10^{20}Hz stochastic gravitational waves from photon spheres of supermassive black holes.

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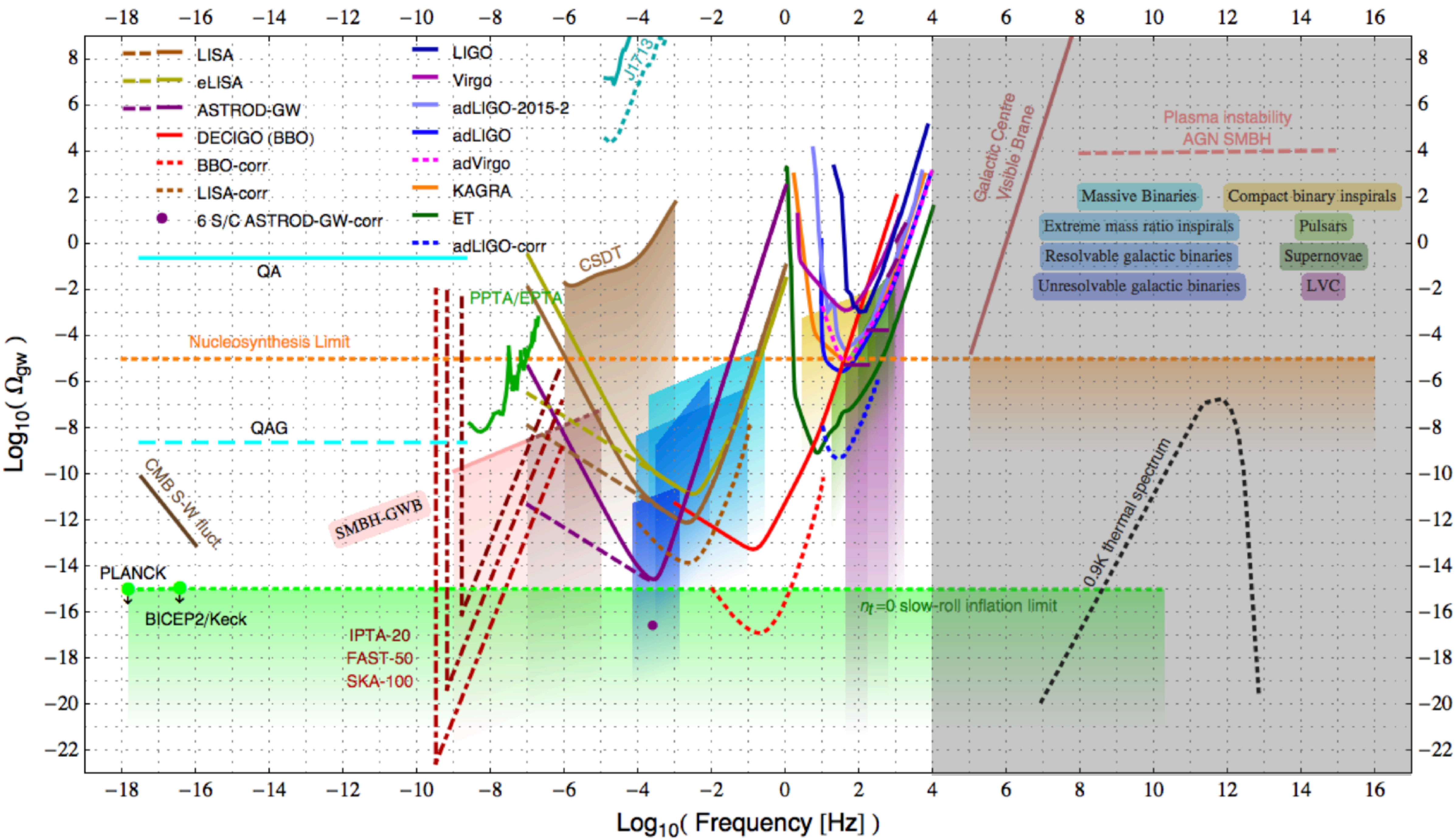
Collaboration with Jiro Soda (Kobe University)

and Hirotaka Yoshino (Osaka Metropolitan University).

