV signals the "**transition**"

additive $\rightarrow$ **multiplicative** renormalization





$$\mu \frac{d}{d\mu} m_H^2 = \frac{\alpha(\mu)}{16\pi^2}\mu^2 + \gamma(\mu)\, m_H^2 \quad \Rightarrow \quad \mu \frac{d}{d\mu} m_{H,r}^2(\mu) = \gamma(\mu)\, m_{H,r}^2(\mu)$$

Change in the usual paradigm

Taking into account **experimental uncertainties**: in the **SM parameter space** there exists a **region** of "**tiny size**" from which **large UV boundary values** of m_H^2 give rise, through the RG flow, to the **measured** (within errors) value of the **Higgs mass**

Region **inherited from the ultimate UV completion** of the SM (or of the yet unknown BSM): the **Theory of Everything**.

Multiplicative renormalization confined to the **IR regime only**. It can be obtained within **different schemes** (DR, heat kernel, ...), but **no physical content** can be related to the choice of a specific scheme.

Change in the usual paradigm

An **interesting question** arises, that might be subject to **experimental investigation** in the (hopefully not too far) future.

No one has observed up to now the **running of the Higgs mass**, but we can consider **processes** that should **test the flow** (as for the running bottom quark mass). **Future experiments** could discriminate between $m_H^2(\mu)$ and $m_{H,r}^2(\mu)$

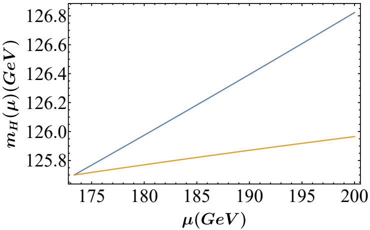


Figure: Blue line: IR flow of $m_H^2(\mu)$. Yellow line: IR flow of $m_{H,r}^2(\mu)$.

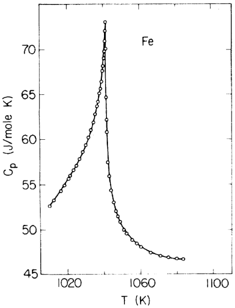
Change in the usual paradigm

Usually **connection** between **QFT** and **Statistical Physics**: correspondence between the request $\xi \gg a$ in the **Theory of Critical Phenomena** ($a =$ lattice spacing, $\xi =$ correlation length) and the request $m^2 \ll \Lambda^2$ in **QFT**

Phrased in RG language $\rightarrow$ **tuning towards the "critical surface"**, achieved through the **subtraction of the "critical mass"**: $m^2_{ren}(\mu) = m^2(\mu) - m^2_{cr}(\mu)$

But $m^2_{ren}(\mu)$ **captures** the **IR final part** of the running of $m^2_{phys}(\mu)$

Flow **physically meaningful** even far from **critical surface** and **fixed points**



Nice Example. Landau-Ginzburg Theory - Ferromagnetic Transition

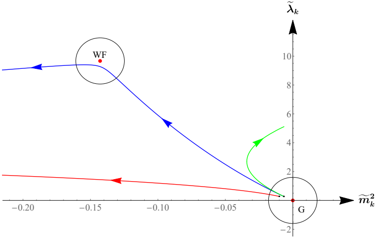
$F_k[\phi] = \int d^3x \left(\frac{1}{2}(\vec{\nabla}\phi)^2 + U_k(\phi) \right) \qquad U_k(\phi) = \frac{1}{2} m_k^2 \phi^2 + \frac{\lambda_k}{4!}\phi^4$

$$k\frac{dm_k^2}{dk} = -\frac{k^3\lambda_k}{4\pi^2\left(k^2+m_k^2\right)} \qquad k\frac{d\lambda_k}{dk} = \frac{3k^3\lambda_k^2}{4\pi^2\left(k^2+m_k^2\right)^2}$$

