



Dynamical Cobordism of a 9d Domain Wall and its companion 7-brane

Based on 2205.09782, with R. Blumenhagen, N. Cribiori, C. Kneißl

Andriana Makridou

Workshop on Holography and the Swampland

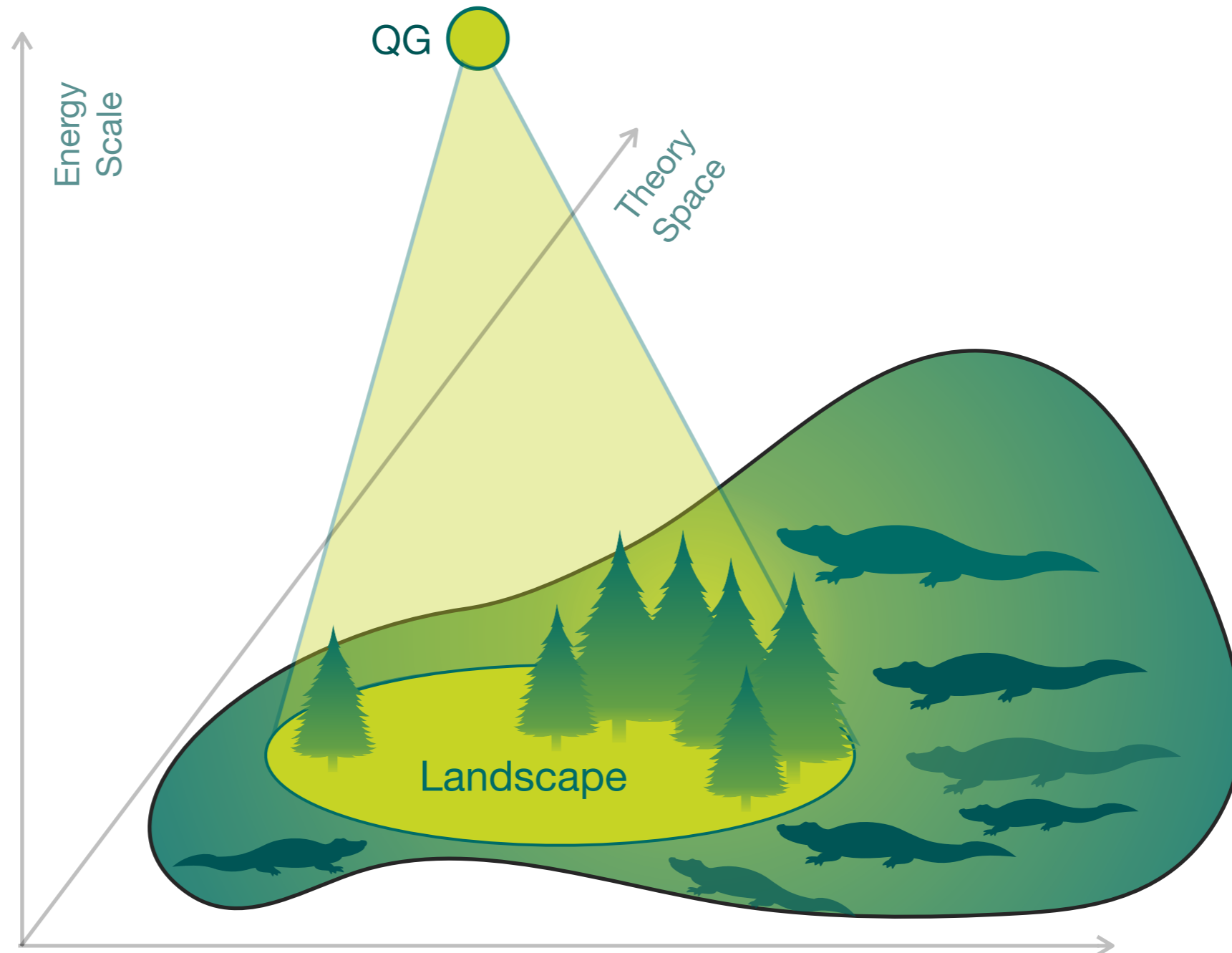
Corfu, September 9th 2022

Outline

- Swampland and the Cobordism Conjecture
- Dynamical Cobordism and Tadpoles
- An explicit case of Breaking:
Dynamical Cobordism for a 9d domain wall and a new 7-brane object
[\[2205.09782, Blumenhagen, Cribiori, Kneißl, AM \]](#)

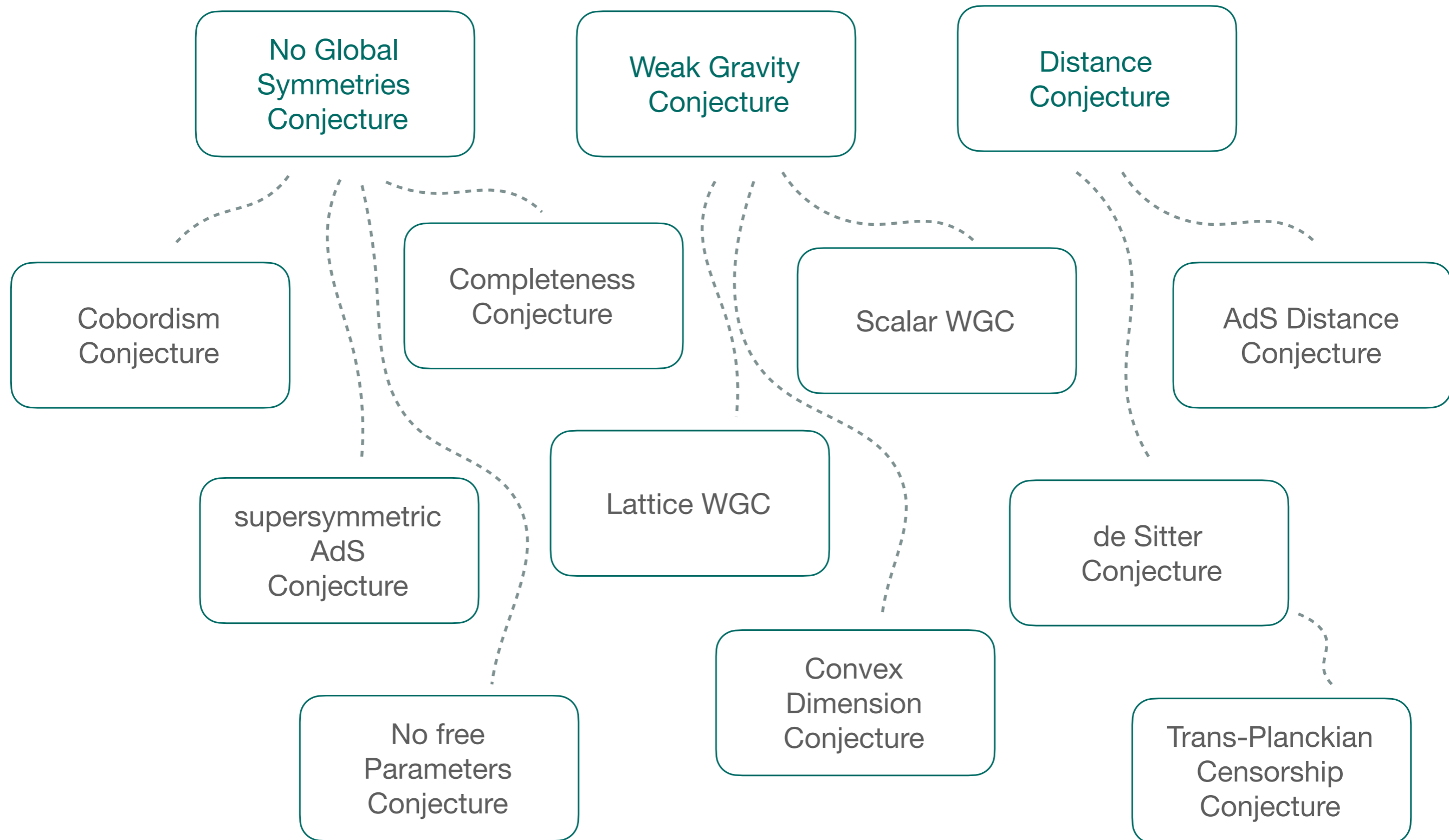
The Swampland Program [Recent reviews: 1903.06239, 2102.01111]

