Grand Unified Theories Chapter 12	Graham G. Ross Grand Unified Theories Chapter 12
nce generating an (approximately) de Sitter eason is that, in de Sitter space, there is a ature, the Hawking temperature, T _H , which atum fluctuations and is given by	tracking the evolution of these inhomogeneities to the present time, show that the amplitude of density fluctuations is $\frac{\delta\rho}{\rho} = \frac{H^2}{\phi} \qquad (12.77)$
are thermal fluctuations for the value of down the potential slope.	SM and inhomogeneous universe
ctuations in the potential energy stored $-V(\phi)$	the horizon. The condition that $\frac{\delta \rho}{\rho} \simeq 10^{-4}$ imposes another constraint on the effective potential. 12.10 <u>Supersymmetric inflationary cosmology</u> To achieve the form of Fig(12.1) requires a many film
for $\phi_b < \phi < \phi_c$ (12.76) y density fluctuations are produced at h scales (much smaller than the horizon)	Superpotential (see section (10.5))
e universe inflates Remembering G particular scale evolves as the scale grows, as the scale becomes so large that it leaves	The superpotential has the form
that time onward, causal physics cannot act the entire scale, and so the fluctuation en" Et Ance ULTY OF PHYSICS I Since the thermalies and when the scale Since the thermalies and so the fluctuations	$(\phi) = \frac{12.78}{M}$ $(\phi - \phi_0)^2$ (12.78) (10.70) $N \text{ ATIONAL SCIENCE CENTRE}$ (10.70)





PLANCK 2012 Conference, Warsaw. Graham Ross, Zygmunt Lalak. Photo: A. M. Kobos.



PLANCK 2012 Conference, Warsaw. Paul Steinhardt, Subir Sarkar, Graham Ross, Zygmunt Lalak, Burt A. Ovrut.





GRANI

Available online at www.sciencedirect.com





Nuclear Physics B 766 (2007) 1-20

Racetrack inflation and assisted moduli stabilisation

Z. Lalak^{a,b,*}, G.G. Ross^{a,c}, S. Sarkar^c

^a Theory Division, CERN, 1211 Geneva 23, Switzerland ^b Institute of Theoretical Physics, University of Warsaw, 00-681 Warsaw, Poland ^c Rudolf Peierls Centre for Theoretical Physics, University of Oxford, 1 Keble Road, Oxford, OX1 3NP

Received 21 April 2005; received in revised form 19 June 2006; accepted 23 August 2006

Available online 25 October 2006

Abstract

We present a model of inflation based on a racetrack model *without* flux stabilization. The initial conditions are set automatically through topological inflation. This ensures that the dilaton is not swept to weak coupling through either thermal effects or fast roll. Including the effect of non-dilaton fields we find that moduli provide natural candidates for the inflaton. The resulting potential generates slow-roll inflation without the need to fine-tune parameters. The energy scale of inflation must be near the GUT scale and the scalar density perturbation generated has a spectrum consistent with WMAP data. © 2006 Published by Elsevier B.V.

Graham G. Ross



Nuclear Physics B Volume 434, Issue 3, 30 January 1995, Pages 675-696



Large scale structure from biased nonequilibrium phase transitions - percolation theory picture

Z. Lalak ^{a, b}, S. Lola ^a, B.A. Ovrut ^c, G.G. Ross ^{a, 1}

Show more \checkmark

😪 Share 📑 Cite

https://doi.org/10.1016/0550-3213(94)00557-U

Get rights and content

Abstract

We give an analytical description of the spatial distribution of domain walls produced during a biased nonequilibrium phase transition in the vacuum state of a light scalar field. We discuss in detail the spectrum of the associated cosmological energy density perturbations. It is shown that the contribution coming from domain walls can enhance the standard cold dark matter spectrum in such a way as to account for the whole range of IRAS data and for the COBE measurement of the microwave background anisotropy. We also demonstrate that in case of a biased phase transition which allows a percolative description, the number of large size domain walls is strongly suppressed. This offers a way of avoiding excessive microwave background distortions due to the gravitational field of domain walls present after decoupling.

