Challenges in supersymmetric cosmology

I. Antoniadis

LPTHE, Sorbonne University, CNRS, Paris

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Problem of scales

- describe high energy (SUSY?) extension of the Standard Model

  unification of all fundamental interactions

- incorporate Dark Energy

  simplest case: infinitesimal (tuneable) +ve cosmological constant

- describe possible accelerated expanding phase of our universe

  models of inflation (approximate de Sitter) [4]

⇒ 3 very different scales besides M_W and M_{Planck} : [11]

Supersymmetry

A well motivated proposal

addressing several open problems of the Standard Model

- natural elementary scalars
- realise unification of the three Standard Model forces
- natural dark matter candidate (lightest supersymmetric particle)
- addressing the hierarchy problem
- prediction of light Higgs ($\lesssim$ 130 GeV)
- soft UV behavior and important ingredient of string theory

But no experimental indication of any BSM physics at LHC

It is likely to be there at some (more) fundamental level

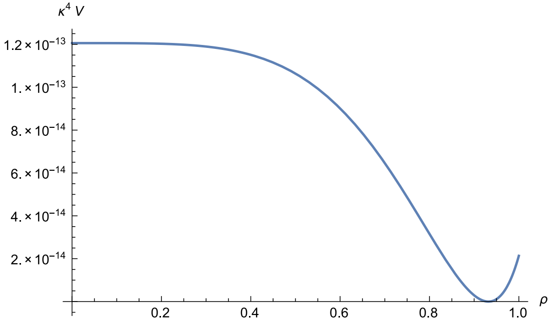
Inflation:

Theoretical paradigm consistent with cosmological observations

But phenomelogical models with not real underlying theory [2]

introduce a new scalar field that drives Universe expansion at early times

Inflaton potential



slow-roll region with V', V'' small compared to the de Sitter curvature

Inflation in supergravity: main problems

Inflaton: part of a chiral superfield X [12]

- slow-roll conditions: the eta problem $\Rightarrow$ fine-tuning of the potential

  $$\eta = V''/V\,, \quad V_F = e^K(|DW|^2 - 3|W|^2)\,, \quad DW = W' + K'W$$

  K: Kähler potential, W: superpotential Planck units: $\kappa = 1$

  canonically normalised field: $K = X\bar{X} \Rightarrow \eta = 1 + \ldots$

- trans-Planckian initial conditions $\Rightarrow$ break validity of EFT

  no-scale type models that avoid the η-problem

  $$K = -3\ln(T + \bar{T})\,;\; W = W_0 \;\Rightarrow\; V_F = 0$$

- stabilisation of the (pseudo) scalar companion of the inflaton

  chiral multiplets $\Rightarrow$ complex scalars

- moduli stabilisation, de Sitter vacuum, ...

Starobinsky model of inflation

$$\mathcal{L} = \frac{1}{2}R + \alpha R^2$$

Lagrange multiplier ϕ : $\mathcal{L} = \frac{1}{2}(1+2\phi)R - \frac{1}{4\alpha}\phi^2$ $\phi = 2\alpha R$

Rescaling the metric to the Einstein frame $\Rightarrow$

equivalent to a scalar field with exponential potential:

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{M^2}{12}\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 \qquad M^2 = \frac{3}{4\alpha}$$

supersymmetric extension: need two Lagrange multipliers

$\Rightarrow$ two chiral superfields

one contains the inflaton ϕ and the other the goldstino [10]

Goldstone fermion of spontaneous supersymmetry breaking

SUSY extension of Starobinsky model

$$K = -3\ln(T + \bar{T} - C\bar{C}) \quad ; \quad W = MC(T - \frac{1}{2})$$

- T contains the inflaton: $\mathrm{Re}\, T = e^{\sqrt{\frac{2}{3}}\phi}$
- $C \sim \mathcal{R}$ is unstable during inflation

  $\Rightarrow$ add higher order terms to stabilize it

  e.g. $C\bar{C} \rightarrow h(C, \bar{C}) = C\bar{C} - \zeta(C\bar{C})^2$ Kallosh-Linde '13