Not all low-energy EFTs can be UV-completed to Quantum Gravity

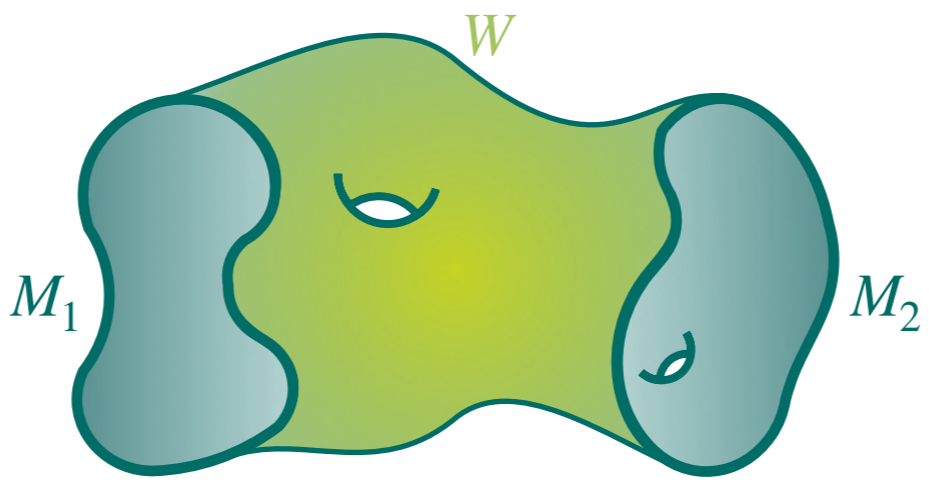


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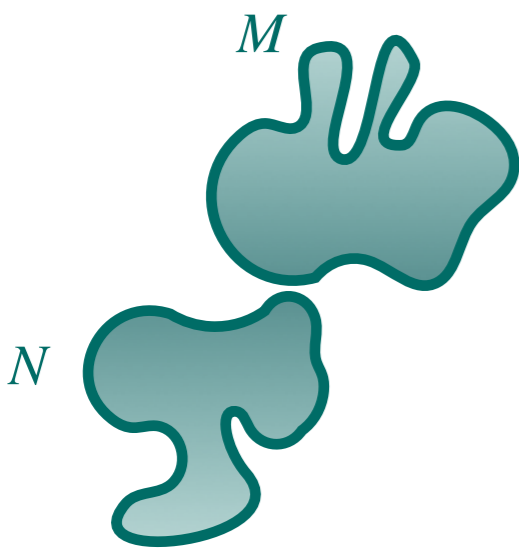
Cobordism



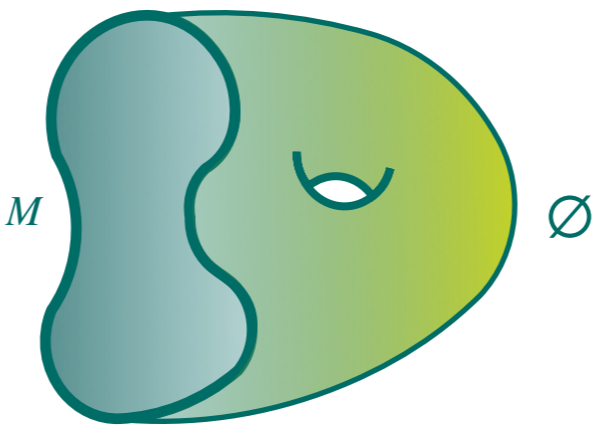
Allowed topology changes
(Encoded in ξ)

$$M_1 \sim M_2 \Leftrightarrow \exists W \text{ s.t. } \partial W = M_1 \sqcup M_2$$

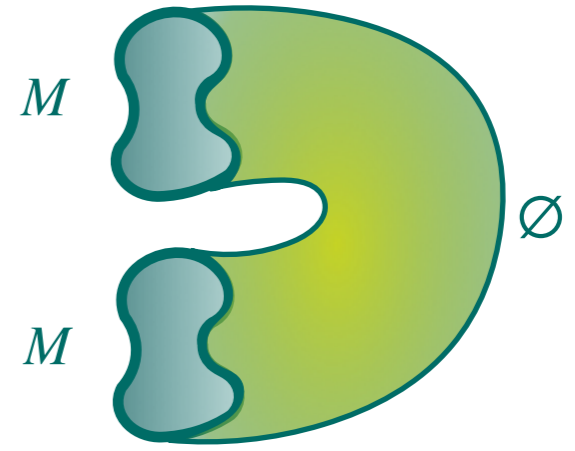
$$\Omega_k^\xi = \{\text{compact, closed, } k\text{-dimensional manifolds}\} / \sim$$



$$[M \sqcup N] = [M] + [N]$$



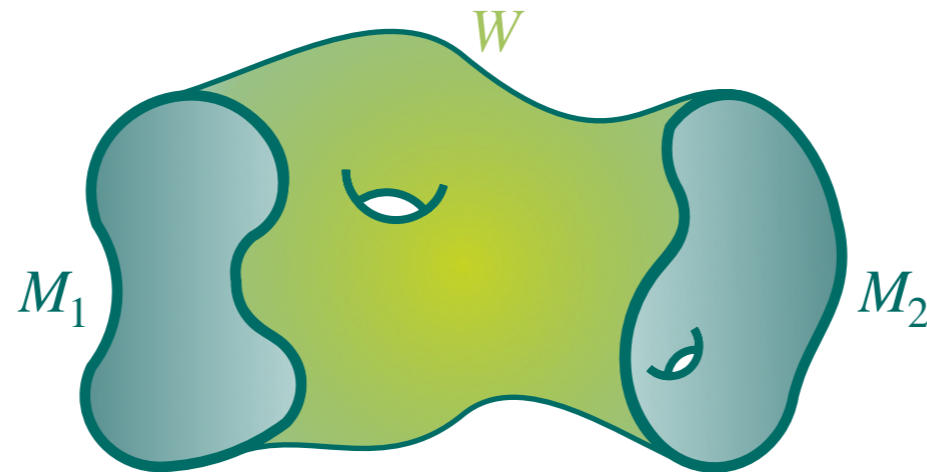
$$[\emptyset] = 0$$



$$[M] + [M] = [M \sqcup M] = 0$$

For oriented manifolds: $[M] + [\bar{M}] = 0$, $[M] + [N] = [M \sqcup \bar{N}]$

Cobordism

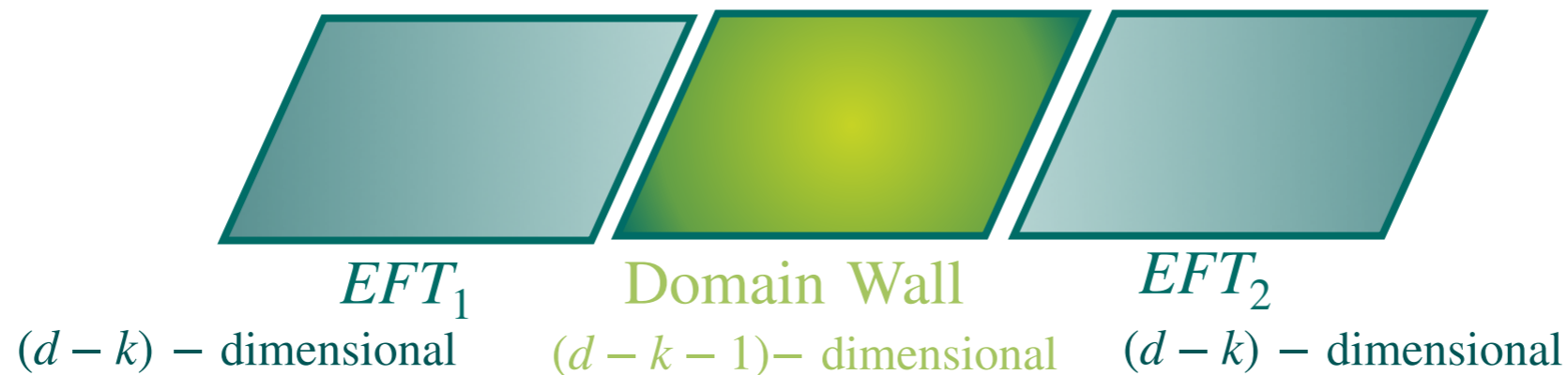


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Compactify d-dimensional theory on M^k down to $D=d-k$ dimensions:



→ Finite-energy transition between EFTs

Cobordism and String Theory

What are the relevant structures for String Theory?

$$\text{Type I} \leftrightarrow \xi = \text{Spin}$$

$$\text{Type II} \leftrightarrow \xi = \text{Spin}^c$$

Full Quantum Gravity $\leftrightarrow \xi = ?$

n	0	1	2	3	4	5	6	7	8	9	10
Ω_n^{Spin}	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2	0	0	0	\mathbb{Z}	0	$2\mathbb{Z}$	$2\mathbb{Z}_2$	$3\mathbb{Z}_2$
$\Omega_n^{\text{Spin}^c}$	\mathbb{Z}	0	\mathbb{Z}	0	$2\mathbb{Z}$	0	$2\mathbb{Z}$	0	$4\mathbb{Z}$	0	$4\mathbb{Z} \oplus \mathbb{Z}_2$