SPIRE	literature	literature find a ross, g and t scale and t invariant or t scale-invariant							Q	
		<u>Literature</u>	Auth	nors	Jobs	Seminars	Conferences	Мо	re	
Date of paper		3 results	[→ cite all						Most Recent	
1		Citation Su	ummary							
		Exclude	self-citations	s (?)						
						Cite	able 🕐		Published	
00			Papers				3			
20162018			Citations				167		16	
Number of authors			h-index ⑦				3			
		Citat	ions/paper (a	avg)			55.7		55	
TO authors or less	3	Papers	Citeable	- Publishe	Ч					
Exclude RPP		2	oncubic	1 abilities	_2_2					
Exclude Deview of Destints Div										
3 Exclude Review of Particle Phy	SICS	1.5								
		1				1 1				
Document Type		0.5								
article	3	0								
published ⑦	3		0	1-9	10-49	50-99	100-249	250-499	500+ Citation:	
		Inflation in	n a scale in	ivariant ur	niverse					
Author		Pedro G. Fe	rreira (Oxford	I U.), Christo	pher T. Hill (Fe	rmilab), Johannes	Noller (Zurich U.), (Graham G. Ros	ss (Oxford U.	
Christopher T. Hill	3	Theor. Phys.	.) (Feb 16, 20)18)	0 400540	D-1-1-1000.0000	0 (astro at 001			
Pedro G. Ferreira	3	Published in	: Pnys.Rev.D	97 (2018) 1	2, 123516 • e-	-Print: 1802.0606	9 [astro-ph.CO]		0	
Graham G. Ross	3	占 pdf	ළ [ා] links	é [,] doi	ite ⊡				45 citation	
Johannes Noller	1	No fifth fo	orce in a sc	ale invaria	ant universe					
		Pedro G. Fe	rreira (Oxford	I U.), Christo	pher T. Hill (Fe	rmilab), Graham G	B. Ross (Oxford U., T	'heor. Phys.) (Dec 9, 2016)	
Subject		Published in	: Phys.Rev.D	95 (2017) 6	6, 064038 ∙ e-P	Print: 1612.03157	[gr-qc]			
Astrophysics	3	ট pdf	ළ links	ି DOI	ite ⊡				➔ 46 citation	
Gravitation and Cosmology	2	West 0	ant Carl	In	Inflation -	Diamata O a a la	O an anati an			
Theory-HEP	2	Weyl Curr	ent, Scale	-invariant	Inflation and	rmilah) Craham (Oct 20 2010	\ \	
Phenomenology-HEP	1	Published in	: Phys.Rev.D	95 (2017) 4	, 043507 • e-P	Print: 1610.09243	[hep-th]	001 20, 2010	,	
		D pdf	2 links	@ DOI	[→ cite				→ 76 citatir	
arXiv Category			C 111173	0 001						

SM + dilaton

with D. Ghilencea, P. Olszewski, P. Michalak

Quantum scale symmetric effective lagrangian





$$\begin{pmatrix} \phi \\ \sigma \end{pmatrix} = M \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}$$
, $V_{\text{eff}} = M^4 W(\theta)$,

$$\blacktriangleright$$
 flat direction in $V_{
m eff}$ \Rightarrow

$$\exists_{\theta=\theta_0} W(\theta_0) = W'(\theta_0) = 0$$

renormalization condition, similar to choosing C.C.

- $W(\theta)$
- Hierarchy of scales via aligning the flat direction $\perp \phi \longrightarrow \theta_0 \approx \frac{\phi_0}{\sigma_0} \ll 1$
- New perspective on naturalness: is this alignement stable wrt. embedding in a UV completion?