- SUSY is broken during inflation with C the goldstino superfield

  $\rightarrow$ model independent treatment in the decoupling sgoldstino limit

  replace C by a constrained superfield X satisfying $X^2 = 0$

  $\Rightarrow$ sgoldstino $=$ (goldstino)$^2/F$

$\Rightarrow$ minimal SUSY extension that evades stability problem

Non-linear supersymmetry $\Rightarrow$ goldstino mode χ

Volkov-Akulov '73

Effective field theory of SUSY breaking at low energies

Analog of non-linear σ-model $\Rightarrow$ constraint superfields

Rocek-Tseytlin '78, Lindstrom-Rocek '79, Komargodski-Seiberg '09

Goldstino: chiral superfield X_{NL} satisfying $X_{NL}^2 = 0 \Rightarrow$

$$X_{NL}(y) = \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F$$

$$\mathcal{L}_{NL} = \int d^4\theta X_{NL}\bar{X}_{NL} - \frac{1}{\sqrt{2}\kappa}\left\{\int d^2\theta X_{NL} + h.c.\right\} = \mathcal{L}_{Volkov-Akulov}$$

R-symmetry with $[\theta]_R = [\chi]_R = 1$ and $[X]_R = 2$ $\qquad F = \frac{1}{\sqrt{2}\kappa} + \dots$

Non-linear SUSY in supergravity

I.A.-Dudas-Ferrara-Sagnotti '14

$$K = X\bar{X} \quad ; \quad W = f\,X + W_0$$

$X \equiv X_{NL}$ nilpotent goldstino superfield

$$X_{NL}^2 = 0 \;\Rightarrow\; X_{NL}(y) = \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F$$

$$\Rightarrow \quad V = |f|^2 - 3|W_0|^2 \quad ; \quad m_{3/2}^2 = |W_0|^2$$

- V can have any sign   contrary to global NL SUSY
- NL SUSY in flat space $\Rightarrow f = \sqrt{3}\, m_{3/2} M_p$
- R-symmetry is broken by W_0

Non-linear Starobinsky supergravity [7][7]

I.A.-Dudas-Ferrara-Sagnotti '14

Non-linear supersymmetry limit: one field decouples: [6]

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{M^2}{12}\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 - \frac{1}{2}e^{-2\sqrt{\frac{2}{3}}\phi}(\partial a)^2 - \frac{M^2}{18}e^{-2\sqrt{\frac{2}{3}}\phi}a^2$$

- no eta-problem but

  initial conditions require trans-planckian values for ϕ $(\phi > 1)$

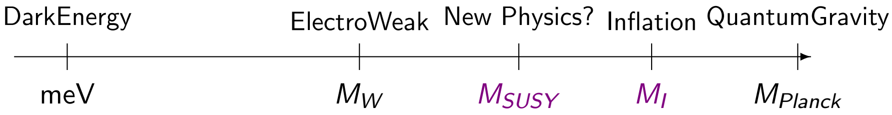
- pseudoscalar a much heavier than ϕ during inflation, decouples:

  $m_\phi = \frac{M}{3}e^{-\sqrt{\frac{2}{3}}\phi_0} << m_a = \frac{M}{3}$

- inflation scale M independent from supersymmetry breaking scale

  $\Rightarrow$ compatible with low energy supersymmetry

Problem of scales: connections



Direct connection of inflation and supersymmetry breaking:

identify the inflaton with the partner of the goldstino

Goldstone fermion of spontaneous supersymmetry breaking

while accommodating observed vacuum energy

Inflation from supersymmetry breaking

I.A.-Chatrabhuti-Isono-Knoops '16, '17, '19

Inflaton : goldstino superpartner in the presence of a gauged R-symmetry

- linear superpotential $W = f\,X \Rightarrow$ no η-problem [5]

$$\begin{aligned} V_F &= e^K \left(|DW|^2 - 3|W|^2\right) \\ &= e^K \left(|1 + K_X X|^2 - 3|X|^2\right)|f|^2 \qquad K = X\bar{X} \\ &= e^{|X|^2} \left(1 - |X|^2 + \mathcal{O}(|X|^4\right)|f|^2 = \mathcal{O}(|X|^4) \quad \Rightarrow \eta = 0 + \dots \end{aligned}$$

  linear W guaranteed by an R-symmetry

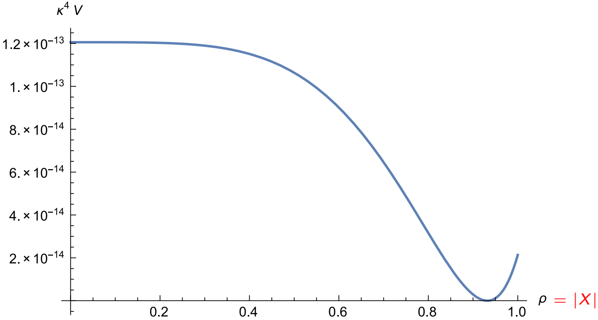
- gauge R-symmetry: (pseudo) scalar absorbed by the $U(1)_R$
- inflation around a maximum of scalar potential (hill-top) $\Rightarrow$ small field

  no large field initial conditions

- vacuum energy at the minimum: tuning between V_F and V_D

Two classes of models

- Case 1: R-symmetry is restored during inflation (at the maximum)