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KO^{-n}	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2	0	0	0	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2
K^{-n}	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}

[See Cribiori's talk]

Cobordism Conjecture

[McNamara, Vafa '19]

Cobordism Group $\Omega_k^\xi \leftrightarrow$ Cobordism Invariant μ_k

For empty set: $\mu_k[\emptyset] = 0$

If cobordism class $[M] \neq 0 \leftrightarrow$ obstruction to decay into “nothing”

$\Omega_k^\xi \neq 0 \Leftrightarrow (d - k - 1)$ -dim. global symmetry
with charges labelled by $[M] \in \Omega_k^\xi$

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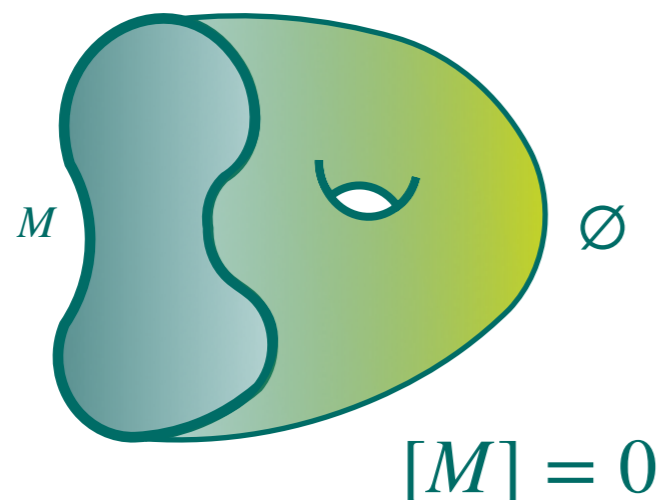
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But: No global symmetries in quantum gravity \rightarrow Cobordism Conjecture
e.g. [Banks, Seiberg '10] [McNamara, Vafa '19]

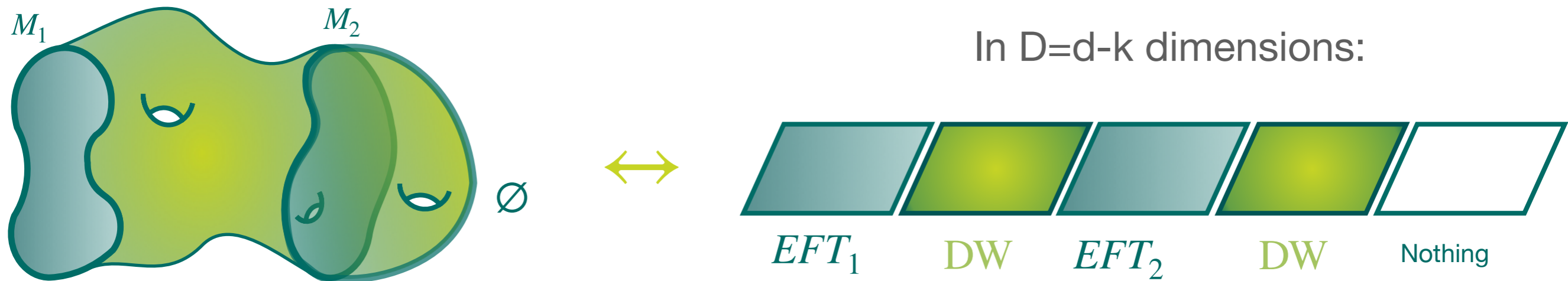


All Cobordism Classes should be trivial

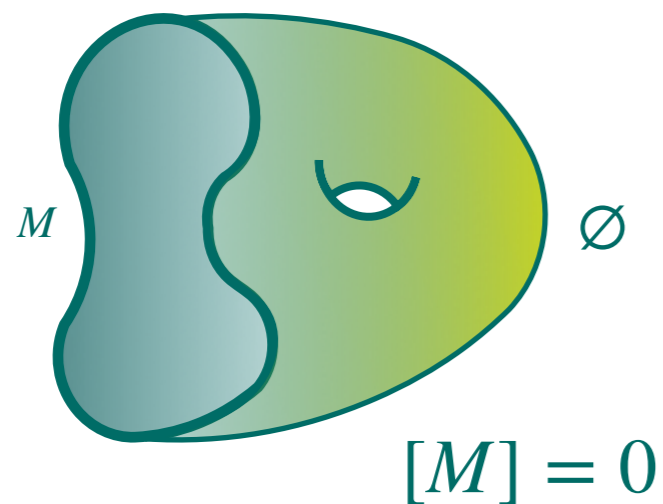
$$\Omega_k^{QG} = 0$$

Cobordism Conjecture

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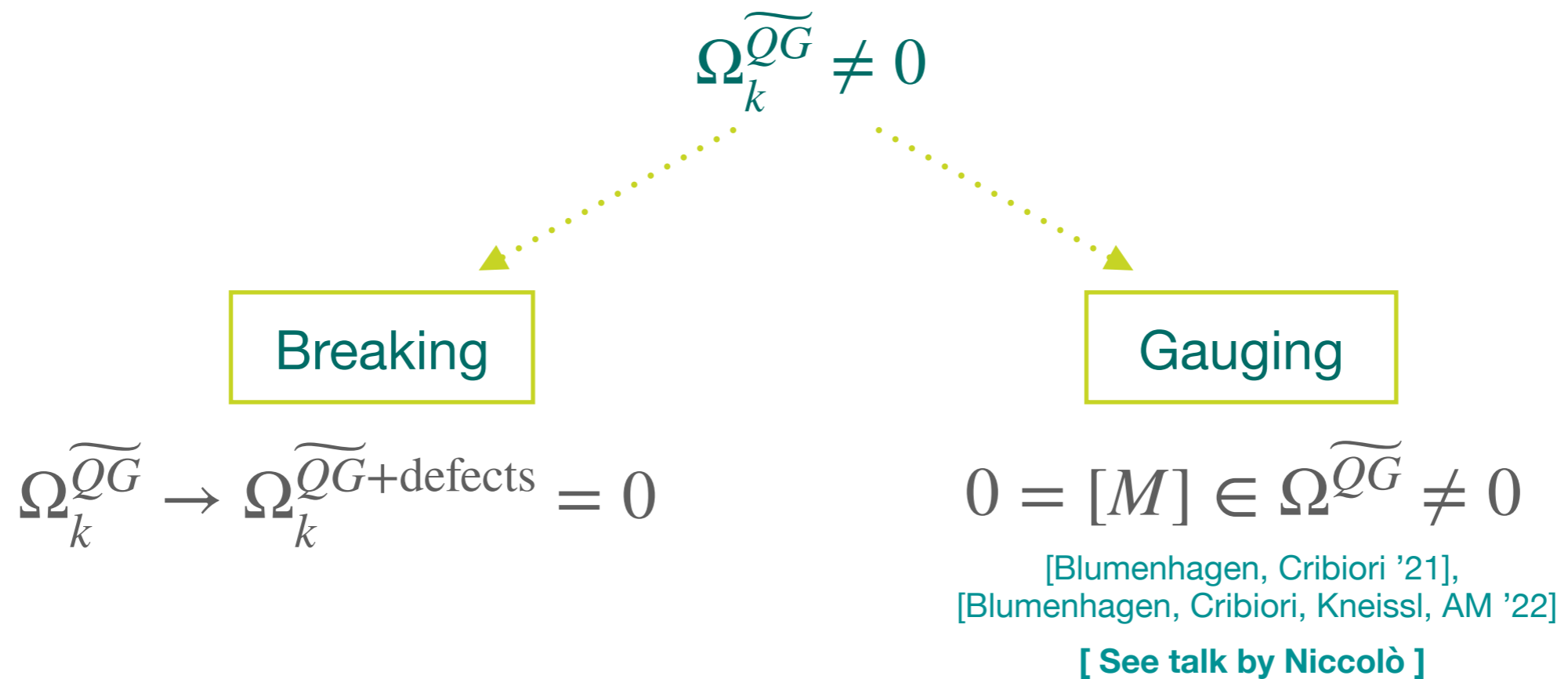
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Cobordism Conjecture Consequences



Cobordism Conjecture Consequences

$$\Omega_k^{\widetilde{QG}} \neq 0$$

Breaking

Gauging

$$\Omega_k^{\widetilde{QG}} \rightarrow \Omega_k^{\widetilde{QG}+\text{defects}} = 0$$

$$0 = [M] \in \Omega_k^{\widetilde{QG}} \neq 0$$

[Blumenhagen, Cribiori '21],
[Blumenhagen, Cribiori, Kneissl, AM '22]

[See talk by Niccolò]



Possibly new defects
detected!