Symmetry breaking at finite temperature

$$\frac{\mathcal{L}}{\sqrt{g}} = -\frac{1}{12} \Big(\xi_0 \phi_0^2 + \xi_1 \phi_1^2 \Big) R + \frac{1}{2} \partial_\mu \phi_0 \partial^\mu \phi_0 + \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 - V(\phi_0, \phi_1),$$

$$V(\phi_0, \phi_1) = \lambda_0 \phi_0^4 + \lambda_1 \phi_0^2 \phi_1^2 + \lambda_2 \phi_1^4.$$

hierarchy of small couplings

 $\lambda_2 \gg |\lambda_1| \gg \lambda_0$

leads to the hierarchy of vets

$$\langle \phi_1^2 \rangle = -\frac{\lambda_1}{2\lambda_2} \langle \phi_0^2 \rangle, \qquad \lambda_0 = \frac{\lambda_1^2}{4\lambda_2}, \qquad \langle R \rangle = 0,$$

Scale symmetric Lagrangian - minimal vn

hierarchy of scales

$$m_H^2 = -4\lambda_1 \Big(1 - \frac{\lambda_1}{2\lambda_2} \Big) \langle \phi_0^2 \rangle, \quad v^2 = \langle \phi_1^2 \rangle = -\frac{\lambda_1}{2\lambda_2} \langle \phi_0^2 \rangle.$$

$$\frac{1}{6} \left(\xi_0 - \frac{\lambda_1}{2\lambda_2} \xi_1 \right) \langle \phi_0^2 \rangle = M_{Planck}^2.$$

Scale symmetric Lagrangian - non-minimal

$$V = \lambda_0 \phi_0^4 + \sum_{n=0}^N \left(\lambda_{2n+1} \phi_n^2 \phi_{n+1}^2 + \lambda_{2n+1} \phi_{n+1}^4 \right)$$

Take N=2
$$\phi_1^2 = -\frac{2\lambda_0}{\lambda_1}\phi_0^2, \ \phi_2^2 = -\frac{\lambda_3}{2\lambda_4}\phi_1^2$$

with tuning of couplings

$$\lambda_0 = \frac{\lambda_1^2}{4\lambda_2 - \lambda_3/\lambda_4}$$

assuming $\lambda_2 \sim \lambda_4 \sim 1$

hierarchy of vevs

$$\phi_1^2 = -\lambda_1 \phi_0^2, \ \phi_2^2 = -\lambda_3 \phi_1^2$$

Scale symmetric Lagrangian - minimal vn thermal corrections

To obtain temperature corrections one adds to potential temperature dependent parts:

$$V(\phi_0, \phi_1) \to V(\phi_0, \phi_1) + \delta V_T(\phi_0, \phi_1, T) + \delta V_{ring}(\phi_0, \phi_1, T).$$

 δV_T stands for standard temperature corrections of first order:

$$\delta V_T(\phi_0, \phi_1, T) = \frac{T^4}{2\pi^2} \left[\sum_{i=\text{bosons}} n_i \cdot J_B\left(\frac{m_i^2(\phi_k)}{T^2}\right) + \sum_{j=\text{fermions}} n_j \cdot J_F\left(\frac{m_j^2(\phi_k)}{T^2}\right) \right],$$

$$\delta V_{ring} = -\frac{T}{12\pi} \Big(m_{eff}(\phi_i, T)^3 - m_i(\phi_i)^3 \Big),$$

$$V_{eff} = V_{T=0} + \frac{1}{2}\phi_1^2 \cdot \left(\lambda_2 + \frac{\lambda_1}{6} + \frac{g_1^2}{16} + \frac{3g_2^2}{16} + \frac{h_t^2}{4}\right)T^2 + \frac{1}{2}\phi_0^2 \cdot \frac{\lambda_1}{6}T^2 = V_{T=0} + \frac{\gamma T^2}{2}\phi_1^2 + \frac{\lambda_1 T^2}{12}\phi_0^2.$$

$$V_{eff} = V_{T=0} + \frac{1}{2}\phi_1^2 \cdot \left(\lambda_2 + \frac{\lambda_1}{6} + \frac{g_1^2}{16} + \frac{3g_2^2}{16} + \frac{h_t^2}{4}\right)T^2 + \frac{1}{2}\phi_0^2 \cdot \frac{\lambda_1}{6}T^2 = V_{T=0} + \frac{\gamma T^2}{2}\phi_1^2 + \frac{\lambda_1 T^2}{12}\phi_0^2.$$

mismatch between eoms gives tree-level tuning

$$\left(\frac{\lambda_2}{12} - \gamma\right) T^2 \neq 0. \tag{41}$$

This is the amount of the scale symmetry breaking by the finite temperature effects. As the result, the only consistent solution to the corrected equations of motion becomes at this order

$$\phi_1 = 0, \qquad \phi_0 = -\frac{\lambda_2}{24\lambda_1}T^2.$$
 (42)