Introduction

For high frequency gravitational wave, there are few observational method.
To encourage experiments, “guaranteed sources” are required in high frequency range.



[K.Kuroda et.al,”arXiv:1511.00231(2015)]

Introduction

We proposed a new source for (stochastic) high frequency gravitational wave, based on photon-graviton mixing(conversion) phenomenon.

[Short summary of talk]

I. Photon and graviton are minimally coupled via $\int d^4x\sqrt{-g}\; g^{\mu\rho}g^{\nu\sigma}\; F_{\mu\nu}F_{\rho\sigma}$.

II. If there is background magnetic field B, photon and graviton are mixing.

The probability of conversion from photon to graviton is roughly given by

$P_{\gamma\to g}(z) \propto \sin^2\left(\frac{B}{M_{pl}}z\right) \sim \left(\frac{B}{M_{pl}}z\right)^2$, where z is distance of propagation.

Long light path in the strong magnetic field makes conversion efficient.

[ME.Gertsenshtein(1962), G.Raffelt and L.Stodolsky(1988)]

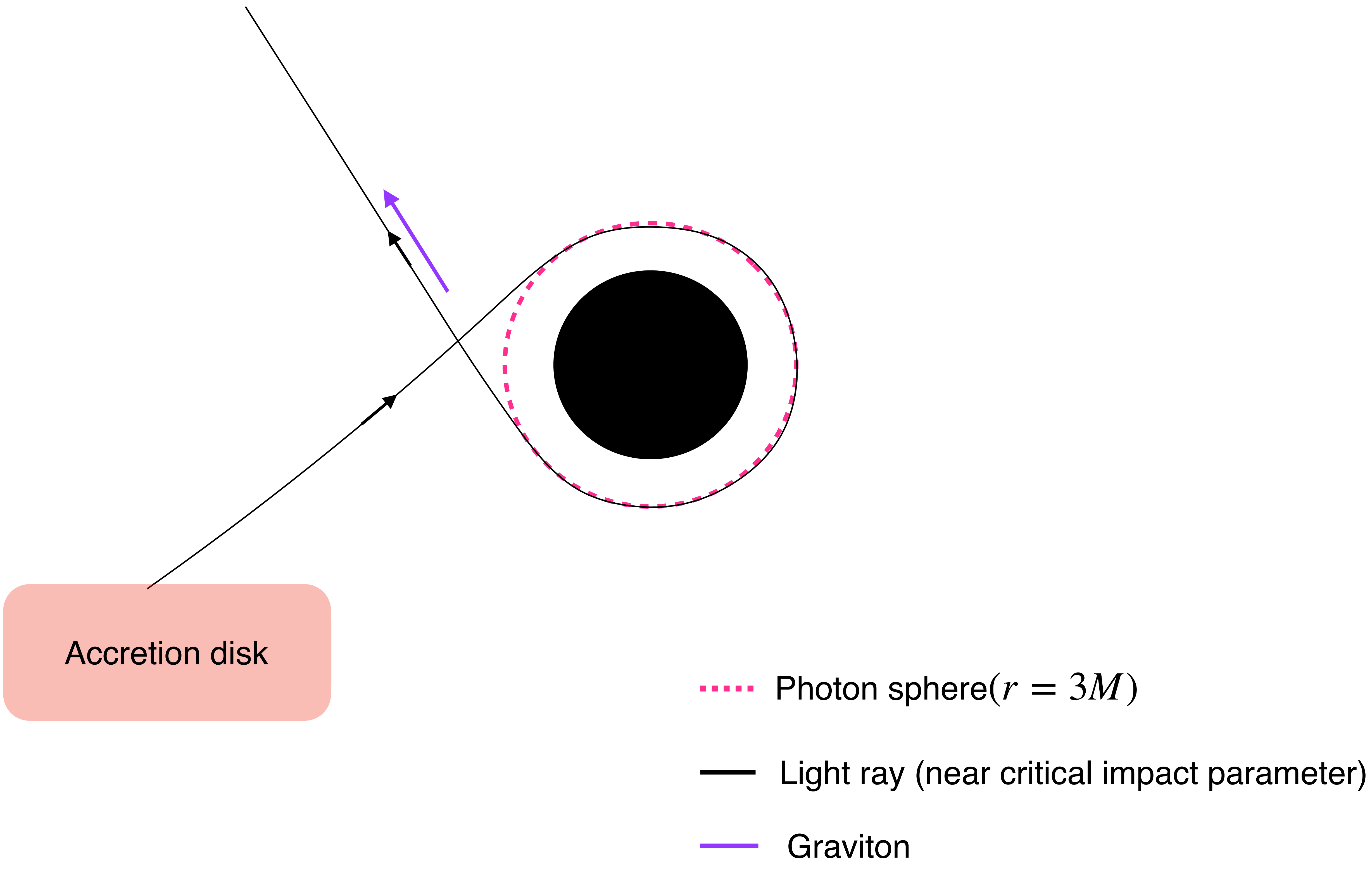
III. We consider photon sphere of SuperMassive BlackHole(SMBH) and its accretion disk.