[McNamara, Vafa '19], [Montero, Vafa '20]

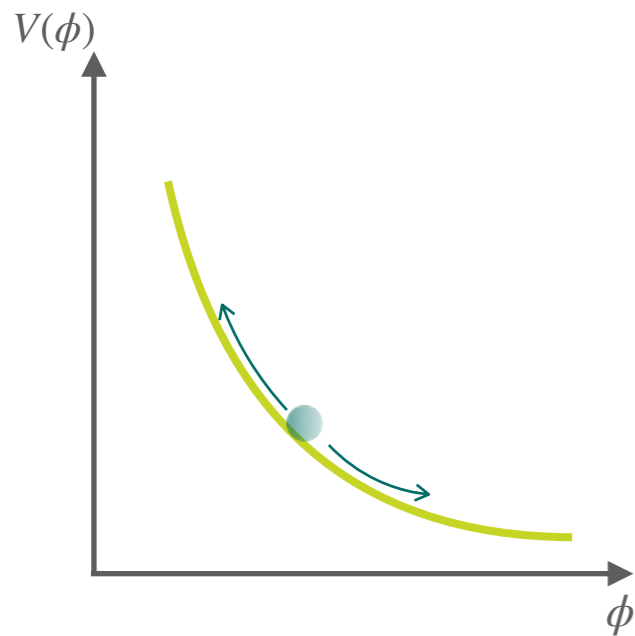
[Dierigl, Heckman '21]

[Debray, Dierigl, Heckman, Montero '21]

$$0 \neq [M] \in \Omega_k^{\widetilde{QG}}$$

$$[M] = 0 = \Omega_k^{\widetilde{QG}+\text{defects}}$$

Tadpoles & Dynamical Cobordism



Dynamical tadpoles (vs RR tadpoles)

Naturally occurring in supersymmetry-breaking potentials

Indicate lack of maximally-symmetric vacuum

[Sugimoto '99]

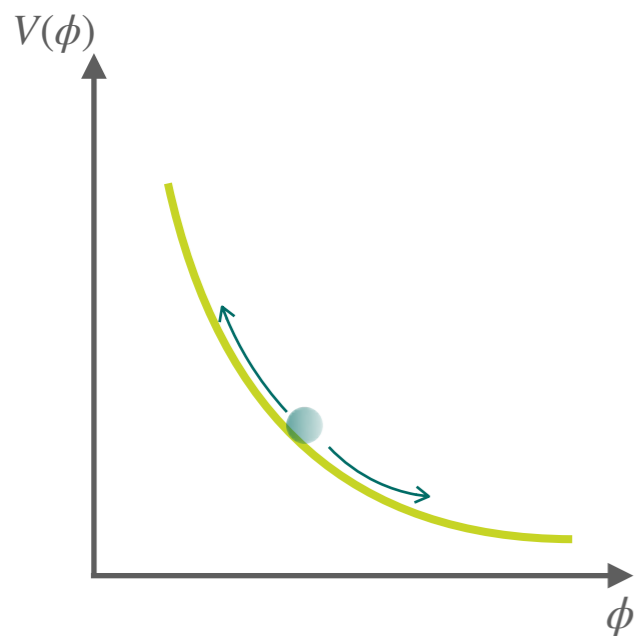
[Antoniadis, Dudas, Sagnotti '99]

[Angelantonj '99]

...

recently : [Raucci '22]

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Example: Sugimoto Model ($USp(N)$ Type I with $N \bar{D}8$ and $N O9_-$)

[Sugimoto '99]

Action:

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left(\mathcal{R} - \frac{1}{2}(\partial\Phi)^2 \right) - T_9 \int d^{10}x \left((N + 32)\sqrt{-G}e^{\frac{3}{2}\Phi} - (N - 32)A_{10} \right) + \dots$$

Solution preserving 9d Poincaré invariance:

[Dudas, Mourad '00]

$$ds_E^2 = |\sqrt{\alpha_E}|^{1/9} e^{-\frac{\alpha_E y^2}{8}} \eta_{\mu\nu} dx^\mu dx^\nu + |\sqrt{\alpha_E y}|^{-1} e^{-\frac{3\phi_0}{2}} e^{-\frac{9\alpha_E}{8}y^2} dy^2, \quad \alpha_E = 64k^2 T_9$$

→ singularities at finite spacetime distance, spontaneous compactification to 9d

Tadpoles & Dynamical Cobordism

Tadpoles & Dynamical Cobordism

Reinterpretation in Cobordism Terms

[Buratti, Delgado, Uranga '21]

[Buratti, Calderon-Infante, Delgado, Uranga '21]

[Angius, Calderon-Infante, Delgado, Huertas, Uranga '22]

[Angius, Delgado, Uranga '22]

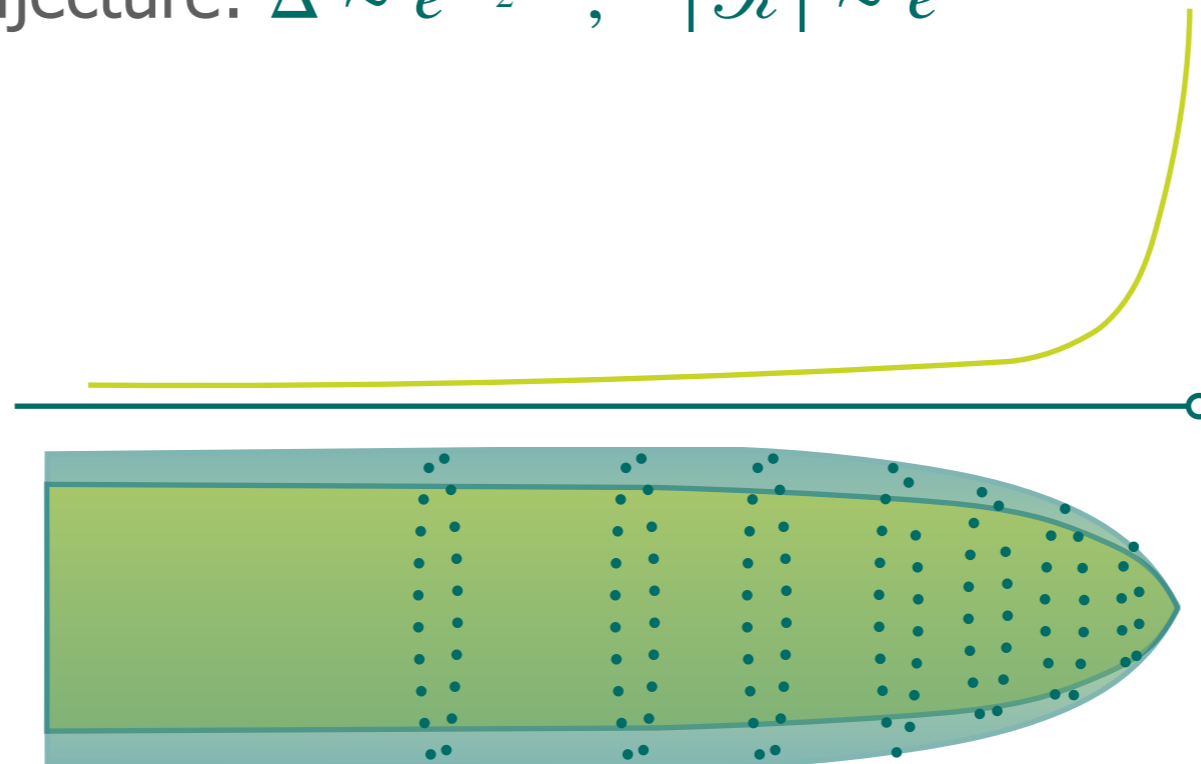
Solution extends in finite spacetime distance Δ , with $\Delta \sim \mathcal{T}^{-n}$

Mechanism: "apparent singularity" = cobordism defect.

