ΕΘΝΙΚΟ ΜΕΤΣΟΒΙΟ ΠΟΛΥΤΕΧΝΕΙΟ

ΣΧΟΛΗ ΕΦΑΡΜΟΣΜΕΝΩΝ ΜΑΘΗΜΑΤΙΚΩΝ ΚΑΙ ΦΥΣΙΚΩΝ ΕΠΙΣΤΗΜΩΝ

ΣΧΟΛΗ ΜΗΧΑΝΟΛΟΓΩΝ ΜΗΧΑΝΙΚΩΝ

ΕΚΕΦΕ «ΔΗΜΟΚΡΙΤΟΣ»

ΙΝΣΤΙΤΟΥΤΟ ΝΑΝΟΕΠΙΣΤΗΜΗΣ ΚΑΙ ΝΑΝΟΤΕΧΝΟΛΟΓΙΑΣ

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Διατμηματικό Πρόγραμμα Μεταπτυχιακών Σπουδών

«Φυσική και Τεχνολογικές Εφαρμογές»

Καμπύλες Σταθερής Φυσικής στο Αβελιανό-Χιγκς Μοντέλο

ΜΕΤΑΠΤΥΧΙΑΚΗ ΔΙΠΛΩΜΑΤΙΚΗ ΕΡΓΑΣΙΑ

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Περίληψη

Στην παρούσα μεταπτυχιακή μελέτη θα ασχοληθούμε με το Αβελιανό-Higgs μοντέλο και εν συνεχεία θα επεκταθούμε σε μη-Αβελιανές αυθόρμητα σπασμένες θεωρίες βαθμίδος, και συγκεκριμένα το Καθιερωμένο Πρότυπο, προκειμένου να εξετάσουμε τον τρόπο με τον οποίο επιδρούν οι κβαντικές διορθώσεις στις φυσικές παραμέτρους των μοντέλων, όπως είναι οι σταθερές σύζευξης και οι μάζες. Συγκεκριμένα, ενώ μπορούμε να διαλέξουμε μια συγκεκριμένη βαθμίδα για να υπολογίσουμε το εμείς κάνουμε όλη την διαδικασία σε δύο ξεχωριστές βαθμίδες. Ειδικότερα, ξεκινάμε τον υπολογισμό με την βαθμίδα Unitary, όπου εκεί υπάρχουν μόνο οι φυσικοί βαθμοί ελευθερίας, και εν συνεχεία χρησιμοποιώντας το υπόβαθρο της διαταρακτικής Κβαντικής Θεωρίας Πεδίου, δηλαδή μέσω των διαγραμμάτων Feynman, εισάγουμε διαράμματα ενός βρόνχου ως κβαντικές διορθώσεις αποκομίζοντας την ενεργό Λαγκραντζιανή την οποία επανακανονικοποιούμε με φυσικό τρόπο. Πλέον οι επανακανονικοποιημένες παράμετροι μεταβάλλονται ως συνάρτηση της ενεργειακής κλίμακας. Εν συνεχεία, πραγματοποιούμε ακριβώς τον ίδιο υπολογισμό και σε μια τυχαία Rξ βαθμίδα όπου και πάλι αποκομίζουμε τις φυσικές ποσότητες. Τώρα όμως δείχνουμε με ποιον τρόπο οι τελευταίες είναι ανεξάρτητες της βαθμίδας και ίδιες με αυτές της Unitary βαθμίδας.

Τώρα, το ενδιαφέρον μας στρέφεται στην επανακανονικοποιημένη μάζα του Higgs μποζονίου, καθώς εκτός από λογαριθμικές κβαντικές διορθώσεις δέχεται και τετραγωνικές, ανάλογες των μαζών των σωματιδίων με τα οποία αλληλεπιδρά το πεδίο Higgs. Αυτό το αποτέλεσμα οδηγεί στο πρόβλημα της Ιεραρχίας και ο βασικός στόχος της παρούσας διδακτορικής διατριβής είναι η μελέτη μιας νέας προσέγγισης του τρόπου διαχείρισης του προβλήματος αλλά και μιας πιθανής κατεύθυνσης επίλυσης του. Προκειμένου αυτό να καταστεί δυνατό, ακολουθούμε μια συγκεκριμένη μεθοδολογία σύμφωνα με την οποία υπολογίζουμε για το Αβελιανό-Higgs μοντέλο και το Καθιερωμένο Πρότυπο στις τέσσερις διαστάσεις, τις ροές της ομάδας επανακανονικοποίησης που αφορούν συγκεκριμένες ανεξάρτητες παραμέτρους. Εν συνεχεία, χρησιμοποιώντας την σχέση μεταξύ γυμνών και επανακανονικοποιημένων παραμέτρων φτιάχνουμε έναν ν-διάστατο χώρο φάσεων για τις ν ανεξάρτητες παραμέτρους. Εκεί σχεδιάζονται οι Γραμμές Σταθερής Φυσικής οι οποίες καταγράφουν την μεταβολή των ανεξάρτητων παραμέτρων ως προς την ενεργειακή κλίμακα απαιτώντας οι φυσικές ποσότητες, όπως η μάζα του Higgs μποζονίου, να παραμένουν σταθερές. Με αυτόν τον τρόπο αποκαλύπτεται το

πρόβλημα της Ιεραρχίας και βλέπουμε την εξέλιξη του μέχρι το πόλο Landau. Το επόμενο βήμα είναι να θεωρήσουμε το ίδιο μοντέλο προερχόμενο από τις πέντε διαστάσεις, με συνοριακές συνθήκες orbifold, εισάγοντας τις καταστάσεις Kaluza-Klein. Αυτό μας οδηγεί πάλι στις τέσσερις διαστάσεις αλλά τώρα έχουμε έναν πύργο Kaluza-Klein σωματιδίων για κάθε ένα από τα είδη υπάρχοντα σωματίδια. Σχεδιάζοντας πάλι τις Γραμμές Σταθερής Φυσικής θα δούμε αν και πώς τα νέα σωματίδια ακυρώνουν τους τετραγωνικούς όρους από την επανακανονικοποιημένη μάζα του Higgs μποζονίου δίνοντας μια διέξοδο στο Πρόβλημα της Ιεραρχίας. Τέλος γνωρίζουμε ότι το πρόβλημα αυτό έγκειται στο θεωρητικό πλαίσιο της Φυσικότητας, οπότε πέρα από το πρόβλημα της Ιεραρχίας εξετάζουμε το αν και πως συνδέονται με την μεθοδολογία μας άλλα προβλήματα αυτού του πλαισίου.

Εισαγωγή

Αδιαμφισβήτητο γεγονός είναι ότι το Καθιερωμένο Πρότυπο των στοιχειωδών σωματιδίων έχει μελετηθεί διεξοδικά και έχει επιβεβαιωθεί από όλα τα μέχρι τώρα πειραματικά δεδομένα. Η τελευταία εξαιρετικά σημαντική επιβεβαίωση του έρχεται από την πρόσφατη πειραματική ανακάλυψη του σωματιδίου Higgs του αντίστοιχου μηχανισμού, ο οποίος αποτελεί τον ακρογωνιαίο λίθο του Καθιερωμένου Προτύπου. Το πρώτο στοιχείο το οποίο αναδεικνύει την σημασία του πεδίου Higgs έγκειται στο γεγονός ότι μέσω της αλληλεπίδρασης του με τα άλλα σωματίδια, τα τελευταία αποκτούν μάζα χωρίς όμως να υπάρχει ρητά σπάσιμο της συμμετρίας βαθμίδος όταν περιλαμβάνονται σωματίδια βαθμίδος με μάζα. Αυτό επιτυγχάνεται όταν το πεδίο Higgs αποκτά μια αναμενόμενη τιμή στο κενό μέσω του αυθόρμητου σπασίματος της συμμετρίας βαθμίδος, διατηρώντας την συμμετρία Lorentz, και το οποίο έχει σαν αποτέλεσμα τα σωματίδια που συζεύγονται μαζί του να αποκτούν μάζα. Στην ουσία η συμμετρία βαθμίδας έχει κρυφτεί και εξακολουθεί να διατηρείται κάτι που καθιστά το Καθιερωμένο Πρότυπο μια επανακανονικοποιήσημη θεωρία. Πέρα από τον μηχανισμό Higgs αυτό που παίζει εξίσου σημαντικό ρόλο στην κατανόηση του φυσικού κόσμου είναι οι παράμετροι που έχει το δυναμικό Higgs, δηλαδή η μάζα του και η σταθερά ζεύξης με τον εαυτό του λ. Ξεκινώντας με την δεύτερη παράμετρο, και πραγματοποιώντας μια ποιοτική ανάλυση, γνωρίζουμε ότι το επανακανονικοποιημένο λ μεταβάλλεται ως συνάρτηση της ενεργειακής κλίμακας επηρεαζόμενο κατά κόρων από την τιμή της μάζας του Higgs σωματιδίου. Για την ακρίβεια εάν η τελευταία είναι πολύ μεγάλη τότε, για μια σταθερή αναμενόμενη τιμή του κενού, το λ έχει μεγάλη τιμή με αποτέλεσμα να οδηγήται προς το πόλο Landau η επανακανονικοποιημένη λ. Άρα μεγαλύτερη μάζα του Higgs σωματιδίου συνεπάγεται μεγαλύτερο λ που οδηγεί τον πόλο Landau σε πειραματικά επιτρεπόμενες περιοχές. Αυτό σημαίνει ότι σε εκείνη την ενεργειακή κλίμακα η θεωρία διαταραχών δεν ισχύει και θα πρέπει να υπάρχει μια νέα μη διαταρακτική θεωρία ισχυρών αλληλεπιδράσεων. Το όριο αυτό για το λ ονομάζεται Τετριμμένο Όριο. Αντιθέτως, εάν η μάζα του Higgs σωματιδίου είναι μικρή τότε εμφανίζεται ένα άλλο ενδιαφέρον όριο όπου το επανακανονικοποιημένο λ επηρεάζεται κυρίως από τα βαρέα κουάρκς, και δει από το top κουάρκ, αποκομίζοντας αρνητικές τιμές στις υψηλές ενέργειες. Αυτό δηλώνει μια αρνητική συνεισφορά στο δυναμικό Higgs που έχει σαν αποτέλεσμα η θεωρία να γίνεται ασταθής και το Καθιερωμένο Πρότυπο να γίνεται μη συνεπές πάνω από μια ενεργειακή κλίμακα. Το όριο αυτό λέγεται Όριο Αστάθειας. Βέβαια η μάζα του Higgs σωματιδίου μετρήθηκε πειραματικά και η τιμή της οδηγεί το λ να είναι ανάμεσα στα δύο όρια, δηλαδή στο Όριο Μετασταθερότητας. Η μελέτη της επανακανονικοποιημένης μάζας του Higgs σωματιδίου είναι εξίσου σημαντική και αποτελεί το βασικό θέμα της παρούσας διδακτορικής διατριβής. Συγκεκριμένα, η τελευταία έχει μια πολύ ενδιαφέρουσα ιδιότητα σύμφωνα με την οποία μεταβάλλεται με την ενεργειακή κλίμακα λογαριθμικά αλλά και τετραγωνικά ως προς τις μάζες των σωματιδίων που αλληλεπιδρούν με το πεδίο Higgs. Αρχικά γνωρίζουμε ότι η φυσική μάζα του πεδίου Higgs είναι μετρήσιμη πειραματικά και είναι ίση με 125 GeV. Οπότε παρατηρήται μια μεγάλη διαφορά μεταξύ των τετραγώνων της φυσικής και της επανακανονικοποιημένης μάζας του Higgs, η οποία είναι ανάλογη των τετραγώνων των μαζών των σωματιδίων που αλληλεπιδρούν με το πεδίο Higgs καθώς και του ίδιου. Άρα από την στιγμή που τα βαρέα σωματίδια δεν αποσυζεύγονται, η μάζα του πεδίου Higgs είναι ευαίσθητη στις υψηλές ενέργειες. Υποθέτοντας ότι το Καθιερωμένο Πρότυπο είναι το όριο στις χαμηλές ενέργειες μιας πιο ολοκληρωμένης θεωρίας που εμφανίζεται πάνω από μια ενεργειακή κλίμακα Λ, τότε η γυμνή μάζα του Higgs σωματιδίου ισούται με το άθροισμα της φυσική μάζας και του τετραγώνου του Λ. Ως συνέπεια αυτού, και αφού η φυσική μάζα είναι ίση με 125 GeV, εάν το Λ ανήκει στην κλίμακα Planck, θα υπάρχει ένα τεράστιο ενεργειακό κενό μεταξύ των δύο. Το γεγονός ότι η φυσική μάζα του Higgs σωματιδίου είναι τόσο μικρότερη από το Λ οδηγεί στο Πρόβλημα της Ιεραρχίας. Η σπουδαιότητα του προβλήματος αυτού έγκειται στο γεγονός ότι απαιτώντας η φυσική μάζα του πεδίου Higgs να παραμένει ίση με την πειραματική της τιμή, θα πρέπει να υπάρχει ακύρωση όλων των ευαίσθητων στις υψηλές ενέργειες τετραγωνικών όρων με τεράστια ακρίβεια, το οποίο δεν προκύπτει ως φυσική διαδικασία της θεωρίας. Το παρόν πρόβλημα έγκειται στο θεωρητικό πλαίσιο της Φυσικότητας, συμφωνα με το οποίο όλες οι παράμετροι μιας στοιχειώδους θεωρίας πρέπει να είναι της τάξης της μονάδας.

Contents

1 Abelian Higgs Model in the Unitary Gauge

In the present work we demonstrate how to perform the on-shell renormalization of the Abelian Higgs Model so as to obtain the physical quantities. In order to do so we start from the classical Lagrangian of the model that we study in a general gauge which reads

$$\mathcal{L}_{AH} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + |D_\mu H|^2 + m^2|H|^2 - \lambda|H|^4 + const. \tag{1}$$

The next step is to consider the spontaneous symmetry breaking of the Z_2 global symmetry of the theory which through the Higgs mechanism will give mass to the gauge boson and will produce several interacting terms. In order to see the Higgs mechanism we should give to the scalar field a vacuum expectation value(vev) and this is done by minimizing the potential that we have here, thus the minimization condition gives

$$\begin{aligned} \frac{dV(H)}{dH} &= -2m^2H + 4\lambda H^3 \Leftrightarrow \\ \frac{dV(H)}{dH} &= 2H\left(-m^2 + 2\lambda H^2\right) = 0 \end{aligned} \tag{2}$$

which shows us that there is a local maximum at $\langle H\rangle = 0$ and, in this specific geometry space, two local minima at $\langle H\rangle = \pm\frac{m}{\sqrt{2\lambda}}$. Therefore the vev that our scalar takes has the form $v_0 = \frac{m}{\sqrt{\lambda}}$.

Now, we can use two different ways in order to insert the vev inside the Lagrangian, namely using the Cartesian or the Polar basis expansion of the scalar field. Here we choose to use the second case which gives

$$H = \frac{\sigma e^{i\frac{\chi}{v_0}}}{\sqrt{2}} \tag{3}$$

and including the vacuum expectation value we obtain the following form

$$H(x) = \frac{(\phi(x) + v_0)\, e^{i\frac{\chi}{v_0}(x)}}{\sqrt{2}} \tag{4}$$

where now $\phi(x)$ is the Higgs field and $\chi(x)$ is the massless Goldstone boson. Thus if we insert (4) in the Lagrangian (1) then we get two sets of multiplying terms which give the following

$$
\begin{aligned}
|D_\mu H|^2 &= \frac{1}{2}\left(\partial_\mu + igA_\mu\right)(\phi + v_0)\, e^{i\frac{\chi}{v_0}} \left(\partial^\mu - igA^\mu\right)(\phi + v_0)\, e^{-i\frac{\chi}{v_0}} \Leftrightarrow \\
|D_\mu H|^2 &= \frac{1}{2}\left[\partial_\mu \phi + igA_\mu \phi + igA_\mu v_0\right]\left[\partial^\mu \phi - igA^\mu \phi - igA^\mu v_0\right] \\
&+ (\phi + v_0)^2 \frac{(\partial_\mu \chi)^2}{v_0^2} + g(\phi + v_0)^2 A_\mu \frac{\partial_\mu \chi}{v_0}
\end{aligned}
\tag{5}
$$

thus doing the calculation we get

$$
\begin{aligned}
|D_\mu H|^2 &= \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - i\frac{1}{2} g \partial_\mu \phi A^\mu \phi - i\frac{1}{2} g v_0 \partial_\mu \phi A^\mu \\
&+ i\frac{1}{2} g \partial^\mu \phi A_\mu \phi + \frac{1}{2} g^2 A^\mu A_\mu \phi^2 + \frac{1}{2} g^2 A^\mu v_0 A_\mu \phi \\
&+ i\frac{1}{2} g \partial^\mu \phi A_\mu v_0 + \frac{1}{2} g^2 A^\mu v_0 A_\mu \phi + \frac{1}{2} g^2 v_0^2 A^\mu A_\mu \\
&+ (\phi + v_0)^2 \frac{(\partial_\mu \chi)^2}{v_0^2} + g(\phi + v_0)^2 A_\mu \frac{\partial_\mu \chi}{v_0} \Leftrightarrow \\
|D_\mu H|^2 &= \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{2} g^2 A^\mu A_\mu \phi^2 + g^2 A^\mu v_0 A_\mu \phi + \frac{1}{2} g^2 v_0^2 A^\mu A_\mu \\
&+ (\phi + v_0)^2 \frac{(\partial_\mu \chi)^2}{v_0^2} + g(\phi + v_0)^2 A_\mu \frac{\partial_\mu \chi}{v_0}\,.
\end{aligned}
\tag{6}
$$

It is very important to mention that from the forth and the sixth term of equation (6) we obtain that our Lagrangian would be proportional to

$$
\mathcal{L} \sim \frac{g^2 v_0^2}{2}\left(A_\mu + \frac{1}{g}\frac{\partial_\mu \chi}{v_0}\right)^2
\tag{7}
$$

which gives a mass term for the physical gauge boson and a cross term between this field and the non-physical Goldstone boson.

Next we consider the $m^2 H^2$ term which reads

$$
\begin{aligned}
m^2 |H|^2 &= \frac{m^2}{2}(\phi + v_0)^2 \Leftrightarrow \\
m^2 |H|^2 &= \frac{m^2}{2}\phi^2 + \frac{m^3}{\sqrt{\lambda}}\phi + \frac{m^4}{2\lambda^2}
\end{aligned}
\tag{8}
$$

where we have used the explicit form $v_0 = \frac{m}{\sqrt{\lambda}}$. Finally we have the term $\lambda|H|^4$ which after the spontaneous symmetry breaking gives

$$\begin{aligned}
\lambda|H|^4 &= \frac{\lambda}{4}\left(v_0^4 + 4v_0^3\phi + 6v_0^2\phi^2 + 4v_0\phi^3 + \phi^4\right) \Leftrightarrow \\
\lambda|H|^4 &= \frac{m^4}{4\lambda^2} + \frac{m^3}{\sqrt{\lambda}}\phi + \frac{3m^2}{2}\phi^2 + \lambda v_0\phi^3 + \frac{\lambda}{4}\phi^4
\end{aligned} \tag{9}$$

where again we have used that $v_0 = \frac{m}{\sqrt{\lambda}}$. Now, we should put the equations (6), (8) and (9) back in the Lagrangian (1). According to what we noticed in (7), doing this will give us non-physical fluctuating degrees of freedom concerning the Goldstone bosons, in addition of the extra non-physical fields corresponding to ghosts. Therefore, in order not to have such states we should work in a physical gauge and thus we choose the unitary gauge. In order to work in this gauge we should demanded that $\xi \to \infty$ and we should perform a gauge transformation of the form

$$\begin{aligned}
A_\mu(x) &\to A_\mu(x) + \frac{1}{g}\partial_\mu\alpha(x) \\
\chi(x) &\to \chi(x) - \alpha(x)v_0
\end{aligned} \tag{10}$$

so as to set $\chi(x) = 0$ which gives only a massive physical gauge boson. Thus combining every thing that we have mentioned before our Lagrangian becomes

$$\begin{aligned}
\mathcal{L}_{AH} = &-\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\left(\partial_\mu\phi_0\right)\left(\partial^\mu\phi_0\right) + \frac{1}{2}m_{Z_0}^2 A_\mu^0 A^{0\mu} + \frac{g^{\mu\nu}}{2}g_0^2\phi_0^2 A_\mu^0 A_\nu^0 \\
&+ g^{\mu\nu}g_0 m_Z \phi A_\mu^0 A_\nu^0 - \frac{1}{2}m_{H_0}^2\phi_0^2 - \lambda_0 v_0\phi_0^3 - \frac{\lambda_0}{4}\phi_0^4 + const.
\end{aligned} \tag{11}$$

where we have defined that the subscript 0 denotes the bare parameters and fields of the model.

Moreover, here we set that $g_0^2 = e_0^2 z_H^2$, where z_H is a parameter that will be defined later, $m_{Z_0}^2 = e_0^2 v_0^2 z_H^2 = g_0^2 v_0^2$ is our gauge field square-mass, $m_{H_0} = \sqrt{2}m$ is the Higgs mass, λ_0 is the quartic Higgs coupling and e_0 is the gauge boson coupling. Finally, $v_0 = \frac{m_{H_0}}{\sqrt{2\lambda_0}}$ is the classical vacuum expectation value of the Higgs field.

Here we will have Feynman rules concerning the gauge and Higgs field propagators and the vertexes that can be made from their combination, as we can see in Appendix A. Fortunately, our Lagrangian does not have fluctuating non-physical degrees of freedom and the only term coming from ghosts is non-dynamical, therefore there would not be neither propagators concerning the ghosts fields $\bar{c}$ and c nor propagators concerning the

Goldstone boson χ. Finally we can express all the parameters of the model as function of the independent ones, using the following notation

$$\begin{aligned} g_0 &= \frac{m_{Z_0}}{m_{H_0}}\sqrt{2\lambda_0} \\ v_0 &= \frac{m_{H_0}}{\sqrt{2\lambda_0}} \\ \lambda_0 v_0 &= \sqrt{\frac{\lambda_0}{2}} m_{H_0} \\ g_0^2 v_0 &= \frac{m_{Z_0}^2}{m_{H_0}}\sqrt{2\lambda_0} \end{aligned} \tag{12}$$

and according to this the only independent parameters left in this model are the Higgs quartic λ_0, the Higgs m_{H_0} and Z-boson mass m_{Z_0}.
Thus the final form of (11) reads

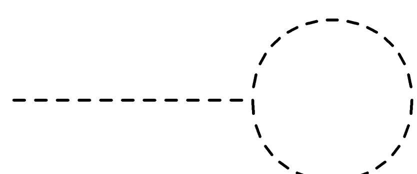
$$\begin{aligned} \mathcal{L}_{AH} &= -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\left(\partial_\mu \phi_0\right)\left(\partial^\mu \phi_0\right) + \frac{1}{2}m_{Z_0}^2 A_\mu^0 A^{0\mu} + g^{\mu\nu}\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}\phi_0^2 A_\mu^0 A_\nu^0 \\ &+ g^{\mu\nu}\frac{\sqrt{2\lambda_0}m_{Z_0}^2}{m_{H_0}}\phi A_\mu^0 A_\nu^0 - \frac{1}{2}m_{H_0}^2\phi_0^2 - \sqrt{\frac{\lambda_0}{2}}m_{H_0}\phi_0^3 - \frac{\lambda_0}{4}\phi_0^4 + const. \end{aligned} \tag{13}$$

2 One-loop Corrections of the Abelian Higgs Model

XXX

2.1 One-point functions

The first quantum corrections at one-loop order come from the one-point functions, namely the one-leg Tadpoles. Such diagrams come from the ϕ^3 and $\phi A_\mu A_\nu$ vertices. Here the first case reads



(14)

and it has the form

$$i\mathcal{T}_H^1 = -6i\mathcal{S}_T^1\sqrt{\frac{\lambda_0}{2}}m_{H_0}\int\frac{d^4k}{(2\pi)^4}\frac{i}{\left(k^2-m_{H_0}^2\right)} \Leftrightarrow \quad (15)$$

$$\mathcal{T}_H^1 = -6\mathcal{S}_T^1\sqrt{\frac{\lambda_0}{2}}m_{H_0}\int\frac{d^4k}{(2\pi)^4}\frac{i}{\left(k^2-m_{H_0}^2\right)} \quad (16)$$

which in d-dimensions could be written as

$$\mathcal{T}_H^1 = 6\mathcal{S}_T^1\frac{\sqrt{\frac{\lambda_0}{2}}m_{H_0}\mu^{4-d}}{(4\pi)^{d/2}}\int\frac{d^dk}{i\pi^{d/2}}\frac{1}{\left(k^2-m_{H_0}^2\right)}.$$

where we have defined that the term $\mathcal{S}_T^1$ corresponds to a symmetry factor which contains the number that multiplies the corresponding vertex. Generally, in every case of Feynman diagrams that we face throughout this work we should consider symmetry factors like the previous one. Therefore, before we move on, we should first declare the way that we evaluate them in order to include them properly in our calculation. Now, the procedure that we follow so as to consider the correct symmetry factors in each case is the following:
Firstly we should consider the number of possibilities to connect outer lines with lines of the vertices, thus we define n_O as the coefficient of this possibility.
Next we define the number of possibilities to connect all the inner lines of the given vertices, and we define this number as n_I.
Now we should consider all the possible lines that are equivalent in a vertex i and this is given by the factor ℓ_i.
Finally, as a last step we define the number of all the equal vertices of type j by v_j.
Thus, having all these definitions in mind we can construct a general form that would give us the symmetry factors of any Feynman diagram and which will read

$$\mathcal{S}_b^a = \eta\frac{n_On_I}{\prod\limits_i\ell_i!\prod\limits_j v_j!} \quad (17)$$

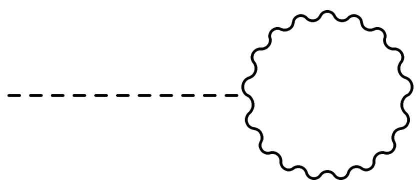
where a, b are indices which indicate the diagram that we study each time. So, for this first case that we consider here, namely $\mathcal{T}_H^1$, we have that it comes from a ϕ^3 vertex which means that it has the following relations

$$
\begin{aligned}
n_O &= 3 \\
n_I &= 1 \\
\ell_1 &= 3 \\
v_1 &= 1
\end{aligned} \tag{18}
$$

therefore, the symmetry factor here reads $\mathcal{S}_T^1 = \frac{1}{2}$.
In addition this integral corresponds to the first case of the equation (454) so it gives

$$(4\pi)^{d/2}\mathcal{T}_H^1 = 3\sqrt{\frac{\lambda_0}{2}} m_{H_0}\mu^{4-d}A_0(m_{H_0}). \tag{19}$$

The second one-leg Tadpole takes the form



(20)

which reads

$$i\mathcal{T}_H^2 = 2i\mathcal{S}_T^2 g^{\mu\nu}\frac{\sqrt{2\lambda_0}m_{Z_0}^2}{m_{H_0}}\int\frac{d^4k}{(2\pi)^4}\frac{i\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_{Z_0}^2}\right)}{\left(k^2-m_{Z_0}^2\right)} \Leftrightarrow \tag{21}$$

$$\mathcal{T}_H^2 = 2(d+\varepsilon)\mathcal{S}_T^2\frac{\sqrt{2\lambda_0}m_{Z_0}^2}{m_{H_0}}\int\frac{d^4k}{(2\pi)^4}\frac{-i}{k^2-m_{Z_0}^2} + 2\frac{d+\varepsilon}{d}\frac{\mathcal{S}_T^2\sqrt{2\lambda_0}}{m_{H_0}}\int\frac{d^4k}{(2\pi)^4}\frac{ik^2}{k^2-m_{Z_0}^2} \tag{22}$$

where we have expanded the numerator and we have done the calculation. Here, and in what follows, we use the fact that in d-dimensions the trace of the metric reads $g_{\mu\nu}g^{\mu\nu} = d+\varepsilon$. Moreover, using the relation $k^\mu k^\nu = \frac{g_{\mu\nu}}{d}k^2$ in d-dimensions the above integral, which is similar with that from the first Tadpole, reads

$$
\begin{aligned}
\mathcal{T}_H^{2\alpha} &= 2\frac{(d+\varepsilon)\mathcal{S}_T^2\frac{\sqrt{2\lambda_0}m_{Z_0}^2}{m_{H_0}}\mu^{4-d}}{(4\pi)^{d/2}}\int\frac{d^dk}{i\pi^{d/2}}\frac{1}{k^2-m_{Z_0}^2} \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{T}_H^{2\alpha} &= 2(d+\varepsilon)\mathcal{S}_T^2\frac{\sqrt{2\lambda_0}m_{Z_0}^2}{m_{H_0}}\mu^{4-d}A_0(m_{Z_0}).
\end{aligned} \tag{23}
$$

The second term of (22) corresponds to the case of equation (513) in the Appendix, thus here we get that

$$
\begin{aligned}
\mathcal{T}_H^{2\beta} &= 2\frac{d+\varepsilon}{d}\frac{\frac{\sqrt{2\lambda_0}m_{Z_0}}{m_{H_0}}\mathcal{S}_T^2}{m_{Z_0}}\frac{m_{Z_0}^4}{m_{Z_0}^4}\int\frac{d^dk}{(2\pi)^d}\frac{ik^2}{k^2-m_{Z_0}^2} \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{T}_H^{2\beta} &= -2\frac{d+\varepsilon}{d}\mathcal{S}_T^2\frac{\sqrt{2\lambda_0}m_{Z_0}^4}{m_{H_0}}\mu^{4-d}U_{\mathcal{T}}(1,m_{Z_0}).
\end{aligned} \tag{24}
$$

Now adding equations (23) and (24) we obtain that

$$
(4\pi)^{d/2}\mathcal{T}_H^2 = 2\mathcal{S}_T^2\mu^{4-d}\left((d+\varepsilon)\frac{\sqrt{2\lambda_0}m_{Z_0}^2}{m_{H_0}}A_0(m_{Z_0}) - \frac{d+\varepsilon}{d}\frac{\sqrt{2\lambda_0}m_{Z_0}^4}{m_{H_0}}U_{\mathcal{T}}(1,m_{Z_0})\right) \tag{25}
$$

moreover in this case we have that the diagram comes from a $\phi A_\mu A_\nu$ vertex which means that it has the following relations

$$
\begin{aligned}
n_O &= 1 \\
n_I &= 1 \\
\ell_1 &= 2 \\
v_1 &= 1
\end{aligned} \tag{26}
$$

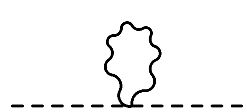
thus, the symmetry factor here reads $\mathcal{S}_T^2 = \frac{1}{2}$ and therefore the full contribution of the Tadpoles using equation (520) reads

$$
(4\pi)^{d/2}\mathcal{T}_H = \mu^{4-d}\left\{3\sqrt{\frac{\lambda_0}{2}}m_{H_0}A_0(m_{H_0}) + 3\frac{\sqrt{2\lambda_0}m_{Z_0}^2}{m_{H_0}}A_0(m_{Z_0}) + \frac{3\sqrt{2\lambda_0}m_{Z_0}^4}{2m_{H_0}}\right\}. \tag{27}
$$

2.2 Two-point functions

Now we move on to the calculation of the one-loop two-point functions of the Higgs boson, which will indicate the nature of the Hierarchy problem.

First we begin with a diagram coming from the $\phi_0^2 A_\mu^0 A_\nu^0$ vertex, which has the form:



$$
= i\mathcal{M}_H^1. \tag{28}
$$

The exact relation of this diagram is

$$i\mathcal{M}_H^1 = 4i\mathcal{S}^1_{\mathcal{M}_H} g^{\mu\nu}\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}\int\frac{d^4k}{(2\pi)^4}\frac{i\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_{Z_0}^2}\right)}{\left(k^2-m_{Z_0}^2\right)} \Leftrightarrow \tag{29}$$

$$\mathcal{M}_H^1 = 4(d+\varepsilon)\mathcal{S}^1_{\mathcal{M}_H}\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}\int\frac{d^4k}{(2\pi)^4}\frac{-i}{k^2-m_{Z_0}^2}+4\frac{d+\varepsilon}{d}\frac{\mathcal{S}^1_{\mathcal{M}_H}\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}}{m_{Z_0}^2}\int\frac{d^4k}{(2\pi)^4}\frac{ik^2}{k^2-m_{Z_0}^2} \tag{30}$$

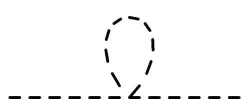
and as we can see except the coefficient of the integral everything else is the same with that of (22), moreover in this case we have that the diagram comes from a $\phi_0^2 A^0_\mu A^0_\nu$ vertex which means that it has the following relations

$$\begin{aligned} n_O &= 2\\ n_I &= 1\\ \ell_1 &= 2\\ \ell_2 &= 2\\ v_1 &= 1 \end{aligned} \tag{31}$$

therefore, the symmetry factor here reads $\mathcal{S}^1_{\mathcal{M}_H} = \frac{1}{2}$. Thus, with that in mind in d-dimensions and using the relation (520) from the Appendix, we obtain the following

$$\begin{aligned} (4\pi)^{d/2}\mathcal{M}_H^1 &= \mu^{d-4}\Big\{2(d+\varepsilon)\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}A_0(m_{Z_0}) - 2\frac{d+\varepsilon}{d}\frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2}U_{\mathcal{T}}(1,m_{Z_0})\Big\} \Leftrightarrow\\ (4\pi)^{d/2}\mathcal{M}_H^1 &= \mu^{d-4}\Big\{6\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}A_0(m_{Z_0}) + 3\frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2}\Big\}. \end{aligned} \tag{32}$$

Next diagram that contributes as a one-loop two-point function comes from the ϕ_0^4 vertex and has the form

 $= i\mathcal{M}_H^2$ (33)

which reads

$$
\begin{aligned}
i\mathcal{M}_H^2 &= -6i\mathcal{S}^2_{\mathcal{M}_H}\lambda_0 \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2_{H_0}} \Leftrightarrow \\
\mathcal{M}_H^2 &= 6\mathcal{S}^2_{\mathcal{M}_H}\lambda_0 \int \frac{d^4k}{(2\pi)^4}\frac{-i}{k^2-m^2_{H_0}}.
\end{aligned} \tag{34}
$$

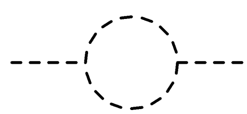
again this is similar with the first one-leg Higgs Tadpole and moreover since that diagram comes from a ϕ^4 vertex it has the following relations

$$
\begin{aligned}
n_O &= 4\cdot 3 \\
n_I &= 1 \\
\ell_1 &= 4 \\
v_1 &= 1
\end{aligned} \tag{35}
$$

therefore, the symmetry factor here reads $\mathcal{S}^2_{\mathcal{M}_H} = \frac{1}{2}$ and thus in d-dimensions we get that

$$
(4\pi)^{d/2}\mathcal{M}_H^2 = 3\lambda_0\mu^{4-d}A_0(m_{H_0}). \tag{36}
$$

Now we move on to the next diagram which can be obtained multiplying the ϕ_0^3 vertex with itself and gives



$$= i\mathcal{M}_H^3 \tag{37}$$

which is equal to

$$
\begin{aligned}
i\mathcal{M}_H^3 &= -18\mathcal{S}^3_{\mathcal{M}_H}\lambda_0 m^2_{H_0}\int \frac{d^4k}{(2\pi)^4}\frac{i}{\left(k^2-m^2_{H_0}\right)}\frac{i}{\left((k+p)^2-m^2_{H_0}\right)} \Leftrightarrow \\
\mathcal{M}_H^3 &= 18\mathcal{S}^3_{\mathcal{M}_H}\lambda m^2_{H_0}\int \frac{d^4k}{(2\pi)^4}\frac{-i}{\left(k^2-m^2_{H_0}\right)\left((k+p)^2-m^2_{H_0}\right)}
\end{aligned} \tag{38}
$$

where we should consider that since that diagram comes from the square of the ϕ^3 vertex it has the following relations

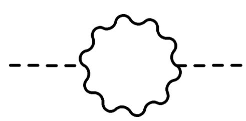
$$
\begin{aligned}
n_O &= 3\cdot 3\\
n_I &= 2\cdot 1\\
\ell_1 &= 3\\
\ell_2 &= 3\\
v_1 &= 1\\
v_2 &= 1
\end{aligned}
\tag{39}
$$

therefore, the symmetry factor here reads $\mathcal{S}^3_{\mathcal{M}_H} = \frac{1}{2}$.
Thus we can write this integral in d-dimensions where it takes the form

$$
\begin{aligned}
\mathcal{M}^3_H &= \frac{9\lambda_0 m^2_{H_0}\mu^{d-4}}{(4\pi)^{d/2}}\int \frac{d^dk}{i\pi^{d/2}}\frac{1}{\left(k^2-m^2_{H_0}\right)\left((k+p)^2-m^2_{H_0}\right)} \Leftrightarrow\\
(4\pi)^{d/2}\mathcal{M}^3_H &= 9\lambda_0 m^2_{H_0}\mu^{d-4}B_0(p,m_{H_0},m_{H_0})
\end{aligned}
\tag{40}
$$

where we have used the exact form of the first case of the equation (455) in Appendix B.