Photon sphere provides long propagate distance in a strong magnetic field.

Huge photon luminosity of accretion disk make conversion effectively efficient.

→ Photon sphere of SMBHs as an efficient factory of graviton.

Introduction



Review on photon graviton conversion

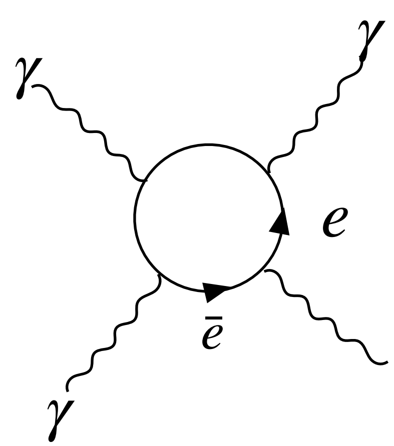
We consider Einstein Maxwell action + electron one loop correction

$$S = \frac{M_{pl}^2}{16\pi}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}\left[-\frac{1}{16\pi}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{16\pi^2}\frac{\alpha^2}{90m_e^4}\left\{(g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right\}\right],$$

in natural cgs-gauss unit. QED coupling: $\alpha \sim \dfrac{1}{137}$

[W.Heisenberg and H.Euler (1936)]

QED correction = $\int \mathscr{D}(\text{electron})$ (



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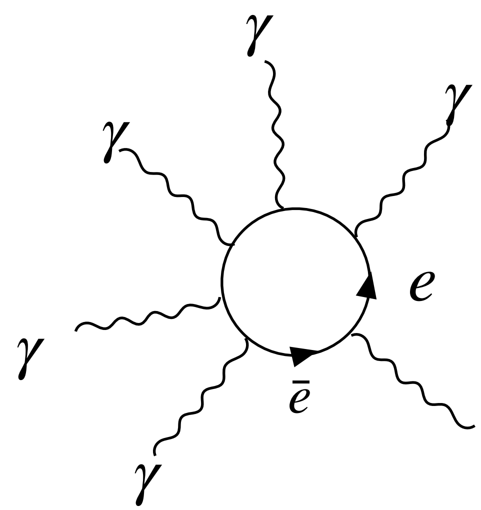
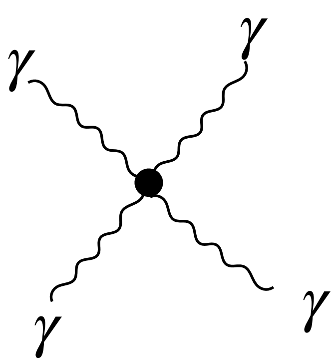