↪ Tadpole strength

For field distance $D \rightarrow \infty$ at singularity: Wall of Nothing/End-of-the-world brane

Cobordism distance conjecture: $\Delta \sim e^{-\frac{1}{2}\delta D}$, $|\mathcal{R}| \sim e^{\delta D}$



9-dimensional Domain Wall

Goal: Backreaction of gauge neutral,
non-supersymmetric 9-dimensional object w/ brane-like dilaton coupling

Physical realisation: non-BPS $\tilde{D}8$ -brane,
non-SUSY stack of $16 \times \bar{D}8 + O8^{++}$ [Blumenhagen, Font '00]

Action:
$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left(\mathcal{R} - \frac{1}{2} (\partial\Phi)^2 \right) - T \int d^{10} \sqrt{-g} e^{\frac{5}{4}\Phi} \delta(r).$$

↑
transverse
direction

Solution Ansatz:
$$ds^2 = e^{2\mathcal{A}(r,y)} ds_8^2 + e^{2\mathcal{B}(r,y)} (dr^2 + dy^2).$$

$$\mathcal{A} = A(r) + U(y)$$

$$\mathcal{B} = B(r) + V(y)$$

$$\Phi = \chi(r) + \psi(y)$$

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$$\Phi = \chi(r) + \psi(y)$$

Solution I

$$A(r) = B(r) = \frac{1}{8} \log \left| \sin \left(8K \left(|r| - \frac{R}{2} \right) \right) \right|$$

$$\chi(r) = -\frac{3}{2} \log \left| \tan \left(4K \left(|r| - \frac{R}{2} \right) \right) \right| + \phi_0$$

$$U(y) = -Ky$$

$$\psi(y) = V(y) = 0$$

Solutions II^\pm

$$A(r) = \frac{1}{8} \log \left| \sin \left(8K \left(|r| - \frac{R}{2} \right) \right) \right|$$

$$\chi(r) = \frac{\alpha^\pm}{8} \log \left| \sin \left(8K \left(|r| - \frac{R}{2} \right) \right) \right| \mp 2 \log \left| \tan \left(4K \left(|r| - \frac{R}{2} \right) \right) \right| + \phi_0$$

$$B(r) = \frac{\mu}{8} \log \left| \sin \left(8K \left(|r| - \frac{R}{2} \right) \right) \right| \mp \frac{\alpha^\pm}{8} \log \left| \tan \left(4K \left(|r| - \frac{R}{2} \right) \right) \right|$$

$$U(y) = \frac{1}{8} \log (\cosh(8Ky))$$

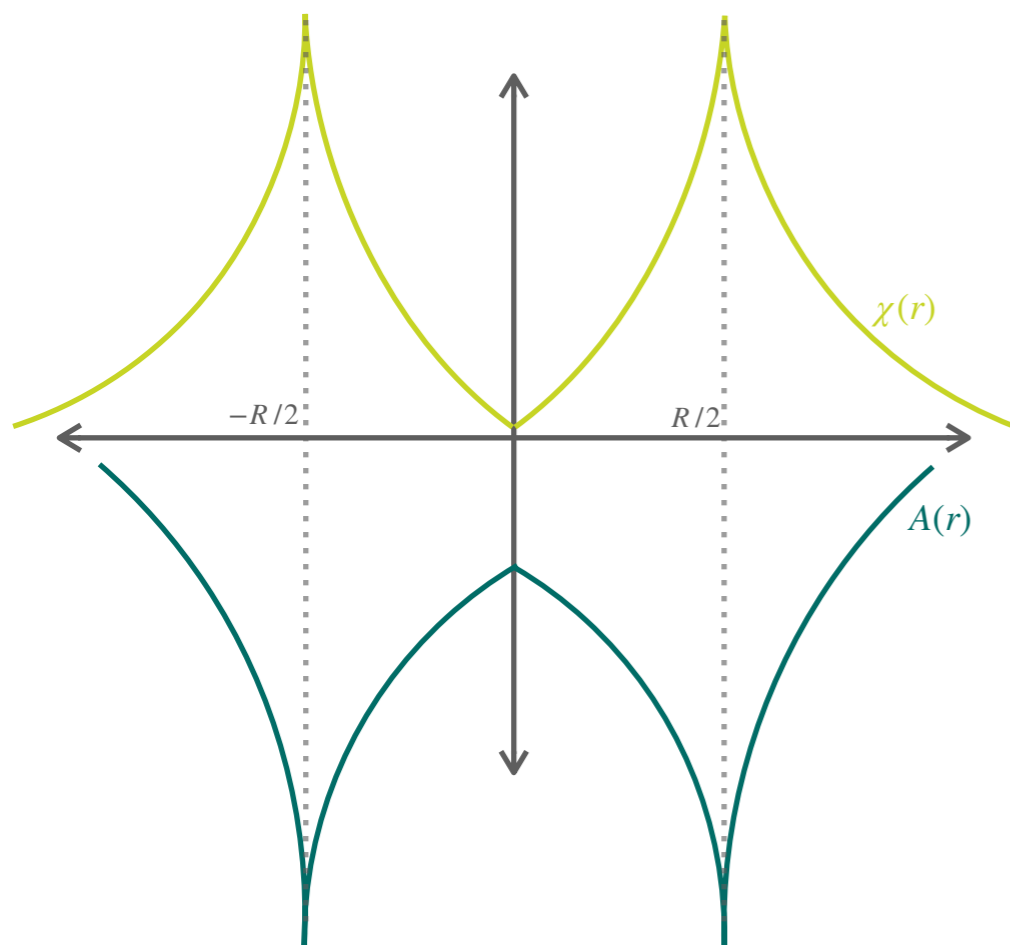
$$\psi(y) = \alpha U(y), \quad V(y) = -\frac{5}{4} U(y)$$

Qualitative behavior

r-direction spontaneously compactified on S^1 with radius R : $e^{\frac{5}{4}\phi_0} \sim \frac{1}{\lambda R}$

Logarithmic singularities at $r = \pm \frac{R}{2}$, string coupling diverges $\uparrow \kappa_{10}^2 T$

y-direction: infinite length in sols I, II^- , becomes **finite interval** in sol II^+

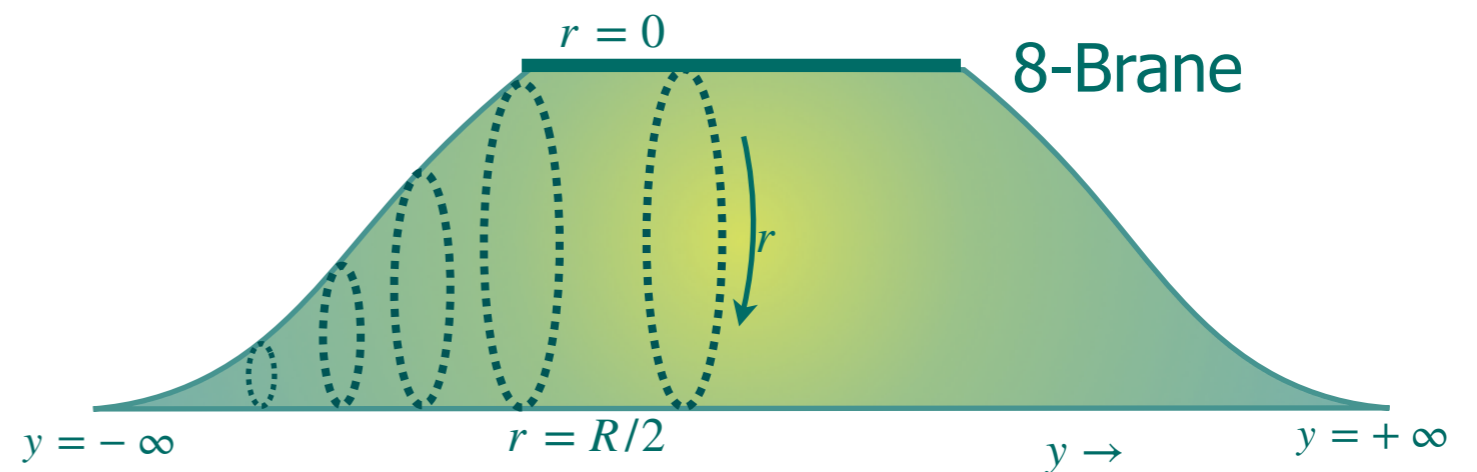
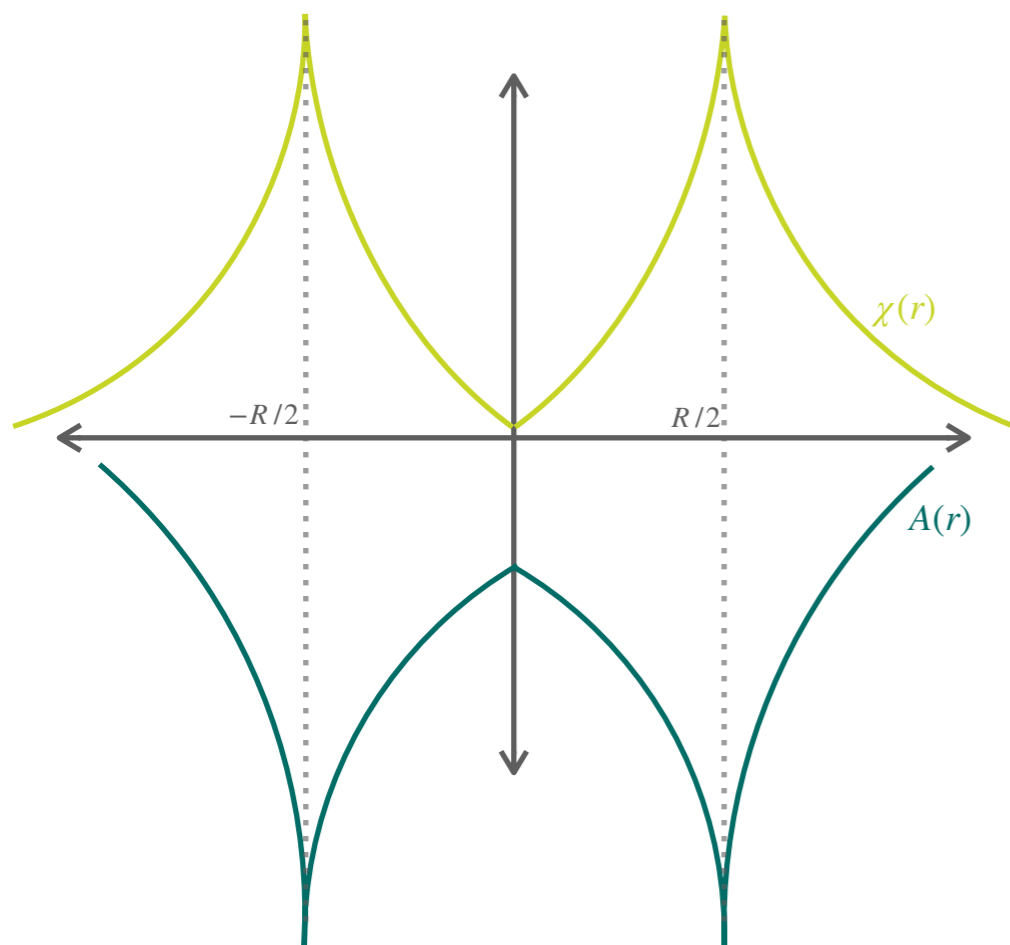