The last two-point one-loop diagram that contributes to the Higgs-mass counterterm comes from the square of the $\phi_0 A^0_\mu A^0_\nu$ vertex and gives

 $= i\mathcal{M}^4_H\,.$ (41)

This diagram corresponds to the following integral

$$
\begin{aligned}
i\mathcal{M}^4_H &= -8\mathcal{S}^4_{\mathcal{M}_H} g^{\mu\nu}g^{\alpha\beta}\frac{\lambda_0 m^4_{Z_0}}{m^2_{H_0}}\int\frac{d^4k}{(2\pi)^4}\frac{i\left(-g_{\mu\alpha}+\frac{k_\mu k_\alpha}{m^2_{Z_0}}\right)}{\left(k^2-m^2_{Z_0}\right)}\frac{i\left(-g_{\nu\beta}+\frac{(k+p)_\nu(k+p)_\beta}{m^2_{Z_0}}\right)}{\left((k+p)^2-m^2_{Z_0}\right)} \Leftrightarrow\\
\mathcal{M}^4_H &= 8\mathcal{S}^4_{\mathcal{M}_H} g^{\mu\nu}g^{\alpha\beta}\frac{\lambda_0 m^4_{Z_0}}{m^2_{H_0}}\int\frac{d^4k}{(2\pi)^4}\frac{-i\left(-g_{\mu\alpha}+\frac{k_\mu k_\alpha}{m^2_{Z_0}}\right)}{\left(k^2-m^2_{Z_0}\right)}\frac{\left(-g_{\nu\beta}+\frac{(k+p)_\nu(k+p)_\beta}{m^2_{Z_0}}\right)}{\left((k+p)^2-m^2_{Z_0}\right)}.
\end{aligned}
\tag{42}
$$

where since it comes from the square of the $\phi_0 A^0_\mu A_\nu$ vertex it has the following relations

$$
\begin{aligned}
n_O &= 1 \cdot 1 \\
n_I &= 2 \cdot 1 \\
\ell_1 &= 2 \\
\ell_2 &= 2 \\
v_1 &= 1 \\
v_2 &= 1
\end{aligned} \tag{43}
$$

therefore, the symmetry factor here reads $\mathcal{S}^4_{\mathcal{M}_H} = \frac{1}{2}$. In order to go on properly we have first to deal with the numerator, namely we should simplify it so as to be able to identify its terms with the relations that we have presented in B. Therefore, we have that the numerator becomes

$$
\begin{aligned}
N &= g^{\mu\nu} g^{\alpha\beta} \left(-g_{\mu\alpha} + \frac{k_\mu k_\alpha}{m^2_{Z_0}}\right) \left(-g_{\nu\beta} + \frac{(k+p)_\nu (k+p)_\beta}{m^2_{Z_0}}\right) \\
&= g^{\mu\nu} \left(-g^\beta_\mu + \frac{k_\mu k^\beta}{m^2_{Z_0}}\right) \left(-g_{\nu\beta} + \frac{(k+p)_\nu (k+p)_\beta}{m^2_{Z_0}}\right) \\
&= \left(-g^{\nu\beta} + \frac{k^\nu k^\beta}{m^2_{Z_0}}\right) \left(-g_{\nu\beta} + \frac{(k+p)_\nu (k+p)_\beta}{m^2_{Z_0}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= g^{\nu\beta} g_{\nu\beta} - \frac{g^{\nu\beta}}{m^2_{Z_0}} \left(k_\nu k_\beta + k_\nu p_\beta + p_\nu k_\beta + p_\nu p_\beta\right) - \frac{k^\nu k^\beta}{m^2_{Z_0}} g_{\nu\beta} + \frac{k^\nu k^\beta}{m^4_{Z_0}} \left(k_\nu k_\beta + k_\nu p_\beta + p_\nu k_\beta + p_\nu p_\beta\right) \\
&= (d+\varepsilon) - \frac{d+\varepsilon}{d} \left\{ \frac{k^2 + 2k \cdot p + p^2}{m^2_{Z_0}} + \frac{k^2}{m^2_{Z_0}} \right\} + \frac{(d+\varepsilon)}{d^2} \frac{k^4 + 2k^2 k \cdot p + k^2 p^2}{m^4_{Z_0}} \, .
\end{aligned} \tag{44}
$$

where we can see that there appears a term proportional to k^4 which actually will give rise to a highly divergent U-integral. In particular, this integral will read

$$
U_{\mathcal{M}4}(p, m_{Z_0}, m_{Z_0}) = \frac{1}{m^4_{Z_0}} \int \frac{d^4k}{(2\pi)^4} \frac{-ik^4}{\left(k^2 - m^2_{Z_0}\right)\left((k+p)^2 - m^2_{Z_0}\right)} \tag{45}
$$

and its solution is given in App.D. Fortunately, here we do not have to evaluate it since the last term of the numerator could be written as

$$\frac{k^4 + 2k^2 k \cdot p + k^2 p^2}{m_{Z_0}^4} = \frac{k^2 (k+p)^2}{m_{Z_0}^4} \tag{46}$$

making the reduction easier.
Thus, putting the numerator that we found in equation (42) we get four terms, namely

$$\begin{aligned}
\mathcal{M}_H^4 &= (d+\varepsilon)\frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2}\int \frac{d^4k}{(2\pi)^4}\frac{-i4}{\left(k^2 - m_{Z_0}^2\right)\left((k+p)^2 - m_{Z_0}^2\right)} \\
&+ 4\frac{(d+\varepsilon)}{d}\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}\int \frac{d^4k}{(2\pi)^4}\frac{ik^2}{\left(k^2 - m_{Z_0}^2\right)\left((k+p)^2 - m_{Z_0}^2\right)} \\
&+ 4\frac{(d+\varepsilon)}{d}\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}\int \frac{d^4k}{(2\pi)^4}\frac{i(k+p)^2}{\left(k^2 - m_{Z_0}^2\right)\left((k+p)^2 - m_{Z_0}^2\right)} \\
&- \frac{4(d+\varepsilon)}{d^2}\frac{\lambda_0}{m_{H_0}^2}\int \frac{d^4k}{(2\pi)^4}\frac{ik^2(k+p)^2}{\left(k^2 - m_{Z_0}^2\right)\left((k+p)^2 - m_{Z_0}^2\right)}
\end{aligned} \tag{47}$$

where we notice that these terms correspond to the relations (455) and (576) from the Appendix, thus we can write $\mathcal{M}_H^4$ in a compactified form as follows

$$\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_H^4 &= \mu^{d-4}\Bigg\{4(d+\varepsilon)\frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2}B_0(p, m_{Z_0}, m_{Z_0}) \\
&- \frac{4(d+\varepsilon)}{d}\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} g_{\mu\nu}B^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) - \frac{4(d+\varepsilon)}{d}\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} g_{\mu\nu}B^{\mu\nu}_{k+p}(p, m_{Z_0}, m_{Z_0}) \\
&+ \frac{4(d+\varepsilon)}{d^2}\frac{\lambda_0}{m_{H_0}^2} m_{Z_0}^2 g_{\mu\nu}B^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) + \frac{4(d+\varepsilon)}{d^2}\frac{\lambda_0}{m_{H_0}^2} m_{Z_0}^2 A_0(m_{Z_0})\Bigg\}
\end{aligned} \tag{48}$$

where we have defined that

$$g_{\mu\nu}B^{\mu\nu}_{k+p}(p, m_{Z_0}, m_{Z_0}) = \int \frac{d^4k}{(2\pi)^4}\frac{-i(k+p)^2}{\left(k^2 - m_{Z_0}^2\right)\left((k+p)^2 - m_{Z_0}^2\right)}\,. \tag{49}$$

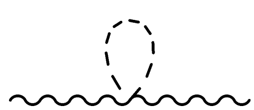
Finally, we should add all the corresponding diagrams in order to compute the complete contribution of the one-loop two-point functions to the Higgs boson propagator, thus adding (32), (36), (40) and (48) we obtain

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_H(p) &= \mu^{d-4}\Bigg\{6\frac{\lambda m_{Z_0}^2}{m_{H_0}^2}A_0(m_{Z_0}) + \frac{3\lambda m_{Z_0}^4}{m_{H_0}^2} + 3\lambda_0 A_0(m_{H_0}) \\
&+ 9\lambda_0 m_{H_0}^2 B_0(p,m_{H_0},m_{H_0}) + \frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}\Big\{4(d+\varepsilon)B_0(p,m_{Z_0},m_{Z_0}) \\
&- \frac{4(d+\varepsilon)}{d}g_{\mu\nu}B^{\mu\nu}(p,m_{Z_0},m_{Z_0}) - \frac{4(d+\varepsilon)}{d}g_{\mu\nu}B^{\mu\nu}_{k+p}(p,m_{Z_0},m_{Z_0}) \\
&+ \frac{4(d+\varepsilon)}{d^2}g_{\mu\nu}B^{\mu\nu}(p,m_{Z_0},m_{Z_0}) + \frac{4(d+\varepsilon)}{d^2}A_0(m_{Z_0})\Big\}\Bigg\}.
\end{aligned}
\tag{50}
$$

Next we should find the reduced form of the above result using the the scalar integrals that we have presented in the Appendix. This would be done for every set of diagrams that we will calculate here. Therefore, using equations (471), (473) we obtain the following form

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_H(p) &= \mu^{d-4}\Bigg\{6\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}A_0(m_{Z_0}) + \frac{3\lambda m_{Z_0}^4}{m_{H_0}^2} + 3\lambda_0 A_0(m_{H_0}) \\
&+ 9\lambda_0 m_{H_0}^2 B_0(p,m_{H_0},m_{H_0}) + 4(d+\varepsilon)\frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2}B_0(p,m_{Z_0},m_{Z_0}) \\
&- \frac{8(d+\varepsilon)}{d}\frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2}\left[B_0(p,m_{Z_0},m_{Z_0}) + \frac{1}{m_{Z_0}^2}A_0(m_{Z_0})\right] \\
&+ \frac{4(d+\varepsilon)}{d^2}\frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2}\left[B_0(p,m_{Z_0},m_{Z_0}) + \frac{2}{m_{Z_0}^2}A_0(m_{Z_0})\right]\Bigg\}.
\end{aligned}
\tag{51}
$$

Now, since we have finished with the first set of the one-loop two-point functions concerning the Higgs field, comes the turn of the Z-boson. Here, we use the same form of Feynman rules with the case of the Higgs field and thus the symmetry factors are straightforward. Fortunately in the second set of the one-loop two-point functions, that we consider here, contribute only two individual diagrams. In particular, the first one comes from the $\phi_0^2 A_\mu^0 A_\nu^0$ vertex and reads

 $= i\mathcal{M}^1_{Z,\mu\nu}$ (52)

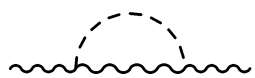
where

$$
\begin{aligned}
i\mathcal{M}^1_{Z,\mu\nu} &= i2g^{\mu\nu}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2_{H_0}} \Leftrightarrow \qquad (53)\\
\mathcal{M}^1_{Z,\mu\nu} &= -2g^{\mu\nu}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{-i}{k^2-m^2_{H_0}} \qquad (54)
\end{aligned}
$$

which is similar with the case of the loop diagram (284) which in d-dimensions reads

$$
(4\pi)^{d/2}\mathcal{M}^1_{Z,\mu\nu} = -2g^{\mu\nu}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}A_0(m_{H_0})\,. \qquad (55)
$$

The next two-point one-loop correction comes from the diagram involving the square of the $\phi_0 A^0_\mu A^0_\nu$ vertex, which gives

 $= i\mathcal{M}^2_{Z,\mu\nu}$ (56)

where

$$
\begin{aligned}
i\mathcal{M}^2_{Z,\mu\nu} &= -8g^{\mu\alpha}g^{\nu\beta}\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{i}{(k+p)^2-m^2_{H_0}}\frac{i\left(-g_{\alpha\beta}+\frac{k_\alpha k_\beta}{m^2_{Z_0}}\right)}{\left(k^2-m^2_{Z_0}\right)} \Leftrightarrow\\
\mathcal{M}^2_{Z,\mu\nu} &= -8g^{\mu\alpha}g^{\nu\beta}\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{i}{(k+p)^2-m^2_{H_0}}\frac{\left(-g_{\alpha\beta}+\frac{k_\alpha k_\beta}{m^2_{Z_0}}\right)}{\left(k^2-m^2_{Z_0}\right)} \Leftrightarrow\\
\mathcal{M}^2_{Z,\mu\nu} &= -8g^{\mu\alpha}g^{\nu\beta}\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{-ig_{\alpha\beta}}{\left(k^2-m^2_{Z_0}\right)\left((k+p)^2-m^2_{H_0}\right)}\\
&- 8g^{\mu\alpha}g^{\nu\beta}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{ik_\alpha k_\beta}{\left(k^2-m^2_{Z_0}\right)\left((k+p)^2-m^2_{H_0}\right)}. \qquad (57)
\end{aligned}
$$

Again the first term of this diagram corresponds to the first case of (455) but here we have to be careful because we have two different masses in the denominators d_1 and d_2. Therefore we get that

$$
\begin{aligned}
\mathcal{M}^{2A}_{Z,\mu\nu} &= -8g^{\mu\alpha}g^{\nu\beta}\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{-ig_{\alpha\beta}}{\left(k^2-m^2_{Z_0}\right)\left((k+p)^2-m^2_{H_0}\right)} \Leftrightarrow \\
\mathcal{M}^{2A}_{Z,\mu\nu} &= -8g^{\mu\nu}\frac{\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}}{(4\pi)^{d/2}}B_0(p,m_{Z_0},m_{H_0}) \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{M}^{2A}_{Z,\mu\nu} &= -8g^{\mu\nu}\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}B_0(p,m_{Z_0},m_{H_0}).
\end{aligned} \tag{58}
$$

Now we move on to the next term of equation (57) which is a little bit more complicated than the previous one. To be more specific, here we have the following

$$
\begin{aligned}
\mathcal{M}^{2B}_{Z,\mu\nu} &= -8g^{\mu\alpha}g^{\nu\beta}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{ik_\alpha k_\beta}{\left(k^2-m^2_{Z_0}\right)\left((k+p)^2-m^2_{H_0}\right)} \Leftrightarrow \\
\mathcal{M}^{2B}_{Z,\mu\nu} &= 8\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{-ik^\mu k^\nu}{\left(k^2-m^2_{Z_0}\right)\left((k+p)^2-m^2_{H_0}\right)} \Leftrightarrow \\
\mathcal{M}^{2B}_{Z,\mu\nu} &= 8\frac{g^{\mu\nu}}{d}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\frac{\mu^{4-d}}{(4\pi)^{d/2}}g_{\mu\nu}B^{\mu\nu}(p,m_{Z_0},m_{H_0}) \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{M}^{2B}_{Z,\mu\nu} &= 8\frac{g^{\mu\nu}}{d}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}g_{\mu\nu}B^{\mu\nu}(p,m_{Z_0},m_{H_0})
\end{aligned} \tag{59}
$$

where we have used the third case of the relation (455). Thus adding the equations (58) and (59) we get

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}^{2}_{Z,\mu\nu} &= -8g^{\mu\nu}\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}B_0(p,m_{Z_0},m_{H_0}) \\
&+ 8\frac{g^{\mu\nu}}{d}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}\left[m^2_{Z_0}B_0(p,m_{Z_0},m_{H_0})+A_0(m_{H_0})\right].
\end{aligned} \tag{60}
$$

Finally, in order to obtain the full result of the one-loop two-point functions concerning the Z-boson that we have, we should add the two diagrams. Therefore we get the final form

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_{Z,\mu\nu} &= g_{\mu\nu}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}\Bigg\{-8m^2_{Z_0}B_0(p,m_{Z_0},m_{H_0})-2A_0(m_{H_0}) \\
&+ \frac{8}{d}m^2_{Z_0}B_0(p,m_{Z_0},m_{H_0})+\frac{8}{d}A_0(m_{H_0})\Bigg\}.
\end{aligned} \tag{61}
$$

Now, as we have done earlier, we should reduce this form using the scalar integrals that we have defined in the Appendix. The difference here is that we need the contracted form of $\mathcal{M}_{Z,\mu\nu}$. To be more specific, The Z-boson vacuum polarization amplitude can be Lorentz-covariantly split into a transverse and a longitudinal part

$$\mathcal{M}_{Z,\mu\nu} = \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)\Pi^T(p^2) + \frac{p_\mu p_\nu}{p^2}\Pi^L(p^2). \tag{62}$$

Contracting with $p_\mu p_\nu$ both sides fixes

$$\Pi^L(p^2) = \frac{p_\mu p_\nu}{p^2}\mathcal{M}_{Z,\mu\nu}. \tag{63}$$

Contracting with $g^{\mu\nu}$ gives on the other hand

$$g^{\mu\nu}\mathcal{M}_{Z,\mu\nu} = -(d-1)\Pi^T + \Pi^L \tag{64}$$

that can be easily solved for the transverse part in $d = 4$

$$\Pi^T = \frac{1}{3}\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)\mathcal{M}_{Z,\mu\nu}\,. \tag{65}$$

Now, the Schwinger-Dyson equation that the dressed Z-propagator

$$G_{\mu\nu} = -g_{\mu\nu}G(p^2) + \frac{p^\mu p^\nu}{m_{Z_0}^2}L(p^2) \tag{66}$$

obeys is written as

$$G_{\mu\nu} = G_{\mu\nu} + D_{\mu\rho}\mathcal{M}^{\rho\sigma}G_{\sigma\nu} \tag{67}$$

with $D_{\mu\rho}$ the tree level gauge boson propagator

$$D_{\mu\rho} = \frac{\left(-g_{\mu\rho} + \frac{p_\mu p_\rho}{m_{Z_0}^2}\right)}{p^2 - m_{Z_0}^2}. \tag{68}$$

So, performing the contractions the Schwinger-Dyson equation becomes

$$-g_{\mu\nu}G+\frac{p^\mu p^\nu}{m_{Z_0}^2}L=\frac{\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{m_{Z_0}^2}\right)}{p^2-m_{Z_0}^2}+\frac{\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{p^2}\right)}{p^2-m_{Z_0}^2}\Pi^T G. \tag{69}$$

Contracting with $p^\mu p^\nu$ we have that

$$-G+\frac{p^2}{m_{Z_0}^2}L=\frac{1}{m_{Z_0}^2}\left[1-\Pi^L(G-L)\right] \tag{70}$$

while contracting with the metric gives

$$-dG+\frac{p^2}{m_{Z_0}^2}L=\frac{-d+\frac{p^2}{m_{Z_0}^2}}{p^2-m_{Z_0}^2}+\frac{-d+1}{p^2-m_{Z_0}^2}\Pi^T G-\frac{1}{m_{Z_0}^2}\Pi^L(G-L). \tag{71}$$

The solution of the above system reads

$$\begin{aligned} G(p^2) &= \frac{1}{p^2-m_{Z_0}^2-\Pi^T(p^2)} \\ L(p^2) &= G(p^2)\left[1-\frac{\Pi^T}{p^2-\Pi^L}\right]. \end{aligned} \tag{72}$$

Finally, for the reason that we have demonstrated in the previous argument, along with the condition that gives the physical Z-mass

$$G(m_Z^2)=\frac{1}{m_Z^2-m_{Z_0}^2-\Pi^T(m_Z^2)} \tag{73}$$

the reduction should be made in the term

$$(4\pi)^{d/2}\mathcal{M}_Z=(4\pi)^{d/2}\frac{1}{3}\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{p^2}\right)\mathcal{M}_{Z,\mu\nu}(p^2=m_Z^2) \tag{74}$$

Therefore, we can split the above equation into two pieces so as to make our calculation easier. In particular, the two terms read

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_{Z_1} &= (4\pi)^{d/2} g_{\mu\nu}\mathcal{M}_{Z,\mu\nu}(p^2 = m_Z^2) \\
(4\pi)^{d/2}\mathcal{M}_{Z_2} &= (4\pi)^{d/2}\frac{p_\mu p_\nu}{p^2}\mathcal{M}_{Z,\mu\nu}(p^2 = m_Z^2)
\end{aligned}
$$

so starting with the first one we and contracting with the metric we get that

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_{Z_1} &= (4\pi)^{d/2} g^{\mu\nu}\mathcal{M}_{Z,\mu\nu} = (d+\varepsilon)\frac{m_{Z_0}^2}{m_{H_0}^2}\lambda_0\mu^{4-d}\Bigg\{-8m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) - 2A_0(m_{H_0}) \\
&+ \frac{8}{d}m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) + \frac{8}{d}A_0(m_{H_0})\Bigg\}.
\end{aligned}
\tag{75}
$$

Now we move on to the $\mathcal{M}_{Z_2}$ term which read

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_{Z_2} &= (4\pi)^{d/2}\frac{p_\mu p_\nu}{p^2}\mathcal{M}_{Z,\mu\nu}(p) \\
&= \frac{m_{Z_0}^2}{m_{H_0}^2}\lambda_0\mu^{4-d}\frac{p^\mu p^\nu}{p^2}g_{\mu\nu}\Bigg\{-8m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) - 2A_0(m_{H_0}) \\
&+ \frac{8}{d}m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) + \frac{8}{d}A_0(m_{H_0})\Bigg\} \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{M}_{Z_2} &= \frac{m_{Z_0}^2}{m_{H_0}^2}\lambda_0\mu^{4-d}\Bigg\{-8m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) - 2A_0(m_{H_0}) \\
&+ \frac{8}{d}m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) + \frac{8}{d}A_0(m_{H_0})\Bigg\}.
\end{aligned}
\tag{76}
$$

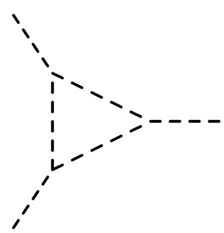
Finally, in order to see the full one-loop contribution on the vacuum polarization of the Z-boson, as it was defined by equation (61), we should evaluate the deference between equations (75) and (76) multiplied by $\frac{1}{3}$. Thus, the full $\mathcal{M}_Z$ reads

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_Z(p^2 = m_Z^2) &= \frac{1}{3}\frac{m_{Z_0}^2}{m_{H_0}^2}\lambda_0\mu^{4-d}\Bigg\{(d+\varepsilon)\Bigg\{8m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) + 2A_0(m_{H_0}) \\
&- \frac{8}{d}m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) - \frac{8}{d}A_0(m_{H_0})\Bigg\} - 8m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) \\
&- 2A_0(m_{H_0}) + \frac{8}{d}m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) + \frac{8}{d}A_0(m_{H_0})\Bigg\}.
\end{aligned}
\tag{77}
$$

For completeness in Unitary gauge we present also the calculation of the three- and four-point functions in the next subsection.

2.3 Three-point functions

In the previous subsection we devoted our work to find and compute all the one-loop two-point functions that can occur from the model that we study. As we mentioned there the corresponding functions would reveal the Hierarchy problem through the physical quantities, which is one of the main motivations of this work. The present subsection is devoted to the calculation of the one-loop three-point functions which we need in order to renormalize properly this model.
As we can see from the vertices that we have, there would be four such diagrams that refer to quantum corrections to the Higgs boson. Therefore we start with the first one which comes from the cubic power of ϕ^3 vertex and reads



$$= i\mathcal{K}_H^1 \tag{78}$$

and has the following explicit form

$$\begin{aligned} i\mathcal{K}_H^1 &= i\mathcal{S}_{\mathcal{K}_H}^1 \lambda^3 v_0^3 \int \frac{d^4k}{(2\pi)^4} \frac{i}{(k^2-m_H^2)} \frac{i}{\left((k+p_1)^2-m_H^2\right)} \frac{i}{\left((k+p_1+p_2)^2-m_H^2\right)} \Leftrightarrow \\ \mathcal{K}_H^1 &= \mathcal{S}_{\mathcal{K}_H}^1 \lambda^3 v_0^3 \int \frac{d^4k}{(2\pi)^4} \frac{-i}{(k^2-m_H^2)\left((k+p_1)^2-m_H^2\right)\left((k+p_1+p_2)^2-m_H^2\right)}. \end{aligned} \tag{79}$$

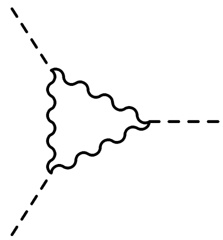
the symmetry factor here reads $\mathcal{S}_{\mathcal{K}_H}^1 = 216 \cdot 3^3 \cdot 4$. Now we can rewrite this integral but this time in d-dimensions where it reads

$$\mathcal{K}_H^1 = \frac{216 \cdot 3^3 \cdot 4\lambda^3 v_0^3 \mu^{d-4}}{(4\pi)^{d/2}} \int \frac{d^dk}{i\pi^{d/2}} \frac{1}{(k^2-m_H^2)\left((k+p_1)^2-m_H^2\right)\left((k+p_1+p_2)^2-m_H^2\right)}$$

and as we can see this integral has the same form with the first case of equation (456) form the Appendix B, thus it takes the final form

$$(4\pi)^{d/2}\mathcal{K}_H^1 = 216 \cdot 3^3 \cdot 4\lambda^3 v_0^3 \mu^{d-4} C_0\left(p_1, p_2, m_H, m_H, m_H\right). \tag{80}$$

Now we move on to the next diagram that corresponds to the cubic power of the $\phi A_\mu A_\nu$ vertex. In particular we have



$$= i\mathcal{K}_H^2 \tag{81}$$

which reads

$$i\mathcal{K}_H^2 = -i\mathcal{S}^2_{\mathcal{K}_H} g^{\mu\nu} g^3 m_Z^3 \int \frac{d^4k}{(2\pi)^4} \frac{i\left(-g_{\mu\alpha} + \frac{k_\mu k_\alpha}{m_Z^2}\right)}{(k^2-m_Z^2)} \frac{i\left(-g_{\nu\beta} + \frac{(k+p_1)_\nu (k+p_1)_\beta}{m_Z^2}\right)}{\left((k+p_1)^2 - m_Z^2\right)} \frac{i\left(-g_{\beta\alpha} + \frac{(k+p_1+p_2)_\beta (k+p_1+p_2)_\alpha}{m_Z^2}\right)}{\left((k+p_1+p_2)^2 - m_Z^2\right)}$$

$$\mathcal{K}_H^2 = -\mathcal{S}^2_{\mathcal{K}_H} g^{\mu\nu} g^3 m_Z^3 \int \frac{d^4k}{(2\pi)^4} \frac{-i\left(-g_{\mu\alpha} + \frac{k_\mu k_\alpha}{m_Z^2}\right)}{(k^2-m_Z^2)} \frac{\left(-g_{\nu\beta} + \frac{(k+p_1)_\nu (k+p_1)_\beta}{m_Z^2}\right)}{\left((k+p_1)^2 - m_Z^2\right)} \frac{\left(-g_{\beta\alpha} + \frac{(k+p_1+p_2)_\beta (k+p_1+p_2)_\alpha}{m_Z^2}\right)}{\left((k+p_1+p_2)^2 - m_Z^2\right)}. \tag{82}$$

Now, since this diagram comes from the cubic power of the $\phi A_\mu A_\nu$ vertex i the symmetry factor here reads $\mathcal{S}^2_{\mathcal{K}_H} = 8 \cdot 4$ In order to calculate it properly we should first divide it into eight individual parts and consider each one separately. Thus we get that

$$\mathcal{K}_{HA}^2 = 8 \cdot 4 g^3 m_Z^3 \int \frac{d^4k}{(2\pi)^4} \frac{-i4}{(k^2-m_Z^2)\left((k+p_1)^2 - m_Z^2\right)\left((k+p_1+p_2)^2 - m_Z^2\right)} \tag{83}$$

$$\mathcal{K}_{HB}^2 = -8 \cdot 4 g^3 m_Z \int \frac{d^4k}{(2\pi)^4} \frac{-ig^{\beta\alpha}(k+p_1+p_2)_\beta (k+p_1+p_2)_\alpha}{(k^2-m_Z^2)\left((k+p_1)^2 - m_Z^2\right)\left((k+p_1+p_2)^2 - m_Z^2\right)} \tag{84}$$

$$\mathcal{K}_{HC}^2 = -8 \cdot 4 g^3 m_Z \int \frac{d^4k}{(2\pi)^4} \frac{-ig^{\nu\beta}(k+p_1)_\nu (k+p_1)_\beta}{(k^2-m_Z^2)\left((k+p_1)^2 - m_Z^2\right)\left((k+p_1+p_2)^2 - m_Z^2\right)} \tag{85}$$

$$\mathcal{K}_{HD}^2 = \frac{8 \cdot 4 g^3}{m_Z} \int \frac{d^4k}{(2\pi)^4} \frac{-ig^{\alpha\nu}(k+p_1)_\nu (k+p_1)_\beta (k+p_1+p_2)_\beta (k+p_1+p_2)_\alpha}{(k^2-m_Z^2)\left((k+p_1)^2 - m_Z^2\right)\left((k+p_1+p_2)^2 - m_Z^2\right)} \tag{86}$$

$$\mathcal{K}_{HE}^2 = -8 \cdot 4 g^3 m_Z \int \frac{d^4k}{(2\pi)^4} \frac{-ig^{\mu\alpha}k_\mu k_\alpha}{(k^2-m_Z^2)\left((k+p_1)^2 - m_Z^2\right)\left((k+p_1+p_2)^2 - m_Z^2\right)} \tag{87}$$

$$\mathcal{K}_{HF}^2 = \frac{8 \cdot 4 g^3}{m_Z} \int \frac{d^4k}{(2\pi)^4} \frac{-ig^{\mu\beta}k_\mu k_\alpha (k+p_1+p_2)_\beta (k+p_1+p_2)_\alpha}{(k^2-m_Z^2)\left((k+p_1)^2 - m_Z^2\right)\left((k+p_1+p_2)^2 - m_Z^2\right)} \tag{88}$$

$$\mathcal{K}_{HG}^2 = \frac{8\cdot 4g^3}{m_Z}\int\frac{d^4k}{(2\pi)^4}\frac{-ig^{\mu\nu}g^{\beta\alpha}k_\mu k_\alpha(k+p_1)_\nu(k+p_1)_\beta}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \tag{89}$$

$$\mathcal{K}_{HI}^2 = -\frac{8\cdot 4g^3}{m_Z^3}\int\frac{d^4k}{(2\pi)^4}\frac{-ig^{\mu\nu}k_\mu k_\alpha(k+p_1)_\beta(k+p_1)_\nu(k+p_1+p_2)_\alpha\,(k+p_1+p_2)_\beta}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \tag{90}$$

so we start our calculation with the first integral, namely $\mathcal{K}_{HA}^2$, which is similar with (80), therefore we get that

$$(4\pi)^{d/2}\mathcal{K}_{HA}^2 = 32\cdot 4g^3m_Z^3\mu^{d-4}C_0\left(p_1,p_2,m_Z,m_Z,m_Z\right). \tag{91}$$

Now we move on to the next integral which is $\mathcal{K}_{HB}^2$ where we should first calculate the numerator which gives

$$\begin{aligned} N &= (k+p_1+p_2)_\beta(k+p_1+p_2)_\alpha \\ &= k_\beta k_\alpha + k_\beta(p_1+p_2)_\alpha + (p_1+p_2)_\beta k_\alpha + (p_1+p_2)_\beta(p_1+p_2)_\alpha \Leftrightarrow \\ g^{\beta\alpha}N &= g^{\beta\alpha}k_\beta k_\alpha + p_1^2 + 2p_1\cdot p_2 + p_2^2 + 2k(p_1+p_2) \end{aligned} \tag{92}$$

thus putting this in (84) we obtain that

$$\begin{aligned} \mathcal{K}_{HB}^2 &= -8\cdot 4g^3m_Z\int\frac{d^4k}{(2\pi)^4}\frac{-ig^{\beta\alpha}k_\beta k_\alpha}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \\ &- 8\cdot 4g^3m_Z\int\frac{d^4k}{(2\pi)^4}\frac{-i(p_1^2+2p_1\cdot p_2+p_2^2)}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \\ &- 8\cdot 4g^3m_Z\int\frac{d^4k}{(2\pi)^4}\frac{-i2(p_1+p_2)}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)}. \end{aligned}$$

Now in d-dimensions and using the first and the second case of the relation (456) we get that

$$
\begin{aligned}
\mathcal{K}_{HB}^2 &= -\frac{8\cdot 4g^3 m_Z \mu^{4-d}}{(4\pi)^{d/2}} \int \frac{d^dk}{\pi^{d/2}} \frac{-ig^{\beta\alpha}k_\beta k_\alpha}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \\
&- \frac{8\cdot 4g^3 m_Z \mu^{4-d}}{(4\pi)^{d/2}} \int \frac{d^dk}{\pi^{d/2}} \frac{-i(p_1^2+2p_1\cdot p_2+p_2^2)}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \\
&- \frac{8\cdot 4g^3 m_Z \mu^{4-d}}{(4\pi)^{d/2}} \int \frac{d^dk}{\pi^{d/2}} \frac{-i2k(p_1+p_2)}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \Leftrightarrow \\
\mathcal{K}_{HB}^2 &= -\frac{8\cdot 4g^3 m_Z \mu^{4-d}}{(4\pi)^{d/2}} g_{\mu\nu} C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&- \frac{8\cdot 4g^3 m_Z \mu^{4-d}}{(4\pi)^{d/2}} (p_1^2+2p_1\cdot p_2+p_2^2) C_0\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&- \frac{8\cdot 4g^3 m_Z \mu^{4-d}}{(4\pi)^{d/2}} (p_1+p_2)_\mu C^{\mu}\left(p_1,p_2,m_Z,m_Z,m_Z\right) \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{K}_{HB}^2 &= -8\cdot 4g^3 m_Z \mu^{4-d} \Bigg\{ g_{\mu\nu} C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right) \qquad (93) \\
&+ 2(p_1+p_2)_\mu C^{\mu}(p_1,p_2,m_Z,m_Z,m_Z) + (p_1^2+2p_1\cdot p_2+p_2^2) C_0\left(p_1,p_2,m_Z,m_Z,m_Z\right) \Bigg\}.
\end{aligned}
$$