$+ \cdots$)

$=$



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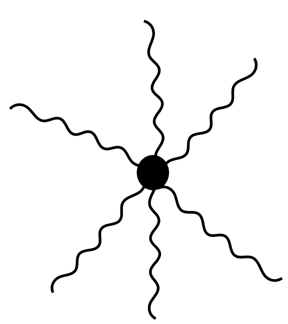


$+ \cdots \sim \boxed{F^4} + \cancel{F^6 + \mathcal{O}(F^8)}$

If $F \ll m_e^2 \sim 10^{13}$G is satisfied.

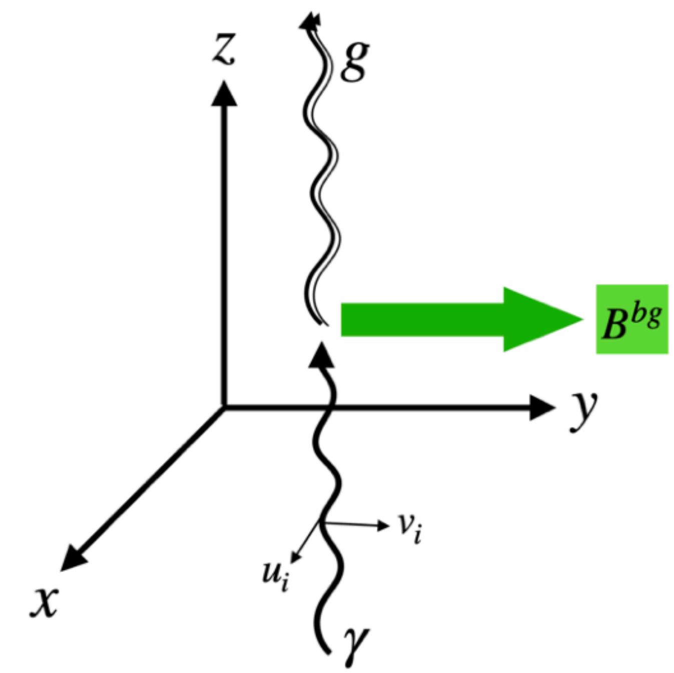
Review on photon graviton conversion

$$G_{\mu\nu} = \frac{2}{M_{pl}^2}\left[g^{\alpha\beta}F_{\alpha\mu}F_{\beta\nu} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right]$$
$$+ \frac{1}{4\pi}\frac{2}{M_{pl}^2}\frac{\alpha^2}{90m_e^4}\left[g_{\mu\nu}(F_{\alpha\beta}F^{\alpha\beta})^2 - 8(F_{\alpha\beta}F^{\alpha\beta})g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} - \frac{7}{4}g_{\mu\nu}(F_{\alpha\beta}\tilde{F}^{\alpha\beta})^2\right]$$

$$\nabla_\mu F^{\mu\nu} = \frac{1}{4\pi}\frac{\alpha^2}{45m_e^4}\nabla_\mu\left[4F_{\alpha\beta}F^{\alpha\beta}F^{\mu\nu} + 7F_{\alpha\beta}\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\right]$$

Consider the light ray(eikonal approxi.) propagating z direction in the uniform background magnetic field B^{bg}.