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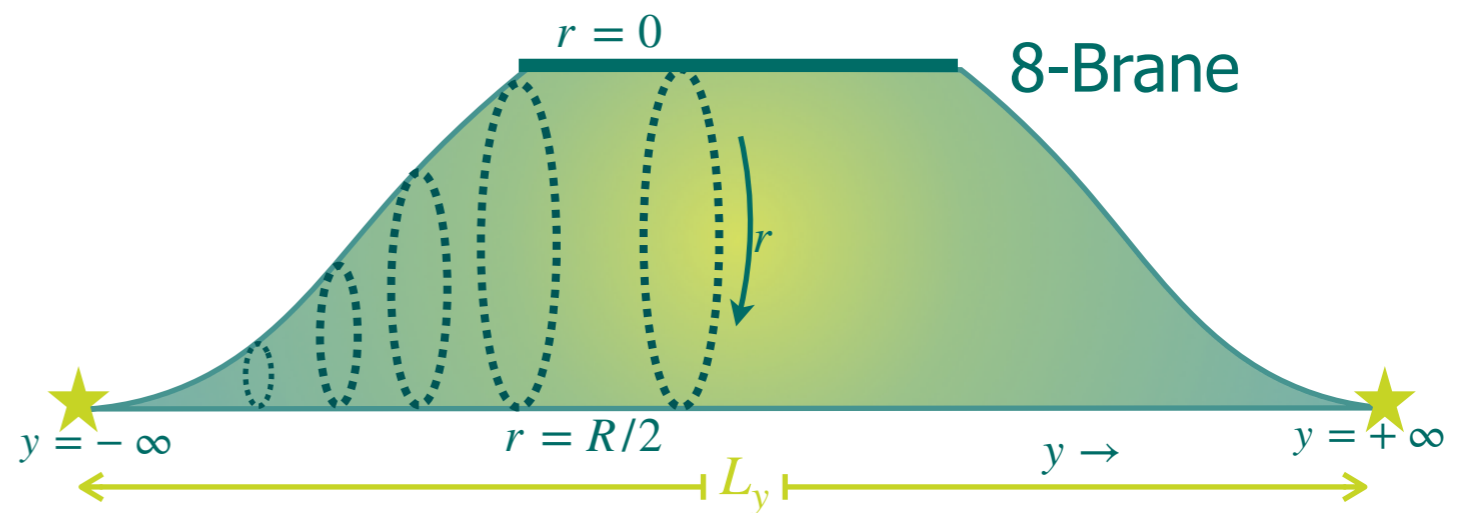
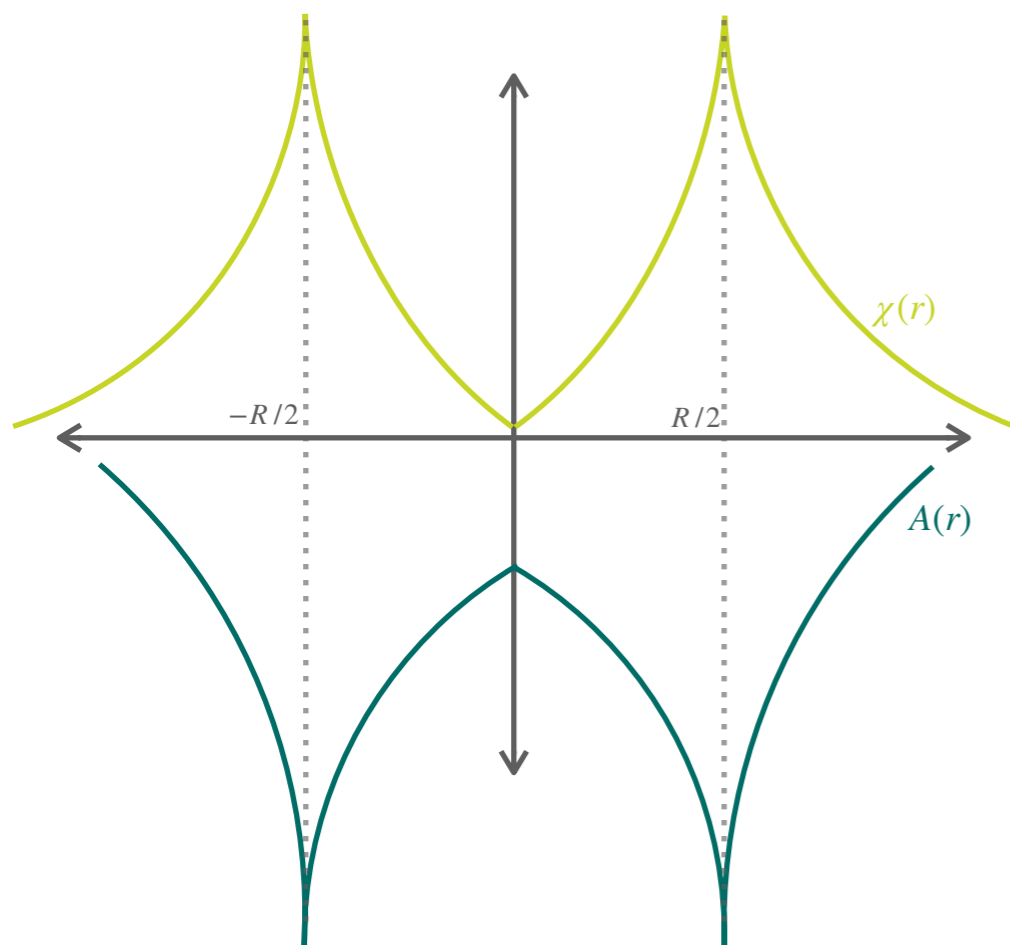