Dimensionless couplings $\widetilde{m}_k^2 \equiv k^{-2} m_k^2$ and $\widetilde{\lambda}_k \equiv k^{-1}\lambda_k$

$$k\frac{d\widetilde{m}_k^2}{dk} = -2\widetilde{m}_k^2 - \frac{\widetilde{\lambda}_k}{4\pi^2(1+\widetilde{m}_k^2)} \qquad k\frac{d\widetilde{\lambda}_k}{dk} = -\widetilde{\lambda}_k + \frac{3\widetilde{\lambda}_k^2}{4\pi^2(1+\widetilde{m}_k^2)^2}$$

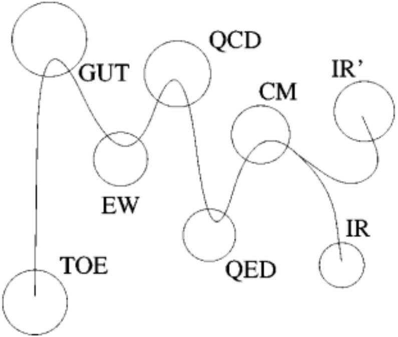
Gaussian G and **Wilson-Fisher** WF **fixed points**. G is IR repulsive (UV attractive)



Blue and **Red** **IR** flows: **Different boundaries** in the UV region around G (Green: linearization)
UV linearly divergent boundary (d=3) crucial for physics at WF: **Ferromagnetic transition**

Fine-tuning Physically needed

From TOE to Condensed Matter and Classical Physics



Alexandre, Branchina, Polonyi, Global Renormalization Group, Phys.Rev.D 58 (1998) 016002

More Slides

Wilsonian RG

versus

Dimensional Regularization

Very useful example: Scalar Theory in d-dimensions

d = integer dimension (no dim reg)

- Wilsonian Effective Action: $S_k[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + V_k(\phi)\right]$

Wilson (Polchinski) RG Equation (LPA)

$$k\frac{\partial}{\partial k} V_k(\phi) = -\frac{k^d}{(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \ln\left(\frac{k^2 + V_k''(\phi)}{k^2}\right)$$

- UV boundary: $V_\Lambda(\phi) \equiv V_0(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\mu^{4-d}\lambda_0}{4!}\phi^4$

Approximating $V_k(\phi)$ in the rhs as $V_k(\phi) \to V_\Lambda(\phi)$

One-loop effective potential

$$V_{1l}(\phi) = V_0(\phi) + \underbrace{\frac{1}{2}\int^{(\Lambda)} \frac{d^d k}{(2\pi)^d} \ln\left(1 + \frac{m_0^2 + \frac{1}{2}\mu^{4-d}\lambda_0\,\phi^2}{k^2}\right)}_{\delta V(\phi)}$$

Lesson: One-loop Effective Potential Approx. of the Wilsonian Potential

Let us focus on the Radiative Correction $\delta V(\phi)$

$$\delta V(\phi) = \frac{1}{2}\int^{(\Lambda)} \frac{d^d k}{(2\pi)^d} \ln\left(1 + \frac{M^2(\phi)}{k^2}\right) \equiv \delta V_1(\phi) + \delta V_2(\phi)$$

where $$M^2(\phi) \equiv m_0^2 + \tfrac{1}{2}\mu^{4-d}\lambda_0\,\phi^2$$

$$\delta V_1(\phi) \equiv \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)}\left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{d}{2}} \int_{\frac{M^2}{M^2+\Lambda^2}}^{1} dt\,(1-t)^{\frac{d}{2}-1}\,t^{-\frac{d}{2}}$$

$$\delta V_2(\phi) \equiv \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)}\left(\frac{\Lambda}{\mu}\right)^{d} \ln\left(1 + \frac{M^2(\phi)}{\Lambda^2}\right)$$

Calculating $\delta V(\phi)$

For any **integer** d:

$$\delta V_1(\phi) = \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{d}{2}} \int_{\frac{M^2}{M^2+\Lambda^2}}^{1} dt\, t^{-\frac{d}{2}}(1-t)^{\frac{d}{2}-1} =$$

$$= \lim_{z\to d} \left[A_1(z) - A_2(z)\right]$$

where z is **complex**, and

$$A_1(z) \equiv F(z)\cdot \overline{B}\left(1-\frac{z}{2}, \frac{z}{2}\right) \qquad A_2(z) \equiv F(z)\cdot \overline{B}_i\left(1-\frac{z}{2}, \frac{z}{2}; \frac{M^2(\phi)}{M^2(\phi)+\Lambda^2}\right)$$

$$F(z) \equiv \frac{\mu^z}{z(4\pi)^{\frac{z}{2}}\Gamma\left(\frac{z}{2}\right)} \left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{z}{2}}$$