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{\sqrt{16\pi}}{M_{pl}}h_{\mu\nu}(t,z), \quad A_\mu = A_\mu^{bg} + \mathscr{A}_\mu(t,z)$$
$$h_{\mu\nu}(t,z) = \hat{h}_{\mu\nu}(z)\ e^{i(kz-\omega t)}, \quad \mathscr{A}(t,z) = \hat{\mathscr{A}}(z)\ e^{i(kz-\omega t)}$$



(Linearized EoM)

Photon graviton mixing

$$i\frac{d}{dz}\begin{bmatrix}\hat{h}\\ \hat{\mathscr{A}}\end{bmatrix} = \begin{bmatrix}0 & \Delta_{g\gamma}\\ \Delta_{g\gamma} & \Delta_\gamma\end{bmatrix}\begin{bmatrix}\hat{h}\\ \hat{\mathscr{A}}\end{bmatrix}$$

$$\Delta_{g\gamma} = \frac{B^{bg}}{M_{pl}}, \qquad \Delta_\gamma = \frac{\omega_p^2}{2\omega} - \frac{5.5\alpha^2}{90\pi m_e^4}\omega(B^{bg})^2,$$

Plasma frequency : $\omega_p^2 = 4\pi\alpha\frac{n_e}{m_e}$

Review on photon graviton conversion

Consider the initial condition $\hat{h}(0) = 0, \; \hat{\mathscr{A}}(0) = 1$ (i.e. Initially Photon).

After propagating distance z from origin the conversion probability is given by

$$P(z) := |\hat{h}(z)|^2 = \left(\frac{2\Delta_{g\gamma}}{\sqrt{\Delta_\gamma^2 + (2\Delta_{g\gamma})^2}}\right)^2 \sin^2\left(\frac{\sqrt{\Delta_\gamma^2 + (2\Delta_{g\gamma})^2}}{2} z\right) \leq 1$$

[ME.Gertsenshtein(1962), G.Raffelt and L.Stodolsky(1988)]

◆ An efficient conversion $\dfrac{2\Delta_{g\gamma}}{\sqrt{\Delta_\gamma^2 + (2\Delta_{g\gamma})^2}} \sim 1$

is possible only when $\Delta_\gamma := \dfrac{\omega_p^2}{2\omega} - \dfrac{5.5\alpha^2}{90\pi m_e^4}\omega(B^{bg})^2 = 0$.

n_e : plasma number density
B^{bg} : magnetic field strength

This determine the resonance frequency as $\omega_r^2 = \dfrac{45\pi m_e^4}{5.5\alpha^2}\dfrac{\omega_p^2}{B^{bg2}} = \dfrac{180\pi^2 m_e^3}{5.5\alpha}\dfrac{n_e}{B^{bg2}}$

At $\omega = \omega_r$, (where ω is frequency of incident photon), $P_{\gamma\to g}(z) \sim \sin^2\left(\dfrac{B}{M_{pl}}z\right) \sim \left(\dfrac{B}{M_{pl}}z\right)^2$.

Remark: Long light trajectory makes probability large, BH's **photon sphere** is nice candidate.

10^{19-20} Hz gravitational wave : Mechanism

◆ **Assumption for disk structure**

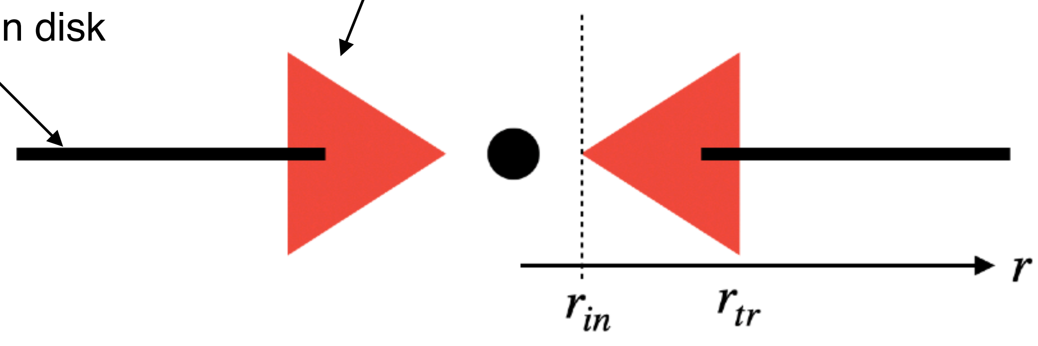
We assume, at the inner edge of disk($\sim 6M$), advection dominated accretion flow(ADAF) exist as a source of high energy photon (hard X ray, gamma ray).