Thermal fluctuations force the dilation vev away from the origin

thermal fluctuations in Higgs direction

$$\langle \phi_1^2 \rangle_{T,p} = T \frac{p^3}{\omega_p^2}$$

result in the negative mass squared term for the dilation

$$\delta_m V = \lambda_1 \phi_0^2 \phi_1^2 \to \lambda_1 T^2 \phi_0^2,$$

and in the repulsive force

$$-\frac{\partial \delta_m V}{\partial \phi_0} = -2\lambda_1 T^2 \phi_0,$$



Figure 1: Plots of $V_{full}(\phi_0, \phi_1, T)$ for different temperatures and $\lambda_1 = -10^{-6}$ value. Orange dashed line marks flat direction $\phi_1^2 = -\frac{\lambda_1}{2\lambda_2}$. It is easy to see that as the temperature increase, the flat direction no longer exists and the scale symmetry is broken.

Thermal equilibrium for the dilation



Figure 5: Ratio $f(T) = (T^3 \langle \sigma v \rangle)/H$ as a function of temperature for different λ_1 . ϕ_0 field can reach thermal equilibrium for sufficiently large $|\lambda_1|$ value.



Figure 7: Evolution of ϕ_i fields and H with time for coupling constants values fulfilling requirements from section 3.1: $\lambda_2 = 0.03125$, $\lambda_1 = -4.37 \cdot 10^{-26}$, $\xi_0 = 10^{10}$, $\xi_1 = 0.1$. Initial conditions: $\phi_0(0) = 8 \cdot 10^{13}$, $\dot{\phi}_0(0) = 5 \cdot 10^{13}$, $\phi_1(0) = 0$, $\dot{\phi}_1(0) = 10$, and two different $H(0) = H_0$ values. The bigger the initial H_0 , the faster ϕ_i fields loose their velocity and settles in lower values. Two plots for $\phi'_0(t)$ are shown, one for the same time range as in the evolution of ϕ_1 and H, one for later times, to show that ϕ_0 indeed loose its velocity and settles in desired value.



Figure 8: Evolution of ϕ_i fields and H with time for non-zero temperature and coupling constants values $\lambda_2 = 0.03125$, $\lambda_1 = -10^{-6}$, $\xi_0 = 10^3$, $\xi_1 = 0.1$. Initial conditions: $\phi_0(0) = 3 \cdot 10^4$, $\dot{\phi_0}(0) = 5 \cdot 10^3$, $\phi_1(0) = 0$, $\dot{\phi_1}(0) = 10$, and two different $H(0) = H_0$ values. Initial temperature $T_0 = 10^4$ GeV. After fields ϕ_i land in flat direction, they start to roll along that flat valley to higher values and they don't stop.

Conclusions I

- Scale symmetry as the underlying symmetry offers a way to understand the origin of scales as expected
- Scale symmetry is broken at finite T with thermal dilaton vev proportional to T
- Cosmologicl evolution can easily lead to large dilaton vev needed to model hierarchy

Quantum scale symmetric SM + σ

 $H = \begin{pmatrix} 0 \\ \frac{\phi}{\sqrt{2}} \end{pmatrix}$ (electroweak vacuum —> electroweak flat direction)

$$\mathcal{L}_{SM}\Big|_{\substack{m^2=0\\\mu=\mu(\sigma)}}^{m^2=0} + \frac{1}{2}\left(\partial\sigma\right)^2 - \lambda_m |H|^2 \sigma^2 - \frac{\lambda_\sigma}{4} \sigma^4 + \sum_{n=0} \lambda_n \frac{|H|^{4+2n}}{\sigma^{2n}}$$

$$V_{\text{eff}}^{\text{SM}}(\phi, \sigma) \approx \frac{1}{4} \lambda_{\text{eff}} \left(\log \frac{\phi}{\sigma} \right) \phi^4 = M^4 \lambda_{\text{eff}} (\log \tan \theta) \frac{\tan^4 \theta}{(1 + \tan^2 \theta)^2}$$