$\overline{B}$ and $\overline{B}_i$ are (the analytic extensions of) the Beta functions

Both $\overline{B}$ and $\overline{B}_i$ have poles in $z = 2, 4, 6, \ldots$

$\delta V_1(\phi)$ finite $\Rightarrow$ the poles of A_1 and A_2 have to cancel each other

Example: $\delta V(\phi)$ in $d = 4$ dimensions

$z \equiv 4 - \epsilon$. Expanding in powers of ϵ and M^2/Λ^2

$$A_1(4-\epsilon) = \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon}+\gamma+\ln\frac{M^2(\phi)}{4\pi\mu^2}-\frac{3}{2}\right)+\cancel{\mathcal{O}(\epsilon)}$$

$$A_2(4-\epsilon) = \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon}+\gamma+\ln\frac{M^2(\phi)}{4\pi\mu^2}-\frac{3}{2}\right)+\cancel{\mathcal{O}(\epsilon)}+\cancel{\mathcal{O}\left(\frac{M^2}{\Lambda^2}\right)}$$
$$-\frac{\mu^{-\epsilon}}{64\pi^2}\left[M^2(\phi)\right]^2\left(\frac{\Lambda^2}{M^2(\phi)}-\log\frac{\Lambda^2}{M^2(\phi)}\right)$$

Remember: $\delta V_1(\phi) = \lim_{\epsilon\to 0}\left[A_1(4-\epsilon)-A_2(4-\epsilon)\right]$. Adding $\delta V_2(\phi)$

$$\delta V(\phi) = \delta V_1 + \delta V_2 = \frac{\Lambda^2 M^2(\phi)}{32\pi^2} - \frac{\left[M^2(\phi)\right]^2}{64\pi^2}\left(\ln\frac{\Lambda^2}{M^2(\phi)}+\frac{1}{2}\right)+\cancel{\mathcal{O}\left(\frac{\phi^6}{\Lambda^2}\right)}$$

$$\Rightarrow \quad V_{1l}(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\lambda_0}{4!}\phi^4 + \frac{\Lambda^2 M^2}{32\pi^2} - \frac{\left(M^2\right)^2}{64\pi^2}\left(\ln\frac{\Lambda^2}{M^2}+\frac{1}{2}\right)$$

No reference whatsoever to ϵ (of course!)

With $\Omega_0 = \Omega + \delta\Omega_\Lambda$, $m_0^2 = m^2 + \delta m_\Lambda^2$, $\lambda_0 = \lambda + \delta\lambda_\Lambda$

and $\delta\Omega_\Lambda = -\frac{m^2\Lambda^2}{32\pi^2} + \frac{m^4}{64\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right]$; $\delta m_\Lambda^2 = -\frac{\lambda\Lambda^2}{32\pi^2} + \frac{\lambda m^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right]$

$$\delta\lambda_\Lambda = \frac{3\lambda^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right]$$

... where $\delta\Omega_\Lambda$ and δm_Λ^2 realize fine-tunings (*) ...

$\Rightarrow$ **Renormalized One-Loop Effective Potential (take $\Omega = 0$)**

$$V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{64\pi^2}\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left[\ln\left(\frac{m^2 + \frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right]$$

(*) Physically ... in the parameter space of the theory we go close to the Critical region, or Critical Surface ...

... Let's move now to Dim Reg ...

Radiative correction $\delta V(\phi)$ in Dim. Reg.