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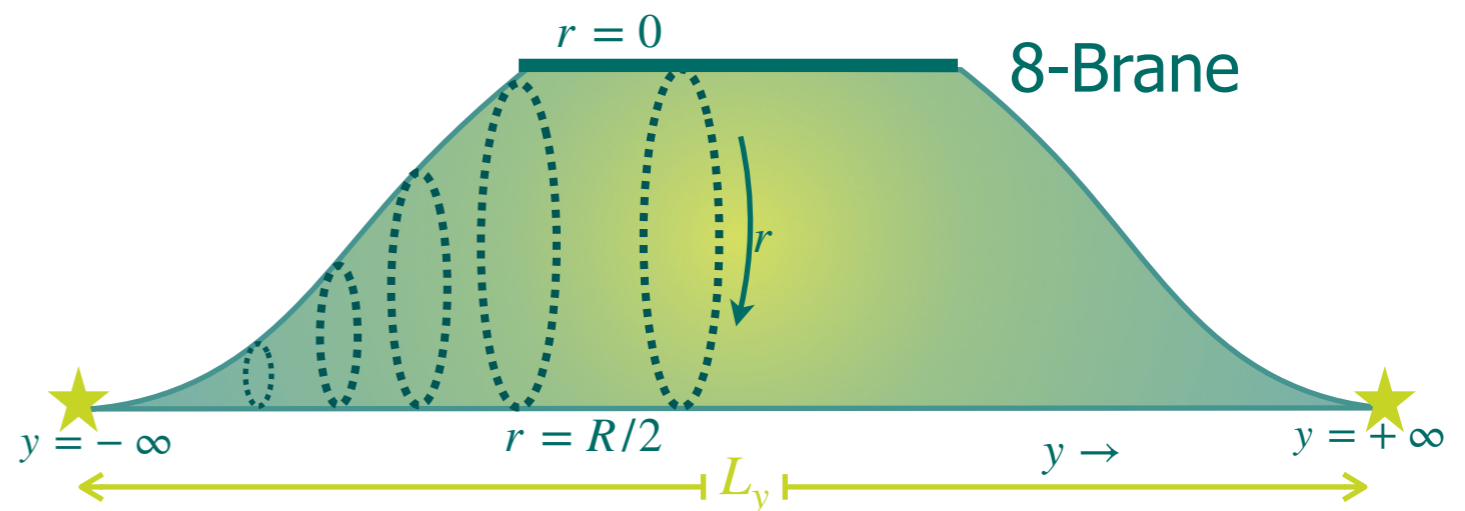
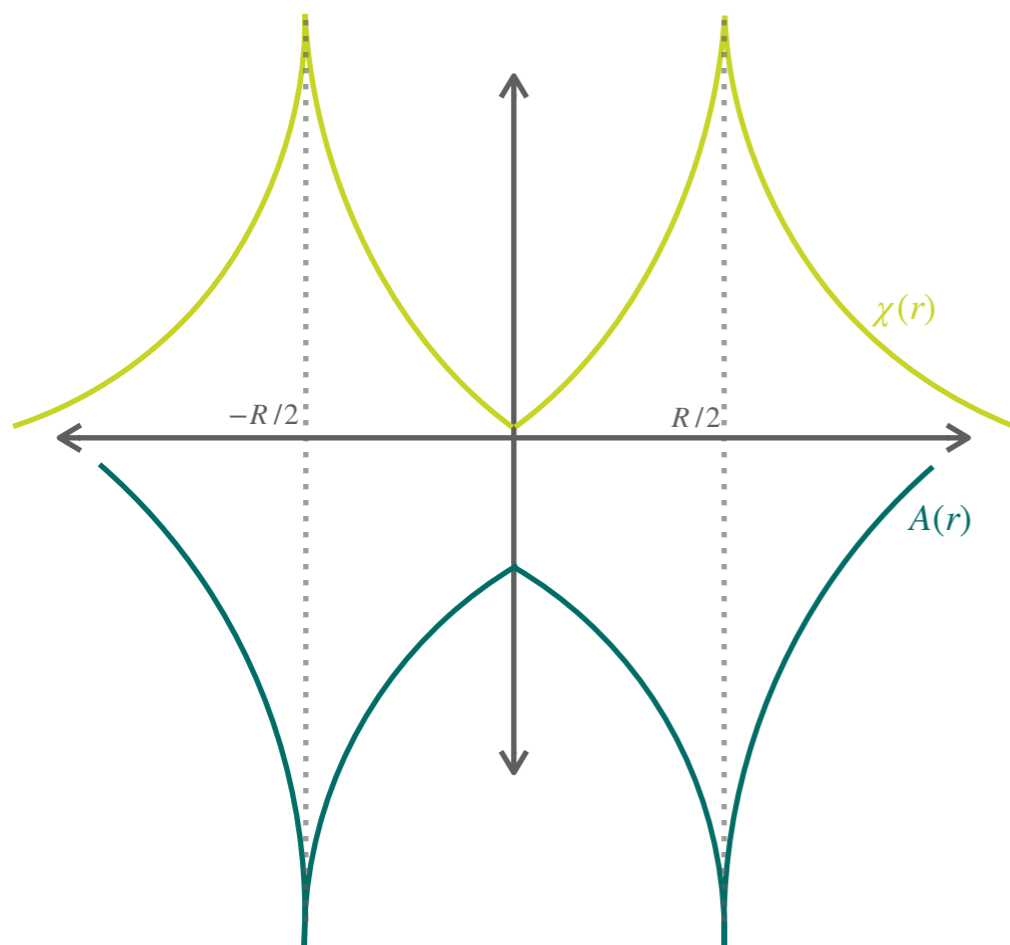


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$$\Delta \sim L_y \sim \mathcal{T}^{-1}$$

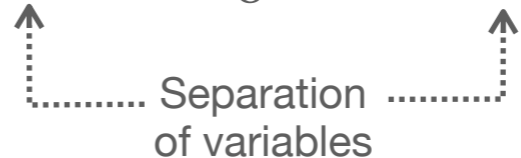
$$\Delta \sim e^{-\sqrt{2}D}$$

$$|\mathcal{R}| \sim e^{2\sqrt{2}D}$$

ETW Defects

Input: 8-dimensional defect : log-singularity, S^1 direction capped off
Poincaré symmetry along the “brane” preserved
2d transversal rotational symmetry broken

Non-Isotropic Solution Ansatz: $ds^2 = e^{2\hat{\mathcal{A}}(\rho,\phi)} ds_8^2 + e^{2\hat{\mathcal{B}}(\rho,\phi)} (d\rho^2 + \rho^2 d\phi^2)$.



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Separation of variables

Solutions: $\hat{A}(\rho) = \frac{1}{8} \log \left| \cosh(8\hat{K} \log(\frac{\rho}{\rho_0})) \right|$ $\hat{B}(\rho) = -\log(\frac{\rho}{\rho_0}) + (\frac{\hat{\alpha}^2}{32} - \frac{7}{2})\hat{A}(\rho)$

$$\hat{\chi}(\rho) = \hat{\alpha}\hat{A}(\rho)$$

⋮

$$\hat{\psi}(\phi) = \frac{\hat{\alpha}}{8} \log \left| \cos(8\hat{K}\phi) \right| \pm 2 \log \left| \tan(4\hat{K}\phi + \frac{\pi}{4}) \right|$$

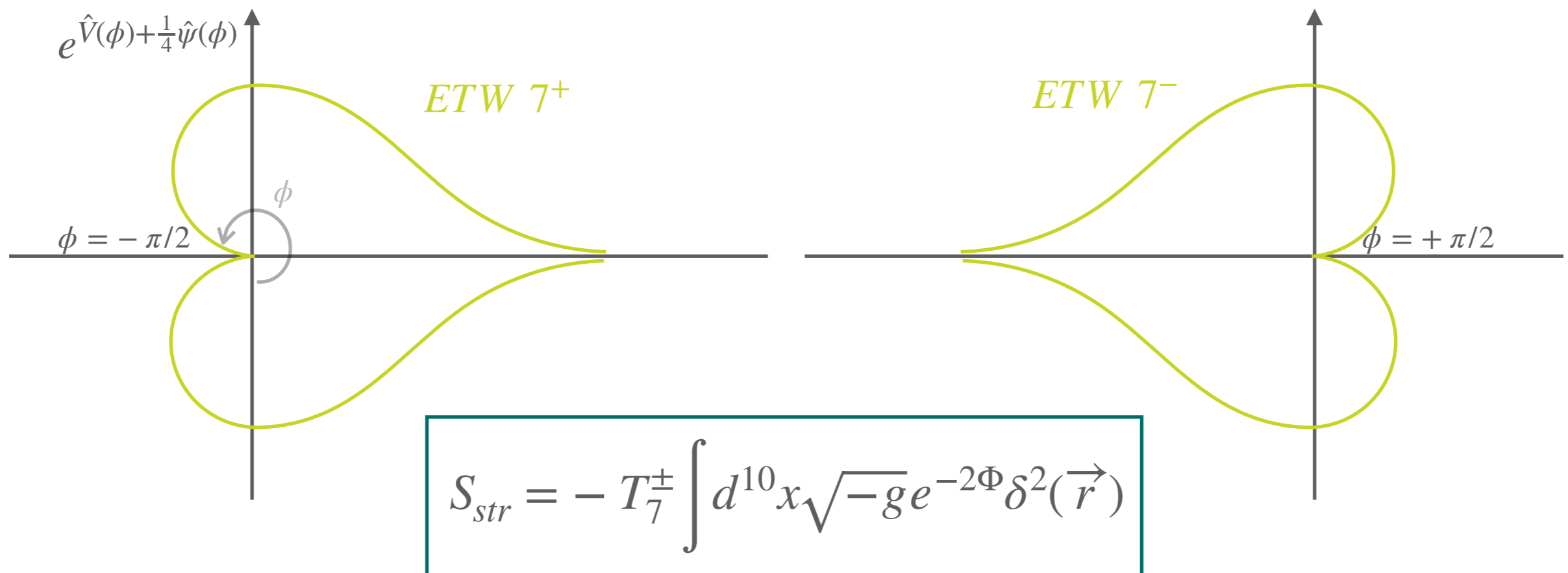
ETW 7^\pm solutions

Qualitative behavior

Logarithmic singularities at $\rho = 0$, string coupling diverges

For appropriate constant ($\hat{\alpha} = \alpha^+$) same scaling as 9d defect

Dynamical Cobordism scaling satisfied: $\Delta \sim e^{-\sqrt{2}D}$, $|\mathcal{R}| \sim e^{2\sqrt{2}D}$