The next integral is $\mathcal{K}_{HC}^2$ which is quite similar with the previous one but witht defferent numerator, namely here we have that

$$
\begin{aligned}
N &= (k+p_1)_\nu (k+p_1)_\beta \\
N &= k_\nu k_\beta + k_\nu p_{1\beta} + p_{1\nu} k_\beta + p_{1\nu} p_{1\beta} \Leftrightarrow \\
g^{\nu\beta} N &= g^{\nu\beta} k_\nu k_\beta + 2kp_1 + p_1^2 \qquad (94)
\end{aligned}
$$

thus we can write straightforward that

$$
\begin{aligned}
\mathcal{K}_{HC}^2 &= -8\cdot 4g^3 m_Z \int \frac{d^4k}{(2\pi)^4} \frac{-ig^{\nu\beta}k_\nu k_\beta}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \\
&- 8\cdot 4g^3 m_Z \int \frac{d^4k}{(2\pi)^4} \frac{-ip_1^2}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \\
&- 8\cdot 4g^3 m_Z \int \frac{d^4k}{(2\pi)^4} \frac{-i2kp_1}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)}.
\end{aligned}
$$

Therefore, in d-dimensions we get that

$$
\begin{aligned}
\mathcal{K}_{HC}^{2} &= -\frac{8\cdot 4g^3 m_Z \mu^{4-d}}{(4\pi)^{d/2}} g_{\mu\nu} C^{\mu\nu}\left(p_1, p_2, m_Z, m_Z, m_Z\right) \\
&- \frac{8g^3 m_Z \mu^{4-d}}{(4\pi)^{d/2}} p_1^2 C_0\left(p_1, p_2, m_Z, m_Z, m_Z\right) \\
&- \frac{8\cdot 4g^3 m_Z \mu^{4-d}}{(4\pi)^{d/2}} p_{1\mu} C^{\mu}\left(p_1, p_2, m_Z, m_Z, m_Z\right) \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{K}_{HC}^{2} &= -8\cdot 4g^3 m_Z \mu^{4-d}\Bigg\{ g_{\mu\nu} C^{\mu\nu}\left(p_1, p_2, m_Z, m_Z, m_Z\right) \\
&+ 2p_{1\mu} C^{\mu}(p_1, p_2, m_Z, m_Z, m_Z) + p_1^2 C_0\left(p_1, p_2, m_Z, m_Z, m_Z\right)\Bigg\}.
\end{aligned}
\tag{95}
$$

Now we move on to the $\mathcal{K}_{HD}^{2}$ integral whose numerator gives the following

$$
\begin{aligned}
N &= (k+p_1)_\nu (k+p_1)^\beta (k+p_1+p_2)_\beta (k+p_1+p_2)_\alpha \\
&= (k+p_1)_\nu (k+p_1+p_2)_\alpha \left(k^2 + k\cdot(2p_1+p_2) + p_1\cdot(p_1+p_2)\right) \Leftrightarrow \\
g^{\nu\alpha} N &= \left(k^2 + k\cdot(2p_1+p_2) + p_1\cdot(p_1+p_2)\right)\left(k^2 + k\cdot(2p_1+p_2) + p_1\cdot(p_1+p_2)\right) \Leftrightarrow \\
g^{\nu\alpha} N &= k^4 + k^2 k\cdot(4p_1+2p_2) + k^2\left[2p_1\cdot(p_1+p_2) + (2p_1+p_2)^2\right] \\
&+ 2k\cdot p_1(p_1+p_2)(2p_1+p_2) + p_1^2\cdot(p_1+p_2)^2 \Leftrightarrow \\
g^{\nu\alpha} N &= k^4 + k^2 k\cdot(4p_1+2p_2) + k^2(6{p_1}^2 + 6p_2 p_1 + {p_2}^2) + 2k\cdot p_1(p_1+p_2)(2p_1+p_2) + p_1^2\cdot(p_1+p_2)^2.
\end{aligned}
\tag{96}
$$

Thus we get that

$$
\begin{aligned}
\mathcal{K}_{HD}^{2} &= \frac{8\cdot 4g^3}{m_Z}\int \frac{d^4k}{(2\pi)^4}\frac{-ik^4}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \\
&+ \frac{8\cdot 4g^3}{m_Z}\int \frac{d^4k}{(2\pi)^4}\frac{-ig_{\mu\nu}k^\mu k^\nu(6{p_1}^2 + 6p_2p_1 + {p_2}^2)}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \\
&+ \frac{8\cdot 4g^3}{m_Z}\int \frac{d^4k}{(2\pi)^4}\frac{-ig_{\mu\nu}g_{\alpha\beta}(4p_1+2p_2)^\beta k^\mu k^\nu k^\alpha}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \\
&+ \frac{8\cdot 4g^3}{m_Z}\int \frac{d^4k}{(2\pi)^4}\frac{-ig_{\mu\nu}p_{1\mu}(p_1+p_2)(2p_1\cdot 4+p_2)^\nu k^\mu}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \\
&+ \frac{8\cdot 4g^3}{m_Z}\int \frac{d^4k}{(2\pi)^4}\frac{-ip_1^2\cdot(p_1+p_2)^2}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)}.
\end{aligned}
\tag{97}
$$

Thus in d-dimensions (97) reads

$$\begin{aligned}
\mathcal{K}^2_{HD} &= \frac{8\cdot 4g^3\mu^{4-d}}{m_Z(4\pi)^{d/2}}(6{p_1}^2+6p_2\cdot p_1+{p_2}^2)g_{\mu\nu}C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \frac{8\cdot 4g^3\mu^{4-d}}{m_Z(4\pi)^{d/2}}g_{\mu\nu}(4p_1+2p_2)_\rho C^{\mu\nu\rho}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \frac{8\cdot 4g^3\mu^{4-d}}{m_Z(4\pi)^{d/2}}(p_1+p_2)(2p_1+p_2)p_{1\mu}C^{\mu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \frac{8\cdot 4g^3\mu^{4-d}}{m_Z(4\pi)^{d/2}}p_1^2\cdot(p_1+p_2)^2C_0\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \frac{8\cdot 4g^3m_Z\mu^{4-d}}{(4\pi)^{d/2}}U_{\mathcal{K}4}(p_1,p_2,m_Z,m_Z,m_Z)
\end{aligned} \tag{98}$$

where we have defined

$$U_{\mathcal{K}4}(p_1,p_2,m_Z,m_Z,m_Z)=\int\frac{d^4k}{(2\pi)^4}\frac{-ik^4}{m_Z^2\left(k^2-m_Z^2\right)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \tag{99}$$

which finally gives that

$$\begin{aligned}
(4\pi)^{d/2}\mathcal{K}^2_{HD} &= \frac{8\cdot 4g^3\mu^{4-d}}{m_Z}\Bigg\{p_1^2\cdot(p_1+p_2)^2C_0\left(p_1,p_2,m_Z,m_Z,m_Z\right)+m_Z^2U_{\mathcal{K}4}(p_1,p_2,m_Z,m_Z,m_Z)\\
&+ (p_1+p_2)(2p_1+p_2)p_{1\mu}C^{\mu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ (6{p_1}^2+6p_2\cdot p_1+{p_2}^2)g_{\mu\nu}C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ g_{\mu\nu}(4p_1+2p_2)_\alpha C^{\mu\nu\alpha}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Bigg\}.
\end{aligned} \tag{100}$$

The next integral that we have to calculate is $\mathcal{K}^2_{HE}$ whose result is known since it corresponds to the second case of equation (456) and it is like the (95), thus it gives

$$(4\pi)^{d/2}\mathcal{K}^2_{HE}=-8\cdot 4g^3m_Z\mu^{4-d}g_{\mu\nu}C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right). \tag{101}$$

The next two integrals that have to be considered are similar with the case of (97), but they differ on the numerator. To be more specific, the numerator of $\mathcal{K}^2_{HF}$ reads

$$\begin{aligned}
N &= k^\beta k^\alpha(k+p_1+p_2)_\beta(k+p_1+p_2)_\alpha\\
&= k\cdot(k+p_1+p_2)\,k\cdot(k+p_1+p_2)\Leftrightarrow\\
N &= k^4+2k^2k\cdot(p_1+p_2)+k^2(p_1+p_2)^2
\end{aligned} \tag{102}$$

thus in d-dimensions the final form of this integral becomes

$$\begin{aligned}(4\pi)^{d/2}\mathcal{K}^2_{HF} &= \frac{8\cdot 4g^3\mu^{4-d}}{m_Z}\Big\{(p_1+p_2)^2 g_{\mu\nu}C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\ &+ 2g_{\mu\nu}(p_1+p_2)_\rho C^{\mu\nu\rho}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\ &+ m_Z^2 U_{\mathcal{K}4}(p_1,p_2,m_Z,m_Z,m_Z)\Big\}. \qquad (103)\end{aligned}$$

Now, for the $\mathcal{K}^2_{HG}$ integral we can see that its numerator is the same with that of $\mathcal{K}^2_{HF}$ if we perform the replacement $(p_1+p_2)\to p_1$, then we take just the following form

$$N = k^4 + 2k^2 k\cdot p_1 + k^2 p_1^2 \qquad (104)$$

therefore we get that

$$\begin{aligned}(4\pi)^{d/2}\mathcal{K}^2_{HG} &= \frac{8\cdot 4g^3\mu^{4-d}}{m_Z}\Big\{p_1^2 g_{\mu\nu}C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\ &+ 2g_{\mu\nu}p_{1\rho} C^{\mu\nu\rho}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\ &+ m_Z^2 U_{\mathcal{K}4}(p_1,p_2,m_Z,m_Z,m_Z)\Big\}. \qquad (105)\end{aligned}$$

Finally, the last integral coming from the diagram (81) is the $\mathcal{K}^2_{HI}$ whose numerator reads

$$\begin{aligned}N &= g^{\mu\nu}k_\mu k_\alpha (k+p_1)_\beta (k+p_1)_\nu (k+p_1+p_2)_\alpha (k+p_1+p_2)_\beta\\ &= k^2\left(k+p_1\right)^2\left(k+p_1+p_2\right)^2 \Leftrightarrow\\ N &= k^6 + 4k^4 k\cdot p_1 + 2(k\cdot p_1)^3 + 2k^4 k\cdot p_2 + 3(k\cdot p_1)^2(k\cdot p_2) + (k\cdot p_1)(k\cdot p_2)^2 + k^4 p_1^2\\ &+ k^2(k\cdot p_1)^2 + k^2(k\cdot(p_1+p_2))^2 + 2k^2(k\cdot p_1)p_1^2 + (k\cdot p_1)^2 p_1^2 + k^2(k\cdot p_2)p_1^2 + (k\cdot p_1)(k\cdot p_2)p_1^2\\ &+ k^4(p_1\cdot p_2) + 2k^2(k\cdot p_1)(p_1\cdot p_2) + (k\cdot p_1)^2(p_1\cdot p_2) + k^2(k\cdot p_2)(p_1\cdot p_2) + (k\cdot p_1)(k\cdot p_2)(p_1\cdot p_2)\,.\end{aligned} \qquad (106)$$

Therefore, we get that the final integral for the present diagram becomes

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{K}^2_{HI} &= -8\cdot 4g^3\mu^{4-d}\Bigg\{\bigg\{\frac{2p_1^4+\left(p_1^2+p_2^2+2p_1\cdot p_2\right)p_1^2}{m_Z^3} \\
&+ \frac{2p_1^4+\left(p_1^2+p_2^2+2p_1\cdot p_2\right)p_2^2}{2m_Z^3}\bigg\}B_0(p,m_Z,m_Z) \\
&+ \left(\frac{\left(-6p_1^2-2p_2^2-4p_1\cdot p_2\right)p_{1\mu}}{m_Z^3}+\frac{\left(-p_1^2-2p_2^2-4p_1\cdot p_2\right)p_{2\mu}}{m_Z^3}\right)B^{\mu}(p,m_Z,m_Z) \\
&+ \left(\frac{4p1^2+2p_2^2+2p1\cdot p_2}{m_Z^3}-\frac{2}{m_Z}\right)g_{\mu\nu}B^{\mu\nu}(p,m_Z,m_Z) \\
&- m_Z U_{\mathcal{K}6}(p_1,p_2,m_Z,m_Z,m_Z)-(4p_{1\mu}+2p_{2\mu})U_{\mathcal{K}5}(p_1,p_2,m_Z,m_Z,m_Z) \\
&+ \left(4m_Z^2-\frac{p_1^2}{m_Z}-\frac{p_1\cdot p_2}{m_Z}\right)U_{\mathcal{K}4}(p_1,p_2,m_Z,m_Z,m_Z) \\
&+ \left(\frac{p_1^4}{m_Z^3}+\frac{p_2^4}{2m_Z^3}+\frac{(p_1\cdot p_2)^2}{m_Z^3}\right)g_{\mu\nu}C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&+ \left(\left[-\frac{p_1^2}{m_Z^3}-\frac{p_1\cdot p_2}{m_Z^3}\right]p_{1\mu}p_{1\nu}+\left[-\frac{p_1^2}{m_Z^3}-\frac{p_1\cdot p_2}{m_Z^3}\right]p_{1\mu}p_{2\nu}\right)C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&+ g_{\mu\nu}\left(\left[-\frac{2p_1^2}{m_Z^3}-\frac{2p_1\cdot p_2}{m_Z^3}\right]p_{1\rho}+\left[-\frac{p_1^2}{m_Z^3}-\frac{p_1\cdot p_2}{m_Z^3}\right]p_{2\rho}\right)C^{\mu\nu\rho}\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&+ \left(-\frac{2}{m_Z^3}p_{1\mu}p_{1\nu}p_{1\rho}-\frac{3}{m_Z^3}p_{1\mu}p_{1\nu}p_{2\rho}-\frac{1}{m_Z^3}p_{1\mu}p_{2\nu}p_{2\rho}\right)C^{\mu\nu\rho}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Bigg\}.
\end{aligned}
\tag{107}
$$

where we have defined that

$$
U_{\mathcal{K}5}(p_1,p_2,m_Z,m_Z,m_Z) = \int\frac{d^4k}{(2\pi)^4}\frac{-ik^4k^{\mu}}{m_Z^3\left(k^2-m_Z^2\right)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)}
\tag{108}
$$

and

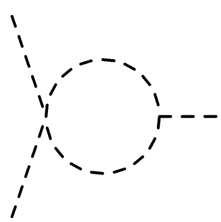
$$
U_{\mathcal{K}6}(p_1,p_2,m_Z,m_Z,m_Z) = \int\frac{d^4k}{(2\pi)^4}\frac{-ik^6}{m_Z^4\left(k^2-m_Z^2\right)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)}
\tag{109}
$$

Now, in order to find the full form of the (81) integral we should add all the previously calculated relations. Therefore adding (91), (93), (95), (100), (101), (103), (105) and (107) we obtain a general and compactified form of (81) which reads

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{K}_H^2 &= 8\cdot 4g^3\mu^{4-d}\Bigg\{\left\{4m_Z^3 - m_Z p_1^2 + \frac{(p_1\cdot P_1)^2}{m_Z} - m_Z P_1^2\right\}C_0\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \left\{-2m_Z p_{1\mu} - 2m_Z P_{1\mu} + \frac{2p_1\cdot P_1}{m_Z}p_{1\mu} + \frac{2p_1\cdot P_1}{m_Z}P_{1\mu}\right\}C^{\mu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \left\{-3m_Z g_{\mu\nu} + \frac{2p_{1\mu}p_{1\nu}}{m_Z} + \frac{2p_{1\mu}P_{1\nu}}{m_Z}\right.\\
&+ \left.\frac{2P_{1\mu}P_{1\nu}}{m_Z} + \frac{2p_1\cdot P_1}{m_Z}g_{\mu\nu} - \frac{p_1\cdot P_1}{m_Z^3}p_{1\mu}P_{1\nu}\right\}C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \left\{-\frac{p_{1\mu}p_{1\nu}}{m_Z^3} - \frac{3p_{1\mu}P_{1\nu}}{m_Z^3} - \frac{P_{1\mu}P_{1\nu}}{m_Z^3}\right\}\left(B^{\mu\nu}(p_1,m_Z,m_Z) + m_Z^2 C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\right)\\
&+ \left\{\frac{4g_{\mu\nu}p_{1\rho}}{m_Z} + \frac{4g_{\mu\nu}P_{1\rho}}{m_Z} - \frac{p_{1\mu}p_{1\nu}P_{1\rho}}{m_Z^3}\right.\\
&- \left.\frac{P_{1\mu}P_{1\nu}p_{1\rho}}{m_Z^3} - \frac{g_{\mu\nu}p_{1\rho}}{m_Z^3}p_1\cdot P_1 - \frac{g_{\mu\nu}P_{1\rho}}{m_Z^3}p_1\cdot P_1\right\}C^{\mu\nu\rho}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \left\{3m_Z - \frac{p_1\cdot P_1}{m_Z}\right\}U_{\mathcal{K}4} + \left\{-2p_{1\mu} - 2P_{1\mu}\right\}U_{\mathcal{K}5} - m_Z U_{\mathcal{K}6}\Bigg\}
\end{aligned}
\tag{110}
$$

where we have defined that $P_{1\mu} = p_{1\mu} + p_{2\mu}$.

Next contribution to the Higgs three-point function comes from the combination of the ϕ^3 and ϕ^4 vertices and gives



$$= i\mathcal{K}_H^3 \tag{111}$$

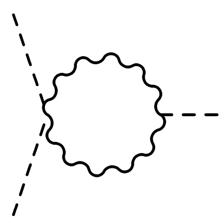
and it reminds us the two-point diagram that we have already calculated in the previous subsection, namely (37). Therefore, considering the appropriate coefficients, the calculation here is straightforward and it gives

$$
\begin{aligned}
i\mathcal{K}_H^3 &= -\mathcal{S}_{\mathcal{K}_H}^3\lambda^2 v_0\int\frac{d^4k}{(2\pi)^4}\frac{i}{(k^2-m_H^2)}\frac{i}{\left((k+p_1+p_2)^2-m_H^2\right)} \Leftrightarrow\\
\mathcal{K}_H^3 &= \mathcal{S}_{\mathcal{K}_H}^3\lambda^2 v_0\int\frac{d^4k}{(2\pi)^4}\frac{-i}{(k^2-m_H^2)\left((k+p_1+p_2)^2-m_H^2\right)}.
\end{aligned}
\tag{112}
$$

Here we should mentioned that these two vertices have the same coefficient multiplying the coupling, thus we obtain that the symmetry factor here reads $\mathcal{S}^3_{\mathcal{K}_H} = 36 \cdot 48$.
Thus we can write this integral in d-dimensions where it takes the form

$$
\begin{aligned}
\mathcal{K}^3_H &= \frac{36 \cdot 48 \lambda^2 v_0 \mu^{d-4}}{(4\pi)^{d/2}} \int \frac{d^d k}{i\pi^{d/2}} \frac{1}{(k^2 - m_H^2)\left((k+p_1+p_2)^2 - m_H^2\right)} \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{K}^3_H &= 36 \cdot 48 \lambda^2 v_0 \mu^{d-4} B_0(p_1, p_2, m_H, m_H) \Leftrightarrow \\
(4\pi)^{d/2}\mu^{d-4}\mathcal{K}^3_H &= (4\pi)^{d/2}\mu^{d-4}\frac{8}{3}\frac{\mathcal{M}^3_H}{v_0}
\end{aligned} \tag{113}
$$

where we have used the exact form of the first case of the equation (455) in Appendix B. Another case like the previous one is that of the diagram that occur from the combination of the $\phi A_\mu A_\nu$ and $\phi^2 A_\mu A_\nu$ vertices, giving



$$= i\mathcal{K}^4_H \tag{114}$$

which is similar with that of (41)

$$
\begin{aligned}
i\mathcal{K}^4_H &= -\mathcal{S}^4_{\mathcal{K}_H} g^{\mu\nu} g^{\alpha\beta} g^3 m_Z \int \frac{d^4 k}{(2\pi)^4} \frac{i\left(-g_{\mu\alpha} + \frac{k_\mu k_\alpha}{m_Z^2}\right)}{(k^2 - m_Z^2)} \frac{i\left(-g_{\nu\beta} + \frac{(k+p_1+p_2)_\nu (k+p_1+p_2)_\beta}{m_Z^2}\right)}{\left((k+p_1+p_2)^2 - m_Z^2\right)} \Leftrightarrow \\
\mathcal{K}^4_H &= \mathcal{S}^4_{\mathcal{K}_H} g^{\mu\nu} g^{\alpha\beta} g^3 m_Z \int \frac{d^4 k}{(2\pi)^4} \frac{-i\left(-g_{\mu\alpha} + \frac{k_\mu k_\alpha}{m_Z^2}\right)}{(k^2 - m_Z^2)} \frac{\left(-g_{\nu\beta} + \frac{(k+p_1+p_2)_\nu (k+p_1+p_2)_\beta}{m_Z^2}\right)}{\left((k+p_1+p_2)^2 - m_Z^2\right)} .
\end{aligned} \tag{115}
$$

where again here the two vertices have the same coefficient multiplying the coupling, thus we obtain that the symmetry factor here reads $\mathcal{S}^4_{\mathcal{K}_H} = 4 \cdot 4$.
Thus, since we have already calculated an integral like this, we get that the final result here is

$$\begin{aligned}
(4\pi)^{d/2}\mathcal{K}_H^4 &= \mu^{d-4}\Bigg\{(64g^3m_Z - \frac{16g^3p^2}{m_Z})B_0(p_1,m_Z,m_Z) - \frac{32g^3\mu^{d-4}}{m_Z}g_{\mu\nu}B^{\mu\nu}(p_1,m_Z,m_Z) \\
&+ \frac{16g^3\mu^{4-d}}{m_Z^3}p_\mu p_\nu B^{\mu\nu}(p_1,m_Z,m_Z) + 16g^3m_Z\mu^{d-4}U_{\mathcal{M}4}(p_1,p_2,m_Z,m_Z)\Bigg\} \Leftrightarrow \\
(4\pi)^{d/2}\mu^{d-4}\mathcal{K}_H^4 &= (4\pi)^{d/2}\mu^{d-4}\frac{2\mathcal{M}_H^4}{v_0} \qquad (116)
\end{aligned}$$

Finally, in order to see the complete contribution of the one-loop three-point functions to the Higgs boson we should add all the corresponding diagrams. Therefore adding (80), (110), (113) and (116) we get that

$$\begin{aligned}
(4\pi)^{d/2}\mathcal{K}_H &= \mu^{4-d}\Bigg\{216\cdot 3^3\cdot 4\lambda^3v_0^3C_0\left(p_1,p_2,m_H,m_H,m_H\right) \\
&+ 8\cdot 4g^3\mu^{4-d}\Bigg\{\left(4m_Z^3 - m_Zp_1^2 + \frac{\left(p_1\cdot P_1\right)^2}{m_Z} - m_ZP_1^2\right)C_0\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&+ \left(-2m_Zp_{1\mu} - 2m_ZP_{1\mu} + \frac{2p_1\cdot P_1}{m_Z}p_{1\mu} + \frac{2p_1\cdot P_1}{m_Z}P_{1\mu}\right)C^\mu\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&+ \Bigg\{-3m_Zg_{\mu\nu} + \frac{2p_{1\mu}p_{1\nu}}{m_Z} + \frac{2p_{1\mu}P_{1\nu}}{m_Z} \\
&+ \frac{2P_{1\mu}P_{1\nu}}{m_Z} + \frac{2p_1\cdot P_1}{m_Z}g_{\mu\nu} - \frac{p_1\cdot P_1}{m_Z^3}p_{1\mu}P_{1\nu}\Bigg\}C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&+ \left(-\frac{p_{1\mu}p_{1\nu}}{m_Z^3} - \frac{3p_{1\mu}P_{1\nu}}{m_Z^3} - \frac{P_{1\mu}P_{1\nu}}{m_Z^3}\right)\left(B^{\mu\nu}(p_1,m_Z,m_Z) + m_Z^2C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\right)
\end{aligned}$$

$$\begin{aligned}
&+ \Bigg\{\frac{4g_{\mu\nu}p_{1\rho}}{m_Z} + \frac{4g_{\mu\nu}P_{1\rho}}{m_Z} - \frac{p_{1\mu}p_{1\nu}P_{1\rho}}{m_Z^3} \\
&- \frac{P_{1\mu}P_{1\nu}p_{1\rho}}{m_Z^3} - \frac{g_{\mu\nu}p_{1\rho}}{m_Z^3}p_1\cdot P_1 - \frac{g_{\mu\nu}P_{1\rho}}{m_Z^3}p_1\cdot P_1\Bigg\}C^{\mu\nu\rho}\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&+ \left(3m_Z - \frac{p_1\cdot P_1}{m_Z}\right)U_{\mathcal{K}4} + \left(-2p_{1\mu} - 2P_{1\mu}\right)U_{\mathcal{K}5} - m_ZU_{\mathcal{K}6}\Bigg\} \\
&+ (64g^3m_Z - \frac{16g^3p^2}{m_Z})B_0(p_1,m_Z,m_Z) + \frac{16g^3}{m_Z}(-2g_{\mu\nu} + \frac{p_\mu p_\nu}{2m_Z^2})B^{\mu\nu}(p_1,m_Z,m_Z) \\
&+ 16g^3m_ZU_{\mathcal{M}4}(p_1,p_2,m_Z,m_Z) + 36\cdot 48\lambda^2v_0B_0(p_1,m_H,m_H)\Bigg\}. \qquad (117)
\end{aligned}$$

or it can be written as

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{K}_H &= \mu^{4-d}\Bigg\{216\cdot 3^3\cdot 4\lambda^3 v_0^3 C_0\left(p_1,p_2,m_H,m_H,m_H\right)\\
&+ 8\cdot 4g^3\mu^{4-d}\Bigg\{\left(4m_Z^3 - m_Z p_1^2 + \frac{\left(p_1\cdot P_1\right)^2}{m_Z} - m_Z P_1^2\right)C_0\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \left(-2m_Z p_{1\mu} - 2m_Z P_{1\mu} + \frac{2p_1\cdot P_1}{m_Z}p_{1\mu} + \frac{2p_1\cdot P_1}{m_Z}P_{1\mu}\right)C^{\mu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \Bigg\{-3m_Z g_{\mu\nu} + \frac{2p_{1\mu}p_{1\nu}}{m_Z} + \frac{2p_{1\mu}P_{1\nu}}{m_Z}\\
&+ \frac{2P_{1\mu}P_{1\nu}}{m_Z} + \frac{2p_1\cdot P_1}{m_Z}g_{\mu\nu} - \frac{p_1\cdot P_1}{m_Z^3}p_{1\mu}P_{1\nu}\Bigg\}C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \left(-\frac{p_{1\mu}p_{1\nu}}{m_Z^3} - \frac{3p_{1\mu}P_{1\nu}}{m_Z^3} - \frac{P_{1\mu}P_{1\nu}}{m_Z^3}\right)\left(B^{\mu\nu}(p_1,m_Z,m_Z) + m_Z^2 C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\right)\\
&+ \Bigg\{\frac{4g_{\mu\nu}p_{1\rho}}{m_Z} + \frac{4g_{\mu\nu}P_{1\rho}}{m_Z} - \frac{p_{1\mu}p_{1\nu}P_{1\rho}}{m_Z^3}\\
&- \frac{P_{1\mu}P_{1\nu}p_{1\rho}}{m_Z^3} - \frac{g_{\mu\nu}p_{1\rho}}{m_Z^3}p_1\cdot P_1 - \frac{g_{\mu\nu}P_{1\rho}}{m_Z^3}p_1\cdot P_1\Bigg\}C^{\mu\nu\rho}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \left(3m_Z - \frac{p_1\cdot P_1}{m_Z}\right)U_{\mathcal{K}4} + \left(-2p_{1\mu} - 2P_{1\mu}\right)U_{\mathcal{K}5} - m_Z U_{\mathcal{K}6}\Bigg\}\\
&+ \frac{8}{3}\frac{\mathcal{M}_H^3}{v_0} + \frac{2\mathcal{M}_H^4}{v_0}\Bigg\}. \qquad (118)
\end{aligned}
$$

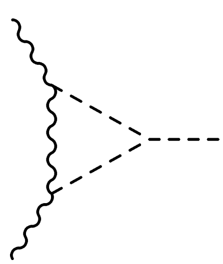
Now has come the time to use the complete reduction formulae that we have presented in the Appendix B. Therefore, using these relations we will be able to express the final result of $\mathcal{K}_H$ as a function of only the scalar integrals A_0, B_0 and C_0. To be more specific, we should use the equations defined in (469), (471), (532), (538), (545) and (551) and then we get that the reduced form of equation (117) reads

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{K}_H \;&=\; \mu^{4-d}\Bigg\{\frac{16g^3m_H^2}{m_Z^3}A_0(m_Z)+36\cdot 24\lambda^2 v_0 B_0(m_H,m_H,m_H)\\
&+\; 216\cdot 3^3\cdot 4\lambda^3 v_0^3 C_0\left(m_H,m_H,m_H,m_H,m_H\right)\\
&+\; g^3\Bigg\{-8m_Z+\left(\frac{38m_H^4}{9m_Z^3}-\frac{52m_H^2}{9m_Z}+\frac{80m_Z}{3}\right)B_{0\{1,2\}}(m_H,m_Z,m_Z)\\
&+\; \left(\frac{-46m_H^4}{9m_Z^3}-\frac{604m_H^2}{9m_Z}+\frac{224m_Z}{3}\right)B_{0\{1,3\}}(m_H,m_Z,m_Z)\\
&+\; \left(\frac{272m_H^4}{9m_Z^3}-\frac{340m_H^2}{9m_Z}-\frac{160m_Z}{3}\right)B_{0\{2,3\}}(m_H,m_Z,m_Z)\\
&+\\
&+\; \left(\frac{4m_H^6}{9m_Z^3}+\frac{352m_H^4}{9m_Z}-\frac{400m_H^2m_Z}{3}+96m_Z^3\right)C_0(m_H,m_H,m_Z,m_Z,m_Z)\Bigg\}\Bigg\}
\end{aligned}
\tag{119}
$$

where the index notation $\{\cdots\}$ refers to the denominators of the calculated integrals. Moreover, we have defined the relation $D = \frac{1}{\det G_2}$ and the G_2 determinant which in on-shell reads

$$
\det G_2 = \frac{3m_H^4}{4}\,. \tag{120}
$$

Until now we have presented the calculation of the three-point one-loop diagrams concerning the Higgs boson and in addition we have shown how we can reduce this contribution using only scalar integrals. Therefore, before we move on to the quantum corrections of the Higgs quartic coupling we should present the corresponding diagram calculation concerning the Higgs-Z three-vertex. These loop-corrections contribute to the renormalization of the gauge coupling, thus it is necessary for us to consider them in order to renormalize properly our Lagrangian. Therefore we start with the three-point diagram which comes from the combination of the $\phi A_\mu A_\nu$ and ϕ^3 vertices which reads



$$
= i\mathcal{K}^1_{HZ,\mu\nu} \tag{121}
$$

and it has the following form

$$
\begin{aligned}
i\mathcal{K}^1_{HZ,\mu\nu} &= i12\times 24 g^2 m_Z^2 \lambda v_0 g^{\mu\alpha} g^{\nu\beta} \int \frac{d^4k}{(2\pi)^4} \frac{i(-g_{\alpha\beta}+\frac{k_\alpha k_\beta}{m_Z^2})}{(k^2-m_Z^2)} \frac{i}{\Big((k+p_1)^2-m_H^2\Big)} \frac{i}{\Big((k+p_1+p_2)^2-m_H^2\Big)} \Leftrightarrow \\
\mathcal{K}^1_{HZ,\mu\nu} &= 12\times 24 g^2 m_Z^2 \lambda v_0 g^{\mu\alpha} g^{\nu\beta} \int \frac{d^4k}{(2\pi)^4} \frac{-i(-g_{\alpha\beta}+\frac{k_\alpha k_\beta}{m_Z^2})}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_H^2\Big)} . \qquad (122)
\end{aligned}
$$

Now, as we have done in previous calculations, we will separate this integral in two terms which we will evaluate them independently, therefore we get that

$$
\begin{aligned}
\mathcal{K}^{1A}_{HZ,\mu\nu} &= -12\times 24 g^2 m_Z^2 \lambda v_0 g^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{-i}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_H^2\Big)} \\
&= -\frac{12\times 24}{(4\pi)^{d/2}} \int \frac{d^dk}{\pi^{d/2}} \frac{-ig^2 m_Z^2 \lambda v_0 g^{\mu\nu}\mu^{4-d}}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_H^2\Big)} \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{K}^{1A}_{HZ,\mu\nu} &= -12\times 24 g^2 m_Z^2 \lambda v_0 g^{\mu\nu}\mu^{4-d} C_0(p_1,p_2,m_Z,m_H,m_H) \qquad (123)
\end{aligned}
$$

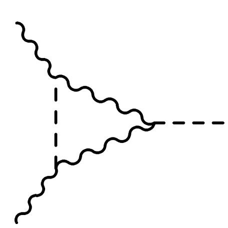
where we have used the first case of the relation (456). Now, the second term we have reads

$$
\begin{aligned}
\mathcal{K}^{1B}_{HZ,\mu\nu} &= 12\times 24 g^2 m_Z^2 \lambda v_0 \int \frac{d^4k}{(2\pi)^4} \frac{-ik^\mu k^\nu}{m_Z^2\,(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_H^2\Big)} \\
&= \frac{12\times 24 g^2 \lambda v_0 \mu^{4-d}}{(4\pi)^{d/2}} \int \frac{d^dk}{\pi^{d/2}} \frac{-ik^\mu k^\nu}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_H^2\Big)} \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{K}^{1B}_{HZ,\mu\nu} &= 12\times 24 g^2 \lambda v_0 \mu^{4-d} C^{\mu\nu}(p_1,p_2,m_Z,m_H,m_H) \qquad (124)
\end{aligned}
$$

thus adding the two contributions we get that $\mathcal{K}^1_{HZ}$ reads

$$
(4\pi)^{d/2}\mathcal{K}^1_{HZ,\mu\nu} = 12\times 24 g^2 \lambda v_0 \mu^{4-d} \Bigg\{ -g^{\mu\nu} m_Z^2 C_0(p_1,p_2,m_Z,m_H,m_H) + C^{\mu\nu}(p_1,p_2,m_Z,m_H,m_H) \Bigg\} . \qquad (125)
$$