ADAF is the accretion flow which has the energy balance equation,

$$Q^+ = Q^- + Q_{\text{adv}} \sim Q_{\text{adv}}$$

Q^+: Viscosity heating, Q^-: Radiative cooling, Q_{adv}: Cooling by matter entropy transfer(advection).

A self similar solution of ADAF gives sound velocity as $c_s \sim v_K$,
where $v_K = \sqrt{GM/r}$ is Keplerian velocity. [Narayan and Yi (1994)]



10^{19-20} Hz gravitational wave : Mechanism

◆ **Estimation of resonance frequency of photon graviton conversion**

$$\omega_r = \sqrt{\frac{180\pi^2}{5.5\alpha}\frac{m_e^3 n_e}{B^2}} \sim 0.84 \times 10^5 \text{eV} \left(\frac{\beta}{0.01}\right)^{1/2} \left(\frac{v_K(3M)}{c_s}\right),$$

where $\beta := \dfrac{p_{gas}}{p_{mag}} = \dfrac{c_s^2 \rho}{B^2/8\pi}$ is pressure ratio and c_s is sound speed.

For inner disk, 3D-GRMHD simulation of non radiative disk suggests $\beta = 0.01 \sim 1$

[De Villiers et al.(2003)]

Therefore $\omega_r \sim 0.84 \times 10^5$eV, equivalently $f_r \sim 2 \times 10^{19}$Hz at photon sphere($r = 3M$).

➡ **Hard X rays emitted from inner disk are relevant to conversion.**
10^{19-20}**Hz gravitational waves from SMBHs photon sphere.**

10^{19-20} Hz gravitational wave : Luminosity

◆ Idea of luminosity calculation

We parametrize light ray near photon sphere by impact parameter to BH.
$b = b_{crit} = 3\sqrt{3}M$ correspond to ray orbiting photon sphere.

$$\frac{dE_{GW}}{dt} = \underbrace{\int db\ \Delta\omega_r}_{\text{Summation}} \frac{d^3N}{dt\,d\omega\,db} \times P_{\gamma g}\left(cT(b)\right) \times \hbar\omega_r$$

Energy per graviton with resonance freq.

Graviton number
produced at given impact parameter b $(b \sim b_{crit.})$.

(comment)
Orbiting time at given impact parameter

$$\text{if } b \sim b_{crit.},$$

$$T(b) := -\frac{3\sqrt{3}}{2c} r_g \log|2(b - b_{crit.})/r_g|$$

[H.Yoshino et.al.(2019)]

◆ We assume Eddington luminosity for X-ray region.

$$L\left(\omega_r \sim 10^{20}\ \text{Hz}\right) \sim \frac{L_{Edd.}(M)}{10^{20}\ \text{Hz}} \sim 10^{24}\left(\frac{M}{10^6 M_\odot}\right) \text{erg sec}^{-1}\,\text{Hz}^{-1}.$$

◆ Poynting flux $\sim$ radiation energy of electro magnetic field

$$\frac{B^2}{8\pi} \times 4\pi\,(3M)^2\,c = L_{Edd.}(M) \quad\rightarrow\quad B \sim 0.241 \times 10^7 \sqrt{\frac{10^6 M_\odot}{M}}\ \text{G}$$

Then we can estimate the luminosity as

$$\frac{dE_{GW}}{dt}(M) \sim \frac{\pi M^2}{A(r_{in}, r_{tr})} \times 10^{25}\,\text{erg sec}^{-1}\left(\frac{M}{10^6 M_\odot}\right)^{5/2}$$

,where $A(r_{in}, r_{tr})$ is emission region of hard Xray.

Observation as a stochastic background.