- $\delta V(\phi)$ in **Dim Reg.** $\boxed{d \to \text{complex}\,,\ d \equiv 4-\epsilon}$

$$\delta V(\phi) \to \delta V_\epsilon(\phi) \equiv -\frac{\mu^{4-\epsilon}}{2(4\pi)^{2-\frac{\epsilon}{2}}}\left(\frac{M^2(\phi)}{\mu^2}\right)^{2-\frac{\epsilon}{2}} \overline{\Gamma}\left(\frac{\epsilon}{2}-2\right)$$

$$= \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon}+\gamma+\ln\frac{M^2(\phi)}{4\pi\mu^2}-\frac{3}{2}\right)+\mathcal{O}(\epsilon)$$

$\overline{\Gamma}(-d/2)$ defined for any complex $d \neq 2,4,6,\ldots$

- Counterterms in $\overline{MS}$ scheme $\left(\overline{\epsilon} \equiv \epsilon\left(1+\frac{\epsilon}{2}\ln\frac{e^\gamma}{4\pi}\right)\right)$:

$$\delta\Omega_\epsilon = \frac{m^4}{32\pi^2\overline{\epsilon}}\mu^{-\epsilon} \quad , \quad \delta m^2_\epsilon = \frac{\lambda m^2}{16\pi^2\overline{\epsilon}} \quad , \quad \delta\lambda_\epsilon = \frac{3\lambda^2}{16\pi^2\overline{\epsilon}}$$

- **Renormalized One-loop Effective Potential (take $\Omega = 0$) as before**

$$V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{64\pi^2}\left(m^2+\frac{\lambda}{2}\phi^2\right)^2\left[\ln\left(\frac{m^2+\frac{\lambda}{2}\phi^2}{\mu^2}\right)-\frac{3}{2}\right]$$

Before going on with our analysis ... Let's hear “news” from the Literature

“Dim Reg” versus “Wilson” (= “successive elimination of modes”)

Views on “Dim Reg” and “Wilson”

1) Typical textbook statement . . . “Dimensional Regularization has no direct physical interpretation” (J. Zinn-Justin - **Quantum field theory of critical phenomena**)

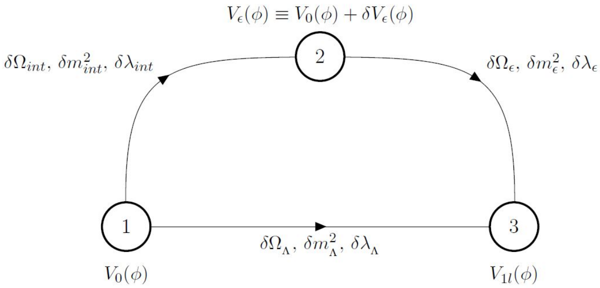
2) Recent ideas (gaining lot of followers)

“Maybe power divergences vanish because the ultimate unknown physical cut-off behaves like dimensional regularization” (M. Farina, D. Pappadopulo and A. Strumia, JHEP 08 (2013) 022)

“Wilsonian computation techniques attribute physical meaning to momentum shells of loop integrals” . . . “The naturalness problem can be more generically formulated as a problem of the **Effective Theory Ideology**” (A. Salvio and A. Strumia, JHEP 06 (2014) 080)

Accordingly **DR** should have special physical properties that make it the **correct way** to calculate the quantum fluctuations ... while **Wilson** ... **incorrect** ...

Dim Reg.: Physical Meaning? ... Special Physical Properties?



$$V_0(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\lambda_0}{4!}\phi^4 = (\Omega + \delta\Omega_\Lambda) + \frac{1}{2}(m^2 + \delta m^2_\Lambda)\,\phi^2 + \frac{1}{4!}(\lambda + \delta\lambda_\Lambda)\phi^4$$

$$= (\Omega + \delta\Omega_{int} + \delta\Omega_\epsilon) + \frac{1}{2}(m^2 + \delta m^2_{int} + \delta m^2_\epsilon)\phi^2 + \frac{1}{4!}(\lambda + \delta\lambda_{int} + \delta\lambda_\epsilon)\phi^4$$

$\Rightarrow$ Ren.Pot. : $$V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{64\pi^2}\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left[\ln\left(\frac{m^2 + \frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right]$$

Dim Reg.: Physical Meaning? ... Special Physical Properties?