Next we move on to the diagram that occurs from the cubic power of the $\phi A_\mu A\nu$ vertex which gives



$$
= i\mathcal{K}^2_{HZ,\mu\nu} \qquad (126)
$$

and it reads

$$
\begin{aligned}
i\mathcal{K}^2_{HZ,\mu\nu} &= -i4\times 8g^3 m_Z^3 g_{\mu\delta}g_{\nu\alpha}g_{\beta\lambda}\int\frac{d^4k}{(2\pi)^4}\frac{i(-g^{\delta\lambda}+\frac{k^\delta k^\lambda}{m_Z^2})}{(k^2-m_Z^2)}\frac{i}{\Big((k+p_1)^2-m_H^2\Big)}\frac{i(-g^{\alpha\beta}+\frac{k^\alpha k^\beta}{m_Z^2})}{\Big((k+p_1+p_2)^2-m_Z^2\Big)}\Leftrightarrow \\
\mathcal{K}^2_{HZ,\mu\nu} &= -4\times 8g^3 m_Z^3 g_{\mu\delta}g_{\nu\alpha}g_{\beta\lambda}\int\frac{d^4k}{(2\pi)^4}\frac{-i(-g^{\delta\lambda}+\frac{k^\delta k^\lambda}{m_Z^2})(-g^{\alpha\beta}+\frac{k^\alpha k^\beta}{m_Z^2})}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\,.
\end{aligned}
\tag{127}
$$

Therefore in order to evaluate this integral we have first to calculate its numerator which, as we expect, will give four independent terms. To be more specific, the numerator becomes

$$
\begin{aligned}
N &= g_{\mu\delta}g_{\nu\alpha}g_{\beta\lambda}(-g^{\delta\lambda}+\frac{k^\delta k^\lambda}{m_Z^2})(-g^{\alpha\beta}+\frac{k^\alpha k^\beta}{m_Z^2}) \\
&= g_{\mu\delta}g_{\nu\alpha}g_{\beta\lambda}\Bigg\{g^{\delta\lambda}g^{\alpha\beta}-g^{\delta\lambda}\frac{(k+p_1+p_2)^\alpha(k+p_1+p_2)^\beta}{m_Z^2}-g^{\alpha\beta}\frac{k^\delta k^\lambda}{m_Z^2} \\
&+ \frac{k^\delta k^\lambda(k+p_1+p_2)^\alpha(k+p_1+p_2)^\beta}{m_Z^4}\Bigg\}\Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
N &= g^{\mu\nu}-\frac{(k+p_1+p_2)^\mu(k+p_1+p_2)^\nu}{m_Z^2}-\frac{k^\mu k^\nu}{m_Z^2} \\
&+ \frac{k^\mu(k+p_1+p_2)^\nu k\cdot(k+p_1+p_2)}{m_Z^4} \\
&= g^{\mu\nu}-\frac{2k^\mu k^\nu}{m_Z^2}-\frac{k^\mu(p_1+p_2)^\nu}{m_Z^2}-\frac{(p_1+p_2)^\mu k^\nu}{m_Z^2}-\frac{(p_1+p_2)^\mu(p_1+p_2)^\nu}{m_Z^2} \\
&+ \frac{k^\mu k^\nu k^2}{m_Z^4}+\frac{k^\mu k^\nu k\cdot(p_1+p_2)}{m_Z^4}+\frac{k^\mu(p_1+p_2)^\nu k^2}{m_Z^4}+\frac{k^\mu(p_1+p_2)^\nu k\cdot(p_1+p_2)}{m_Z^4}
\end{aligned}
\tag{128}
$$

and putting this in the relation for the $\mathcal{K}^2_{HZ,\mu\nu}$ we get the following

$$
\begin{aligned}
\mathcal{K}^2_{HZ,\mu\nu} &= -4\times 8g^3m_Z^3\int\frac{d^4k}{(2\pi)^4}\frac{-ig^{\mu\nu}}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\\
&+ 4\times 16g^3m_Z\int\frac{d^4k}{(2\pi)^4}\frac{-ik^\mu k^\nu}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\\
&+ 4\times 8g^3m_Z\int\frac{d^4k}{(2\pi)^4}\frac{-ik^\mu (p_1+p_2)^\nu}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\\
&+ 4\times 8g^3m_Z\int\frac{d^4k}{(2\pi)^4}\frac{-i(p_1+p_2)^\mu k^\nu}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\\
&+ 4\times 8g^3m_Z\int\frac{d^4k}{(2\pi)^4}\frac{-i(p_1+p_2)^\mu (p_1+p_2)^\nu}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\\
&- \frac{4\times 8g^3}{m_Z}\int\frac{d^4k}{(2\pi)^4}\frac{-ik^\mu k^\nu k^2}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\\
&- \frac{4\times 8g^3}{m_Z}\int\frac{d^4k}{(2\pi)^4}\frac{-ik^\mu k^\nu k\cdot(p_1+p_2)}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\\
&- \frac{4\times 8g^3}{m_Z}\int\frac{d^4k}{(2\pi)^4}\frac{-ik^\mu (p_1+p_2)^\nu k^2}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\\
&- \frac{4\times 8g^3}{m_Z}\int\frac{d^4k}{(2\pi)^4}\frac{-ik^\mu (p_1+p_2)^\nu k\cdot(p_1+p_2)}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\,.
\end{aligned}
\tag{129}
$$

This terms could be reduced to scalar integrals according to the procedure that we follow throughout this work, therefore writing them in d-dimensions we get that

$$
\begin{aligned}
\mathcal{K}^2_{HZ,\mu\nu} &= -\frac{4\times 8g^3m_Z^3\mu^{d-4}}{(4\pi)^{d/2}}\int\frac{d^dk}{\pi^{d/2}}\frac{-ig^{\mu\nu}}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\\
&+ \frac{4\times 16g^3m_Z\mu^{d-4}}{(4\pi)^{d/2}}\int\frac{d^dk}{\pi^{d/2}}\frac{-ik^\mu k^\nu}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\\
&+ \frac{4\times 8g^3m_Z\mu^{d-4}}{(4\pi)^{d/2}}\int\frac{d^dk}{\pi^{d/2}}\frac{-ik^\mu (p_1+p_2)^\nu}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\\
&+ \frac{4\times 8g^3m_Z\mu^{d-4}}{(4\pi)^{d/2}}\int\frac{d^dk}{\pi^{d/2}}\frac{-i(p_1+p_2)^\mu k^\nu}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{4\times 8g^3 m_Z \mu^{d-4}}{(4\pi)^{d/2}} \int \frac{d^dk}{\pi^{d/2}} \frac{-i(p_1+p_2)^\mu (p_1+p_2)^\nu}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)} \\
&- \frac{4\times 8g^3 \mu^{d-4}}{m_Z(4\pi)^{d/2}} \int \frac{d^dk}{\pi^{d/2}} \frac{-ik^\mu k^\nu k^2}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)} \\
&- \frac{4\times 8g^3 \mu^{d-4}}{m_Z(4\pi)^{d/2}} \int \frac{d^dk}{\pi^{d/2}} \frac{-ik^\mu k^\nu k\cdot(p_1+p_2)}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)} \\
&- \frac{4\times 8g^3 \mu^{d-4}}{m_Z(4\pi)^{d/2}} \int \frac{d^dk}{\pi^{d/2}} \frac{-ik^\mu (p_1+p_2)^\nu k^2}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)} \\
&- \frac{4\times 8g^3 \mu^{d-4}}{m_Z(4\pi)^{d/2}} \int \frac{d^dk}{\pi^{d/2}} \frac{-ik^\mu (p_1+p_2)^\nu k\cdot(p_1+p_2)}{(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}
\end{aligned} \tag{130}
$$

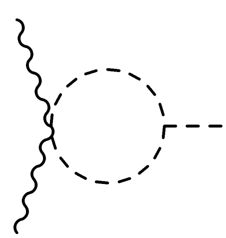
and now if we recall the relations (456) from the Appendix, then we get that $\mathcal{K}^2_{HZ,\mu\nu}$ takes its final form which reads

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{K}^2_{HZ,\mu\nu} &= 4\times 8g^3\mu^{d-4}\Bigg\{ -m_Z^3 g^{\mu\nu} C_0(p_1,p_2,m_Z,m_H,m_Z) + 2m_Z C^{\mu\nu}(p_1,p_2,m_Z,m_H,m_Z) \\
&+ m_Z(p_1+p_2)^\nu C^\mu(p_1,p_2,m_Z,m_H,m_Z) \\
&+ m_Z(p_1+p_2)^\mu C^\nu(p_1,p_2,m_Z,m_H,m_Z) + m_Z(p_1+p_2)^\mu(p_1+p_2)^\nu C_0(p_1,p_2,m_Z,m_H,m_Z) \\
&- m_Z U^{\mu\nu}_{\mathcal{K}4} - \frac{(p_1+p_2)_\rho}{m_Z} C^{\mu\nu\rho}(p_1,p_2,m_Z,m_H,m_Z) - \frac{(p_1+p_2)_\nu}{m_Z} C^{\mu\rho\sigma}(p_1,p_2,m_Z,m_H,m_Z) \\
&- \frac{(p_1+p_2)_\nu (p_1+p_2)_\rho}{m_Z} C^{\mu\rho}(p_1,p_2,m_Z,m_H,m_Z) \Bigg\}.
\end{aligned} \tag{131}
$$

Moreover, we have defined the dimensionless integral $U^{\mu\nu}_{\mathcal{K}4}$ as

$$
U^{\mu\nu}_{\mathcal{K}4} = \int \frac{d^4k}{(4\pi)^4} \frac{-ik^\mu k^\nu k^2}{m_Z^2\,(k^2-m_Z^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_Z^2\Big)}\,. \tag{132}
$$

Next comes a diagram that occurs from the combination of the $\phi^2 A_\mu A_\nu$ and ϕ^3 vertices which gives



$$
= i\mathcal{K}^3_{HZ,\mu\nu} \tag{133}
$$

which is very easy to see that it is connected with the diagram (111) which is connected with the diagram (37). Therefore we get that

$$\begin{aligned}
(4\pi)^{d/2}\mathcal{K}^3_{HZ,\mu\nu} &= \frac{-g^2 g^{\mu\nu}}{6\times 3\lambda}(4\pi)^{d/2}\mu^{d-4}\mathcal{K}^3_H = \frac{-g^2 g^{\mu\nu}}{6\times 3v_0\lambda}(4\pi)^{d/2}\mu^{d-4}\mathcal{M}^3_H \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{K}^3_{HZ,\mu\nu} &= -8\times 12 g^2 g^{\mu\nu}\lambda v_0 \mu^{d-4} B_0(p_1, m_H, m_H)
\end{aligned} \tag{134}$$

Finally, in order to evaluate the full contribution of these diagrams to the Higgs-Z vertex we should add equations (125), (131) and (134), obtaining

$$\begin{aligned}
(4\pi)^{d/2}\mathcal{K}_{HZ,\mu\nu} &= -8\times 12 g^2 g^{\mu\nu}\lambda v_0\mu^{d-4}B_0(p_1, m_H, m_H) \\
&+ 4\times 8 g^2\mu^{4-d}\Bigg\{3\times 3\lambda v_0\Big\{-g^{\mu\nu}m_Z^2 C_0(p_1,p_2,m_Z,m_H,m_H) \\
&+ C^{\mu\nu}(p_1,p_2,m_Z,m_H,m_H)\Big\} \\
&+ g\Big\{(m_Z(p_1+p_2)^\mu(p_1+p_2)^\nu - m_Z^3 g^{\mu\nu})C_0(p_1,p_2,m_Z,m_H,m_Z) \\
&+ 2m_Z C^{\mu\nu}(p_1,p_2,m_Z,m_H,m_Z) + m_Z(p_1+p_2)^\nu C^\mu(p_1,p_2,m_Z,m_H,m_Z) \\
&+ m_Z(p_1+p_2)^\mu C^\nu(p_1,p_2,m_Z,m_H,m_Z) - m_Z U^{\mu\nu}_{\mathcal{K}4} \\
&- \frac{(p_1+p_2)_\rho}{m_Z}C^{\mu\nu\rho}(p_1,p_2,m_Z,m_H,m_Z) - \frac{(p_1+p_2)_\nu}{m_Z}C^{\mu\rho\sigma}(p_1,p_2,m_Z,m_H,m_Z) \\
&- \frac{(p_1+p_2)_\nu(p_1+p_2)_\rho}{m_Z}C^{\mu\rho}(p_1,p_2,m_Z,m_H,m_Z)\Big\}\Bigg\}.
\end{aligned} \tag{135}$$

As we can see this result has many common features with the result that we have obtained from the one-loop three-point functions concerning the Higgs field. On the other hand it seems more difficult for $\mathcal{K}_{HZ,\mu\nu}$ to be reduced to scalar integrals since it has tensor form. Fortunately, this is not the case since for the on-shell renormalization that we discuss later, we should consider the contracted with the metric case of this result, namely $g_{\mu\nu}\mathcal{K}_{HZ,\mu\nu}$. Therefore here the reduction into scalar integrals of the Higgs-Z vertex correction will concern its contacted version. Therefore, using the equations (493), (494), (499) and (538) we get the reduced form

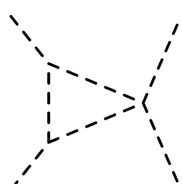
$$
\begin{aligned}
(4\pi)^{d/2} g^{\mu\nu} \mathcal{K}_{HZ,\mu\nu} &= \mu^{4-d} \Bigg\{ 8g^2 \lambda v_0 B_0(m_Z, m_H, m_H) + \frac{4\times 8g^3}{m_Z} A_0(m_H) - \frac{32g^3}{3m_Z} A_0(m_Z) \\
&+ \frac{4g^3(-7m_H^2 + 16m_Z^2)}{9m_Z} B_0(m_Z, m_H, m_Z) \\
&+ \frac{g^3(35Dm_H^8 + 32m_H^2 m_Z^2 - 136Dm_H^6 m_Z^2)}{6m_Z} C_0(m_Z, m_Z, m_Z, m_H, m_Z) \\
&+ \frac{g^3(-80m_Z^4 + 144Dm_H^4 m_Z^4 - 64Dm_H^2 m_Z^6)}{18m_Z} C_0(m_Z, m_Z, m_Z, m_H, m_Z) \\
&- 24g^2 m_Z^2 \lambda v_0 C_0(m_Z, m_Z, m_Z, m_H, m_H) \Bigg\} .
\end{aligned} \tag{136}
$$

where we have defined the relation $D = \frac{1}{\det G_2}$ and the G_2 determinant which in on-shell reads

$$
\det G_2 = 2m_H^2 m_Z^2 - \frac{m_H^4}{4} - m_Z^4 \,. \tag{137}
$$

2.4 Four-point functions

In the present subsection we deal with an other important set of one-loop quantum corrections corresponding to the four-point functions, namely the box diagrams. The first set of these diagrams contributes to the correction of the quartic coupling of the Higgs boson and corresponds to four external Higgs fields. In particular we start with a diagram coming from the combination of the ϕ^3 and ϕ^4 vertices which gives



$$
= i\mathcal{B}_H^1 \tag{138}
$$

and as we can see if we could replace one of the external Higgs fields with v_0 then we would obtain exactly the same diagram with that of (78) except the symmetry factor. Generally in what follows we will consider diagrams similar with that of the previous section but with different symmetry factors. Therefore the only thing left is to evaluate these factors, so for the current diagram, following the reasoning that we have developed in the previous cases, we have that

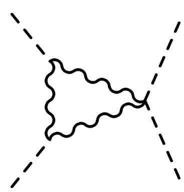
the symmetry factor here reads $\mathcal{S}^1_{\mathcal{BK}_H} = 216 \cdot 3^3 \cdot 16$.
thus it is straightforward to write here that

$$(4\pi)^{d/2}\mathcal{B}^1_H = 216 \cdot 3^3 \cdot 16\lambda^3 v_0^2 \mu^{d-4} C_0\left(p_1, p_2, m_H, m_H, m_H\right) \Leftrightarrow \tag{139}$$

otherwise, we have that

$$\mathcal{K}^1_H = \frac{v_0}{4}\mathcal{B}^1_H\,. \tag{140}$$

Same arguments are true for the case of the next diagram coming from the combination of the $\phi A_\mu A_\nu$ and $\phi^2 A_\mu A_\nu$ which reads



$$= i\mathcal{B}^2_H \tag{141}$$

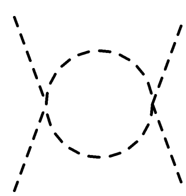
and it is similar with that of (81). Again here following the known procedure for calculating the symmetry factor we get $\mathcal{S}^2_{\mathcal{BK}_H} = 8 \cdot 8$.
Thus its final form here becomes

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}_H^2 &= 8\cdot 8 g^4\mu^{4-d}\Bigg\{\left(4m_Z^2 - p_1^2 - (p_1+p_2)^2 + \frac{1}{m_Z^2}\left(p_1^2 + p_1\cdot p_2\right)^2\right) C_0\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \left(\left[-4 + \frac{4p_1^2}{m_Z^2} + \frac{4p_1\cdot p_2}{m_Z^2}\right] p_{1\mu} + \left[-2 + \frac{2p_1^2}{m_Z^2} + \frac{2p_1\cdot p_2}{m_Z^2}\right] p_{2\mu}\right) C^{\mu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \left(-3 + \frac{2p_1^2}{m_Z^2} + \frac{2p_1\cdot p_2}{m_Z^2}\right) g_{\mu\nu} C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \left(\left[\frac{6}{m_Z^2} - \frac{p_1^2}{m_Z^4} - \frac{p_1\cdot p_2}{m_Z^4}\right] p_{1\mu}p_{1\nu} + \left[\frac{6}{m_Z^2} - \frac{p_1^2}{m_Z^4} - \frac{p_1\cdot p_2}{m_Z^4}\right] p_{1\mu}p_{2\nu}\right) C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \frac{2}{m_Z^2} p_{2\mu}p_{2\nu} C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ g_{\mu\nu}\left(\left[\frac{8}{m_Z^2} - \frac{2p_1^2}{m_Z^4} - \frac{2p_1\cdot p_2}{m_Z^4}\right] p_{1\rho} + \left[\frac{4}{m_Z^2} - \frac{p_1^2}{m_Z^4} - \frac{p_1\cdot p_2}{m_Z^4}\right] p_{2\rho}\right) C^{\mu\nu\rho}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \left(-\frac{2}{m_Z^4} p_{1\mu}p_{1\nu}p_{1\rho} - \frac{3}{m_Z^4} p_{1\mu}p_{1\nu}p_{2\rho} - \frac{1}{m_Z^4} p_{1\mu}p_{2\nu}p_{2\rho}\right) C^{\mu\nu\rho}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&+ \left(3 - \frac{p_1^2}{m_Z^2} - \frac{p_1\cdot p_2}{m_Z^2}\right) U_{\mathcal{K}4}(p_1,p_2,m_Z,m_Z,m_Z)\\
&+ -U_{\mathcal{K}6}(p_1,p_2,m_Z,m_Z,m_Z) - \left(\frac{4p_{1\mu}}{m_Z} + \frac{2p_{2\mu}}{m_Z}\right) U_{\mathcal{K}5}(p_1,p_2,m_Z,m_Z,m_Z)\Bigg\}
\end{aligned}
\tag{142}
$$

which gives that

$$
\mathcal{K}_H^2 = \frac{v_0}{2}\mathcal{B}_H^2\,. \tag{143}
$$

Next we deal with two more Box diagrams which are related with the Triangular case in the same way that we have described previously. To be more specific, the first one that we consider here comes from the square of the ϕ^4 vertex and gives



$$
= i\mathcal{B}_H^3 \tag{144}
$$

which is exactly the same with (111) divided by v_0 but its symmetry factor is different. To be more specific, here the ϕ^4 vertex gives $\mathcal{S}^3_{\mathcal{B}\mathcal{K}_H} = \frac{1}{2}$.

Moreover, in this particular set of four point functions, we should take into consideration the existence of u- and t-channels in addition of the s-channel that we have presented here. Thus, we have to add two more contributions coming from these channels. Here s, u and t represent the Mandelstam variables and in the center-of-mass frame are defined as

$$\begin{aligned} s &= (p_1 + p_2)^2 = 4E^2 = E_{CM}^2 && (145) \\ t &= (p_1 - p_3)^2 = m_1^2 + m_2^2 - 2E^2 + 2\vec{k}\cdot\vec{p} && (146) \\ u &= (p_2 - p_4)^2 = m_1^2 + m_2^2 - 2E^2 - 2\vec{k}\cdot\vec{p}. && (147) \end{aligned}$$

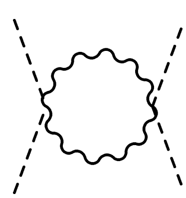
These relations, as we will see later in this work, help us to specify the kinematics of the Feynman diagrams which plays a very important role in the evaluation of the physical quantities. Thus if we suppose that the external legs of this diagram have indices i, j, k and l, then the combination of these channels will give that this diagram reads

$$(4\pi)^{d/2}\mathcal{B}_H^3 = 18\lambda^2\mu^{d-4}(\delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk})B_0(p_1, m_H, m_H). \qquad (148)$$

since every contribution of the s-, u- and t-channels is identical, with only difference in the way that the external legs combine each other. Therefore for each pair of ij and kl we get that

$$\mathcal{K}_H^3 = \frac{v_0}{6}\mathcal{B}_H^3\,. \qquad (149)$$

Now, the second diagram comes from the square of the $\phi^2 A_\mu A_\nu$ vertex and reads



$$= i\mathcal{B}_H^4 \qquad (150)$$

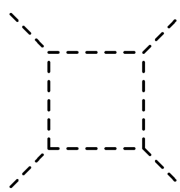
thus, using the same arguments with diagram (144), we get $\mathcal{S}^4_{\mathcal{B}\mathcal{K}_H} = \frac{1}{2}$ and so we obtain that

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}_H^4 &= (\delta^{ij}\delta^{kl}+\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk})\mu^{d-4}\Bigg\{6\frac{\lambda m_{Z_0}^2}{m_{H_0}^2}A_0(m_{Z_0})+\frac{3\lambda m_{Z_0}^4}{m_{H_0}^2}+3\lambda_0 A_0(m_{H_0}) \\
&+ 9\lambda_0 m_{H_0}^2 B_0(p,m_{H_0},m_{H_0})+\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}\Big\{4(d+\varepsilon)B_0(p,m_{Z_0},m_{Z_0}) \\
&- \frac{4(d+\varepsilon)}{d}g_{\mu\nu}B^{\mu\nu}(p,m_{Z_0},m_{Z_0})-\frac{4(d+\varepsilon)}{d}g_{\mu\nu}B^{\mu\nu}_{k+p}(p,m_{Z_0},m_{Z_0}) \\
&+ \frac{4(d+\varepsilon)}{d^2}g_{\mu\nu}B^{\mu\nu}(p,m_{Z_0},m_{Z_0})+\frac{4(d+\varepsilon)}{d^2}A_0(m_{Z_0})\Big\}\Bigg\} \qquad (151)
\end{aligned}
$$

which means that

$$
\mathcal{K}_H^4=\frac{v_0}{2}\mathcal{B}_H^4 \qquad (152)
$$

Now we move on to the final two Box diagrams that contribute to the Higgs quartic coupling. These diagrams do not have any relation with Triangles as the cases that we have previously dealt with. The first of these new diagrams comes from the fourth power of the ϕ^3 vertex and gives



$$
= i\mathcal{B}_H^5 \qquad (153)
$$

which reads

$$
\begin{aligned}
i\mathcal{B}_H^5 &= \int\frac{d^4k}{(2\pi)^4}\frac{i}{(k^2-m_H^2)}\frac{i}{\Big((k+p_1)^2-m_H^2\Big)}\frac{i\lambda^4 v_0^4\mathcal{S}^1_{\mathcal{B}_H}}{\Big((k+p_1+p_2)^2-m_H^2\Big)}\frac{i}{\Big((k+p_1+p_2+p_3)^2-m_H^2\Big)} \\
\mathcal{B}_H^5 &= \int\frac{d^4k}{(2\pi)^4}\frac{-i\lambda^4 v_0^4\mathcal{S}^1_{\mathcal{B}_H}}{(k^2-m_H^2)\Big((k+p_1)^2-m_H^2\Big)\Big((k+p_1+p_2)^2-m_H^2\Big)\Big((k+p_1+p_2+p_3)^2-m_H^2\Big)}.
\end{aligned}
$$

Here the symmetry factor is given by using the same technic with the previous set, thus we get that the symmetry factor here reads $\mathcal{S}^1_{\mathcal{B}_H}=6^4\cdot 81\cdot 8$.

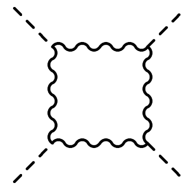
Now, writing this integral in d-dimensions we see that it reads

$$
\mathcal{B}_H^5 = 6^4\cdot 81\cdot 8\frac{\lambda^4 v_0^4\mu^{d-4}}{(4\pi)^{d/2}}\int\frac{d^dk}{i\pi^{d/2}}\frac{-i}{d_1d_2d_3d_4}
$$

where $d_i = \left(k + \sum_{k=1}^{i-1} p_k\right)^2 - m_i + i\varepsilon$ with $, i = 1, .., 4$. Thus, recalling the first case of the relation (457) from the Appendix B we obtain the following

$$(4\pi)^{d/2}\mathcal{B}_H^5 = 6^4 \cdot 81 \cdot 8\lambda^4 v_0^4 \mu^{d-4} D_0\left(p_1, p_2, p_{3,} m_H, m_H, m_H, m_H\right). \tag{154}$$

The last diagram that we have to consider here comes from the fourth power of the $\phi A_\mu A_\nu$ vertex and it has the form



$$= i\mathcal{B}_H^6 \tag{155}$$

with symmetry factor $\mathcal{S}^2_{\mathcal{B}_H}$ which is given by $\mathcal{S}^2_{\mathcal{B}_H} = 16 \cdot 8$. Therefore this diagram reads

$$\begin{aligned}
i\mathcal{B}_H^6 &= 16 \cdot 8 g^4 m_Z^4 g_{\mu\delta} g_{\nu\gamma} \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{i\left(-g^{\mu\alpha} + \frac{k^\mu k^\alpha}{m_Z^2}\right)}{(k^2 - m_Z^2)} \frac{i\left(-g^{\alpha\beta} + \frac{(k+p_1)^\alpha (k+p_1)^\beta}{m_Z^2}\right)}{\left((k+p_1)^2 - m_Z^2\right)} \right. \\
&\times \left. \frac{i\left(-g^{\beta\nu} + \frac{(k+p_1+p_2)^\beta (k+p_1+p_2)^\nu}{m_Z^2}\right)}{\left((k+p_1+p_2)^2 - m_Z^2\right)} \frac{i\left(-g^{\gamma\delta} + \frac{(k+p_1+p_2+p_3)^\gamma (k+p_1+p_2+p_3)^\delta}{m_Z^2}\right)}{\left((k+p_1+p_2+p_3)^2 - m_Z^2\right)} \right\} \Leftrightarrow \\
\mathcal{B}_H^6 &= 16 \cdot 8 g^4 m_Z^4 g_{\mu\delta} g_{\nu\gamma} \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{-i\left(-g^{\mu\alpha} + \frac{k^\mu k^\alpha}{m_Z^2}\right)}{(k^2 - m_Z^2)} \frac{\left(-g^{\alpha\beta} + \frac{(k+p_1)^\alpha (k+p_1)^\beta}{m_Z^2}\right)}{\left((k+p_1)^2 - m_Z^2\right)} \right. \\
&\times \left. \frac{\left(-g^{\beta\nu} + \frac{(k+p_1+p_2)^\beta (k+p_1+p_2)^\nu}{m_Z^2}\right)}{\left((k+p_1+p_2)^2 - m_Z^2\right)} \frac{\left(-g^{\gamma\delta} + \frac{(k+p_1+p_2+p_3)^\gamma (k+p_1+p_2+p_3)^\delta}{m_Z^2}\right)}{\left((k+p_1+p_2+p_3)^2 - m_Z^2\right)} \right\}.
\end{aligned} \tag{156}$$

Before we start our calculation using the reduction formula as we have done till now, we should notice that the numerator of this Box diagram has four parenthesis which give sixteen independent terms. The general form of calculating such integrals which we have followed throughout this work says that we should fully expand the numerator and consider each term separately. Nevertheless, here we can change a little bit the procedure, namely we can expand only the last parenthesis in the numerator. Doing this, including $g_{\mu\delta} g^\delta_\varepsilon$, we obtain two terms for the numerator which read

$$
\begin{aligned}
N &= N_1 + N_2 \\
&= -g_{\mu\nu}\left(-g^{\mu\alpha} + \frac{k^\mu k^\alpha}{m_Z^2}\right)\left(-g^{\alpha\beta} + \frac{(k+p_1)^\alpha (k+p_1)^\beta}{m_Z^2}\right)\left(-g^{\beta\nu} + \frac{(k+p_1+p_2)^\beta (k+p_1+p_2)^\nu}{m_Z^2}\right) \\
&+ \frac{(k+p_1+p_2+p_3)_\mu (k+p_1+p_2+p_3)_\nu}{m_Z^2}\left(-g^{\mu\alpha} + \frac{k^\mu k^\alpha}{m_Z^2}\right)\left(-g^{\alpha\beta} + \frac{(k+p_1)^\alpha (k+p_1)^\beta}{m_Z^2}\right) \\
&\times \left(-g^{\beta\nu} + \frac{(k+p_1+p_2)^\beta (k+p_1+p_2)^\nu}{m_Z^2}\right). \qquad (157)
\end{aligned}
$$

Comparing this with the Triangular integral (82) we notice that we have its numerator multiplied with two different terms. Thus for our case here we can calculate separately the sixteen terms coming from the two numerators N_1 and N_2 using the results from the corresponding Triangle diagram. To be more specific, from the numerator N_1 we get exactly the terms that we have in (81), thus the first eight of them are $\mathcal{B}^6_{HA}$, $\mathcal{B}^6_{HB}$, $\mathcal{B}^6_{HC}$, $\mathcal{B}^6_{HD}$, $\mathcal{B}^6_{HE}$, $\mathcal{B}^6_{HF}$, $\mathcal{B}^6_{HG}$ and $\mathcal{B}^6_{HI}$ the first one gives

and gives that

$$
(4\pi)^{d/2}\mathcal{B}^6_{HA} = 64 \cdot 8 g^4 m_Z^4 \mu^{4-d} D_0(p_1, p_2, p_3, m_Z, m_Z, m_Z). \qquad (158)
$$

The second integral has

$$
N = g_{\mu\nu} k^\mu k^\nu + {p_1}^2 + 2p_1 p_2 + {p_2}^2 + 2k(p_1 + p_2) \qquad (159)
$$

and gives that

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^6_{HB} &= -16 \cdot 8 g^4 m_Z^2 \mu^{4-d} \Big\{ g_{\mu\nu} D^{\mu\nu}(p_1, p_2, p_3, m_Z, m_Z, m_Z) \\
&+ ({p_1}^2 + 2p_1 p_2 + {p_2}^2) D_0(p_1, p_2, p_3, m_Z, m_Z, m_Z) + 2(p_1+p_2)_\mu D^\mu(p_1, p_2, p_3, m_Z, m_Z, m_Z) \Big\}.
\end{aligned} \qquad (160)
$$

The next integral is the same with the previous one if we replace $(p_1 + p_2) \to p_1$, thus it reads

$$(4\pi)^{d/2}\mathcal{B}^6_{HC} = -16\cdot 8g^4 m_Z^2\mu^{4-d}\Big\{ g_{\mu\nu}D^{\mu\nu}(p_1,p_2,p_3,m_Z,m_Z,m_Z) + {p_1}^2 D_0(p_1,p_2,p_3,m_Z,m_Z,m_Z) + 2p_{1\mu}D^{\mu}(p_1,p_2,p_3,m_Z,m_Z,m_Z)\Big\}. \tag{161}$$

The fourth integral in the row is more complicated since it has bigger mass dimension than the previous three integrals. However, we know that its numerator becomes

$$N = k^4 + k^2 k\cdot(4p_1+2p_2) + k^2(6{p_1}^2+6p_2p_1+{p_2}^2) + 2k\cdot p_1(p_1+p_2)(2p_1+p_2) + p_1^2\cdot(p_1+p_2)^2 \tag{162}$$

so using this relation we obtain that

$$\begin{aligned}(4\pi)^{d/2}\mathcal{B}^6_{HD} &= 16\cdot 8g^4\mu^{4-d}\Big\{ p_1^2(p_1+p_2)^2 D_0\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) + U_{\mathcal{B}4}(p_1,p_2,p_3,m_Z,m_Z,m_Z)\\ &+ [p_1(p_1+p_2)(2p_1+p_2)]_\mu D^\mu\,(p_1,p_2,p_3,m_Z,m_Z,m_Z)\\ &+ (6{p_1}^2+6p_2\cdot p_1+{p_2}^2)g_{\mu\nu}D^{\mu\nu}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z)\\ &+ g_{\mu\nu}(4p_1+2p_2)_\alpha D^{\mu\nu\alpha}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z)\Big\}.\end{aligned} \tag{163}$$

where we have defined

$$U_{\mathcal{B}4}(p_1,p_2,p_3,m_Z,m_Z,m_Z) = \int\frac{d^4k}{(2\pi)^4}\Big\{\frac{-ik^4}{(k^2-m_Z^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \times \frac{1}{((k+p_1+p_2+p_3)^2-m_Z^2)}\Big\}. \tag{164}$$

The next integral correspond to the second case of equation (457) and we can write straightforward that

$$(4\pi)^{d/2}\mathcal{B}^6_{HE} = -16\cdot 8g^4 m_Z^2\mu^{4-d}g_{\mu\nu}D^{\mu\nu}(p_1,p_2,p_3,m_Z,m_Z,m_Z). \tag{165}$$

Now we move on to the sixth integral which is similar with (163) and whose numerator according to (102) reads

$$N = k^4 + 2k^2 k\cdot(p_1+p_2) + k^2(p_1+p_2)^2 \tag{166}$$

so in d-dimensions this integral gives

$$\begin{aligned}(4\pi)^{d/2}\mathcal{B}^6_{HF} &= 16\cdot 8g^4\mu^{4-d}\Bigg\{(p_1+p_2)^2 g_{\mu\nu}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\ &+ 2g_{\mu\nu}(p_1+p_2)_\alpha D^{\mu\nu\alpha}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\ &+ U_{\mathcal{B}4}(p_1,p_2,p_3,m_Z,m_Z,m_Z)\Bigg\}. \end{aligned}\tag{167}$$

The next integral corresponds to $\mathcal{B}^6_{HG}$ and as we already know is exactly the same with $\mathcal{B}^6_{HF}$ if we perform the replacement $(p_1+p_2)\to p_1$, then we obtain the following

$$\begin{aligned}(4\pi)^{d/2}\mathcal{B}^6_{HG} &= 16\cdot 8g^4\mu^{4-d}\Bigg\{p_1^2 g_{\mu\nu}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\ &+ 2g_{\mu\nu}p_{1\alpha} D^{\mu\nu\alpha}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\ &+ U_{\mathcal{B}4}(p_1,p_2,p_3,m_Z,m_Z,m_Z)\Bigg\}. \end{aligned}\tag{168}$$