with $\kappa_{10}^2 T_7 = 2\pi$

Cobordism Interpretation

Backreaction of 9d Domain Wall

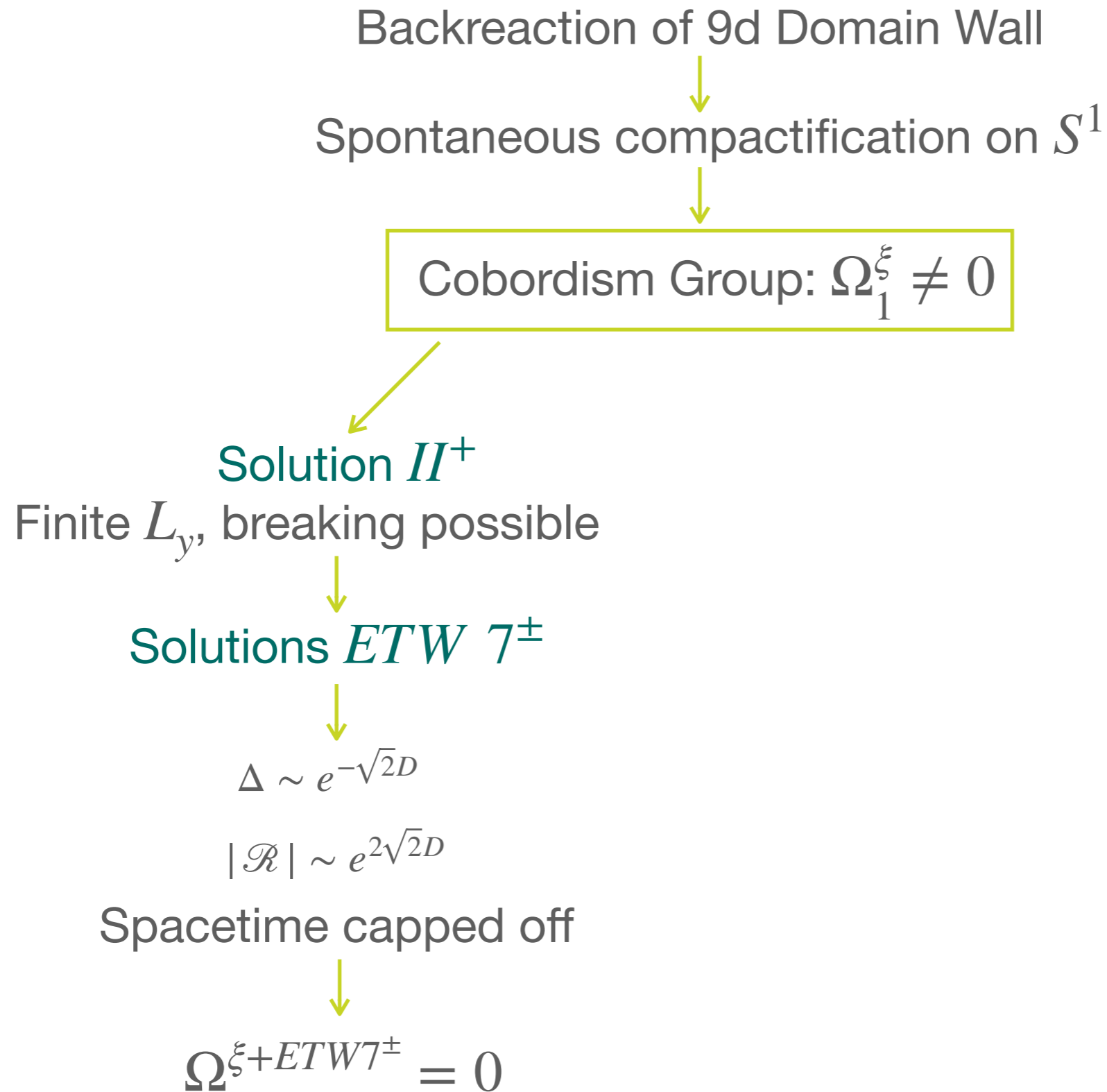


Spontaneous compactification on S^1

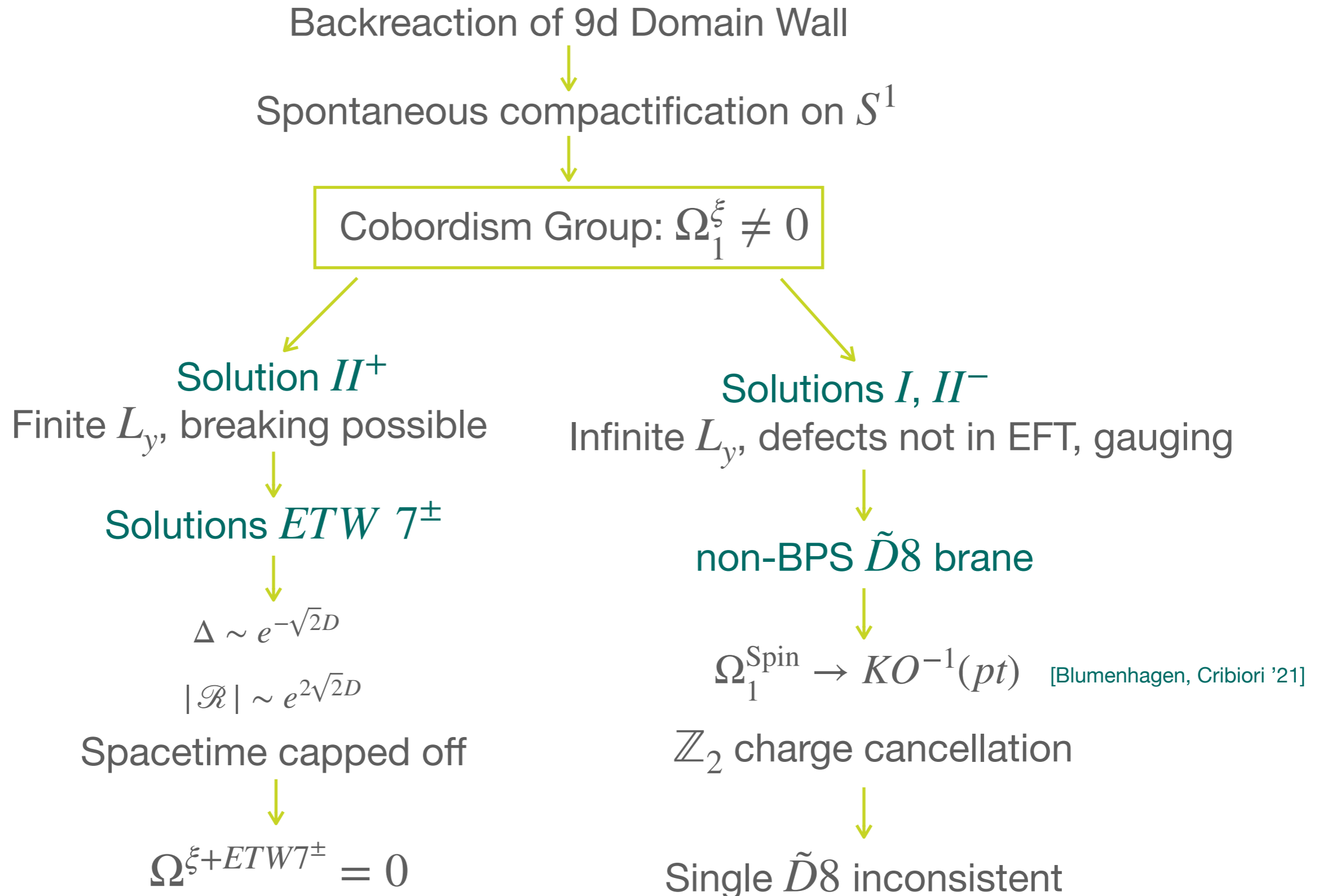


$$\text{Cobordism Group: } \Omega_1^\xi \neq 0$$

Cobordism Interpretation



Cobordism Interpretation



Summary and Outlook

We have provided:

Concrete example for physical realisation of dynamical cobordism

Explanation of singularities in preexisting solution, expected scalings satisfied

Eom for defect solved \rightarrow new 7-brane defect

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