DR secretly realizes the fine-tuning:

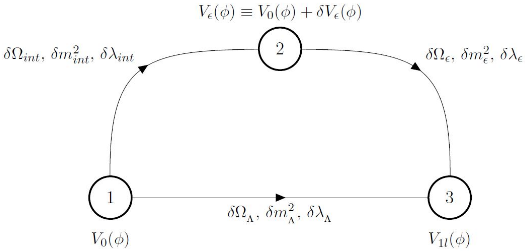
$$\delta\Omega_{int} = -\frac{m^2\Lambda^2}{32\pi^2} + \frac{m^4}{64\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right] - \frac{m^4}{32\pi^2\overline{\epsilon}}\mu^{-\epsilon}$$

$$\delta m^2_{int} = -\frac{\lambda\Lambda^2}{32\pi^2} + \frac{\lambda m^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right] - \frac{\lambda m^2}{16\pi^2\overline{\epsilon}}$$

$$\delta\lambda_{int} = \frac{3\lambda^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right] - \frac{3\lambda^2}{16\pi^2\overline{\epsilon}}$$

DR has a Physical Meaning but No Special Physical Properties. It implements the Wilsonian iterative elimination of modes for including the quantum fluctuations in the Effective Theory, and **secretly** realizes the fine-tuning

Summary on DR



DR setting, "Bubble (2)", obtained by introducing an intermediate step, (1) → (2), in the process of obtaining the Renormalized Potential, "Bubble (3)".

DR provides a shortcut: "Bubble (3)" is reached starting from "Bubble (2)". The fine-tuning step "Bubble (1)" → "Bubble (2)" is skipped (secretly realized)

Lesson: DR is a way to implement the Wilson's strategy in the perturbative regime, where the *fine-tuning* (in the Wilsonian language: tuning toward the critical regime, critical surface) is secretly performed.

Naturalness and Dimensional Regularization

What should we then say on those attempts to solve the Naturalness/Hierarchy problem with DR?

- **Classically Scale Invariant BSM**. The theory does not possess mass or length scales $\Rightarrow$ only dimension four operators
- Dimensional Regularization used $\Rightarrow$ Scale Invariance only softly broken $\Rightarrow$ apparently **no fine-tuning needed** ... seems good ...
- ... But ... we have just shown ... DR **secretly realizes the fine-tuning**

$\Rightarrow$ **No way to solve the Naturalness/Hierarchy problem with DR**

Flourishing literature

- K. A. Meissner and H. Nicolai, Phys. Lett. B **648**, 312-317 (2007).
- K. A. Meissner and H. Nicolai, Phys. Lett. B **660**, 260-266 (2008).
- R. Foot, A. Kobakhidze, K. L. McDonald and R. R. Volkas, Phys. Rev. D **77**, 035006 (2008).
- F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659**, 703-706 (2008).
- M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671**, 162-166 (2009).
- L. Alexander-Nunneley and A. Pilaftsis, JHEP **09**, 021 (2010).
- L. Boyle, S. Farnsworth, J. Fitzgerald and M. Schade, arXiv:1111.0273 [hep-ph].
- C. D. Carone and R. Ramos, Phys. Rev. D **88**, 055020 (2013).
- A. Farzinnia, H. J. He and J. Ren, Phys. Lett. B **727**, 141-150 (2013).
- Y. Kawamura, PTEP **2013**, no.11, 113B04 (2013).
- I. Bars, P. Steinhardt and N. Turok, Phys. Rev. D **89**, no.4, 043515 (2014).
- M. Heikinheimo, A. Racioppi, M. Raidal, *et al*, Mod. Phys. Lett. A **29**, 1450077 (2014).
- A. Salvio and A. Strumia, JHEP **06**, 080 (2014).
- T. G. Steele, Z. W. Wang, *et al.*, Phys. Rev. Lett. **112**, no.17, 171602 (2014).
- F. Bezrukov, J. Rubio and M. Shaposhnikov, Phys. Rev. D **92**, no.8, 083512 (2015).
- J. Guo and Z. Kang, Nucl. Phys. B **898**, 415-430 (2015).
- D. M. Ghilencea, Phys. Rev. D **93**, no.10, 105006 (2016).
- D. M. Ghilencea, Z. Lalak and P. Olszewski, Eur. Phys. J. C **76**, no.12, 656 (2016).
- Z. W. Wang, T. G. Steele, T. Hanif and R. B. Mann, JHEP **08**, 065 (2016).
- D. M. Ghilencea, Z. Lalak and P. Olszewski, Phys. Rev. D **96**, no.5, 055034 (2017).
- I. Oda, Eur. Phys. J. C **78**, no.10, 798 (2018).
- S. Mooij, M. Shaposhnikov and T. Voumard, Phys. Rev. D **99**, no.8, 085013 (2019).
- M. Farina, D. Pappadopulo and A. Strumia, JHEP **08**, 022 (2013).
- I. Brivio and M. Trott, Phys. Rept. **793**, 1-98 (2019).