The last integral coming from the first eight terms that we calculate here is $\mathcal{B}^6_{HI}$ and its numerator according to (106) reads

$$N = k^6 + k^4 k\cdot(4p_1+2p_2) + k^4(6{p_1}^2+6p_2p_1+{p_2}^2) + 2k^2k\cdot p_1(p_1+p_2)(2p_1+p_2) + k^2p_1^2\cdot(p_1+p_2)^2 \tag{169}$$

and it gives that

$$\begin{aligned}(4\pi)^{d/2}\mathcal{B}^6_{HI} &= -\frac{16\cdot 8g^4\mu^{4-d}}{m_Z^2}\Bigg\{(6p_1^2+6p_1\cdot p_2+p_2^2)U_{\mathcal{B}4}(p_1,p_2,p_3,m_Z,m_Z,m_Z)\\ &+ m_Z^2U_{\mathcal{B}6}(p_1,p_2,p_3,m_Z,m_Z,m_Z) + m_Z(4p_1+2p_2)_\mu U_{\mathcal{B}5}(p_1,p_2,p_3,m_Z,m_Z,m_Z)\\ &+ p_1^2(p_1+p_2)^2 g_{\mu\nu}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\ &+ g_{\mu\nu}(4p_1^3+6p_1^2p_2+2p_1p_2^2)_\alpha D^{\mu\nu\alpha}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\Bigg\}. \end{aligned}\tag{170}$$

where we have defined the following

$$U_{\mathcal{B}5}(p_1,p_2,p_3,m_Z,m_Z,m_Z) = \int \frac{d^4k}{(2\pi)^4}\Bigg\{\frac{-ik^4k^\mu}{m_Z\left(k^2-m_Z^2\right)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \times \frac{1}{((k+p_1+p_2+p_3)^2-m_Z^2)}\Bigg\} \qquad (171)$$

and

$$U_{\mathcal{B}6}(p_1,p_2,p_3,m_Z,m_Z,m_Z) = \int \frac{d^4k}{(2\pi)^4}\Bigg\{\frac{-ik^6}{m_Z^2\left(k^2-m_Z^2\right)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \times \frac{1}{((k+p_1+p_2+p_3)^2-m_Z^2)}\Bigg\}. \qquad (172)$$

Now that we have finished with the calculation of the first parenthesis integrals we move on to the next eight integrals that occur from the N_2 numerator which are $\mathcal{B}^6_{HJ}$, $\mathcal{B}^6_{HK}$, $\mathcal{B}^6_{HL}$, $\mathcal{B}^6_{HM}$, $\mathcal{B}^6_{HN}$, $\mathcal{B}^6_{HO}$, $\mathcal{B}^6_{HP}$, $\mathcal{B}^6_{HQ}$. As we can recall, the difference here is that we have the numerator of the (82) multiplied with a zero-mass dimension term. Thus, in order to evaluate these integrals we have first to calculate the corresponding numerators something that we should do for each case separately. As an example we give the first integral of the second set where we have that its numerator reads

$$\begin{aligned} N &= (k+p_1+p_2+p_3)\cdot(k+p_1+p_2+p_3) \Leftrightarrow \\ N &= k^2+2k(p_1+p_2+p_3)+(p_1+p_2+p_3)^2 \end{aligned} \qquad (173)$$

and now using the same arguments as in each previous case that we have faced, we get that

$$(4\pi)^{d/2}\mathcal{B}^6_{HJ} = -16\cdot 8g^4m_Z^2\mu^{4-d}\Bigg\{g_{\mu\nu}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) + 2(p_1+p_2+p_3)_\mu D^\mu\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)+(p_1+p_2+p_3)^2D_0\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\Bigg\}. \qquad (174)$$

Next we have $\mathcal{B}^6_{HK}$ whose numerator becomes

$$
\begin{aligned}
N &= (k+p_1+p_2+p_3)\cdot(k+p_1+p_2+p_3)\left(k^2+(p_1+p_2)^2\right) \Leftrightarrow \\
N &= k^4+2k^2k(2p_1+2p_2+p_3)+k^2[(p_1+p_2+p_3)^2 \\
&+ (p_1+p_2)^2+4(p_1+p_2+p_3)(p_1+p_2)]+2k[(p_1+p_2+p_3)^2(p_1+p_2)+(p_1+p_2)^2(p_1+p_2+p_3)] \\
&+ (p_1+p_2+p_3)^2(p_1+p_2)^2
\end{aligned} \tag{175}
$$

so putting this in the corresponding integral it gives

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^6_{HK} &= 16\cdot 8g^4\mu^{4-d}\Big\{U_{\mathcal{B}4}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ (p_1+p_2+p_3)^2(p_1+p_2)^2 D_0\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ [(p_1+p_2+p_3)^2(p_1+p_2)+(p_1+p_2)^2(p_1+p_2+p_3)]_\mu D^\mu\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ (6p_1^2+6p_2^2+6p_2\cdot p_3+p_3^2+6p_1\cdot(2p_2+p_3))g_{\mu\nu}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ g_{\mu\nu}(4p_1+4p_2+2p_3)_\alpha D^{\mu\nu\alpha}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\Big\}.
\end{aligned} \tag{176}
$$

Similar with the previous arguments, the next integral occurs by replacing $(p_1+p_2)\rightarrow p_1$. therefore we get that

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^6_{HL} &= 16\cdot 8g^4\mu^{4-d}\Big\{U_{\mathcal{B}4}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)+(p_1+p_2+p_3)^2p_1^2D_0\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ [(p_1+p_2+p_3)^2p_1+p_1^2(p_1+p_2+p_3)]_\mu D^\mu\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ (6p_1^2+6p_1\cdot(p_2+p_3)+(p_2+p_3)^2)g_{\mu\nu}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ g_{\mu\nu}(4p_1+4p_2+2p_3)_\alpha D^{\mu\nu\alpha}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\Big\}.
\end{aligned} \tag{177}
$$

Next we have the integral $\mathcal{B}^6_{HM}$ so we follow the same procedure as before, which finally gives that

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^6_{HM} &= -\frac{16\cdot 8g^4\mu^{4-d}}{m_Z^2}\Big\{ m_Z^2 U_{\mathcal{B}6}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ m_Z((4p_1+2p_2)^2+(8p_1+4p_2)(p_1+p_2+p_3))U_{\mathcal{B}5}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ (6p_1^2+6p_1\cdot p_2+p_2^2+(p_1+p_2+p_3)^2)U_{\mathcal{B}4}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ (p_1+p_2+p_3)^2(p_1+p_2)^2p_1^2 D_0\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ [2(p_1+p_2)^2p_1^2(p_1+p_2+p_3) \\
&+ 2p_1(p_1+p_2)(2p_1+p_2)(p_1+p_2+p_3)^2]_\mu D^\mu\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ ((p_1+p_2)^2p_1^2+(6p_1^2+6p_1\cdot p_2+p_2^2)(p_1+p_2+p_3)^2)g_{\mu\nu}D^{\mu\nu}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ 4[(p_1+p_2)(2p_1+p_2)]_\nu(p_1+p_2+p_3)_\mu D^{\mu\nu}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ 2g_{\mu\nu}[(p_1+p_2)p_1(2p_1+p_2)+(6p_1^2+6p_1\cdot p_2 \\
&+ p_2^2)(p_1+p_2+p_3)+(4p_1+2p_2)(p_1+p_2+p_3)^2]_\alpha D^{\mu\nu\alpha}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z)\Big\} \qquad (178)
\end{aligned}
$$

The next integral that we face is $\mathcal{B}^6_{HN}$ and its numerator has much simplier form than the previous one. This case reads

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^6_{HN} &= 16\cdot 8g^4\mu^{4-d}\Big\{ U_{\mathcal{B}4}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ (p_1+p_2+p_3)^2 g_{\mu\nu}D^{\mu\nu}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ g_{\mu\nu}(2p_1+2p_2+2p_3)_\alpha D^{\mu\nu\alpha}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z)\Big\}. \qquad (179)
\end{aligned}
$$

Now we move on to the next two integrals, namely we have $\mathcal{B}^6_{HO}$ and $\mathcal{B}^6_{HP}$ which are similar with the integral (178) but with much simpler results. To be more specific here we have that

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^6_{HO} &= -\frac{16\cdot 8g^4\mu^{4-d}}{m_Z^2}\Big\{ m_Z^2 U_{\mathcal{B}6}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ m_Z(4p_1+4p_2+2p_3)U_{\mathcal{B}5}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ [(p_1+p_2)^2+(p_1+p_2+p_3)^2]U_{\mathcal{B}4}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ (p_1+p_2)^2(p_1+p_2+p_3)^2 g_{\mu\nu}D^{\mu\nu}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ 2g_{\mu\nu}[(p_1+p_2)(p_1+p_2+p_3)^2+(p_1+p_2)^2(p_1+p_2+p_3)]_\alpha D^{\mu\nu\alpha}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z) \\
&+ g_{\mu\nu}(p_1+p_2)_\alpha(p_1+p_2+p_3)_\beta D^{\mu\nu\alpha\beta}\,(p_1,p_2,p_3,m_Z,m_Z,m_Z)\Big\} \qquad (180)
\end{aligned}
$$

and moreover we now know that $\mathcal{B}^6_{HP}$ could be obtained from $\mathcal{B}^6_{HO}$ by doing the replacement $(p_1+p_2) \to p_1$, therefore we get that

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^6_{HP} &= -\frac{16\cdot 8g^4\mu^{4-d}}{m_Z^2}\Big\{ m_Z^2 U_{\mathcal{B}6}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ m_Z(4p_1+2p_2+2p_3)U_{\mathcal{B}5}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ [p_1^2+(p_1+p_2+p_3)^2]U_{\mathcal{B}4}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ p_1^2(p_1+p_2+p_3)^2 g_{\mu\nu}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ 2g_{\mu\nu}[p_1(p_1+p_2+p_3)^2+p_1^2(p_1+p_2+p_3)]_\alpha D^{\mu\nu\alpha}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ g_{\mu\nu}p_{1\alpha}(p_1+p_2+p_3)_\beta D^{\mu\nu\alpha\beta}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\Big\}. \qquad (181)
\end{aligned}
$$

Finally, here we consider the most complicated integral from these that we have faced until now, namely the $\mathcal{B}^6_{HQ}$ integral. Its numerator occurs using (169) in the same way that we have treated the previous cases and thus we obtain that the final form of this integral becomes

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^6_{HQ} &= \frac{16\cdot 8g^4\mu^{4-d}}{m_Z^4}\Big\{ m_Z^4 U_{\mathcal{B}8}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ m_Z^3(6p_1+4p_2+2p_3)_\mu U_{\mathcal{B}7}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ m_Z^2[(6p_1^2+6p_1\cdot p_2+p_2^2)+(8p_1+4p_2)\cdot(p_1+p_2+p_3) \qquad (182) \\
&+ (p_1+p_2+p_3)^2]U_{\mathcal{B}6}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ m_Z^2[(p_1^2+p_1\cdot p_2)_\mu(2p_1+p_2)_\nu+(4p_1+2p_2)_\mu(p_1+p_2+p_3)^2_\nu \\
&+ (2p_1^2+3p_1\cdot p_2+p_2^2)_\mu(p_1+p_2+p_3)_\nu]U^{\mu\nu}_{\mathcal{B}6}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ m_Z(6p_1^2+6p_1\cdot p_2+p_2^2)(p_1+p_2+p_3)_\mu U_{\mathcal{B}5}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ [p_1^2(p_1+p_2)^2+(6p_1^2+6p_1\cdot p_2+p_2^2)(p_1+p_2+p_3)^2]U_{\mathcal{B}4}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ (p_1^2(p_1+p_2)^2(p_1+p_2+p_3)^2)g_{\mu\nu}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ 2g_{\mu\nu}[p_1\cdot(p_1+p_2)(2p_1+p_2)(p_1+p_2+p_3)^2 \\
&+ (p_1+p_2+p_3)p_1^2(p_1+p_2)^2]_\alpha D^{\mu\nu\alpha}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\Big\}. \qquad (183)
\end{aligned}
$$

where we have defined the zero-mass dimension integrals

$$U^{\mu\nu}_{\mathcal{B}6}(p_1,p_2,p_3,m_Z,m_Z,m_Z) = \int \frac{d^4k}{(2\pi)^4}\Bigg\{\frac{-ik^4k^\mu k^\nu}{m_Z^2\left(k^2-m_Z^2\right)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \times \frac{1}{((k+p_1+p_2+p_3)^2-m_Z^2)}\Bigg\} \quad (184)$$

$$U_{\mathcal{B}7}(p_1,p_2,p_3,m_Z,m_Z,m_Z) = \int \frac{d^4k}{(2\pi)^4}\Bigg\{\frac{-ik^6k^\mu}{m_Z^3\left(k^2-m_Z^2\right)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \times \frac{1}{((k+p_1+p_2+p_3)^2-m_Z^2)}\Bigg\} \quad (185)$$

and

$$U_{\mathcal{B}8}(p_1,p_2,p_3,m_Z,m_Z,m_Z) = \int \frac{d^4k}{(2\pi)^4}\Bigg\{\frac{-ik^8}{m_Z^4\left(k^2-m_Z^2\right)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \times \frac{1}{((k+p_1+p_2+p_3)^2-m_Z^2)}\Bigg\}. \quad (186)$$

In order to move on we should sum up all the sixteen results which we have found and which constitute the box diagram (156). In addition we can use the similar notation with that used in the case of the Triangle diagrams so as to obtain a general and compactified form for the current diagram. Moreover here it should be cleared that we use a similar calculation technic with that of the evaluation of the U-integrals in App. D. To be more specific, in the calculation of the $\mathcal{B}_H^6$ we deal with highly divergent integrals which in this diagram correspond to the $D^{\mu\nu\rho}$, $D^{\mu\nu\rho\alpha}$ and $D^{\mu\nu\rho\alpha\beta}$ terms. Thus, these cases are treated using the following relations

$$\left\langle\frac{k^2k^\mu\cdots k^\alpha}{d_1d_2d_3d_4}\right\rangle = \left\langle\frac{\left(k^2-m_Z^2\right)k^\mu\cdots k^\alpha}{d_1d_2d_3d_4}\right\rangle + \left\langle\frac{m_Z^2k^\mu\cdots k^\alpha}{d_1d_2d_3d_4}\right\rangle \quad (187)$$

$$\left\langle\frac{p_i\cdot kk^\mu\cdots k^\alpha}{d_1d_2d_3d_4}\right\rangle = \left\langle\frac{\frac{1}{2}\left(f_i+d_{i+1}-d_i\right)k^\mu\cdots k^\alpha}{d_1d_2d_3d_4}\right\rangle \quad (188)$$

where we have used the equation (478) from the Appendix .
Therefore, considering the above equations each time that we face the corresponding highly divergent integrals, (156) reads

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}_H^6 &= 16\cdot 8g^4\mu^{4-d}\Bigg\{U_{\mathcal{B}8}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
&+ \Big\{2p_{1\mu}+2P_{1\mu}+2P_{2\mu}\Big\}U_{\mathcal{B}7}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
&+ \Big\{-4+\frac{p_1\cdot P_1}{m_Z^2}+\frac{P_1\cdot P_2}{m_Z^2}\Big\}U_{\mathcal{B}6}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
&+ \Big\{\frac{4p_{1\mu}P_{2\nu}}{m_Z^2}+\frac{3P_{1\mu}P_{2\nu}}{m_Z^2}+\frac{P_{2\mu}P_{2\nu}}{m_Z^2}\Big\}U_{\mathcal{B}6}^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
&+ \Big\{-\frac{6p_{1\mu}}{m_Z}-\frac{6P_{1\mu}}{m_Z}-\frac{6P_{2\mu}}{m_Z}+\frac{p_1\cdot P_1}{m_Z^3}p_{1\mu}+\frac{2p_1\cdot P_1}{m_Z^3}P_{2\mu}+\frac{2P_1\cdot P_2}{m_Z^3}p_{1\mu}\\
&+ \frac{P_1\cdot P_2}{m_Z^3}P_{1\mu}+\frac{P_1\cdot P_2}{m_Z^3}P_{2\mu}\Big\}U_{\mathcal{B}5}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
&+ \Big\{6-\frac{2p_1\cdot P_1}{m_Z^2}-\frac{2p_1\cdot P_2}{m_Z^2}-\frac{2P_1\cdot P_2}{m_Z^2}+\frac{p_1\cdot P_2P_1\cdot P_2}{m_Z^4}\Big\}U_{\mathcal{B}4}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
&+ \Big\{4m_Z^4-m_Z^2p_1^2+(p_1\cdot P_1)^2+(p_1\cdot P_2)^2-m_Z^2P_1\cdot P_1-\frac{p_1\cdot P_2P_1\cdot P_2p_1\cdot P_1}{m_Z^2}\\
&+ (P_1\cdot P_2)^2-m_Z^2P_2\cdot P_2\Big\}D_0\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
&+ \Big(-2m_Z^2p_{1\mu}-2m_Z^2P_{1\mu}-2m_Z^2P_{2\mu}+2p_1\cdot P_1P_{1\mu}\Big)D^{\mu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
&+ \Big(-4m_Z^2g_{\mu\nu}+3p_{1\mu}p_{1\nu}+2p_{1\mu}P_{1\nu}+3P_{1\mu}P_{1\nu}+2p_{1\mu}P_{2\nu}\Big)D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
&+ \Big\{6g_{\mu\nu}-\frac{6}{m_Z^2}p_{1\mu}P_{1\nu}-\frac{3P_{1\mu}P_{1\nu}}{m_Z^2}-\frac{6}{m_Z^2}p_{1\mu}P_{2\nu}\\
&- \frac{6}{m_Z^2}P_{1\mu}P_{2\nu}-\frac{3P_{2\mu}P_{2\nu}}{m_Z^2}\Big\}\Big(m_Z^2D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)+C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big)\\
&+ \Big\{6P_{2\mu}-\frac{2p_1\cdot P_1}{m_Z^2}p_{1\mu}\\
&- \frac{2p_1\cdot P_1}{m_Z^2}P_{1\mu}\Big\}\Big(m_Z^2D^{\mu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)+C^{\mu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big)\\
&+ \Big\{6g_{\mu\nu}+\frac{p_{1\mu}P_{1\nu}}{m_Z^2}-\frac{P_{1\mu}P_{1\nu}}{m_Z^2}-\frac{2P_{2\mu}P_{2\nu}}{m_Z^2}\Big\}\Big\{\frac{f_1}{2}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)
\end{aligned}
$$

$$
\begin{aligned}
+ &\quad \frac{1}{2}C^{\mu\nu}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)-\frac{1}{2}C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
+ &\quad \Big\{-2\frac{p_{1\mu}p_{1\nu}}{m_Z^2}-2\frac{P_{1\mu}P_{2\nu}}{m_Z^2}+\frac{P_{1\mu}P_{2\nu}}{m_Z^4}\frac{f_1}{2}-\frac{P_{2\mu}P_{2\nu}}{m_Z^2}+\frac{P_{2\mu}P_{2\nu}}{m_Z^4}\frac{f_1}{2}\Big\}\Big\{\frac{f_1}{2}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
- &\quad \frac{1}{2}C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)+\frac{f_2}{2}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
+ &\quad \frac{1}{2}C^{\mu\nu}_{\{1,2,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
+ &\quad \Big\{-\frac{3g_{\mu\nu}}{m_Z^2}\Big\}\Big\{\frac{f_1^2}{4}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)+\frac{f_1}{4}C^{\mu\nu}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
- &\quad \frac{f_1}{4}C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)+\frac{p_{1\rho}}{2}C^{\mu\nu\rho}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)-\frac{p_{1\rho}}{2}C^{\mu\nu\rho}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
+ &\quad \Big\{\frac{p_{1\mu}p_{1\nu}P_{1\rho}}{m_Z^4}+\frac{p_{1\mu}P_{1\nu}P_{1\rho}}{m_Z^4}\\
+ &\quad \frac{2p_{1\mu}p_{1\nu}P_{2\rho}}{m_Z^4}+\frac{p_{1\mu}P_{1\nu}P_{1\rho}}{m_Z^4}+\frac{4p_{1\mu}P_{1\nu}P_{2\rho}}{m_Z^4}+\frac{4P_{2\mu}P_{2\nu}p_{1\rho}}{m_Z^4}+\frac{P_{2\mu}P_{2\nu}P_{1\rho}}{m_Z^4}\Big\}C^{\mu\nu\rho}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
+ &\quad \Big\{-\frac{2p_{1\mu}P_{2\nu}}{m_Z^2}+\frac{2p_{1\mu}P_{2\nu}}{m_Z^2}\Big\}\Big\{\frac{f_1}{2}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
+ &\quad \frac{1}{2}C^{\mu\nu}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)-\frac{1}{2}C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
+ &\quad \Big\{\frac{2P_{1\mu}P_{2\nu}}{m_Z^2}+\frac{2P_{2\mu}P_{2\nu}}{m_Z^2}+\frac{P_{2\mu}P_{2\nu}f_1}{2m_Z^4}+\frac{p_1\cdot P_1}{m_Z^4}P_{1\mu}P_{2\nu}+\frac{p_1\cdot P_1}{m_Z^4}P_{2\mu}P_{2\nu}+\frac{P_1\cdot P_2}{m_Z^4}p_{1\mu}P_{2\nu}\\
+ &\quad \frac{P_1\cdot P_2}{m_Z^4}P_{1\mu}P_{2\nu}\Big\}\Big\{\frac{f_1}{2}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
+ &\quad \frac{1}{2}C^{\mu\nu}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)-\frac{1}{2}C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
+ &\quad \Big\{\frac{P_{1\mu}P_{2\nu}}{m_Z^4}\Big\}\Big\{\frac{f_1^2}{4}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)+\frac{f_1}{4}C^{\mu\nu}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
- &\quad \frac{f_1}{4}C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
+ &\quad \Big\{\frac{P_{1\mu}P_{1\nu}p_{1\rho}}{m_Z^4}+\frac{P_{1\mu}P_{1\nu}P_{2\rho}}{m_Z^4}\\
+ &\quad \frac{P_{2\mu}P_{2\nu}p_{1\rho}}{m_Z^4}+\frac{P_{2\mu}P_{2\nu}P_{1\rho}}{m_Z^4}\Big\}\Big\{\frac{1}{2}C^{\mu\nu\rho}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)-\frac{1}{2}C^{\mu\nu\rho}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \Big\{2P_{1\mu}P_{2\nu} + 3P_{2\mu}P_{2\nu} + 2p_1\cdot P_1 g_{\mu\nu} - \frac{2p_1\cdot P_1}{m_Z^2}p_{1\mu}P_{1\nu} - \frac{p_1\cdot P_1}{m_Z^2}p_{1\mu}P_{2\nu} \\
&- \quad \frac{p_1\cdot P_1}{m_Z^2}P_{1\mu}P_{2\nu} - \frac{p_1\cdot P_1}{m_Z^2}P_{2\mu}P_{2\nu} + 2p_1\cdot P_1 g_{\mu\nu} - \frac{p_1\cdot P_2 p_{1\mu}P_{1\nu}}{m_Z^2} \\
&- \quad \frac{p_1\cdot P_2 P_{1\mu}P_{1\nu}}{m_Z^2} - \frac{2p_1\cdot P_2}{m_Z^2}p_{1\mu}P_{2\nu} - \frac{p_1\cdot P_2}{m_Z^2}P_{1\mu}P_{2\nu} - \frac{p_1\cdot P_2 p_1\cdot P_1}{m_Z^2}g_{\mu\nu} + 2P_1\cdot P_2 g_{\mu\nu} - \frac{P_1\cdot P_2}{m_Z^2}p_{1\mu}p_{1\nu} \\
&- \quad \frac{P_1\cdot P_2}{m_Z^2}p_{1\mu}P_{1\nu} - \frac{P_1\cdot P_2}{m_Z^2}p_{1\mu}P_{2\nu} - \frac{2P_1\cdot P_2}{m_Z^2}P_{1\mu}P_{2\nu} - \frac{p_1\cdot P_2 P_1\cdot P_2}{m_Z^2}g_{\mu\nu} \\
&+ \quad \frac{p_1\cdot P_1 P_1\cdot P_2}{m_Z^4}p_{1\mu}P_{2\nu} - \frac{p_1\cdot P_2 P_1\cdot P_2}{m_Z^2}g_{\mu\nu}\Big\} D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ \quad \Big\{2p_1\cdot P_1 p_{1\mu} + 2p_1\cdot P_1 p_{1\mu} + \frac{2p_1\cdot P_2 P_{2\mu}}{m_Z^2} - \frac{p_1\cdot P_2 p_1\cdot P_1}{m_Z^2}P_{1\mu} - \frac{p_1\cdot P_2 p_1\cdot P_1}{m_Z^2}gP_{2\mu} + 2P_1\cdot P_2 P_{1\mu} \\
&+ \quad 2P_1\cdot P_2 P_{2\mu} - \frac{p_1\cdot P_2 P_1\cdot P_2}{m_Z^2}p_{1\mu} - \frac{p_1\cdot P_1 P_1\cdot P_2}{m_Z^2}P_{2\mu} \\
&- \quad \frac{p_1\cdot P_2 P_1\cdot P_2}{m_Z^2}p_{1\mu} - \frac{p_1\cdot P_2 P_1\cdot P_2}{m_Z^2}P_{1\mu}\Big\} D^{\mu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ \quad \Big\{\frac{p_1\cdot P_1}{m_Z^4}p_{1\mu}P_{1\nu} + \frac{2p_1\cdot P_1}{m_Z^2}p_{1\mu}P_{2\nu} + \frac{p_1\cdot P_1}{m_Z^4}P_{1\mu}P_{2\nu} + \frac{p_1\cdot P_1}{m_Z^4}P_{2\mu}P_{2\nu} + \frac{P_1\cdot P_2}{m_Z^4}p_{1\mu}p_{1\nu} \\
&+ \quad \frac{P_1\cdot P_2}{m_Z^4}p_{1\mu}P_{1\nu} + \frac{2P_1\cdot P_2}{m_Z^4}p_{1\mu}P_{2\nu} \\
&+ \quad \frac{P_1\cdot P_2}{m_Z^4}P_{1\mu}P_{2\nu}\Big\}\left(m_Z^2 D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) + C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\right) \\
&+ \quad \Big\{-\frac{2p_1\cdot P_1}{m_Z^2}P_{2\mu} - \frac{2p_1\cdot P_1 p_{1\mu}}{m_Z^2} - \frac{2p_1\cdot P_1 P_{1\mu}}{m_Z^2} - \frac{2p_1\cdot P_2 P_{2\nu}}{m_Z^2} - \frac{2P_1\cdot P_2}{m_Z^2}p_{1\mu} - \frac{2P_1\cdot P_2}{m_Z^2}P_{1\mu} \\
&- \quad \frac{2P_1\cdot P_2}{m_Z^2}P_{1\nu} + \frac{p_1\cdot P_1 P_1\cdot P_2}{m_Z^4}p_{1\mu} \\
&+ \quad \frac{p_1\cdot P_1 P_1\cdot P_2}{m_Z^4}P_{2\mu}\Big\}\left(m_Z^2 D^{\mu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) + C^{\mu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\right)\Bigg\}.
\end{aligned}
\tag{189}
$$

Finally, in order to see the complete quantum corrections of the one-loop four-point diagrams to the Higgs quartic coupling, we should add all the results that we have presented here. In particular, we should sum the results from equations (139), (142), (148), (151), (154) and (189). Therefore, the final result coming from the box diagrams reads

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}_H &= \mu^{4-d}\Big\{216\cdot 3^3\cdot 16\lambda^3 v_0^2 C_0\left(p_1,p_2,m_H,m_H,m_H\right) \\
&+ 8\cdot 8g^4\Big\{\left(4m_Z^2-(2p_1^2+2p_1\cdot p_2+p_2^2)+\frac{p_1^2\left(p_1+p_2\right)^2}{m_Z^2}\right)C_0\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&+ [\frac{(2p_1^2+3p_1\cdot p_2+p_1p_2^2)}{m_Z}-4p_1-2p_2]_\mu C^\mu\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&+ \left(-2+\frac{7p_1^2+6p_1\cdot p_2+p_2^2+(p_1+p_2)^2}{m_Z^2}-\frac{p_1^2(p_1+p_2)^2}{m_Z^4}\right)g_{\mu\nu}C^{\mu\nu}\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&+ g_{\mu\nu}[\frac{(8p_1+3p_2)}{m_Z^2}-\frac{(4p_1^2+6p_1\cdot p_2+2p_2^2)p_1}{m_Z^4}]_\rho C^{\mu\nu\rho}\left(p_1,p_2,m_Z,m_Z,m_Z\right) \\
&+ \left(3-\frac{6{p_1}^2+6p_2\cdot p_1+{p_2}^2}{m_Z^2}\right)U_{\mathcal{K}4}(p_1,p_2,m_Z,m_Z,m_Z) \\
&- U_{\mathcal{K}6}(p_1,p_2,m_Z,m_Z,m_Z)-\frac{(4p_1+2p_2)_\mu}{m_Z}U_{\mathcal{K}5}(p_1,p_2,m_Z,m_Z,m_Z)\Big\} \\
&+ 36\cdot 6\cdot 48\lambda^2(\delta^{ij}\delta^{kl}+\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk})B_0(p_1,m_H,m_H) \\
&+ 4\cdot 8(\delta^{ij}\delta^{kl}+\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk})\Big\{(4g^4-\frac{g^4p^2}{m_Z^2})B_0(p_1,m_Z,m_Z)-g^4U_{\mathcal{M}4}(p_1,p_2,m_Z,m_Z) \\
&+ \frac{g^4}{m_Z^4}p_\mu p_\nu B^{\mu\nu}(p_1,m_Z,m_Z)-\frac{2g^4}{m_Z^2}g_{\mu\nu}B^{\mu\nu}(p_1,m_Z,m_Z)\Big\} \\
&+ 6^4\cdot 81\cdot 8\lambda^4 v_0^4 D_0\left(p_1,p_2,p_3,m_H,m_H,m_H,m_H\right) \\
&+ 16\cdot 8g^4\Big\{U_{\mathcal{B}8}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ \Big\{2p_{1\mu}+2P_{1\mu}+2P_{2\mu}\Big\}U_{\mathcal{B}7}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ \Big\{-4+\frac{p_1\cdot P_1}{m_Z^2}+\frac{P_1\cdot P_2}{m_Z^2}\Big\}U_{\mathcal{B}6}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ \Big\{\frac{4p_{1\mu}P_{2\nu}}{m_Z^2}+\frac{3P_{1\mu}P_{2\nu}}{m_Z^2}+\frac{P_{2\mu}P_{2\nu}}{m_Z^2}\Big\}U^{\mu\nu}_{\mathcal{B}6}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ \Big\{-\frac{6p_{1\mu}}{m_Z}-\frac{6P_{1\mu}}{m_Z}-\frac{6P_{2\mu}}{m_Z}+\frac{p_1\cdot P_1}{m_Z^3}p_{1\mu}+\frac{2p_1\cdot P_1}{m_Z^3}P_{2\mu}+\frac{2P_1\cdot P_2}{m_Z^3}p_{1\mu} \\
&+ \frac{P_1\cdot P_2}{m_Z^3}P_{1\mu}+\frac{P_1\cdot P_2}{m_Z^3}P_{2\mu}\Big\}U_{\mathcal{B}5}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ \Big\{6-\frac{2p_1\cdot P_1}{m_Z^2}-\frac{2p_1\cdot P_2}{m_Z^2}-\frac{2P_1\cdot P_2}{m_Z^2}+\frac{p_1\cdot P_2P_1\cdot P_2}{m_Z^4}\Big\}U_{\mathcal{B}4}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) \\
&+ \Big\{4m_Z^4-m_Z^2p_1^2+(p_1\cdot P_1)^2+(p_1\cdot P_2)^2-m_Z^2P_1\cdot P_1-\frac{p_1\cdot P_2P_1\cdot P_2p_1\cdot P_1}{m_Z^2}
\end{aligned}
$$