$$W(\theta)$$

$$W($$



FIG. 2. Contour plots of the effective potentials $-V_{SM+\sigma}(\phi, \sigma)$ for various choices of $\langle \sigma \rangle$. Lower green dashed line marks the electroweak vacuum-direction, higher green dashed line marks the direction of greatest instability. Red continuous line is a plot of the bounce configuration (ϕ_B, σ_B). (Note that, mainly due to varying contribution of the nonrenormalizable interaction from one plot to another, the plots present differing potentials and it would be misleading to plot the bounce configurations in a single frame.)

Summary

SM + dilaton

- 1) You may use a field as the scale μ in Dim-Reg to preserve scale symmetry at the quantum level.
- 2) The price to pay: infinitely many nonpolynomial ϕ/σ operators and corresponding couplings: **nonrenormalizability**.
- 3) Minimal subtraction scheme involves evanescent interactions.
- 4) Presence of a **flat direction** \leftarrow tuning.
- 5) Naturalness: aligning the flat direction perpendicular to Higgs
- 6) Instability = unboundedness below

Domain walls

T. Krajewski, M. Lewicki

Network of walls prefers the true vacuum!



Models of interest

 Radiatively generated minima (eg SM at large field strength)



 Run-away potentials (moduli of stringy models), Quantum Scale Symmetric SM



 Models of strong first-order phase transitions - colliding bubbles (thermal effects play a role)

Models of interest

• Monodromy axion models, relaxion

$$V_{\text{monodromy}}(\phi) = m^2 \phi^2 + \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right) \right]$$
$$V_{\text{relaxion}}(\phi) = g\phi + \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right) \right]$$



Generic potential

$$V_{AS}(\phi) = \frac{V_0}{60}\phi \left(15\phi^3 \left(e^2 \left(2d(a+b+c)+ab+ac+bc+d^2\right)+1\right) - 60abc \left(d^2e^2+1\right)\right) - 20\phi^2 \left(e^2 \left(d^2(a+b+c)+2d(a(b+c)+bc)+abc\right)+a+b+c\right) - 12e^2\phi^4(a+b+c+2d) + 30\phi \left(de^2(ad(b+c)+2abc+bcd)+ab+ac+bc\right) + 10e^2\phi^5\right).$$
(3.6)

$$\frac{\partial^3 V_{AS}}{\partial \phi^3}(\phi) = 2V_0 \left(e^2 (a - \phi)(\phi - b)(c + 2d - 3\phi) + (-a - b + 2\phi) \left(e^2 (d - \phi)(2c + d - 3\phi) + 1 \right) + (\phi - c) \left(e^2 (d - \phi)^2 + 1 \right) \right)$$
(3)

a, b - positions of minima, c - position of maximum

$$\delta V = V_{AS}(b) - V_{AS}(a),$$

 $d3V = rac{\partial^3 V}{\partial \phi^3}(c),$
 $5 = w,$

31

$$\Omega_{GW}(\eta) := \frac{1}{\rho_c(\eta)} \frac{d\rho_{GW}}{d\log|k|}(\eta, k).$$

$$\Omega_{GW}(\eta_{dec})|_{peak} = \frac{\tilde{\epsilon}_{GW} \mathcal{A}^2 \sigma_{wall}^2}{24\pi H_{dec}^2 M_{Pl}^4},$$

$$\Omega_{GW}(\eta_0) = \left(\frac{a(\eta_{dec})}{a(\eta_0)}\right)^4 \left(\frac{H(\eta_{dec})}{H(\eta_0)}\right)^4 \Omega_{GW}(\eta_{dec})$$