Consider now attempts to solve the NH problem in a RG framework

Flourishing literature

- C. Wetterich, Phys. Lett. B **104**, 269-276 (1981).
- C. Wetterich, Phys. Lett. B **140**, 215-222 (1984).
- C. Wetterich, Z. Phys. C **48**, 693-705 (1990).
- C. Wetterich, Z. Phys. C **57**, 451-470 (1993).
- S. Bornholdt and C. Wetterich, Phys. Lett. B **282**, 399-405 (1992).
- M. Shaposhnikov and C. Wetterich, Phys. Lett. B **683**, 196-200 (2010).
- C. Wetterich, Phys. Lett. B **718**, 573-576 (2012).
- M. Holthausen, K. S. Lim and M. Lindner, JHEP **02**, 037 (2012).
- G. F. Giudice, PoS **EPS-HEP2013**, 163 (2013).
- M. Holthausen, J. Kubo, K. S. Lim and M. Lindner, JHEP **12** (2013), 076.
- S. Iso and Y. Orikasa, PTEP **2013** (2013), 023B08.
- L. E. Ibanez and I. Valenzuela, JHEP **05** (2013), 064.
- I. Masina and M. Quiros, Phys. Rev. D **88** (2013), 093003.
- M. Hashimoto, S. Iso and Y. Orikasa, Phys. Rev. D **89** (2014) no.1, 016019.
- N. Haba, K. Kaneta and R. Takahashi, JHEP **04** (2014), 029.
- M. Hashimoto, S. Iso and Y. Orikasa, Phys. Rev. D **89** (2014) no.5, 056010.
- P. H. Chankowski, A. Lewandowski, K. A. Meissner and H. Nicolai, Mod. Phys. Lett. A **30**, 1550006 (2015).
- C. Wetterich and M. Yamada, Phys. Lett. B **770**, 268-271 (2017)
- J. M. Pawlowski, M. Reichert, C. Wetterich and M. Yamada, Phys. Rev. D **99**, no.8, 086010 (2019).
- S. Mooij and M. Shaposhnikov, arXiv:2110.05175.
- S. Mooij and M. Shaposhnikov, arXiv:2110.15925.

"Wilson" versus "Perturbatively-Renormalized" RG Equations

Scalar Theory : $\mathcal{L}_\Lambda = \frac{1}{2}(\partial_\mu \phi_\Lambda)^2 + \frac{1}{2} m_\Lambda^2 \phi_\Lambda^2 + \frac{\lambda_\Lambda}{4!}\phi_\Lambda^4$

Wilson-Polchinski RG Equations

$$\mu \frac{d\Omega}{d\mu} = -\frac{m^2\mu^2}{16\pi^2} + \frac{m^4}{32\pi^2} \quad ; \quad \mu \frac{dm^2}{d\mu} = -\frac{\lambda\mu^2}{16\pi^2} + \frac{\lambda m^2}{16\pi^2} \quad ; \quad \mu \frac{d\lambda}{d\mu} = \frac{3\lambda^2}{16\pi^2}$$

$\mu \in [0, \Lambda]$ is the running scale. Λ is the UV boundary (physical cut-off)