$$
\begin{aligned}
+ &\quad (P_1\cdot P_2)^2 - m_Z^2 P_2\cdot P_2\Big\} D_0\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
+ &\quad \Big(-2m_Z^2 p_{1\mu} - 2m_Z^2 P_{1\mu} - 2m_Z^2 P_{2\mu} + 2p_1\cdot P_1 P_{1\mu}\Big) D^{\mu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
+ &\quad \Big(-4m_Z^2 g_{\mu\nu} + 3p_{1\mu}p_{1\nu} + 2p_{1\mu}P_{1\nu} + 3P_{1\mu}P_{1\nu} + 2p_{1\mu}P_{2\nu}\Big) D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
+ &\quad \Big\{6g_{\mu\nu} - \frac{6}{m_Z^2}p_{1\mu}P_{1\nu} - \frac{3P_{1\mu}P_{1\nu}}{m_Z^2} - \frac{6}{m_Z^2}p_{1\mu}P_{2\nu}\\
- &\quad \frac{6}{m_Z^2}P_{1\mu}P_{2\nu} - \frac{3P_{2\mu}P_{2\nu}}{m_Z^2}\Big\}\Big(m_Z^2 D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) + C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big)\\
+ &\quad \Big\{6P_{2\mu} - \frac{2p_1\cdot P_1}{m_Z^2}p_{1\mu}\\
- &\quad \frac{2p_1\cdot P_1}{m_Z^2}P_{1\mu}\Big\}\Big(m_Z^2 D^{\mu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) + C^{\mu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big)\\
+ &\quad \Big\{6g_{\mu\nu} + \frac{p_{1\mu}P_{1\nu}}{m_Z^2} - \frac{P_{1\mu}P_{1\nu}}{m_Z^2} - \frac{2P_{2\mu}P_{2\nu}}{m_Z^2}\Big\}\Big\{\frac{f_1}{2}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
+ &\quad \frac{1}{2}C^{\mu\nu}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right) - \frac{1}{2}C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
+ &\quad \Big\{-2\frac{p_{1\mu}p_{1\nu}}{m_Z^2} - 2\frac{P_{1\mu}P_{2\nu}}{m_Z^2} + \frac{P_{1\mu}P_{2\nu}}{m_Z^4}\frac{f_1}{2} - \frac{P_{2\mu}P_{2\nu}}{m_Z^2} + \frac{P_{2\mu}P_{2\nu}}{m_Z^4}\frac{f_1}{2}\Big\}\Big\{\frac{f_1}{2}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
- &\quad \frac{1}{2}C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right) + \frac{f_2}{2}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
+ &\quad \frac{1}{2}C^{\mu\nu}_{\{1,2,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
+ &\quad \Big\{-\frac{3g_{\mu\nu}}{m_Z^2}\Big\}\Big\{\frac{f_1^2}{4}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right) + \frac{f_1}{4}C^{\mu\nu}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
- &\quad \frac{f_1}{4}C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right) + \frac{p_{1\rho}}{2}C^{\mu\nu\rho}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right) - \frac{p_{1\rho}}{2}C^{\mu\nu\rho}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
+ &\quad \Big\{\frac{p_{1\mu}p_{1\nu}P_{1\rho}}{m_Z^4} + \frac{p_{1\mu}P_{1\nu}P_{1\rho}}{m_Z^4}\\
+ &\quad \frac{2p_{1\mu}p_{1\nu}P_{2\rho}}{m_Z^4} + \frac{p_{1\mu}P_{1\nu}P_{1\rho}}{m_Z^4} + \frac{4p_{1\mu}P_{1\nu}P_{2\rho}}{m_Z^4} + \frac{4P_{2\mu}P_{2\nu}p_{1\rho}}{m_Z^4} + \frac{P_{2\mu}P_{2\nu}P_{1\rho}}{m_Z^4}\Big\}C^{\mu\nu\rho}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
+ &\quad \Big\{-\frac{2p_{1\mu}P_{2\nu}}{m_Z^2} + \frac{2p_{1\mu}P_{2\nu}}{m_Z^2}\Big\}\Big\{\frac{f_1}{2}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
+ &\quad \frac{1}{2}C^{\mu\nu}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right) - \frac{1}{2}C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
+ &\quad \Big\{\frac{2P_{1\mu}P_{2\nu}}{m_Z^2} + \frac{2P_{2\mu}P_{2\nu}}{m_Z^2} + \frac{P_{2\mu}P_{2\nu}f_1}{2m_Z^4} + \frac{p_1\cdot P_1}{m_Z^4}P_{1\mu}P_{2\nu} + \frac{p_1\cdot P_1}{m_Z^4}P_{2\mu}P_{2\nu} + \frac{P_1\cdot P_2}{m_Z^4}p_{1\mu}P_{2\nu}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{P_1\cdot P_2}{m_Z^4}P_{1\mu}P_{2\nu}\Big\}\Big\{\frac{f_1}{2}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
&+ \quad \frac{1}{2}C^{\mu\nu}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)-\frac{1}{2}C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
&+ \quad \Big\{\frac{P_{1\mu}P_{2\nu}}{m_Z^4}\Big\}\Big\{\frac{f_1^2}{4}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)+\frac{f_1}{4}C^{\mu\nu}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\\
&- \quad \frac{f_1}{4}C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
&+ \quad \Big\{\frac{P_{1\mu}P_{1\nu}p_{1\rho}}{m_Z^4}+\frac{P_{1\mu}P_{1\nu}P_{2\rho}}{m_Z^4}\\
&+ \quad \frac{P_{2\mu}P_{2\nu}p_{1\rho}}{m_Z^4}+\frac{P_{2\mu}P_{2\nu}P_{1\rho}}{m_Z^4}\Big\}\Big\{\frac{1}{2}C^{\mu\nu\rho}_{\{1,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)-\frac{1}{2}C^{\mu\nu\rho}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\Big\}\\
&+ \quad \Big\{2P_{1\mu}P_{2\nu}+3P_{2\mu}P_{2\nu}+2p_1\cdot P_1g_{\mu\nu}-\frac{2p_1\cdot P_1}{m_Z^2}p_{1\mu}P_{1\nu}-\frac{p_1\cdot P_1}{m_Z^2}p_{1\mu}P_{2\nu}\\
&- \quad \frac{p_1\cdot P_1}{m_Z^2}P_{1\mu}P_{2\nu}-\frac{p_1\cdot P_1}{m_Z^2}P_{2\mu}P_{2\nu}+2p_1\cdot P_1g_{\mu\nu}-\frac{p_1\cdot P_2p_{1\mu}P_{1\nu}}{m_Z^2}\\
&- \quad \frac{p_1\cdot P_2P_{1\mu}P_{1\nu}}{m_Z^2}-\frac{2p_1\cdot P_2}{m_Z^2}p_{1\mu}P_{2\nu}-\frac{p_1\cdot P_2}{m_Z^2}P_{1\mu}P_{2\nu}-\frac{p_1\cdot P_2p_1\cdot P_1}{m_Z^2}g_{\mu\nu}+2P_1\cdot P_2g_{\mu\nu}-\frac{P_1\cdot P_2}{m_Z^2}p_{1\mu}p_{1\nu}\\
&- \quad \frac{P_1\cdot P_2}{m_Z^2}p_{1\mu}P_{1\nu}-\frac{P_1\cdot P_2}{m_Z^2}p_{1\mu}P_{2\nu}-\frac{2P_1\cdot P_2}{m_Z^2}P_{1\mu}P_{2\nu}-\frac{p_1\cdot P_2P_1\cdot P_2}{m_Z^2}g_{\mu\nu}\\
&+ \quad \frac{p_1\cdot P_1P_1\cdot P_2}{m_Z^4}p_{1\mu}P_{2\nu}-\frac{p_1\cdot P_2P_1\cdot P_2}{m_Z^2}g_{\mu\nu}\Big\}D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
&+ \quad \Big\{2p_1\cdot P_1p_{1\mu}+2p_1\cdot P_1p_{1\mu}+\frac{2p_1\cdot P_2P_{2\mu}}{m_Z^2}-\frac{p_1\cdot P_2p_1\cdot P_1}{m_Z^2}P_{1\mu}-\frac{p_1\cdot P_2p_1\cdot P_1}{m_Z^2}gP_{2\mu}+2P_1\cdot P_2P_{1\mu}\\
&+ \quad 2P_1\cdot P_2P_{2\mu}-\frac{p_1\cdot P_2P_1\cdot P_2}{m_Z^2}p_{1\mu}-\frac{p_1\cdot P_1P_1\cdot P_2}{m_Z^2}P_{2\mu}\\
&- \quad \frac{p_1\cdot P_2P_1\cdot P_2}{m_Z^2}p_{1\mu}-\frac{p_1\cdot P_2P_1\cdot P_2}{m_Z^2}P_{1\mu}\Big\}D^{\mu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)\\
&+ \quad \Big\{\frac{p_1\cdot P_1}{m_Z^4}p_{1\mu}P_{1\nu}+\frac{2p_1\cdot P_1}{m_Z^2}p_{1\mu}P_{2\nu}+\frac{p_1\cdot P_1}{m_Z^4}P_{1\mu}P_{2\nu}+\frac{p_1\cdot P_1}{m_Z^4}P_{2\mu}P_{2\nu}+\frac{P_1\cdot P_2}{m_Z^4}p_{1\mu}p_{1\nu}\\
&+ \quad \frac{P_1\cdot P_2}{m_Z^4}p_{1\mu}P_{1\nu}+\frac{2P_1\cdot P_2}{m_Z^4}p_{1\mu}P_{2\nu}\\
&+ \quad \frac{P_1\cdot P_2}{m_Z^4}P_{1\mu}P_{2\nu}\Big\}\left(m_Z^2D^{\mu\nu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)+C^{\mu\nu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\right)\\
&+ \quad \Big\{-\frac{2p_1\cdot P_1}{m_Z^2}P_{2\mu}-\frac{2p_1\cdot P_1p_{1\mu}}{m_Z^2}-\frac{2p_1\cdot P_1P_{1\mu}}{m_Z^2}-\frac{2p_1\cdot P_2P_{2\nu}}{m_Z^2}-\frac{2P_1\cdot P_2}{m_Z^2}p_{1\mu}-\frac{2P_1\cdot P_2}{m_Z^2}P_{1\mu}\\
&- \quad \frac{2P_1\cdot P_2}{m_Z^2}P_{1\nu}+\frac{p_1\cdot P_1P_1\cdot P_2}{m_Z^4}p_{1\mu}\\
&+ \quad \frac{p_1\cdot P_1P_1\cdot P_2}{m_Z^4}P_{2\mu}\Big\}\left(m_Z^2D^{\mu}\left(p_1,p_2,p_3,m_Z,m_Z,m_Z\right)+C^{\mu}_{\{2,3,4\}}\left(p_1,p_2,m_Z,m_Z,m_Z\right)\right)\Big\}\Bigg\}. \qquad (190)
\end{aligned}
$$

Since we have obtained the full one-loop corrections to the Higgs four-point function,

we should proceed similarly with the previous two subsections. To be more specific, we should reduce the result with the help of the scalar integrals. Here we have to consider six Box-diagrams, but as we have already mentioned in this section, the first four are connected with the corresponding Triangle-diagrams. Therefore we can use their relation in order to obtain the reduced form of this set of Box-diagrams. The remaining two, which are not included in the previous case, have been treated separately.
An interesting point that could be noticed here is that the reduced form of $\mathcal{B}_H$ depends on the diagram kinematics. This result plays a crucial role to the understanding of the physical quantities as we will explain in the section where we obtain the physical quartic coupling. In this section this dependence just makes our results very complicated and difficult to be written. So we choose, only for now, a specific allowed value for the kinematics, namely $c_s = 1$ and $c_t = 1$, and therefore the reduced form of $\mathcal{B}_H$ reads

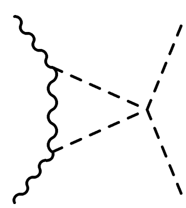
$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}_H &= \mu^{4-d}\Bigg\{ 48g^4 + (-\frac{744g^4m_H^2}{m_Z^4} - \frac{384g^4}{m_Z^2})A_0(m_Z) \\
&+ (-1600g^4m_H^2 + \frac{16g^4m_H^6}{3m_Z^4} + \frac{1408g^4m_H^4}{3m_Z^2} + 1152g^4m_Z^2)C_0[m_Z,m_Z,m_Z] \\
&+ (\frac{96g^4m_H^6}{m_Z^2} + 1152g^4m_H^2m_Z^2 + 3072g^4m_Z^4)D_0[m_Z,m_Z,m_Z,m_Z] \\
&+ 31104\lambda^2B_0[m_H,m_H,0,m_H] + 559872v_0^2\lambda^3C_0[m_H,m_H,m_H] \\
&+ 2519424v_0^4\lambda^4D_0[m_H,m_H,m_H,m_H] \\
&+ (-32g^4 + \frac{8g^4m_H^4}{3m_Z^4} + \frac{128g^4m_H^2}{3m_Z^2} - \frac{512g^4m_Z^2}{m_H^2})B_0[m_H,m_H,0,m_H] \\
&+ (192g^4 + \frac{24g^4m_H^4}{m_Z^4} - \frac{96g^4m_H^2}{m_Z^2})B_0[m_H,m_H,0,m_H] \\
&+ (-416g^4 - \frac{184g^4m_H^4}{3m_Z^4} + \frac{176g^4m_H^2}{3m_Z^2})B_0[m_H,m_Z,0,m_H] \\
&+ (-1472g^4 + \frac{1088g^4m_H^2}{m_Z^2})B_0[m_H,m_Z,0,m_H] \\
&+ (-192g^4 + \frac{56g^4m_H^2}{m_Z^2} - \frac{128g^4m_Z^2}{m_H^2})B_0[m_H,m_Z,0,m_H] \\
&+ (-\frac{3008g^4}{3} + \frac{1592g^4m_H^4}{3m_Z^4} - \frac{5944g^4m_H^2}{3m_Z^2} + \frac{128g^4m_Z^2}{m_H^2})B_0[m_H,m_Z,m_H,m_H] \\
&+ (\frac{8992g^4}{3} - \frac{24g^4m_H^4}{m_Z^4} - \frac{208g^4m_H^2}{m_Z^2} + \frac{384g^4m_Z^2}{m_H^2})B_0[m_H,m_Z,m_H,m_H]
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad (2944g^4 + \frac{1280g^4m_H^2}{m_Z^2})B_0[m_Z, m_Z, m_H, m_H] \\
&+ \quad (-\frac{4736g^4}{3} - \frac{756g^4m_H^4}{m_Z^4} - \frac{4024g^4m_H^2}{m_Z^2} + \frac{128g^4m_Z^2}{m_H^2}B_0[m_Z, m_Z, m_H, m_H] \\
&+ \quad (160g^4m_H^2 + \frac{104g^4m_H^4}{m_Z^2} - 512g^4m_Z^2)C_0[m_H, m_H, m_Z, 0, m_H, m_H] \\
&+ \quad (-\frac{72g^4m_H^4}{m_Z^2} - 384g^4m_Z^2)C_0[m_H, m_H, m_Z, 0, m_H, m_H] \\
&+ \quad (3904g^4m_H^2 - \frac{1216g^4m_H^4}{m_Z^2})C_0[m_H, m_Z, m_Z, 0, m_H, m_H] \\
&+ \quad (-224g^4m_H^2 - \frac{344g^4m_H^4}{m_Z^2} - 384g^4m_Z^2)C_0[m_H, m_Z, m_Z, 0, m_H, m_H] \\
&+ \quad (-2912g^4m_H^2 - \frac{80g^4m_H^6}{m_Z^4} + \frac{5464g^4m_H^4}{3m_Z^2} + 4736g^4m_Z^2)C_0[m_H, m_Z, m_Z, m_H, m_H, m_H]\Bigg\}
\end{aligned}
\tag{191}
$$

where we have defined and used that L is the inverse of the determinant of the G_3 matrix which reads

$$
L \quad = \quad \frac{1}{\det G_3} = \frac{-4}{m_H^6} \tag{192}
$$

Before we move on to the renormalization of the Abelian Higgs model that we study here, we should consider the contribution of the one-loop four point functions with two external gauge bosons, to the gauge coupling g. Here, the procedure that we follow is quite the same with that of the Higgs Box diagrams, thus we can start with the first contribution coming from the combination of the square of $\phi A_\mu A_\nu$ vertex with the ϕ^4 vertex. The resulting diagram is

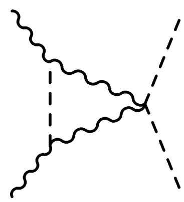


$$
= i\mathcal{B}^1_{HZ,\mu\nu} \tag{193}
$$

where we can notice that putting on of the external Higgs legs equal to v_0 we get the result of diagram (121). Therefore we can write the following relation

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^1_{HZ,\mu\nu} &= (4\pi)^{d/2}\mu^{4-d}\frac{\mathcal{K}^1_{HZ,\mu\nu}}{v_0} \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{B}^1_{HZ,\mu\nu} &= 24\sigma^1_{\mathcal{K}HZ}g^2\lambda\mu^{4-d}\Bigg\{-g^{\mu\nu}m_Z^2C_0(p_1,p_2,m_Z,m_H,m_H)+C^{\mu\nu}(p_1,p_2,m_Z,m_H,m_H)\Bigg\}.
\end{aligned}
\tag{194}
$$

The next integral which we consider here comes from the square of $\phi A_\mu A_\nu$ vertex with the $\phi^2 A_\mu A_\nu$ vertex and it reads

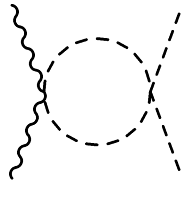


$$= i\mathcal{B}^2_{HZ,\mu\nu} \tag{195}$$

therefore following the same reasoning with the previous diagram, namely comparing $\mathcal{B}^2_{HZ,\mu\nu}$ with (126) we get that

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^2_{HZ,\mu\nu} &= (4\pi)^{d/2}\mu^{4-d}\frac{\mathcal{K}^2_{HZ,\mu\nu}}{v_0} \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{B}^2_{HZ,\mu\nu} &= \frac{8\sigma^2_{\mathcal{K}HZ}g^4\mu^{d-4}}{v_0}\Bigg\{-m_Z^2g^{\mu\nu}C_0(p_1,p_2,m_Z,m_H,m_Z)+2C^{\mu\nu}(p_1,p_2,m_Z,m_H,m_Z) \\
&+ (p_1+p_2)^\nu C^\mu(p_1,p_2,m_Z,m_H,m_Z) \\
&+ (p_1+p_2)^\mu C^\nu(p_1,p_2,m_Z,m_H,m_Z)+(p_1+p_2)^\mu(p_1+p_2)^\nu C_0(p_1,p_2,m_Z,m_H,m_Z) \\
&- U^{\mu\nu}_{\mathcal{K}4}-\frac{(p_1+p_2)_\rho}{m_Z^2}C^{\mu\nu\rho}(p_1,p_2,m_Z,m_H,m_Z)-\frac{(p_1+p_2)_\nu}{m_Z^2}C^{\mu\rho\sigma}(p_1,p_2,m_Z,m_H,m_Z) \\
&- \frac{(p_1+p_2)_\nu(p_1+p_2)_\rho}{m_Z^2}C^{\mu\rho}(p_1,p_2,m_Z,m_H,m_Z)\Bigg\}
\end{aligned}
\tag{196}
$$

Now we move on to the next diagram which occur from the combination of the $\phi^2 A_\mu A_\nu$ and ϕ^4 vertices. To be more specific, this contribution reads

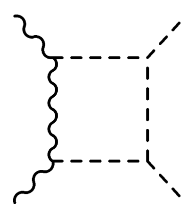


$$= i\mathcal{B}^3_{HZ,\mu\nu} \tag{197}$$

which is exactly the same with the Triangle diagram (133) divide with v_0. Therefore we get that

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^3_{HZ,\mu\nu} &= (4\pi)^{d/2}\mu^{4-d}\frac{\mathcal{K}^3_{HZ,\mu\nu}}{v_0} \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{B}^3_{HZ,\mu\nu} &= \frac{-g^2g^{\mu\nu}}{3\lambda v_0}(4\pi)^{d/2}\mu^{d-4}\mathcal{K}^3_H = \frac{-g^2g^{\mu\nu}}{3v_0^2\lambda}(4\pi)^{d/2}\mu^{d-4}\mathcal{M}^3_H \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{B}^3_{HZ,\mu\nu} &= -6g^2g^{\mu\nu}\lambda\mu^{d-4}B_0(p_1, m_H, m_H)\,.
\end{aligned} \tag{198}
$$

Now it is time to move on to the Box diagrams corresponding to the Higgs-Z interactions. Therefore, we start with the diagram which occurs from the combination of the square of $\phi A_\mu A_\nu$ vertex with the square of the $\phi^2 A_\mu A_\nu$ vertex, giving



$$= i\mathcal{B}^4_{HZ,\mu\nu} \tag{199}$$

which reads

$$
\begin{aligned}
i\mathcal{B}^4_{HZ,\mu\nu} &= 144g^{\mu\alpha}g^{\nu\beta}g^2m_Z^2\lambda^2v_0^2\int\frac{d^4k}{(2\pi)^4}\frac{i}{(k^2-m_H^2)}\frac{i\left(-g_{\alpha\beta}+\frac{(k+p_1)_\alpha(k+p_1)_\beta}{m_Z^2}\right)}{\left((k+p_1)^2-m_Z^2\right)} \\
&\times \frac{i}{\left((k+p_1+p_2)^2-m_H^2\right)}\frac{i}{\left((k+p_1+p_2+p_3)^2-m_H^2\right)} \Leftrightarrow \\
\mathcal{B}^4_{HZ,\mu\nu} &= \int\frac{d^4k}{(2\pi)^4}\frac{-i144g^{\mu\alpha}g^{\nu\beta}g^2m_Z^2\lambda^2v_0^2\left(-g_{\alpha\beta}+\frac{(k+p_1)_\alpha(k+p_1)_\beta}{m_Z^2}\right)}{(k^2-m_H^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_H^2\right)\left((k+p_1+p_2+p_3)^2-m_H^2\right)}\,.
\end{aligned} \tag{200}
$$

Here, similarly with all the other previous cases, we separate this integral into to pieces and we calculate them independently. So, we start with the first one which reads

$$
\mathcal{B}^{4A}_{HZ,\mu\nu} = -\int\frac{d^4k}{(2\pi)^4}\frac{-i144g^{\mu\nu}g^2m_Z^2\lambda^2v_0^2}{(k^2-m_H^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_H^2\right)\left((k+p_1+p_2+p_3)^2-m_H^2\right)}
$$

where considering this in d-dimensions and using the first case of equation (457) we obtain that

$$(4\pi)^{d/2}\mathcal{B}^{4A}_{HZ,\mu\nu} = -144 g^{\mu\nu} g^2 m_Z^2 \lambda^2 v_0^2 \mu^{d-4} D_0(p_1,p_1,p_1,m_H,m_Z,m_H,m_H)\,. \tag{201}$$

The second integral that we consider here has the following explicit form

$$\mathcal{B}^{4B}_{HZ,\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \frac{-i144 g^{\mu\alpha} g^{\nu\beta} g^2 \lambda^2 v_0^2 (k+p_1)_\alpha (k+p_1)_\beta}{(k^2-m_H^2)\left((k+p_1)^2-m_Z^2\right)\left((k+p_1+p_2)^2-m_H^2\right)\left((k+p_1+p_2+p_3)^2-m_H^2\right)}$$

where its numerator reads

$$\begin{aligned} N &= g^{\mu\alpha} g^{\nu\beta} (k+p_1)_\alpha (k+p_1)_\beta \Leftrightarrow \\ N &= k^\mu k^\nu + k^\nu p_1^\mu + k^\mu p_1^\nu + p_1^\mu p_1^\nu\,. \end{aligned} \tag{202}$$

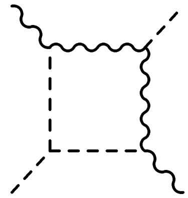
Thus, using the above result along with equation (457) we obtain that

$$\begin{aligned} (4\pi)^{d/2}\mathcal{B}^{4B}_{HZ,\mu\nu} &= 144 g^2 \lambda^2 v_0^2 \mu^{d-4} \Big\{ D^{\mu\nu}(p_1,p_2,p_3,m_H,m_Z,m_H) + p_1^\nu D^\mu(p_1,p_2,p_3,m_H,m_Z,m_H) \\ &+ p_1^\mu D^\nu(p_1,p_2,p_3,m_H,m_Z,m_H) + p_1^\mu p_1^\nu D_0(p_1,p_2,p_3,m_H,m_Z,m_H) \Big\}\,. \end{aligned} \tag{203}$$

Therefore adding equations (201) and (203) we get the final form for $\mathcal{B}^4_{HZ,\mu\nu}$ which gives

$$\begin{aligned} (4\pi)^{d/2}\mathcal{B}^{4}_{HZ,\mu\nu} &= 144 \sigma^1_{\mathcal{B}HZ} g^2 \lambda^2 v_0^2 \mu^{d-4} \Big\{ -g^{\mu\nu} m_Z^2 D_0(p_1,p_2,p_3,m_H,m_Z,m_H) \\ &+ D^{\mu\nu}(p_1,p_2,p_3,m_H,m_Z,m_H) + p_1^\nu D^\mu(p_1,p_2,p_3,m_H,m_Z,m_H) \\ &+ p_1^\mu D^\nu(p_1,p_2,p_3,m_H,m_Z,m_H) + p_1^\mu p_1^\nu D_0(p_1,p_2,p_3,m_H,m_Z,m_H) \Big\}\,. \end{aligned} \tag{204}$$

where $\sigma^1_{\mathcal{B}HZ}$ is a symmetry factor as usual. The next diagram comes from the combination of the third power of the $\phi A_\mu A_\nu$ vertex along with the ϕ^3 vertex, specifically it gives



$$= i\mathcal{B}^{5}_{HZ,\mu\nu} \tag{205}$$

and its explicit form reads

$$\begin{aligned}
i\mathcal{B}^{5}_{HZ,\mu\nu} &= 24 g^{\mu\alpha} g^{\nu\gamma} \frac{\lambda m_Z^4}{v_0} \int \frac{d^4k}{(2\pi)^4} \frac{i\left(-g_{\alpha\beta} + \frac{k_\alpha k_\beta}{m_Z^2}\right)}{(k^2 - m_Z^2)} \frac{i}{\left((k+p_1)^2 - m_H^2\right)} \frac{i}{\left((k+p_1+p_2)^2 - m_H^2\right)} \\
&\times \frac{i\left(-g_{\gamma\beta} + \frac{(k+p_1+p_2+p_3)_\gamma (k+p_1+p_2+p_3)_\beta}{m_Z^2}\right)}{\left((k+p_1+p_2+p_3)^2 - m_Z^2\right)} \Leftrightarrow \\
\mathcal{B}^{5}_{HZ,\mu\nu} &= 24 g^{\mu\alpha} g^{\nu\gamma} \frac{\lambda m_Z^4}{v_0} \int \frac{d^4k}{(2\pi)^4} \Bigg\{ \frac{-i\left(-g_{\alpha\beta} + \frac{k_\alpha k_\beta}{m_Z^2}\right)}{(k^2 - m_Z^2)\left((k+p_1)^2 - m_H^2\right)\left((k+p_1+p_2)^2 - m_H^2\right)} \\
&\times \frac{\left(-g_{\gamma\beta} + \frac{(k+p_1+p_2+p_3)_\gamma (k+p_1+p_2+p_3)_\beta}{m_Z^2}\right)}{\left((k+p_1+p_2+p_3)^2 - m_Z^2\right)} \Bigg\}.
\end{aligned} \tag{206}$$

The numerator of this integral has two parenthesis and therefore after expanding them we obtain the following relation

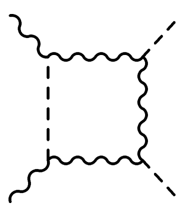
$$\begin{aligned}
N &= g^{\mu\alpha} g^{\nu\gamma} m_Z^4 \left(-g_{\alpha\beta} + \frac{k_\alpha k_\beta}{m_Z^2}\right)\left(-g_{\beta\gamma} + \frac{(k+p_1+p_2+p_3)_\beta (k+p_1+p_2+p_3)_\gamma}{m_Z^2}\right) \Leftrightarrow \\
N &= m_Z^4 g^{\mu\nu} - 2m_Z^2 k_\mu k_\nu + k\cdot k k_\mu k_\nu + (p_1+p_2+p_3)\cdot k k_\mu k_\nu - m_Z^2 (p_1+p_2+p_3)\cdot k \\
&- m_Z^2 (p_1+p_2+p_3)^\nu k^\mu + (p_1+p_2+p_3)^\nu k^\mu k\cdot k + k^\mu (p_1+p_2+p_3)^\nu (p_1+p_2+p_3)\cdot k \\
&- m_Z^2 (p_1+p_2+p_3)^\nu (p_1+p_2+p_3)^\mu .
\end{aligned} \tag{207}$$

Now putting this expression in the (206) integral we obtain the following final form

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^5_{HZ,\mu\nu} &= \frac{24\sigma^2_{\mathcal{B}HZ}\lambda\mu^{d-4}}{v_0}\Big\{m_Z^4 g^{\mu\nu}D_0(p_1,p_2,p_3,m_Z,m_H,m_H) \\
&- 2m_Z^2 D^{\mu\nu}(p_1,p_2,p_3,m_Z,m_H,m_H) + U^{\mu\nu}_{\mathcal{B}4} \\
&+ (p_1+p_2+p_3)_\rho D^{\mu\nu\rho}(p_1,p_2,p_3,m_Z,m_H,m_H) \\
&- m_Z^2(p_1+p_2+p_3)^\mu D^\nu(p_1,p_2,p_3,m_Z,m_H,m_H,m_Z) \\
&- m_Z^2(p_1+p_2+p_3)^\nu D^\mu(p_1,p_2,p_3,m_Z,m_H,m_H) \\
&+ (p_1+p_2+p_3)^\nu g_{\rho\sigma}D^{\mu\rho\sigma}(p_1,p_2,p_3,m_Z,m_H,m_H) \\
&+ (p_1+p_2+p_3)^\nu(p_1+p_2+p_3)_\rho D^{\mu\rho}(p_1,p_2,p_3,m_Z,m_H,m_H) \\
&- m_Z^2(p_1+p_2+p_3)^\nu(p_1+p_2+p_3)^\mu D_0(p_1,p_2,p_3,m_Z,m_H,m_H)\Big\}
\end{aligned}
\tag{208}
$$

where $\sigma^2_{\mathcal{B}HZ}$ is again a symmetry factor.

Finally, the last diagram that we should consider here comes from the fourth power of the $\phi A_\mu A_\nu$ vertex, giving



$$
= i\mathcal{B}^6_{HZ,\mu\nu} \tag{209}
$$

which has the explicit form

$$
\begin{aligned}
i\mathcal{B}^6_{HZ,\mu\nu} &= 16ig^{\mu\alpha}g^{\nu\beta}g^4m_Z^4\int\frac{d^4k}{(2\pi)^4}\frac{i\left(-g_{\delta\alpha}+\frac{k_\delta k_\alpha}{m_Z^2}\right)}{(k^2-m_Z^2)}\frac{i}{\left((k+p_1)^2-m_H^2\right)}\frac{i\left(-g_{\beta\gamma}+\frac{(k+p_1+p_2)_\beta(k+p_1+p_2)_\gamma}{m_Z^2}\right)}{\left((k+p_1+p_2)^2-m_Z^2\right)} \\
&\times\ \frac{i\left(-g_{\gamma\delta}+\frac{(k+p_1+p_2+p_3)_\gamma(k+p_1+p_2+p_3)_\delta}{m_Z^2}\right)}{\left((k+p_1+p_2+p_3)^2-m_Z^2\right)}\Leftrightarrow \\
\mathcal{B}^6_{HZ,\mu\nu} &= 16g^{\mu\alpha}g^{\nu\beta}g^4m_Z^4\int\frac{d^4k}{(2\pi)^4}\Bigg\{\frac{-i\left(-g_{\delta\alpha}+\frac{k_\delta k_\alpha}{m_Z^2}\right)\left(-g_{\beta\gamma}+\frac{(k+p_1+p_2)_\beta(k+p_1+p_2)_\gamma}{m_Z^2}\right)}{(k^2-m_Z^2)\left((k+p_1)^2-m_H^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)} \\
&\times\ \frac{\left(-g_{\gamma\delta}+\frac{(k+p_1+p_2+p_3)_\gamma(k+p_1+p_2+p_3)_\delta}{m_Z^2}\right)}{\left((k+p_1+p_2+p_3)^2-m_Z^2\right)}\Bigg\}.
\end{aligned}
\tag{210}
$$

As we can recall integrals like this one have occurred again in this section and in particular in the case of (156). Therefore, we are familiar with integrals like these one no matter how complicated they are. So we begin this calculation by obtaining the corresponding numerator which in our case reads

$$
\begin{aligned}
N &= g^{\mu\alpha}g^{\nu\beta}m_Z^4\left(-g_{\delta\alpha}+\frac{k_\delta k_\alpha}{m_Z^2}\right)\left(-g_{\beta\gamma}+\frac{(k+p_1+p_2)_\beta(k+p_1+p_2)_\gamma}{m_Z^2}\right)\\
&\times \left(-g_{\gamma\delta}+\frac{(k+p_1+p_2+p_3)_\gamma(k+p_1+p_2+p_3)_\delta}{m_Z^2}\right)\Leftrightarrow\\
N &= -m_Z^4g_{\mu\nu}+3m_Z^2k_\mu k_\nu-3k^2k_\mu k_\nu+\frac{k^4k_\mu k_\nu}{m_Z^2}-2k\cdot(p_1+p_2)k_\mu k_\nu\\
&+ \frac{2k^2k\cdot(p_1+p_2)k_\mu k_\nu}{m_Z^2}+\frac{(k\cdot(p_1+p_2))^2k_\mu k_\nu}{m_Z^2}-2k\cdot(p_1+p_2+p_3)k_\mu k_\nu+\frac{k^2k\cdot(p_1+p_2+p_3)k_\mu k_\nu}{m_Z^2}\\
&+ \frac{k\cdot(p_1+p_2)k\cdot(p_1+p_2+p_3)k_\mu k_\nu}{m_Z^2}-k_\mu k_\nu(p_1+p_2)\cdot(p_1+p_2+p_3)\\
&+ \frac{k\cdot(p_1+p_2)k_\mu k_\nu(p_1+p_2)\cdot(p_1+p_2+p_3)}{m_Z^2}+\frac{k^2k_\mu k_\nu(p_1+p_2)\cdot(p_1+p_2+p_3)}{m_Z^2}\\
&+ m_Z^2k_\mu(p_1+p_2)_\nu-k^2k_\mu(p_1+p_2)_\nu-k\cdot(p_1+p_2)k_\mu(p_1+p_2)_\nu+m_Z^2k_\mu(p_1+p_2)_\nu-k^2k_\mu(p_1+p_2)_\nu\\
&- k\cdot(p_1+p_2)k_\mu(p_1+p_2)_\nu-k\cdot(p_1+p_2+p_3)k_\mu(p_1+p_2)_\nu-k_\mu(p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2)_\nu\\
&+ m_Z^2(p_1+p_2)_\mu(p_1+p_2)_\nu+m_Z^2k_\nu(p_1+p_2+p_3)_\mu-2k^2k_\mu(p_1+p_2+p_3)_\nu+\frac{k^4k_\nu(p_1+p_2+p_3)_\mu}{m_Z^2}\\
&- k\cdot(p_1+p_2)k_\nu(p_1+p_2+p_3)_\mu+\frac{2k^2k\cdot(p_1+p_2)k_\nu(p_1+p_2+p_3)_\mu}{m_Z^2}+\frac{(k\cdot(p_1+p_2))^2k_\nu(p_1+p_2+p_3)_\mu}{m_Z^2}\\
&- 2k\cdot(p_1+p_2+p_3)k_\nu(p_1+p_2+p_3)_\mu+\frac{k^2k\cdot(p_1+p_2+p_3)k_\nu(p_1+p_2+p_3)_\mu}{m_Z^2}\\
&+ \frac{k\cdot(p_1+p_2)k\cdot(p_1+p_2+p_3)k_\nu(p_1+p_2+p_3)_\mu}{m_Z^2}-k_\nu(p_1+p_2+p_3)_\mu(p_1+p_2)\cdot(p_1+p_2+p_3)\\
&+ \frac{k^2k_\nu(p_1+p_2+p_3)_\mu(p_1+p_2)\cdot(p_1+p_2+p_3)}{m_Z^2}-k^2(p_1+p_2)_\nu(p_1+p_2+p_3)_\mu\\
&- k\cdot(p_1+p_2)(p_1+p_2)_\nu(p_1+p_2+p_3)_\mu-k\cdot(p_1+p_2+p_3)(p_1+p_2)_\nu(p_1+p_2+p_3)_\mu\\
&+ (p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2)_\nu(p_1+p_2+p_3)_\mu+m_Z^2k_\mu(p_1+p_2+p_3)_\nu\\
&+ m_Z^2(p_1+p_2)_\nu(p_1+p_2+p_3)_\mu+\frac{k_\nu(p_1+p_2+p_3)_\mu k\cdot(p_1+p_2)(p_1+p_2)\cdot(p_1+p_2+p_3)}{m_Z^2}.
\end{aligned}
\tag{211}
$$

Therefore, now that we have evaluated the numerator of this integral according to previous calculations, we have that $\mathcal{B}^5_{HZ,\mu\nu}$ takes the final form