All SMBHs bright with $\sim 10^{19-20}$Hz gravitational waves regardless its mass.

$$\omega_r = \sqrt{\frac{180\pi^2}{5.5\alpha}\frac{m_e^3 n_e}{B^2}} \sim 0.84 \times 10^5 \text{eV} \left(\frac{\beta}{0.01}\right)^{1/2} \left(\frac{v_K(3M)}{c_s}\right), \qquad \beta := \frac{p_{gas}}{p_{mag}}, \qquad c_s\text{: sound speed.}$$

→ We cannot resolve each contributions.
We proposed that there are 10^{20}Hz Stochastic background.

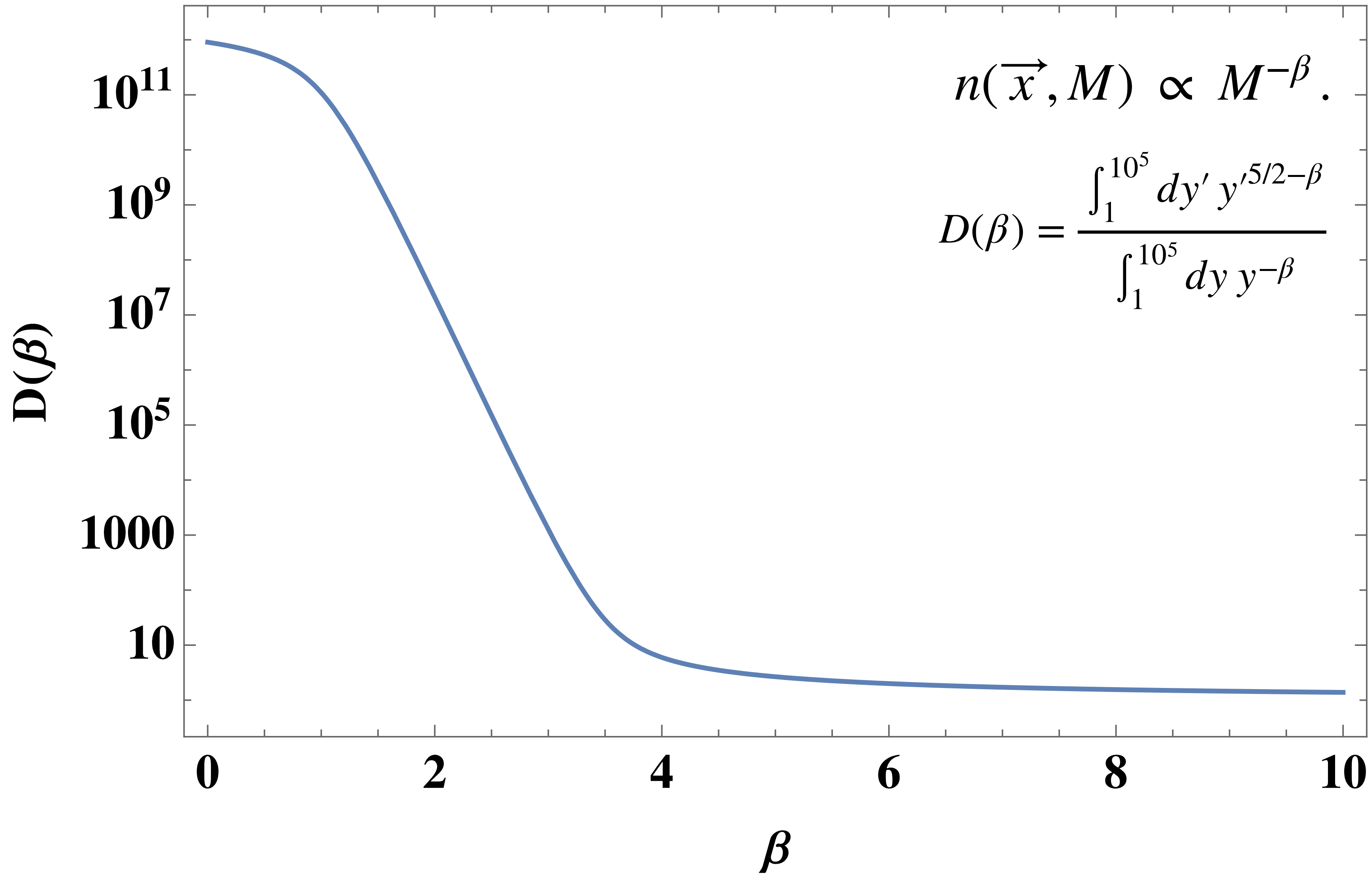
Let us sum up contributions from all SMBHs.
We assume spatially uniform and power law distribution for SMBHs number density as