$\tilde{\epsilon}_{GW}$ efficiency parameter between 0.7 and 1

 $\sigma_{walls}, \eta_{dec}$ - taken from simulations

$$egin{aligned} & rac{A}{V} = rac{a(t)S_{wall}}{H^{-3}} \propto rac{a(t)}{t}. \ & rac{A}{V} = \mathcal{A}\eta^{-1}, \end{aligned}$$

stable DW: \mathcal{A} in the range 0.8 ± 0.1

more generally

$$\log\left(\frac{A}{V}\right) = -\nu\log\eta + \log\mathcal{A}$$

scaling regime: obtained ν ranges from 0.81 to 1.0

meta-stable DW: \mathcal{A} in the range 0.08 - 0.34

Scaling regime



Figure 9: The evolution of conformal surface area of domain walls per unit volume $\frac{A}{V}$ in function of conformal time η (blue) and the fitted scaling behavior defined by eq. (5.8) (orange) for the best (left panel) and the worst (right panel) fits obtained by procedure described in the main text. Vertical dashed lines correspond to the estimated beginning and end of the scaling regime.

$$\Omega_{GW}(\eta_0)|_{peak} = 4.6 \times 10^{-81} \mathcal{A}^2 \left(\frac{\text{GeV}}{H_{dec}}\right)^2 \left(\frac{\sigma_{wall}}{\text{GeV}^3}\right)^2 h^{-2} \left(\frac{100}{g_*(\eta_{dec})}\right)^{\frac{1}{3}}$$

$$f_0|_{peak} = \frac{a(\eta_{dec})}{a(\eta_0)} H_{dec} = 1.63 \times 10^2 \left(\frac{H_{dec}}{\text{GeV}}\right)^{\frac{1}{2}} \text{Hz}_{1}$$

$$\begin{split} \Omega_{GW}(\eta_0)|_{peak} &= 0.29 \times 10^{-77} \mathcal{A}^2 \left(\frac{\eta_{dec}}{w}\right)^4 \left(\frac{\sigma_{wall}}{w^{-3}}\right)^2 \left(\frac{\text{GeV}^{-1}}{w}\right)^4,\\ f_0|_{peak} &= 3.3 \times 10^1 \left(\frac{w}{\eta_{dec}}\right) \left(\frac{\text{GeV}^{-1}}{w}\right)^{\frac{1}{2}} \text{ Hz}, \end{split}$$

36

We have estimated overall factors present in eqs. (6.7) and (6.7) basing on values of \mathcal{A} , η_{dec} obtained in simulations in which networks entered scaling regime and previously computed σ_{wall} . The maximal value of the prefactor in eq. (6.7) obtained in this way is equal to:

$$\Omega_{GW}^{max}(\eta_0)|_{peak} = 0.1 \times 10^{-66} \left(\frac{1\frac{\hbar c}{\text{GeV}}}{w}\right)^4, \qquad f_0^{max}|_{peak} = 0.7 \left(\frac{1\frac{\hbar c}{\text{GeV}}}{w}\right)^{\frac{1}{2}} \text{ Hz}, \qquad (6.9)$$

where the frequency of the peak for this network is denoted as f_0^{max} . On the other hand, the minimal prefactor computed from data from simulations is equal to:

$$\Omega_{GW}^{min}(\eta_0)\big|_{peak} = 0.6 \times 10^{-68} \left(\frac{1\frac{\hbar c}{\text{GeV}}}{w}\right)^4, \qquad f_0^{min}\big|_{peak} = 1.3 \left(\frac{1\frac{\hbar c}{\text{GeV}}}{w}\right)^{\frac{1}{2}} \text{ Hz.}$$
(6.10)



Figure 10: Hypothetical peak amplitudes of GWs emitted from cosmological domain walls as a function of the peak frequency f compared to predicted sensitivities of current and planned detectors LIGO [59–62], LISA [63, 64], AEDGE [65], AION-1km [66], ET [67, 68] as well as upper bound induced by the CMB/BBN [69, 70].

Summary II

- For a strong signal and a low frequency peak a period of stable evolution is needed
- Bias of the initial distribution easily destabilises the network
- Asymmetry of the potential destabilises the network for symmetric distributions
- Short living networks may give a strong signal if the energy scale is very large - but this produces a high frequency peak, beyond current sensitivity
- Decaying networks of domain walls produce a signal in the form of gravitational waves - too weak to be detected anytime soon - if a signal is detected then either finetuning or non-standard cosmology have occurred



Thanks Graham!