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}^6_{HZ,\mu\nu} &= 16\sigma^3_{\mathcal{B}HZ} g^4 \mu^{d-4} \Bigg\{ -3U^{\mu\nu}_{\mathcal{B}4HZ} + \frac{(p_1+p_2+p_3)^\mu}{m_Z} U^{\mu\nu}_{\mathcal{B}5HZ} \\
&+ U^{\mu\nu}_{\mathcal{B}6HZ} + \Big\{ -m_Z^4 g^{\mu\nu} + m_Z^2 (p_1+p_2)^\mu (p_1+p_2)^\nu + m_Z^2 (p_1+p_2)^\nu (p_1+p_2)^\mu \\
&+ (p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2)^\nu (p_1+p_2+p_3)^\mu \Big\} D_0(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \Big\{ 2m_Z^2 (p_1+p_2)^\nu - (p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2)^\nu \\
&+ m_Z^2 (p_1+p_2+p_3)^\nu \Big\} D^\mu(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \Big\{ m_Z^2 (p_1+p_2+p_3)^\mu \\
&- (p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2+p_3)^\mu \Big\} D^\nu(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \Big\{ -(p_1+p_2+p_3)^\mu (p_1+p_2)^\nu (p_1+p_2)_\rho \\
&- (p_1+p_2+p_3)_\rho (p_1+p_2+p_3)^\mu (p_1+p_2)^\nu \Big\} D^\rho(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \Big\{ 3m_Z^2 - (p_1+p_2)\cdot(p_1+p_2+p_3) \Big\} D^{\mu\nu}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&- (p_1+p_2+p_3)^\mu (p_1+p_2)^\nu g_{\rho\sigma} D^{\rho\sigma}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \Big\{ -2(p_1+p_2)^\nu (p_1+p_2)_\rho - (p_1+p_2)^\nu (p_1+p_2+p_3)_\rho \\
&+ \frac{(p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2+p_3)^\nu (p_1+p_2)_\rho}{m_Z^2} \Big\} D^{\mu\rho}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \Big\{ -(p_1+p_2+p_3)^\nu (p_1+p_2)_\rho \\
&- 2(p_1+p_2+p_3)^\nu (p_1+p_2+p_3)_\rho \Big\} D^{\mu\rho}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \Big\{ \frac{(p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2)_\rho}{m_Z^2} - 2(p_1+p_2)_\rho \\
&- 2(p_1+p_2+p_3)_\rho \Big\} D^{\mu\nu\rho}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \Big\{ -2(p_1+p_2)^\nu g_{\rho\sigma} - 2(p_1+p_2+p_3)^\nu g_{\rho\sigma} \Big\} D^{\mu\rho\sigma}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \Big\{ \frac{(p_1+p_2)_\rho (p_1+p_2)_\sigma (p_1+p_2+p_3)^\mu}{m_Z^2} + \frac{(p_1+p_2)_\sigma (p_1+p_2+p_3)_\rho (p_1+p_2+p_3)^\mu}{m_Z^2} \\
&+ \frac{g_{\rho\sigma}(p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2+p_3)^\mu}{m_Z^2} \Big\} D^{\nu\rho\sigma}(p_1,p_2,p_3,m_Z,m_H,m_Z)
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \left\{\frac{(p_1+p_2)_\rho(p_1+p_2)_\sigma}{m_Z^2} + \frac{(p_1+p_2)_\rho(p_1+p_2+p_3)_\sigma}{m_Z^2}\right.\\
&+ \quad \left.\frac{(p_1+p_2)\cdot(p_1+p_2+p_3)}{m_Z^2} g_{\rho\sigma}\right\} D^{\mu\nu\rho\sigma}(p_1,p_2,p_3,m_Z,m_H,m_Z)\\
&+ \quad \left\{\frac{2g_{\sigma\lambda}(p_1+p_2)_\rho(p_1+p_2+p_3)^\mu}{m_Z^2}\right.\\
&+ \quad \left.\left.\frac{2g_{\rho\sigma}(p_1+p_2+p_3)_\lambda(p_1+p_2+p_3)^\mu}{m_Z^2}\right\} D^{\nu\rho\sigma\lambda}(p_1,p_2,p_3,m_Z,m_H,m_Z)\right\}
\end{aligned}
\tag{212}
$$

where $\sigma^3_{\mathcal{B}HZ}$ is a symmetry factor. Moreover, we have defined the following relations

$$
U^{\mu\nu}_{\mathcal{B}4HZ} = \int \frac{d^4k}{(2\pi)^4} \frac{-ik^2k^\mu k^\nu}{(k^2-m_Z^2)\left((k+p_1)^2-m_H^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)\left((k+p_1+p_2+p_3)^2-m_Z^2\right)}
\tag{213}
$$

$$
U^{\mu\nu}_{\mathcal{B}5HZ} = \int \frac{d^4k}{(2\pi)^4} \frac{-ik^4k^\nu}{m_Z\,(k^2-m_Z^2)\left((k+p_1)^2-m_H^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)\left((k+p_1+p_2+p_3)^2-m_Z^2\right)}
\tag{214}
$$

and

$$
U^{\mu\nu}_{\mathcal{B}6HZ} = \int \frac{d^4k}{(2\pi)^4} \frac{-ik^4k^\mu k^\nu}{m_Z^2\,(k^2-m_Z^2)\left((k+p_1)^2-m_H^2\right)\left((k+p_1+p_2)^2-m_Z^2\right)\left((k+p_1+p_2+p_3)^2-m_Z^2\right)}\,.
\tag{215}
$$

Finally, in order to obtain the full contribution of the one-loop four-point functions to the gauge coupling we should add the results of equations (194), (196), (198), (204), (208) and (212). Thus we get that

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{B}_{HZ,\mu\nu} &= \mu^{4-d}\Bigg\{24\sigma^1_{\mathcal{K}HZ}g^2\lambda\Big\{-g^{\mu\nu}m_Z^2C_0(p_1,p_2,m_Z,m_H,m_H)+C^{\mu\nu}(p_1,p_2,m_Z,m_H,m_H)\Big\}\\
&+ \frac{8\sigma^2_{\mathcal{K}HZ}g^4}{v_0}\Big\{-m_Z^2g^{\mu\nu}C_0(p_1,p_2,m_Z,m_H,m_Z)+2C^{\mu\nu}(p_1,p_2,m_Z,m_H,m_Z)\\
&+ (p_1+p_2)^\nu C^\mu(p_1,p_2,m_Z,m_H,m_Z)\\
&+ (p_1+p_2)^\mu C^\nu(p_1,p_2,m_Z,m_H,m_Z)+(p_1+p_2)^\mu(p_1+p_2)^\nu C_0(p_1,p_2,m_Z,m_H,m_Z)\\
&- U^{\mu\nu}_{\mathcal{K}4}-\frac{(p_1+p_2)_\rho}{m_Z^2}C^{\mu\nu\rho}(p_1,p_2,m_Z,m_H,m_Z)-\frac{(p_1+p_2)_\nu}{m_Z^2}C^{\mu\rho\sigma}(p_1,p_2,m_Z,m_H,m_Z)\\
&- \frac{(p_1+p_2)^\nu(p_1+p_2)_\rho}{m_Z^2}C^{\mu\rho}(p_1,p_2,m_Z,m_H,m_Z)\Big\}-6g^2g^{\mu\nu}\lambda B_0(p_1,m_H,m_H)\\
&+ 144\sigma^1_{\mathcal{B}HZ}g^2\lambda^2v_0^2\Big\{-g^{\mu\nu}m_Z^2D_0(p_1,p_2,p_3,m_H,m_Z,m_H)\\
&+ D^{\mu\nu}(p_1,p_2,p_3,m_H,m_Z,m_H)+p_1^\nu D^\mu(p_1,p_2,p_3,m_H,m_Z,m_H)\\
&+ p_1^\mu D^\nu(p_1,p_2,p_3,m_H,m_Z,m_H,m_H)+p_1^\mu p_1^\nu D_0(p_1,p_2,p_3,m_H,m_Z,m_H)\Big\}\\
&+ \frac{24\sigma^2_{\mathcal{B}HZ}\lambda\mu^{d-4}}{v_0}\Big\{m_Z^4g^{\mu\nu}D_0(p_1,p_2,p_3,m_Z,m_H,m_H)\\
&- 2m_Z^2D^{\mu\nu}(p_1,p_2,p_3,m_Z,m_H,m_H)+U^{\mu\nu}_{\mathcal{B}4}\\
&+ (p_1+p_2+p_3)_\rho D^{\mu\nu\rho}(p_1,p_2,p_3,m_Z,m_H,m_H)\\
&- m_Z^2(p_1+p_2+p_3)^\mu D^\nu(p_1,p_2,p_3,m_Z,m_H,m_H,m_Z)\\
&- m_Z^2(p_1+p_2+p_3)^\nu D^\mu(p_1,p_2,p_3,m_Z,m_H,m_H)\\
&+ (p_1+p_2+p_3)^\nu g_{\rho\sigma}D^{\mu\rho\sigma}(p_1,p_2,p_3,m_Z,m_H,m_H)\\
&+ (p_1+p_2+p_3)^\nu(p_1+p_2+p_3)_\rho D^{\mu\rho}(p_1,p_2,p_3,m_Z,m_H,m_H)\\
&- m_Z^2(p_1+p_2+p_3)^\nu(p_1+p_2+p_3)^\mu D_0(p_1,p_2,p_3,m_Z,m_H,m_H)\Big\}\\
&+ 16\sigma^3_{\mathcal{B}HZ}g^4\Bigg\{-3U^{\mu\nu}_{\mathcal{B}4HZ}+\frac{(p_1+p_2+p_3)^\mu}{m_Z}U^{\mu\nu}_{\mathcal{B}5HZ}+U^{\mu\nu}_{\mathcal{B}6HZ}\\
&+ \Big\{-m_Z^4g^{\mu\nu}+m_Z^2(p_1+p_2)^\mu(p_1+p_2)^\nu+m_Z^2(p_1+p_2)^\nu(p_1+p_2)^\mu\\
&+ (p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2)^\nu(p_1+p_2+p_3)^\mu\Big\}D_0(p_1,p_2,p_3,m_Z,m_H,m_Z)\\
&+ \Big\{2m_Z^2(p_1+p_2)^\nu-(p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2)^\nu\\
&+ m_Z^2(p_1+p_2+p_3)^\nu\Big\}D^\mu(p_1,p_2,p_3,m_Z,m_H,m_Z)\\
&+ \Big\{m_Z^2(p_1+p_2+p_3)^\mu\\
&- (p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2+p_3)^\mu\Big\}D^\nu(p_1,p_1,p_1,m_Z,m_H,m_Z,m_Z)
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \Big\{-(p_1+p_2+p_3)^\mu (p_1+p_2)^\nu (p_1+p_2)_\rho \\
&- \quad (p_1+p_2+p_3)_\rho (p_1+p_2+p_3)^\mu (p_1+p_2)^\nu \Big\} D^\rho(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{3m_Z^2 - (p_1+p_2)\cdot(p_1+p_2+p_3)\Big\} D^{\mu\nu}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{-2(p_1+p_2)^\nu (p_1+p_2)_\rho - (p_1+p_2)^\nu (p_1+p_2+p_3)_\rho \\
&+ \quad \frac{(p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2+p_3)^\nu (p_1+p_2)_\rho}{m_Z^2}\Big\} D^{\mu\rho}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{-(p_1+p_2+p_3)^\mu (p_1+p_2)_\rho - 2(p_1+p_2+p_3)^\mu (p_1+p_2+p_3)_\rho\Big\} D^{\mu\rho}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{\frac{(p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2)_\rho}{m_Z^2} - 2(p_1+p_2)_\rho \\
&- \quad 2(p_1+p_2+p_3)_\rho\Big\} D^{\mu\nu\rho}(p_1,p_1,p_1,m_Z,m_H,m_Z,m_Z) \\
&+ \quad \Big\{-2(p_1+p_2)^\nu g_{\rho\sigma} - 2(p_1+p_2+p_3)^\nu g_{\rho\sigma}\Big\} D^{\mu\rho\sigma}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{\frac{(p_1+p_2)_\rho (p_1+p_2)_\sigma (p_1+p_2+p_3)^\mu}{m_Z^2} + \frac{(p_1+p_2)_\sigma (p_1+p_2+p_3)_\rho (p_1+p_2+p_3)^\mu}{m_Z^2} \\
&+ \quad \frac{g_{\rho\sigma}(p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2+p_3)^\mu}{m_Z^2}\Big\} D^{\nu\rho\sigma}(p_1,p_1,p_1,m_Z,m_H,m_Z,m_Z) \\
&+ \quad \Big\{\frac{(p_1+p_2)_\rho (p_1+p_2)_\sigma}{m_Z^2} + \frac{(p_1+p_2)_\rho (p_1+p_2+p_3)_\sigma}{m_Z^2} \\
&+ \quad \frac{(p_1+p_2)\cdot(p_1+p_2+p_3)}{m_Z^2} g_{\rho\sigma}\Big\} D^{\mu\nu\rho\sigma}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{\frac{2g_{\sigma\lambda}(p_1+p_2)_\rho (p_1+p_2+p_3)^\mu}{m_Z^2} \\
&+ \quad \frac{2g_{\rho\sigma}(p_1+p_2+p_3)_\lambda (p_1+p_2+p_3)^\mu}{m_Z^2}\Big\} D^{\nu\rho\sigma\lambda}(p_1,p_1,p_1,m_Z,m_H,m_Z,m_Z)\Big\}\Big\}.
\end{aligned}
\tag{216}
$$

Again here, in order to obtain the reduced form of the Box diagrams with two gauge-boson and two Higgs-boson external propagators we have to consider the contracted version of $\mathcal{B}_{HZ,\mu\nu}$ with the metric. Therefore we get that

$$
\begin{aligned}
(4\pi)^{d/2} g^{\mu\nu}\mathcal{B}_{HZ,\mu\nu} &= \mu^{4-d}\Bigg\{24\sigma^1_{\mathcal{K}HZ} g^2\lambda\Big\{-dm_Z^2 C_0(p_1,p_2,m_Z,m_H,m_H) + m_Z^2 C_0(p_1,p_2,m_Z,m_H,m_H) \\
&+ B_0(p_2,m_Z,m_H)\Big\} \\
&+ \frac{8\sigma^2_{\mathcal{K}HZ} g^4}{v_0}\Big\{-m_Z^2 dC_0(p_1,p_2,m_Z,m_H,m_Z) + 2m_Z^2 C_0(p_1,p_2,m_Z,m_H,m_H) \\
&+ 2B_0(p_2,m_Z,m_H) \\
&+ 2(p_1+p_2)_\mu C^\mu(p_1,p_2,m_Z,m_H,m_Z) + (p_1+p_2)^\mu(p_1+p_2)_\mu C_0(p_1,p_2,m_Z,m_H,m_Z) \\
&- U_{\mathcal{K}4} - 2\frac{(p_1+p_2)_\rho}{m_Z^2} g_{\mu\nu} C^{\mu\nu\rho}(p_1,p_2,m_Z,m_H,m_Z) \\
&- \frac{(p_1+p_2)_\mu (p_1+p_2)_\rho}{m_Z^2} C^{\mu\rho}(p_1,p_2,m_Z,m_H,m_Z)\Big\} - 6g^2 g^{\mu\nu}\lambda B_0(p_1,m_H,m_H) \\
&+ 144\sigma^1_{\mathcal{B}HZ} g^2\lambda^2 v_0^2\Big\{-dm_Z^2 D_0(p_1,p_2,p_3,m_H,m_Z,m_H) \\
&+ g_{\mu\nu} D^{\mu\nu}(p_1,p_2,p_3,m_H,m_Z,m_H) \\
&+ 2p_{1\mu} D^\mu(p_1,p_2,p_3,m_H,m_Z,m_H) + p_1^2 D_0(p_1,p_2,p_3,m_H,m_Z,m_H)\Big\} \\
&+ \frac{24\sigma^2_{\mathcal{B}HZ}\lambda\mu^{d-4}}{v_0}\Big\{m_Z^4 dD_0(p_1,p_2,p_3,m_Z,m_H,m_H) \\
&- 2m_Z^2 g_{\mu\nu} D^{\mu\nu}(p_1,p_2,p_3,m_Z,m_H,m_H) + U_{\mathcal{B}4} \\
&+ (p_1+p_2+p_3)_\rho g_{\mu\nu} D^{\mu\nu\rho}(p_1,p_2,p_3,m_Z,m_H,m_H) \\
&- 2m_Z^2(p_1+p_2+p_3)_\mu D^\mu(p_1,p_2,p_3,m_Z,m_H,m_H) \\
&+ g_{\rho\sigma}(p_1+p_2+p_3)_\mu D^{\mu\rho\sigma}(p_1,p_2,p_3,m_Z,m_H,m_H) \\
&+ (p_1+p_2+p_3)_\mu (p_1+p_2+p_3)_\rho D^{\mu\rho}(p_1,p_2,p_3,m_Z,m_H,m_H) \\
&- m_Z^2(p_1+p_2+p_3)_\mu (p_1+p_2+p_3)^\mu D_0(p_1,p_2,p_3,m_Z,m_H,m_H)\Big\} \\
&+ 16\sigma^3_{\mathcal{B}HZ} g^4\Bigg\{-3U_{\mathcal{B}4HZ} + \frac{(p_1+p_2+p_3)_\mu}{m_Z} U^\mu_{\mathcal{B}5HZ} + U_{\mathcal{B}6HZ} \\
&+ \Big\{-m_Z^4 d + 2m_Z^2(p_1+p_2)_\mu (p_1+p_2)^\mu \\
&+ (p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2)_\mu (p_1+p_2+p_3)^\mu\Big\} D_0(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \Big\{2m_Z^2(p_1+p_2)_\mu - 2(p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2)_\mu + 2m_Z^2(p_1+p_2+p_3)_\mu \\
&+ (p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2+p_3)_\mu\Big\} D^\mu(p_1,p_2,p_3,m_Z,m_H,m_Z)
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \Big\{-(p_1+p_2+p_3)_\mu(p_1+p_2)^\mu(p_1+p_2)_\rho \\
&- \quad (p_1+p_2+p_3)_\rho(p_1+p_2+p_3)_\mu(p_1+p_2)^\mu\Big\}D^\rho(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{3m_Z^2-(p_1+p_2)\cdot(p_1+p_2+p_3)\Big\}g_{\mu\nu}D^{\mu\nu}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{-2(p_1+p_2)_\mu(p_1+p_2)_\rho-(p_1+p_2)_\mu(p_1+p_2+p_3)_\rho \\
&+ \quad \frac{(p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2+p_3)_\mu(p_1+p_2)_\rho}{m_Z^2}\Big\}D^{\mu\rho}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{-(p_1+p_2+p_3)_\mu(p_1+p_2)_\rho-2(p_1+p_2+p_3)_\mu(p_1+p_2+p_3)_\rho\Big\}D^{\mu\rho}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{\frac{(p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2)_\rho}{m_Z^2}-2(p_1+p_2)_\rho \\
&- \quad 2(p_1+p_2+p_3)_\rho\Big\}g_{\mu\nu}D^{\mu\nu\rho}(p_1,p_1,p_1,m_Z,m_H,m_Z,m_Z) \\
&+ \quad \Big\{-2(p_1+p_2)_\mu g_{\rho\sigma}-2(p_1+p_2+p_3)_\mu g_{\rho\sigma}\Big\}D^{\mu\rho\sigma}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{\frac{(p_1+p_2)_\rho(p_1+p_2)_\sigma(p_1+p_2+p_3)_\mu}{m_Z^2}+\frac{(p_1+p_2)_\sigma(p_1+p_2+p_3)_\rho(p_1+p_2+p_3)_\mu}{m_Z^2} \\
&+ \quad \frac{g_{\rho\sigma}(p_1+p_2)\cdot(p_1+p_2+p_3)(p_1+p_2+p_3)_\mu}{m_Z^2}\Big\}D^{\mu\rho\sigma}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{\frac{(p_1+p_2)_\rho(p_1+p_2)_\sigma}{m_Z^2}+\frac{(p_1+p_2)_\rho(p_1+p_2+p_3)_\sigma}{m_Z^2} \\
&+ \quad \frac{(p_1+p_2)\cdot(p_1+p_2+p_3)}{m_Z^2}g_{\rho\sigma}\Big\}g_{\mu\nu}D^{\mu\nu\rho\sigma}(p_1,p_2,p_3,m_Z,m_H,m_Z) \\
&+ \quad \Big\{\frac{2g_{\sigma\lambda}(p_1+p_2)_\rho(p_1+p_2+p_3)_\mu}{m_Z^2} \\
&+ \quad \frac{2g_{\rho\sigma}(p_1+p_2+p_3)_\lambda(p_1+p_2+p_3)_\mu}{m_Z^2}\Big\}D^{\mu\rho\sigma\lambda}(p_1,p_2,p_3,m_Z,m_H,m_Z)\Big\}\Big\}.
\end{aligned}
\tag{217}
$$

Finally, lets see which is the reduced into scalar integrals form of $g^{\mu\nu}\mathcal{B}_{HZ,\mu\nu}$. Similarly with the Box diagrams of the Higgs sector, here we will need the help of some specific functions. The reason why we should do that is because we have a huge result of the above integral. Therefore, we have that

$$
\begin{aligned}
(4\pi)^{d/2} g^{\mu\nu} \mathcal{B}_{HZ,\mu\nu} &= \mu^{4-d} \Bigg\{ 8g^2 \lambda B_0(m_Z, m_H, m_H) + \frac{8g^4}{m_Z^2} A_0(m_H) - \frac{32g^4}{3m_Z^2} A_0(m_Z) \\
&+ \frac{4g^4(-7m_H^2 + 16m_Z^2)}{9m_Z^2} B_0(m_Z, m_H, m_Z) \\
&+ \frac{g^4(35Dm_H^8 + 32m_H^2 m_Z^2 - 136Dm_H^6 m_Z^2)}{6m_Z^2} C_0(m_Z, m_Z, m_Z, m_H, m_Z) \\
&+ \frac{g^4(-80m_Z^4 + 144Dm_H^4 m_Z^4 - 64Dm_H^2 m_Z^6)}{18m_Z^2} C_0(m_Z, m_Z, m_Z, m_H, m_Z) \\
&- 24g^2 m_Z^2 \lambda C_0(m_Z, m_Z, m_Z, m_H, m_H) \\
&+ 36g^2 \lambda^2 v_0^2 \Big\{ -\frac{7}{32} DLm_H^6 m_Z^2 - \frac{3}{16} DLm_H^4 m_Z^4 \\
&+ \frac{1}{2} DLm_H^2 m_Z^6 + DLm_Z^8 \Big\} B_0(m_Z, m_H, m_Z) \\
&+ 36g^2 \lambda^2 v_0^2 \Big\{ \frac{7}{32} DLm_H^6 m_Z^2 + \frac{3}{16} DLm_H^4 m_Z^4 - \frac{1}{2} DLm_H^2 m_Z^6 - DLm_Z^8 \Big\} B_0(m_Z, m_H, m_H) \\
&+ 36g^2 \lambda^2 v_0^2 \Big\{ 2 - \frac{15Lm_H^6}{4} + \frac{9}{2} Lm_H^4 m_Z^2 - \frac{1}{4} DLm_H^8 m_Z^2 - \frac{15}{16} L^2 m_H^{10} m_Z^2 + 6Lm_H^2 m_Z^4 \\
&- \frac{1}{16} DLm_H^6 m_Z^4 + \frac{3}{4} L^2 m_H^8 m_Z^4 - \frac{5Lm_Z^6}{2} + \frac{13}{16} DLm_H^4 m_Z^6 + \frac{31}{16} L^2 m_H^6 m_Z^6 \\
&+ \frac{3}{8} DLm_H^2 m_Z^8 - \frac{13}{16} L^2 m_H^4 m_Z^8 - DLm_Z^{10} \Big\} C_0(m_Z, m_Z, m_Z, m_H, m_Z) \\
&+ 36g^2 \lambda^2 v_0^2 \Big\{ \frac{15Lm_H^6}{4} - \frac{17}{4} Lm_H^4 m_Z^2 + \frac{3}{32} DLm_H^8 m_Z^2 + \frac{15}{16} L^2 m_H^{10} m_Z^2 - 6Lm_H^2 m_Z^4 \\
&+ \frac{11}{32} DLm_H^6 m_Z^4 - \frac{3}{4} L^2 m_H^8 m_Z^4 + 2Lm_Z^6 - \frac{7}{16} DLm_H^4 m_Z^6 - \frac{31}{16} L^2 m_H^6 m_Z^6 \\
&- \frac{1}{2} DLm_H^2 m_Z^8 + \frac{13}{16} L^2 m_H^4 m_Z^8 + DLm_Z^{10} \Big\} C_0(m_Z, m_Z, m_Z, m_H, m_H) \\
&+ 36g^2 \lambda^2 v_0^2 \Big\{ \frac{25Lm_H^8}{4} + 4m_Z^2 - \frac{49}{4} Lm_H^6 m_Z^2 + \frac{25}{16} L^2 m_H^{12} m_Z^2 - \frac{15}{4} Lm_H^4 m_Z^4 \\
&- \frac{39}{16} L^2 m_H^{10} m_Z^4 + 6Lm_H^2 m_Z^6 - \frac{35}{16} L^2 m_H^8 m_Z^6 - Lm_Z^8 \\
&+ \frac{31}{16} L^2 m_H^6 m_Z^8 - \frac{5}{16} L^2 m_H^4 m_Z^{10} \Big\} D_0(m_Z, m_Z, m_Z, m_H, m_Z, m_H) \\
&+ \frac{6\lambda}{v_0} \Big\{ 1 - \frac{69}{128DLm_H^{10}} - \frac{63}{32} DLm_H^8 m_Z^2 - \frac{59}{128} DLm_H^6 m_Z^4 \\
&+ \frac{281}{64} DLm_H^4 m_Z^4 + \frac{45}{8} DLm_H^2 m_Z^8 + \frac{9}{4} DLm_Z^{10} \Big\} B_0(m_Z, m_H, m_Z) \\
&+ \frac{6\lambda}{v_0} \Big\{ -\frac{69}{128DLm_H^{10}} + \frac{63}{32} DLm_H^8 m_Z^2 + \frac{59}{128} DLm_H^6 m_Z^4 \\
&- \frac{281}{64} DLm_H^4 m_Z^4 - \frac{45}{8} DLm_H^2 m_Z^8 - \frac{9}{4} DLm_Z^{10} \Big\} B_0(m_Z, m_H, m_H) \\
&+ \frac{6\lambda}{v_0} \Big\{ 2m_H^2 - \frac{13Lm_H^8}{4} - \frac{23}{32} DLm_H^{12} - \frac{15L^2 m_H^{14}}{128} + 4m_Z^2 + \frac{19}{8} Lm_H^6 m_Z^2 - \frac{75}{64} DLm_H^{10} m_Z^2
\end{aligned}
$$

$$
\begin{aligned}
- &\quad \frac{41}{128}L^2m_H^{12}m_Z^2 + \frac{69}{8}Lm_H^4m_Z^4 + \frac{147}{64}DLm_H^8m_Z^4 + \frac{53}{32}L^2m_H^{10}m_Z^4 + \frac{3}{4}Lm_H^2m_Z^6 \\
+ &\quad \frac{303}{64}DLm_H^6m_Z^6 + \frac{269}{128}L^2m_H^8m_Z^6 - \frac{25Lm_Z^8}{8} + \frac{11}{64}DLm_H^4m_Z^8 - \frac{87}{128}L^2m_H^6m_Z^8 \\
- &\quad \frac{125}{32}DLm_H^2m_Z^{10} - \frac{53}{64}L^2m_H^4m_Z^{10} - \frac{9}{4}DLm_Z^{12}\Big\}C_0(m_Z,m_Z,m_Z,m_H,m_Z) \\
+ &\quad \frac{6\lambda}{v_0}\Big\{\frac{15Lm_H^8}{4} + \frac{69}{128}DLm_H^{12} + \frac{15L^2m_H^{14}}{128} + \frac{13}{8}Lm_H^6m_Z^2 + \frac{147}{128}DLm_H^{10}m_Z^2 \\
+ &\quad \frac{41}{128}L^2m_H^{12}m_Z^2 + \frac{77}{8}Lm_H^4m_Z^4 - \frac{133}{128}DLm_H^8m_Z^4 - \frac{53}{32}L^2m_H^{10}m_Z^4 - 3Lm_H^2m_Z^6 \\
- &\quad \frac{421}{128}DLm_H^6m_Z^6 - \frac{269}{128}L^2m_H^8m_Z^6 + 2Lm_Z^8 - \frac{7}{64}DLm_H^4m_Z^8 + \frac{87}{128}L^2m_H^6m_Z^8 \\
- &\quad \frac{29}{8}DLm_H^2m_Z^{10} + \frac{53}{64}L^2m_H^4m_Z^{10} + \frac{9}{4}DLm_Z^{12}\Big\}C_0(m_Z,m_Z,m_Z,m_H,m_H) \\
+ &\quad \frac{6\lambda}{v_0}\Big\{\frac{Lm_H^{10}}{25} + \frac{L^2m_H^{16}}{25} - \frac{149}{16}Lm_H^8m_Z^2 + \frac{115}{64}L^2m_H^{14}m_Z^2 + 3m_Z^4 \\
- &\quad \frac{63}{4}Lm_H^6m_Z^4 - \frac{335}{128}L^2m_H^{12}m_Z^4\frac{69}{16}Lm_H^4m_Z^6 - \frac{97}{32}L^2m_H^{10}m_Z^6 + 4Lm_H^2m_Z^8 + \frac{5}{4}L^2m_H^8m_Z^8 \\
- &\quad Lm_Z^{10} + \frac{47}{64}L^2m_H^6m_Z^{10} - \frac{13}{64}L^2m_H^4m_Z^{12}\Big\}D_0(m_Z,m_Z,m_Z,m_Z,m_H,m_H) \\
+ &\quad 4g^4\Big\{\Big\{3 - \frac{Dm_H^4}{2} - \frac{69}{64}DLm_H^{10} - \frac{7m_H^2}{2m_Z^2} + \frac{7Dm_H^6}{4m_Z^2} - \frac{9DLm_H^{12}}{8m_Z^2} - \frac{5Dm_H^2m_Z^2}{4} \\
+ &\quad \frac{101}{16}DLm_H^8m_Z^2 + \frac{39DLm_H^6m_Z^4}{16} - \frac{27DLm_H^4m_Z^6}{8} - 8DLm_H^2 - 6DLm_Z^{10}\Big\}B_0(m_Z,m_H,m_Z) \\
+ &\quad \Big\{\frac{69}{64}DLm_H^{10} + \frac{9DLm_H^{12}}{8m_Z^2} - \frac{101}{16}DLm_H^8m_Z^2 \\
- &\quad \frac{39DLm_H^6m_Z^4}{16} + \frac{27DLm_H^4m_Z^6}{8} + 8DLm_H^2 + 6DLm_Z^{10}\Big\}B_0(m_Z,m_H,m_H) \\
+ &\quad \Big\{-\frac{21m_H^2}{2} + \frac{41Dm_H^6}{4} + \frac{155Lm_H^8}{16} + \frac{3D^2m_H^{10}}{4} + 3DLm_H^{12} + \frac{603L^2m_H^{14}}{256} + \frac{9m_H^4}{8m_Z^2} + \frac{3Dm_H^8}{m_Z^2} \\
+ &\quad \frac{117Lm_H^{10}}{64m_Z^2} - \frac{57DLm_H^{14}}{32m_Z^2} - \frac{63L^2m_H^{16}}{64m_Z^2} - 18m_Z^2 - \frac{29}{4}Dm_H^4m_Z^2 + \frac{451Lm_H^6m_Z^2}{16} - 6D^2m_H^8m_Z^2 \\
+ &\quad \frac{263DLm_H^{10}m_Z^2}{64} + \frac{275L^2m_H^{12}m_Z^2}{128} - \frac{13Dm_H^2m_Z^4}{2} - \frac{93Lm_H^4m_Z^4}{2} + \frac{3D^2m_H^6m_Z^4}{4} - \frac{125DLm_H^8m_Z^4}{64} \\
+ &\quad \frac{181L^2m_H^{10}m_Z^4}{32} + 7Dm_Z^6 - 42Lm_H^2m_Z^6 + 12D^2m_H^4m_Z^6 - \frac{203DLm_H^6m_Z^6}{32} + \frac{563L^2m_H^8m_Z^6}{32} + 27Lm_Z^8 \\
+ &\quad \frac{3D^2m_H^2m_Z^8}{2} - \frac{45DLm_H^4m_Z^8}{8} - \frac{23L^2m_H^6m_Z^8}{8} \\
- &\quad 6D^2m_Z^{10} + \frac{17DLm_H^2m_Z^{10}}{4} + \frac{21L^2m_H^4m_Z^{10}}{4} + 6DLm_Z^{12}\Big\}C_0(m_Z,m_Z,m_Z,m_H,m_Z) \\
+ &\quad \Big\{\frac{-363Lm_H^8}{32} - \frac{3DLm_H^{12}}{4} - \frac{603L^2m_H^{14}}{256} \\
+ &\quad \frac{9DLm_H^{14}}{8m_Z^2} + \frac{63L^2m_H^{16}}{64m_Z^2} - \frac{481Lm_H^6m_Z^2}{16} - 6D^2m_H^8m_Z^2 - \frac{125DLm_H^{10}m_Z^2}{32} - \frac{275L^2m_H^{12}m_Z^2}{128}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{371Lm_H^4m_Z^4}{8}+\frac{53DLm_H^8m_Z^4}{32}-\frac{181L^2m_H^{10}m_Z^4}{32}+\frac{89Lm_H^2m_Z^6}{2}+\frac{25}{16}DLm_H^6m_Z^6-\frac{563L^2m_H^8m_Z^6}{32}\\
&- \quad 24Lm_Z^8+\frac{29DLm_H^4m_Z^8}{8}+\frac{23L^2m_H^6m_Z^8}{8}\\
&+ \quad 2DLm_H^2m_Z^{10}-\frac{21L^2m_H^4m_Z^{10}}{4}-6DLm_Z^{12}\Big\}C_0(m_Z,m_Z,m_Z,m_H,m_H)\\
&+ \quad \Big\{\frac{9m_H^4}{4}-\frac{509Lm_H^{10}}{16}-\frac{1063L^2m_H^{16}}{256}+\frac{441Lm_H^{12}}{64}+\frac{147L^2m_H^{18}}{64m_Z^2}-5m_H^2m_Z^2-\frac{801Lm_H^8m_Z^2}{16}\\
&- \quad \frac{523L^2m_H^{14}m_Z^2}{64}-23m_Z^4+\frac{1237Lm_H^6m_Z^4}{8}-\frac{191L^2m_H^{12}m_Z^4}{64}+\frac{71Lm_H^4m_Z^6}{4}\\
&+ \quad \frac{681L^2m_H^{10}m_Z^6}{16}-\frac{185Lm_H^2m_Z^8}{2}-\frac{191L^2m_H^8m_Z^8}{16}\\
&+ \quad 24Lm_Z^{10}-\frac{53L^2m_H^6m_Z^{10}}{4}+\frac{9L^2m_H^4m_Z^{12}}{2}\Big\}D_0(m_Z,m_Z,m_Z,m_Z,m_H,m_Z)\Big\}\Bigg\}
\end{aligned}
\tag{218}
$$

where we have defined that L and D are the inverse determinant of the G_3 and G_2 matrix respectively and to be more specific they have the following form

$$
\begin{aligned}
L &= \frac{1}{-\frac{1}{4}m_H^4m_Z^2-\frac{m_H^2m_Z^4}{2}-\frac{m_Z^6}{4}}\\
D &= \frac{1}{2m_H^2m_Z^2-\frac{m_H^4}{4}-m_Z^4}\,.
\end{aligned}
\tag{219}
$$

3 Abelian Higgs Lagrangian in R_ξ gauge

Since we have finished with the calculation of all the one-loop corrections in the of the Abelian Higgs Model in Unitary gauge, we proceed with the same calculation in the R_ξ-gauge. There, the gauge-fixing term is present and moreover we have both the physical and the un-physical degrees of freedom. The reason why we are dealing with this, is that in the renormalization section we clarify the arguments which demand that the physical quantities should be gauge independent. Thus, we need the above calculation in order to compare the physical results in the two gauges, and that will help us understand how and why there would be a ξ-cancelation from the physical quantities.

$$
\begin{aligned}
\mathcal{L}_{R_\xi} &= -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\left(\partial_\mu\phi_0\right)\left(\partial^\mu\phi_0\right) + \frac{1}{2}\left(\partial_\mu\chi_0\right)\left(\partial^\mu\chi_0\right) + \frac{1}{2}m_{Z_0}^2 A_\mu^0 A^{0\mu} \\
&+ m_{Z_0}\partial_\mu\chi_0 A_\mu^0 + 2\frac{m_{Z_0}}{m_{H_0}}\sqrt{2\lambda_0}A_\mu^0\partial_\mu\chi_0\phi_0 + \frac{\sqrt{2\lambda_0}}{m_{H_0}}(\partial_\mu\chi_0)^2\phi_0 + \frac{2\lambda_0 m_{Z_0}}{m_{H_0}^2}A_\mu^0\partial_\mu\chi_0\phi_0^2 \\
&+ \frac{\lambda_0}{m_{H_0}^2}(\partial_\mu\chi_0)^2\phi_0^2 + g^{\mu\nu}\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}A_\mu^0 A_\nu^0\phi_0^2 + g^{\mu\nu}\frac{m_{Z_0}^2}{m_{H_0}}\sqrt{2\lambda_0}\phi_0 A_\mu^0 A_\nu^0 \\
&- \frac{1}{2}m_{H_0}^2\phi_0^2 - \frac{\lambda_0}{4}\phi_0^4 - \sqrt{\frac{\lambda_0}{2}}m_{H_0}\phi_0^3 + const.
\end{aligned}
\tag{220}
$$

where we have used a specific notation for the parameters of the model in order to clarify exactly which of them are independent and which are not. Thus we make the following replacements

$$
\begin{aligned}
g_0 &= \frac{m_{Z_0}}{m_{H_0}}\sqrt{2\lambda_0} \\
v_0 &= \frac{m_{H_0}}{\sqrt{2\lambda_0}} \\
\lambda_0 v_0 &= \sqrt{\frac{\lambda_0}{2}}m_{H_0} \\
g_0^2 v_0 &= \frac{m_{Z_0}^2}{m_{H_0}}\sqrt{2\lambda_0}
\end{aligned}
\tag{221}
$$

and according to this the only independent parameters left in this model are the Higgs quartic λ_0, the Higgs m_{H_0} and Z-boson mass m_{Z_0}.