Define:

$$m_{\rm cr}^2(\mu) \equiv \frac{\lambda(\mu)}{16\pi^2}\,\mu\,\delta\mu \quad \text{and} \quad \widetilde{m}^2(\mu - \delta\mu) \equiv m^2(\mu-\delta\mu) - m_{\rm cr}^2(\mu)$$

$$\Omega_{\rm cr}(\mu) \equiv \frac{\widetilde{m}^2(\mu)}{16\pi^2}\,\mu\,\delta\mu \quad \text{and} \quad \widetilde{\Omega}(\mu - \delta\mu) \equiv \Omega(\mu-\delta\mu) - \Omega_{\rm cr}(\mu)$$

Perturbatively-Renormalized RG Equations $(\delta\mu \to 0)$

$$\mu \frac{d\widetilde{\Omega}}{d\mu} = \frac{\widetilde{m}^4}{32\pi^2} = \beta_\Omega \quad ; \quad \mu \frac{d\widetilde{m}^2}{d\mu} = \frac{\lambda \widetilde{m}^2}{16\pi^2} = \widetilde{m}^2 \gamma_m \quad ; \quad \mu \frac{d\lambda}{d\mu} = \frac{3\lambda^2}{16\pi^2} = \beta_\lambda$$

The Perturbatively-Renormalized RG Equations **contain the fine-tuning**

Physically: **Tuning towards the Critical Surface**

Perturbatively-Renormalized RG equations in the Standard Model

Well-known Standard Model perturbative RG equations (*)

$$\mu \frac{d}{d\mu}\lambda_i = \beta_{\lambda_i} \qquad \mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

λ_i $(i = 1, \ldots, 5)$ are the SM couplings

$(*)$ similarly for SM extensions

Perturbatively-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 1 : Quantum Gravity "**miracle**" G.F. Giudice, PoS EPS-HEP2013, 163 (2013)

$$m_H^2(\Lambda) \ll \Lambda^2$$

With the SM perturbative γ_m $(\gamma_m \ll 1)$ $\Rightarrow$

Apparently no Hierarchy Problem : $m_H^2(\Lambda) \sim m_H^2(\mu_F)$

. . . But . . . remember . . . in the above RG Equation m_H^2 is the **tuned mass** $\Rightarrow$

Fine-tuning encoded in the RG Equation above

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Perturbatively-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 2 : "**Self-organized criticality**"

J. M. Pawlowski, M. Reichert, C. Wetterich and M. Yamada, Phys. Rev. D **99**, 086010 (2019)

Assumes Quantum Gravity might give a non-perturbative $\gamma_m \sim 2 \;\Rightarrow$

Hierarchy can be tolerated : $m_H^2(\Lambda) \gg m_H^2(\mu_F)$

. . . But . . . remember . . . m_H^2 is the **tuned mass** $\Rightarrow$

Fine-tuning encoded in the above RG Equation

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Perturbatively-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 3 : $m_H^2(\mu)$ from $\lambda(\mu)$ and $v(\mu)$ . . .

P. H. Chankowski, A. Lewandowski, K. A. Meissner and H. Nicolai, Mod. Phys. Lett. A **30**, 1550006 (2015)

M. Holthausen, K. S. Lim and M. Lindner, JHEP **02**, 037 (2012)

Apparently no large corrections : $m_H^2(\mu_F) \sim 125\,\mathrm{GeV}$

... However ... same problem as before . . . Tuning encoded in the RG equation for the vev $v(\mu)$ (equivalent to the above RG equation for $m_H^2(\mu)$)

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Perturbative-Renormalized RG equations in the Standard Model

Attempt 4 : "Finite formulation" of QFT using RG equations *à la* Callan-Symanzik for the Green's functions . . .

S. Mooij and M. Shaposhnikov, arXiv:2110.05175

S. Mooij and M. Shaposhnikov, arXiv:2110.15925

Apparently no quadratic corrections for the mass m^2 of scalar particles

However . . . Tuning encoded in taking derivatives with respect to m^2 of the Green's functions, until they become finite

Callan has shown that this is just a way of implement the subtraction of Λ^2 and $\log \Lambda$ terms C. G. Callan, Jr., Conf. Proc. C **7507281**, 41-77 (1975)