$$n(\vec{x}, M) \propto M^{-\beta}, \qquad \int_{10^6 M_\odot}^{10^{11} M_\odot} dM \int d^3\vec{x}\, n(\vec{x}, M) = N_{galaxy} \sim 10^{12}.$$

$$h_0{}^2\Omega_{gw} = \frac{h_0^2}{\rho_c} \times \int_{10^6 M_\odot}^{10^{11} M_\odot} dM \frac{dE}{dt}(M) \int_{|\vec{x}| \le c H_0{}^{-1}} d^3\vec{x}\, \frac{n(\vec{x}, M)}{|\vec{x}|^2} \sim \frac{34 M^2}{A(r_{in}, r_{tr})} h_0^2 \times 10^{-23} D(\beta)$$

,where $D(\beta) = \dfrac{\int_1^{10^5} dy'\, y'^{5/2-\beta}}{\int_1^{10^5} dy\, y^{-\beta}}$.

Observation as a stochastic background.



$$h_0{}^2\Omega_{gw} \sim \frac{34M^2}{A(r_{in}, r_{tr})} h_0^2 \times 10^{-23} D(\beta) \lesssim 10^{-12}$$

Upper bound strongly depend on the mean area of Xray emission region.

Conclusion and Discussion

[**Conclusion**]
Photon sphere of SMBHs may provide the efficient factory of graviton via photon graviton conversion.

Essentially, there is no reason to restrict our discussion to graviton production.
Since the axion also have coupling to photon, photon sphere also emits axion.

$$S = \int d^4x \left[-\frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu} + \frac{1}{16\pi^2}\frac{\alpha^2}{90 m_e^4}\left((F_{\rho\sigma}F^{\rho\sigma})^2 + \frac{7}{4}(F_{\rho\sigma}\tilde{F}^{\rho\sigma})^2 \right) - \frac{1}{2}\left((\partial_\mu \phi)^2 + m_\phi^2 \phi^2 \right) - \frac{1}{16\pi} g_{\phi\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} \right],$$

In our prototype estimation, if our universe has axion with $g_{\phi\gamma} = 10^{-12}\text{GeV}^{-1}$, $m_\phi \ll 10^{-8}\text{eV}$, photon sphere of M87 emits sufficient axion. We roughly estimated the axion luminosity from photon sphere of M87. The result is

$$\frac{dE_\phi}{dt} = \frac{0.7M^2}{A(r_{in}, r_{tr})}\left(\int_{10^5\text{eV}}^{10^6\text{eV}} d\omega L_\omega \right)\left(\frac{M}{6.5\times 10^9 M_\odot} \right)^2 \left(\frac{g_{\phi\gamma}}{10^{-12}\text{GeV}^{-1}} \right)^2 \left(\frac{\bar{B}}{6.8\text{G}} \right)^2,$$

where $A(r_{in}, r_{tr})$ is emission region of hard Xray, L_ω is specific luminosity of disk.

[Kimihiro Nomura, Kaishu Saito, Jiro Soda, in preparation]

[Event Horizon Telescope, First M87 Event HorizonTelescope Results.V. (2019)]

We predict that, if there is axion, there may be hard X-ray extinction in the M87 core.