- Case 2: R-symmetry is (spontaneously) broken everywhere

and restored at infinity   example: $S = \ln X$

Case 1: R-symmetry restored during inflation

maximum at the origin with small η by a correction to the Kähler potential

$$\mathcal{K}(X,\bar{X}) = \kappa^{-2}X\bar{X} + \kappa^{-4}A\,(X\bar{X})^2 \qquad A > 0 \qquad \text{[16][19]}$$

$$W(X) = \kappa^{-3}f\,X \qquad \Rightarrow$$

$$f(X) = 1 \qquad (+\beta \ln X \text{ to cancel anomalies but } \beta \text{ very small})$$

$$\mathcal{V} = \mathcal{V}_F + \mathcal{V}_D$$

$$\mathcal{V}_F = \kappa^{-4}f^2 e^{X\bar{X}(1+AX\bar{X})}\left[-3X\bar{X} + \frac{\left(1+X\bar{X}(1+2AX\bar{X})\right)^2}{1+4AX\bar{X}}\right]$$

$$\mathcal{V}_D = \kappa^{-4}\frac{q^2}{2}\left[1+X\bar{X}(1+2AX\bar{X})\right]^2$$

Assume inflation happens around the maximum $|X| \equiv \rho \simeq 0 \quad \Rightarrow$

Predictions

slow-roll parameters $(q \simeq 0)$

$$\eta = \frac{1}{\kappa^2}\left(\frac{V''}{V}\right) = -4A + \mathcal{O}(\rho^2)$$

$$\epsilon = \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2 = 16A^2\rho^2 + \mathcal{O}(\rho^4) \simeq \eta^2\rho^2 << |\eta|$$

η naturally small since A is a correction

inflation starts with an initial condition for $\phi = \phi_*$ near the maximum

and ends when $|\eta| = 1$

$\Rightarrow$ number of e-folds $N = \int_{end}^{start} \frac{V}{V'} = \kappa \int \frac{1}{\sqrt{2\epsilon}} \simeq \frac{1}{|\eta_*|} \ln\left(\frac{\rho_{\rm end}}{\rho_*}\right)$

Planck '15 data : $\eta \simeq -0.02 \Rightarrow N \gtrsim 50$ naturally

Predictions

$$\text{amplitude of density perturbations} \quad A_s = \frac{\kappa^2 H_*^2}{8\pi^2 \epsilon_*}$$

$$\text{spectral index} \quad n_s \simeq 1 + 2\eta_*$$

$$\text{tensor} - \text{to} - \text{scalar ratio} \quad r = 16\epsilon_*$$

Planck '15 data : $\eta_* \simeq -0.02,\ A_s \simeq 2.2 \times 10^{-9},\ N \gtrsim 50$

$$\Rightarrow r \lesssim 10^{-4},\ H_* \lesssim 10^{12}\ \text{GeV}$$

Question: can a 'nearby' minimum exist with a tiny +ve vacuum energy?

Answer: Yes [14]

need an extra correction to the kinetic terms

$$\mathcal{K}(X,\bar{X}) : +\kappa^{-6} B\,(X\bar{X})^3$$

Microscopic Model

Fayet-Iliopoulos model based on a $U(1)$ R-symmetry in supergravity

two chiral multiplets $\Phi_\pm$ of charges $q_\pm$ and mass m and FI parameter ξ

$$W = m\,\Phi_+\Phi_-$$

R-symmetry $\Rightarrow$ $q_+ + q_- \neq 0$

Higgs phase: $\langle\Phi_-\rangle = v \neq 0$

Limit of small SUSY breaking compared to the $U(1)$ mass: $m^2 << q_-^2 v^2$

integrate out gauge superfield $\rightarrow$ EFT for the goldstino superfield Φ_+

$$W = mv\Phi_+ \quad ; \quad K = \bar\Phi_+\Phi_+ + A\,(\bar\Phi_+\Phi_+)^2 + B\,(\bar\Phi_+\Phi_+)^3 + \cdots$$

parameter space allows realistic inflation

and a nearby minimum with tuneable energy

Coupling with MSSM

I.A.-Aldabergenov-Chatrabhuti-Isono '21

Two distinct cases:

- Standard Model superfields ϕ neutral under $U(1)_R \Rightarrow (\kappa = 1)$

$$W(X,\phi) = [f + w(\phi)]\,X \qquad w(\phi) : \mathrm{MSSM\,superpotential}$$

- SM particles neutral and superpartners charged $\Rightarrow U(1)_R \supset R$-parity:

$$W(X,\phi) = f\,X + w(\phi)$$

I.A.-Knoops '16

Both cases lead to similar results [21]

Kinetic terms: $K(X,\bar{X},\phi,\bar{\phi}) = \sum \left[1 + \Delta_\phi(X\bar{X})\right]\phi\bar{\phi} + J(X\bar{X})$

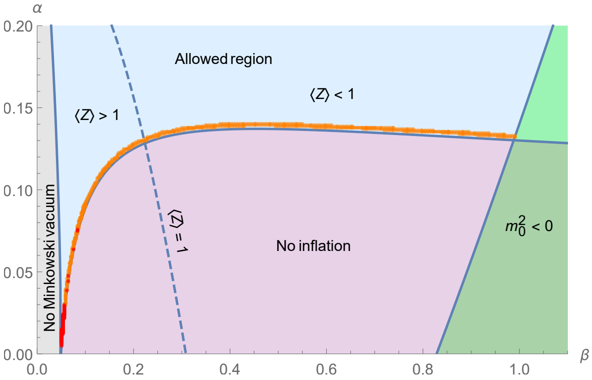
$$J = X\bar{X} + \alpha(X\bar{X})^2 + \beta(X\bar{X})^3$$

Coupling with MSSM: constraints

- Viable inflation and (nearly vanishing) vacuum energy $\Rightarrow q \gtrsim 0.8f$ [14]
- Positive scalar masses: $m_0^2 = m_{3/2}^2 - \frac{1}{2}\langle 1 + \Delta_\phi \rangle \langle \mathcal{D}_R \rangle^2 \geq 0$

$\Rightarrow \langle \Delta_\phi \rangle \lesssim 0.15$

$Z \equiv X$



Spectrum

Gaugino masses from $U(1)_R$ anomaly cancellation:

$$e^{-1}\mathcal{L} \supset \frac{1}{8} \sum_{A=R,1,2,3} \mathrm{Im}(f_A)\, F^A \tilde{F}^A \quad ; \quad f_A = 1 + \beta_A \ln X$$

with $\beta_R = -\frac{g^2}{3\pi^2}\ , \quad \beta_1 = -\frac{11 g_1^2}{8\pi^2}\ , \quad \beta_2 = -\frac{5 g_2^2}{8\pi^2}\ , \quad \beta_3 = -\frac{3 g_3^2}{8\pi^2}$

Typical spectrum:

$$\alpha = 0.139\ , \quad \beta = 0.6\ , \quad g/f = 0.7371\ , \quad f = 2.05 \times 10^{-7} \quad \Rightarrow$$

$m_z\,, m_R$	m_ζ	$m_{3/2}$	m_0	m_1	m_2	m_3
1.25×10^{12}	6.15×10^{11}	7.51×10^{11}	2.68×10^{11}	1.03×10^{10}	6.54×10^{9}	5.84×10^{9}

masses (in GeV) of inflaton, inflatino, gravitino, and MSSM sparticles

$(H_{\rm inf} = 3 \times 10^{11}$ GeV$)$

$\{m_z, m_{3/2}, m_0\} > \{m_1, m_2, m_3\}$

Inflaton decay and reheating

Dominant decay to scalars and inflatino

$$\Gamma^{\rm tot}_{z\to\phi\phi} = 5.8\times10^{-3}\ {\rm GeV}$$

$$\Gamma^{\rm tot}_{z\to\lambda\lambda}\approx\Gamma_{z\to\zeta\zeta} = 4.7\times10^{-4}\ {\rm GeV}$$

$\Rightarrow T_{\rm reh}\simeq\sqrt{M_P\Gamma_{\rm tot}}=1.26\times10^{8}\ {\rm GeV}$

Possible dark matter candidate: superheavy LSP

$m_{\rm LSP}\sim10^{10}$ GeV with $T_{\rm reh}/m_{\rm DM}\sim10^{-3}$ Chung-Kolb-Riotto '99

Conclusions

General class of models with inflation from SUSY breaking:

identify inflaton with goldstino superpartner

- (gauged) R-symmetry restored

  small field, avoids the η-problem, no (pseudo) scalar companion

  a nearby minimum can have tuneable positive vacuum energy

- inflaton sector can be coupled to MSSM

  with gauge $U(1)_R$ containing the R-parity

- D-term inflation is also possible using a new FI term

  it can lead to large r of primordial gravitational waves

Open question: string theory realisation