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Backup Slides

Wilsonian - Polchinski RG equations

- Flow of the theory parameters:

$$\Lambda \frac{d}{d\Lambda}\Omega_0 = -\frac{m_0^2\Lambda^2}{16\pi^2} + \frac{m_0^4}{32\pi^2} \qquad \Lambda \frac{d}{d\Lambda} m_0^2 = -\frac{\lambda_0\Lambda^2}{16\pi^2} + \frac{\lambda_0 m_0^2}{16\pi^2} \qquad \Lambda \frac{d}{d\Lambda}\lambda_0 = \frac{3\lambda_0^2}{16\pi^2}$$

- From the Wegner-Houghton equation for $d = 4$, inserting the expansion $U_k(\phi) = \Omega_k + \frac{1}{2}m_k^2\phi^2 + \frac{1}{4!}\lambda_k\phi^4 + \frac{1}{6!}\lambda_k^{(6)}\phi^6 + \ldots$ we have the flow equations:

$$k\frac{\partial\Omega_k}{\partial k} = -\frac{k^4}{16\pi^2}\log\left(\frac{k^2+m_k^2}{k^2}\right)$$

$$k\frac{\partial m_k^2}{\partial k} = -\frac{k^4}{16\pi^2}\frac{\lambda_k}{k^2+m_k^2}$$

$$k\frac{\partial\lambda_k}{\partial k} = \frac{k^4}{16\pi^2}\frac{3\lambda_k^2}{(k^2+m_k^2)^2}$$

- Under the condition $k^2 \gg m_k^2$, i.e. in the UV regime, they reduce to the bare parameters flow equations.

Critical term

- Finite difference RG equation for the mass:

$$m_0^2\left(\Lambda-\delta\Lambda\right)=m_0^2\left(\Lambda\right)+\frac{\delta\Lambda}{\Lambda}\,\frac{\lambda_0\left(\Lambda\right)}{16\pi^2}\,\Lambda^2-\frac{\delta\Lambda}{\Lambda}\,\frac{\lambda_0\left(\Lambda\right)m_0^2\left(\Lambda\right)}{16\pi^2}+\mathcal{O}\left(\frac{\delta\Lambda^2}{\Lambda^2}\right)$$

- Subtracted mass parameter at the scale $\Lambda-\delta\Lambda$

$$\widetilde{m}^2(\Lambda-\delta\Lambda)\equiv m_0^2(\Lambda-\delta\Lambda)-m_{\rm cr}^2(\Lambda)$$

where the *critical mass* $m_{\rm cr}^2$, and the boundary at Λ are given by

$$m_{\rm cr}^2(\Lambda)\equiv\frac{\lambda_0\left(\Lambda\right)}{16\pi^2}\,\Lambda\,\delta\Lambda\qquad\widetilde{m}^2(\Lambda)=m_0^2(\Lambda)$$

- In the limit $\delta\Lambda\to 0$ we recover the perturbative RG equations:

$$\beta_\Omega=\mu\frac{d\Omega}{d\mu}=\frac{m^4}{32\pi^2}\qquad\gamma_m=\frac{1}{m^2}\left(\mu\frac{dm^2}{d\mu}\right)=\frac{\lambda}{16\pi^2}\qquad\beta_\lambda=\mu\frac{d\lambda}{d\mu}=\frac{3\lambda^2}{16\pi^2}$$

- The renormalized RG equations **contain the fine-tuning**: physically, this corresponds to a *tuning towards the critical surface*.

Perturbative-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 5 : hierarchy between M_P and μ_F generated by an instanton configuration contributing to the vev of the Higgs field ...

M. Shaposhnikov and A. Shkerin, Phys. Lett. B **783**, 253 (2018)

M. Shaposhnikov and A. Shkerin, JHEP **10**, 024 (2018)

Apparently Hierarchy explained

however ...quantum corrections calculated with DR, and flow of the parameters studied with the perturbative RG flows ...same problems as before

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Gauge theories

Attempts to a gauge invariant Wilsonian RG

- V. Branchina, K. Meissner and G. Veneziano, The Price of an exact, gauge invariant RG flow equation, Phys. Lett. B **574**, 319-324 (2003)
- S.P. de Alwis, Exact RG Flow Equations and Quantum Gravity, JHEP **03**, 118 (2018)