The above Lagrangian is not yet complete since we can not extract the gauge boson propagator, by inverting the operator acting on the gauge field kinetic term. Mathematically the reason is that we get a zero determinant. On the other hand, from the physics point of view, gauge invariance prevent us from having a uniq definition of the gauge propagator. Thus, we have to insert a term which will break gauge invariance without affecting the physical quantities. Now, according to the path integral formulation we have that

$$
Z\left[0\right] = \int DA_\mu D\phi e^{i\int d^4x\mathcal{L}(A,\phi)} \tag{222}
$$

where $\mathcal{L}\left(A,\phi\right)$ is the Lagrangian coming from Eq.(220). In order to break gauge invariance we do a gauge fixing by picking some element of the equivalence class of the gauge field. So, the fields in that gauge will follow a constrain given by $G[A]=0$. In our

case we have that the constrain reads $G \equiv G[A,\chi]$.
Now, we can insert in the above relation the unit written as follows

$$1 = \int Da\delta\left(G\left[A + \frac{1}{g_0}\partial_\mu a, \chi - a\left(v_0 + \phi\right)\right]\right)\det\left(\frac{\delta G\left[A - \partial_\mu a, \chi + a\left(v_0 + \phi\right)\right]}{\delta a}\right) \quad (223)$$

so choosing the constrain to be $G[A,\chi] = \frac{1}{\sqrt{\xi}}(\partial_\mu A_\mu - \xi g_0 v_0 \chi)$ and the gauge transformation of the fields as

$$\begin{aligned} A_\mu &\rightarrow A_\mu + \frac{1}{g_0}\partial_\mu a \\ \chi &\rightarrow \chi - a(v_0 + \phi) \end{aligned} \quad (224)$$

we get that

$$\det\left(\frac{\delta G\left[A + \frac{1}{g_0}\partial_\mu a, \chi - a\left(v_0 + \phi\right)\right]}{\delta a}\right) = \frac{1}{g_0\sqrt{\xi}}\det\left[\partial_\mu\partial^\mu + \xi m_{Z_0}^2\left(1 + \frac{\phi}{v_0}\right)\right]. \quad (225)$$

Therefore, Eq.(222) becomes

$$Z[0] = \int Da \int DA_\mu D\phi\delta\left(G\left[A,\chi\right]\right)\det\left(\frac{\delta G\left[A + \frac{1}{g_0}\partial_\mu a, \chi - a\left(v_0 + \phi\right)\right]}{\delta a}\right)_{a\rightarrow 0} e^{i\int d^4x\mathcal{L}(A,\phi)} \quad (226)$$

where the a integral is just an infinite constant. As we can see, if we shift G by a constant the determinant does not change and therefore we can average over a Gaussian-weighted selection of shifts using

$$\int Dke^{-i\int d^4x\frac{k^2}{2\xi}}\delta\left(G\left[A,\chi\right] - k\right) = e^{-i\int d^4x\frac{G^2}{2\xi}} \quad (227)$$

which makes Eq.(226) as follows

$$Z[0] = Const. \times \int DA_\mu D\phi\det\left(\frac{\delta G\left[A + \frac{1}{g_0}\partial_\mu a, \chi - a\left(v_0 + \phi\right)\right]}{\delta a}\right)_{a\rightarrow 0} e^{i\int d^4x\mathcal{L}(A,\phi) - \frac{G^2[A_\mu,\chi]}{2\xi}} \quad (228)$$

where $Const.$ is a constant containing the four dimensional volume.
Now, as final step, we use a known general path integral formulation concerning the anti-commuting fields, which gives that

$$\det[\mathcal{O}] = \int Dc D\bar{c} e^{-i\int d^4x \bar{c}(\mathcal{O})c} \tag{229}$$

and therefore combining all the previous arguments we get that in our case the path integral reads

$$Z[0] = Const. \times \int D\bar{c} Dc DA_\mu D\phi e^{i\int d^4x \mathcal{L}(A,\phi) - \frac{G^2[A_\mu,\chi]}{2\xi} + \bar{c}[-\partial_\mu\partial^\mu - \xi m_{Z_0}^2 (1+\frac{\phi}{v_0})]c} . \tag{230}$$

Thus the Lagrangian in Eq.(220) takes its final form which reads

$$\begin{aligned}
\mathcal{L}_{R_\xi} &= -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu\phi_0)(\partial^\mu\phi_0) + \frac{1}{2}(\partial_\mu\chi_0)(\partial^\mu\chi_0) - \frac{1}{2\xi}\left(\partial_\mu A_\mu^0\right)^2 + \frac{1}{2}m_{Z_0}^2 A_\mu^0 A^{0\mu} \\
&+ 2\frac{m_{Z_0}}{m_{H_0}}\sqrt{2\lambda_0} A_\mu^0 \partial_\mu \chi_0 \phi_0 + \frac{\sqrt{2\lambda_0}}{m_{H_0}}(\partial_\mu\chi_0)^2\phi_0 + \frac{2\lambda_0 m_{Z_0}}{m_{H_0}^2} A_\mu^0 \partial_\mu \chi_0 \phi_0^2 + \frac{\lambda_0}{m_{H_0}^2}(\partial_\mu\chi_0)^2\phi_0^2 \\
&+ g^{\mu\nu}\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} A_\mu^0 A_\nu^0 \phi_0^2 + g^{\mu\nu}\frac{m_{Z_0}^2}{m_{H_0}}\sqrt{2\lambda_0}\phi_0 A_\mu^0 A_\nu^0 - \frac{1}{2}\xi m_{Z_0}^2 \chi_0^2 \\
&- \frac{1}{2}m_{H_0}^2\phi_0^2 - \frac{\lambda_0}{4}\phi_0^4 - \sqrt{\frac{\lambda_0}{2}} m_{H_0}\phi_0^3 + \mathcal{L}_{ghost} + const.
\end{aligned} \tag{231}$$

where

$$\mathcal{L}_{ghost} = (\partial_\mu \bar{c})(\partial^\mu c) - \xi m_{Z_0}^2 \bar{c}c - \sqrt{2\lambda_0}\frac{\xi m_{Z_0}^2}{m_{H_0}}\phi\bar{c}c . \tag{232}$$

An important notation is that by inserting this specific gauge fixing term in the Lagrangian we have get rid of the mixing term between gauge and Goldstone bosons. On the other hand, the gauge fixing term gave birth to a mass term for the Goldstone boson which is defined as $m_{\chi_0} = \xi m_{Z_0}$.
Moreover, with our gauge fixing choice the ghost fields have a kinetic term, a mass term equal to the Goldstone boson's and they are coupled to the Higgs field but not to the gauge boson. So, the ghosts are not completely decoupled and unfortunately, an other un-physical field has been inserted to our calculations.
Now, in order to calculate the n-loop quantum corrections of the tree level procedures we

need the Feynman diagrams and rules coming from the above Lagrangian. Therefore, in this case they read

Gauge boson propagator

$$= \frac{i\left(-g^{\mu\nu} + \frac{(1-\xi)k^{\mu}k^{\nu}}{k^2-\xi m_{Z_0}^2}\right)}{k^2 - m_{Z_0}^2 + i\varepsilon} \tag{233}$$

Higgs boson propagator

$$= \frac{i}{k^2 - m_{H_0}^2 + i\varepsilon} \tag{234}$$

Goldstone boson propagator

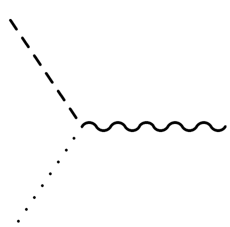
$$= \frac{i}{k^2 - \xi m_{Z_0}^2 + i\varepsilon} \tag{235}$$

Ghost filed propagator

$$= \frac{i}{k^2 - \xi m_{Z_0}^2 + i\varepsilon} \tag{236}$$

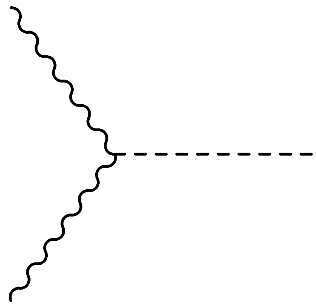
and now we go on to present the Feynman rules for the trilinear vertices:

Higgs-χ-Z vertex



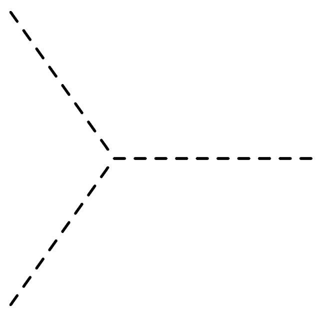
$$= -\frac{2m_{Z_0}}{m_{H_0}}\sqrt{2\lambda_0}k^{\mu} \tag{237}$$

Higgs-Z-Z vertex



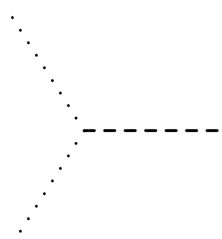
$$= 2ig^{\mu\nu}\frac{m_{Z_0}^2}{m_{H_0}}\sqrt{2\lambda_0} \tag{238}$$

Three-Higgs vertex



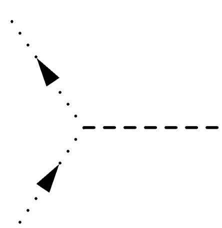
$$= -6i\sqrt{\frac{\lambda_0}{2}}m_{H_0} \tag{239}$$

Higgs-χ-χ vertex



$$= 2i\frac{\sqrt{2\lambda_0}}{m_{H_0}}k\cdot(k+p) \tag{240}$$

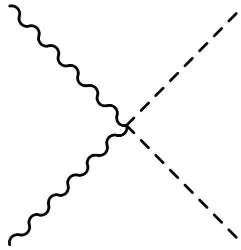
Higgs-$\bar{c}$-c vertex



$$= -i\sqrt{2\lambda_0}\frac{\xi m_{Z_0}^2}{m_{H_0}} \tag{241}$$

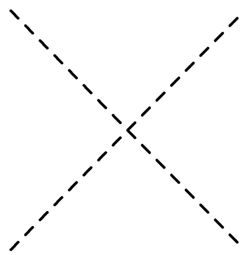
next we have the Feynman rules of the quadrilinear vertices:

Higgs-Higgs-Z-Z vertex



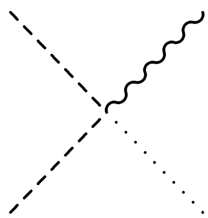
$$= 4i\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} g^{\mu\nu} \tag{242}$$

Four-Higgs vertex



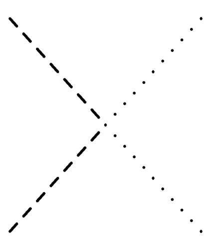
$$= -6i\lambda_0 \tag{243}$$

Higgs-Higgs-χ-Z vertex



$$= -4\frac{\lambda_0 m_{Z_0}}{m_{H_0}^2} k^{\mu} \tag{244}$$

χ-χ-Higgs-Higgs vertex

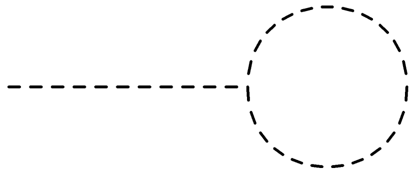


$$= 4i\frac{\lambda_0}{m_{H_0}^2} k \cdot (k + p_1 + p_2) \tag{245}$$

where here, we have defined that the momenta k and p_i, with $i = 1, 2$, correspond to the Goldstone boson and the Higgs boson respectively, assuming that one Goldstone boson gets in and the other gets out of the vertex.

3.1 Tadpoles in R_ξ gauge

Here we present the calculation of the Higgs Tadpoles which are playing a crucial role in our analysis, since they contribute as a shift of the vacuum and as a correction to the Higgs mass. The first Tadpole which contributes here reads



(246)

and it has the form

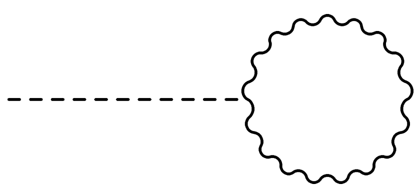
$$i\mathcal{T}_H^{1R_\xi} = -i6\mathcal{S}_T^1\sqrt{\frac{\lambda_0}{2}}m_{H_0}\int\frac{d^4k}{(2\pi)^4}\frac{i}{\left(k^2-m_{H_0}^2\right)} \Leftrightarrow \tag{247}$$

$$\mathcal{T}_H^{1R_\xi} = -3\sqrt{\frac{\lambda_0}{2}}m_{H_0}\int\frac{d^4k}{(2\pi)^4}\frac{i}{\left(k^2-m_{H_0}^2\right)} \tag{248}$$

where its symmetry factor is $\mathcal{S}_T^1 = \frac{1}{2}$. In d-dimensions this integral takes the form

$$(4\pi)^{d/2}\mathcal{T}_H^{1R_\xi} = 3\sqrt{\frac{\lambda_0}{2}}m_{H_0}\mu^{4-d}A_0(m_{H_0})\,. \tag{249}$$

The next tadpole comes from the contribution of the gauge boson and thus it has the following form



(250)

which reads

$$
\begin{aligned}
i\mathcal{T}_H^{2R_\xi} &= 2i\mathcal{S}_T^2 g^{\mu\nu}\frac{m_{Z_0}^2}{m_{H_0}}\sqrt{2\lambda_0}\int\frac{d^4k}{(2\pi)^4}\frac{i\left(-g_{\mu\nu}+(1-\xi)\frac{k_\mu k_\nu}{k^2-\xi m_{Z_0}^2}\right)}{\left(k^2-m_{Z_0}^2\right)} \Leftrightarrow \\
\mathcal{T}_H^{2R_\xi} &= (d+\varepsilon)\frac{m_{Z_0}^2}{m_{H_0}}\sqrt{2\lambda_0}\int\frac{d^4k}{(2\pi)^4}\frac{-i}{k^2-m_{Z_0}^2}+(1-\xi)\frac{m_{Z_0}^2}{m_{H_0}}\sqrt{2\lambda_0}g_{\mu\nu}\int\frac{d^4k}{(2\pi)^4}\frac{ik^\mu k^\nu}{\left(k^2-m_{Z_0}^2\right)\left(k^2-\xi m_{Z_0}^2\right)}
\end{aligned} \tag{251}
$$

with symmetry factor $\mathcal{S}_T^2=\frac{1}{2}$. Here, and in what follows, we use the fact that in d-dimensions the trace of the metric reads $g_{\mu\nu}g^{\mu\nu}=d+\varepsilon$. Moreover, using the relation $k^\mu k^\nu=\frac{g_{\mu\nu}}{d}k^2$ in d-dimensions the above integral reads

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{T}_H^{2R_\xi} &= \frac{m_{Z_0}^2}{m_{H_0}}\sqrt{2\lambda_0}\mu^{4-d}\Bigg\{4A_0(m_{Z_0})-(1-\xi)A_0(\sqrt{\xi}m_{Z_0})-(1-\xi)m_{Z_0}^2B_0(m_{Z_0},\sqrt{\xi}m_{Z_0}) \\
&+ (1+\xi)m_{Z_0}^2\Bigg\}
\end{aligned} \tag{252}
$$

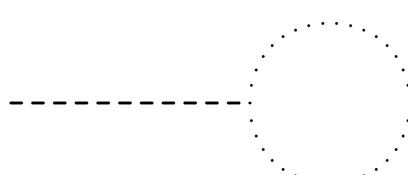
where we can see that the argument of the B_0 scalar integral corresponds to the specific case where $p^2=0$. As a consequence B_0 does not have its usual form, since it is completely symmetric under the interchange of its arguments, and thus it obtains the following symmetrical relation

$$
B_0(1,2)=\frac{A_0(1)-A_0(2)}{m_1^2-m_2^2}\,. \tag{253}
$$

Therefore, replacing this relation to Eq. (252) we obtain the following

$$
(4\pi)^{d/2}\mathcal{T}_H^{2R_\xi} = \frac{m_{Z_0}^2}{m_{H_0}}\sqrt{2\lambda_0}\mu^{4-d}\Bigg\{3A_0(m_{Z_0})+\xi A_0(\sqrt{\xi}m_{Z_0})+(1+\xi)m_{Z_0}^2\Bigg\}\,. \tag{254}
$$

The next Tadpole comes from the interaction of the Higgs boson to the un-physical Goldstone boson and reads



(255)

and it has the form

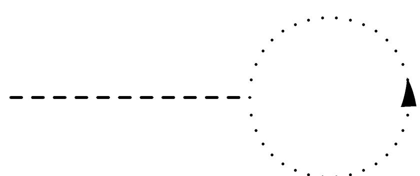
$$i\mathcal{T}_H^{3R_\xi} = 2i\mathcal{S}_T^3\frac{\sqrt{2\lambda_0}}{m_{H_0}}\int\frac{d^4k}{(2\pi)^4}\frac{ik^2}{\left(k^2-m_{\chi_0}^2\right)} \Leftrightarrow \tag{256}$$

$$\mathcal{T}_H^{3R_\xi} = -\frac{\sqrt{2\lambda_0}}{m_{H_0}}m_{\chi_0}^2\int\frac{d^4k}{(2\pi)^4}\frac{-i}{\left(k^2-m_{\chi_0}^2\right)} \tag{257}$$

where its symmetry factor is $\mathcal{S}_T^3 = \frac{1}{2}$. This is essentially a U-integral, similar with the integrals that we face in the Unitary gauge. This fact, as we comment in the next section, comes from our choice to use Polar basis in order to express the scalar field as a function of the physical Higgs. Now, in d-dimensions and using the Eq.(513) this integral takes the form

$$(4\pi)^{d/2}\mathcal{T}_H^{3R_\xi} = -\frac{\sqrt{2\lambda_0}}{m_{H_0}}m_{\chi_0}^2\mu^{4-d}A_0(m_{\chi_0})\,. \tag{258}$$

Finally, the last Higgs Tadpole that contributes to the shift of the vacuum comes from the interaction of the Higgs field with the un-physical ghost field which has been inserted to the Lagrangian. As a consequence, we get the following diagram



(259)

which has the following specific form

$$i\mathcal{T}_H^{4R_\xi} = -i(-1)\mathcal{S}_T^4\sqrt{2\lambda_0}\frac{\xi m_{Z_0}^2}{m_{H_0}}\int\frac{d^4k}{(2\pi)^4}\frac{i}{\left(k^2-m_{\chi_0}^2\right)} \Leftrightarrow \tag{260}$$

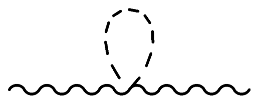
$$\mathcal{T}_H^{4R_\xi} = \sqrt{2\lambda_0}\frac{\xi m_{Z_0}^2}{m_{H_0}}\int\frac{d^4k}{(2\pi)^4}\frac{i}{\left(k^2-m_{\chi_0}^2\right)} \tag{261}$$

where we have inserted a necessary factor (-1) in the loop, since the ghosts are anti-commuting bosons. In addition, the symmetry factor of the above diagram is $\mathcal{S}_T^4 = 1$ and in d-dimensions this integral takes the form

$$(4\pi)^{d/2}\mathcal{T}_H^{4R_\xi} = -\sqrt{2\lambda_0}\frac{\xi m_{Z_0}^2}{m_{H_0}}\mu^{4-d}A_0(m_{\chi_0})\,. \tag{262}$$

3.2 Gauge boson two point functions

Here the one-loop corrections to gauge boson propagator in R_ξ gauge are calculated. The contributing diagrams are the following



$$= i\mathcal{M}^{1R_\xi}_{Z,\mu\nu} \tag{263}$$

where

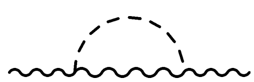
$$i\mathcal{M}^{1R_\xi}_{Z,\mu\nu} = 4i\mathcal{S}^1_g g^{\mu\nu}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2_{H_0}} \Leftrightarrow \tag{264}$$

$$\mathcal{M}^{1R_\xi}_{Z,\mu\nu} = -2g^{\mu\nu}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{-i}{k^2-m^2_{H_0}} \tag{265}$$

where the symmetry factor is $\mathcal{S}^1_g = \frac{1}{2}$. In d-dimensions this integral reads

$$(4\pi)^{d/2}\mathcal{M}^{1R_\xi}_{Z,\mu\nu} = -2g^{\mu\nu}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}A_0(m_{H_0})\,. \tag{266}$$

Now, the next one-loop correction to the gauge boson propagator coming from



$$= i\mathcal{M}^{2R_\xi}_{Z,\mu\nu} \tag{267}$$

where

$$i\mathcal{M}^{2R_\xi}_{Z,\mu\nu} = -8\mathcal{S}^3_g g^{\mu\alpha}g^{\nu\beta}\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{i}{(k+p)^2-m^2_{H_0}}\frac{i\left(-g_{\alpha\beta}+\frac{(1-\xi)k_\alpha k_\beta}{k^2-\xi m^2_{Z_0}}\right)}{\left(k^2-m^2_{Z_0}\right)} \Leftrightarrow$$

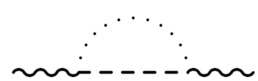
$$\mathcal{M}^{2R_\xi}_{Z,\mu\nu} = -8g^{\mu\alpha}g^{\nu\beta}\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{i}{(k+p)^2-m^2_{H_0}}\frac{\left(-g_{\alpha\beta}+\frac{(1-\xi)k_\alpha k_\beta}{k^2-\xi m^2_{Z_0}}\right)}{\left(k^2-m^2_{Z_0}\right)} \Leftrightarrow$$

$$
\begin{aligned}
\mathcal{M}^{2R_\xi}_{Z,\mu\nu} &= -8g^{\mu\alpha}g^{\nu\beta}\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{-ig_{\alpha\beta}}{\left(k^2-m^2_{Z_0}\right)\left((k+p)^2-m^2_{H_0}\right)} \\
&- 8g^{\mu\alpha}g^{\nu\beta}\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{i(1-\xi)k_\alpha k_\beta}{\left(k^2-m^2_{Z_0}\right)\left((k+p)^2-m^2_{H_0}\right)\left(k^2-\xi m^2_{Z_0}\right)} . \quad (268)
\end{aligned}
$$

where here the symmetry factor is $\mathcal{S}^2_g = 1$. Now, using the Veltman-Passarino reduction formula in d-dimensions we obtain the following

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}^{2R_\xi}_{Z,\mu\nu} &= 8\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}\Bigg\{-g^{\mu\nu}B_0(p,m_{Z_0},m_{H_0}) \\
&+ (1-\xi)\frac{g_{\mu\nu}}{d}\left[m^2_{Z_0}C_0(m_{Z_0},\sqrt{\xi}m_{Z_0},m_{H_0})+B_0(p,m_{\chi_0},m_{H_0})\right]\Bigg\} .
\end{aligned}
\quad (269)
$$

Finally, the last diagram that contributes to the one-loop corrections of the gauge boson propagator reads


$$= i\mathcal{M}^{3R_\xi}_{Z,\mu\nu} \quad (270)$$

$$
\begin{aligned}
i\mathcal{M}^{3R_\xi}_{Z,\mu\nu} &= -8\mathcal{S}^4_g\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{ik^\mu}{k^2-\xi m^2_{Z_0}}\frac{ik^\nu}{(k+p)^2-m^2_{H_0}} \Leftrightarrow \\
\mathcal{M}^{3R_\xi}_{Z,\mu\nu} &= 8\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{-ik^\mu k^\nu}{(k^2-\xi m^2_{Z_0})((k+p)^2-m^2_{H_0})} . \quad (271)
\end{aligned}
$$

where here the symmetry factor is $\mathcal{S}^3_g = 1$. Therefore, with the help of the integral notation developed from the Veltman-Passarino reduction formula, in d-dimensions the final form of $\mathcal{M}^{3R_\xi}_{Z,\mu\nu}$ reads

$$
(4\pi)^{d/2}\mathcal{M}^{3R_\xi}_{Z,\mu\nu} = 8\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}\frac{g^{\mu\nu}}{d}\Bigg\{m_{\chi_0}B_0(p,m_{\chi_0},m_{H_0})+A_0(m_{H_0})\Bigg\} . \quad (272)
$$

Thus, since we have finished with the evaluation of the one-loop diagrams of the gauge boson two-point function, we should add all of them in order to calculate the full corrections of the corresponding propagator. So we get that

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}^{R_\xi}_{Z,\mu\nu} &= \frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}\Bigg\{-2g^{\mu\nu}A_0(m_{H_0}) - 8g^{\mu\nu}B_0(p,m_{Z_0},m_{H_0}) \\
&+ 8(1-\xi)\frac{g_{\mu\nu}}{d}\left[m^2_{Z_0}C_0(m_{Z_0},\sqrt{\xi}m_{Z_0},m_{H_0}) + B_0(p,m_{\chi_0},m_{H_0})\right] \\
&+ 8\frac{g^{\mu\nu}}{d}m^2_{\chi_0}B_0(p,m_{\chi_0},m_{H_0}) + 8\frac{g^{\mu\nu}}{d}A_0(m_{H_0})\Bigg\}
\end{aligned}
\tag{273}
$$

and for completeness here we present the corresponding one-loop corrections to the gauge boson propagator calculated in the Unitary gauge. The exact form reads

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}^{U}_{Z,\mu\nu} &= g_{\mu\nu}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}\Bigg\{-8m^2_{Z_0}B_0(p,m_{Z_0},m_{H_0}) - 2A_0(m_{H_0}) \\
&+ \frac{8}{d}m^2_{Z_0}B_0(p,m_{Z_0},m_{H_0}) + \frac{8}{d}A_0(m_{H_0})\Bigg\}.
\end{aligned}
\tag{274}
$$

Actually the relation of $\mathcal{M}^{R_\xi}_{Z,\mu\nu}$ is far from being complete. As we should recall from the first section, a gauge boson propagator can be split into a transverse part and a longitudinal part as follows

$$
G_{\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\mathcal{M}^{TR_\xi}_{Z,\mu\nu} + \frac{p_\mu p_\nu}{p^2}\mathcal{M}^{LR_\xi}_{Z,\mu\nu}\,. \tag{275}
$$

In addition, if the mass of the propagators running the loop is zero or smaller than the external field mass, then we are allowed to ignore the $p^\mu p^\nu$ terms from Eqs.(275). Generally this is not the case but here for simplicity and in order to see some qualitative results, we will consider only the contracted with the metric term of $G_{\mu\nu}(p)$. And now we can use the Veltman-Passarino reduction formula so as to express Eqs.(273) as a function of the scalar integrals. Now we present some useful relations that we should use in order to manage to reduce our expressions to scalar integrals

$$
\begin{aligned}
f_1 &= m^2_2 - m^2_1 - p^2 \\
p_\mu B^\mu(1,2) &= \frac{f_1}{2}B_0(1,2) + \frac{1}{2}A_0(1) - \frac{1}{2}A_0(2) \\
g_{\mu\nu}B^{\mu\nu}(1,2) &= m^2_1 B_0(1,2) + A_0(2)
\end{aligned}
$$

$$
\begin{aligned}
p_\mu p_\nu B^{\mu\nu}(1,2) &= \frac{m_2^2 - m_1^2 - p^2}{4} A_0(1) + \frac{m_1^2 - m_2^2 + 3p^2}{4} A_0(2) \\
&+ \left(\frac{m_1^4 + m_2^4 - 2m_1^2 m_2^2}{4} + \frac{p^2(m_1^2 - m_2^2 + p^2)}{4} \right) B_0(1,2) \\
g_{\mu\nu} C^{\mu\nu}(1,2,3) &= m_1^2 C_0(1,2,3) + B_0(2,3) \\
p_\mu p_\nu C^{\mu\nu}(1,2,3) &= \frac{(m_2^2 - m_1^2 - p^2)^2}{4} C_0(1,2,3) + \frac{m_3^2 + m_2^2 - 2m_1^2 - 2p^2}{4} B_0(1,3) \\
&+ \frac{-m_3^2 + m_1^2 + 2p^2}{4} B_0(2,3) + \frac{1}{4} A_0(1) - \frac{1}{4} A_0(2)
\end{aligned}
\tag{276}
$$

so as an example, we can use these relations obtaining that

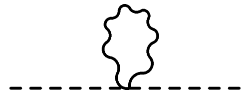
$$
\begin{aligned}
g_{\mu\nu} \mathcal{M}^{R_\xi}_{Z,\mu\nu} &= (d+\varepsilon) \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \frac{\mu^\varepsilon}{16\pi^2} \Bigg\{ -2A_0(m_{H_0}) - 8B_0(p, m_{Z_0}, m_{H_0}) \\
&+ 8(1-\xi)\frac{1}{d} \left[m_{Z_0}^2 C_0(m_{Z_0}, \sqrt{\xi} m_{Z_0}, m_{H_0}) + B_0(p, m_{\chi_0}, m_{H_0}) \right] \\
&+ 8\frac{1}{d} m_{\chi_0}^2 B_0(p, m_{\chi_0}, m_{H_0}) + 8\frac{1}{d} A_0(m_{H_0}) \Bigg\}.
\end{aligned}
\tag{277}
$$

The corresponding contribution of the contracted with the metric $\mathcal{M}^U_{Z,\mu\nu}$ gives

$$
\begin{aligned}
g_{\mu\nu} \mathcal{M}^U_{Z,\mu\nu} &= (d+\varepsilon) \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \frac{\mu^\varepsilon}{16\pi^2} \Bigg\{ -8m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) - 2A_0(m_{H_0}) \\
&+ \frac{8}{d} m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) + \frac{8}{d} A_0(m_{H_0}) \Bigg\}.
\end{aligned}
\tag{278}
$$

3.3 Corrections to the Higgs mass

Since we have finished with the corrections of the gauge propagator in both R_ξ and Unitary gauge, we move on to the demonstration of the same procedure for the Higgs propagator, namely to the corrections of the Higgs mass. In this case the corrections coming from the following contributions



$$= i\mathcal{M}_H^{1R_\xi}. \tag{279}$$

$$\begin{aligned}
i\mathcal{M}_H^{1R_\xi} &= 4i\mathcal{S}^1_{\mathcal{M}_H} g^{\mu\nu} \frac{m^2_{Z_0}}{m^2_{H_0}} \lambda_0 \int \frac{d^4k}{(2\pi)^4} \frac{i\left(-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2-\xi m^2_{Z_0}}\right)}{\left(k^2 - m^2_{Z_0}\right)} \Leftrightarrow \\
\mathcal{M}_H^{1R_\xi} &= 2(d+\varepsilon)\frac{m^2_{Z_0}}{m^2_{H_0}} \lambda_0 \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2 - m^2_{Z_0}} + \frac{2(1-\xi)m^2_{Z_0}}{m^2_{H_0}} \lambda_0 g_{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{ik^\mu k^\nu}{\left(k^2 - m^2_{Z_0}\right)\left(k^2 - \xi m^2_{Z_0}\right)}
\end{aligned} \tag{280}$$

where the symmetry factor here is $\mathcal{S}^1_{\mathcal{M}_H} = \frac{1}{2}$. Now in d-dimensions using the Veltman-Passarino reduction formula and the Eqs.(276), the above integral reads

$$\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_H^{1R_\xi} &= \mu^{4-d}\left\{2(d+\varepsilon)\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0 A_0(m_{Z_0}) - 2\frac{(d+\varepsilon)}{d}(1-\xi)\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0 g_{\mu\nu} B^{\mu\nu}(m_{Z_0}, \sqrt{\xi}m_{Z_0})\right\} \Leftrightarrow \\
(4\pi)^{d/2}\mathcal{M}_H^{1R_\xi} &= \frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\mu^{4-d}\left\{8A_0(m_{Z_0}) - 2(1-\xi)A_0(\sqrt{\xi}m_{Z_0}) - 2(1-\xi)m^2_{Z_0}B_0(m_{Z_0}, \sqrt{\xi}m_{Z_0})\right. \\
&+ \left. 2(1+\xi)m^2_{Z_0}\right\}.
\end{aligned} \tag{281}$$

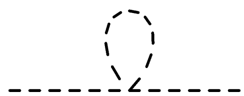
where again here we can see that the argument of the B_0 scalar integral corresponds to the specific case where $p^2 = 0$. As a consequence this B_0 does not have its usual form so in every time that we face an integral like that, we will denote it as $B_0^1(m_1, m_2)$ *i.e.* without a p argument. Finally, since this scalar integral is completely symmetric under the interchange of its arguments, it obtains the following symmetrical relation

$$B_0^1(m_1, m_2) = \frac{A_0(1) - A_0(2)}{m_1^2 - m_2^2}\,. \tag{282}$$

Now, applying the above formula to our case in Eq. (281) we get that

$$(4\pi)^{d/2}\mathcal{M}_H^{1R_\xi} \quad = \quad \frac{m_{Z_0}^2}{m_{H_0}^2}\lambda_0\mu^{4-d}\left\{6A_0(m_{Z_0}) + 2\xi A_0(m_{\chi_0}) + 2(1+\xi)m_{Z_0}^2\right\}. \tag{283}$$

The next contribution comes from the diagram



$$= i\mathcal{M}_H^{2R_\xi} \tag{284}$$

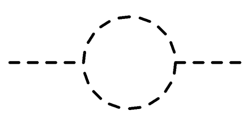
which has the following explicit form

$$\begin{aligned} i\mathcal{M}_H^{2R_\xi} &= -6i\mathcal{S}^2_{\mathcal{M}_H}\lambda_0 \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2 - m_{H_0}^2} \Leftrightarrow \\ \mathcal{M}_H^{2R_\xi} &= 3\lambda_0 \int \frac{d^4k}{(2\pi)^4}\frac{-i}{k^2 - m_{H_0}^2}\,. \end{aligned} \tag{285}$$

Its symmetry factor reads $\mathcal{S}^2_{\mathcal{M}_H} = \frac{1}{2}$ and thus in d-dimensions we get that

$$(4\pi)^{d/2}\mathcal{M}_H^{2R_\xi} = 3\lambda_0\mu^{4-d}A_0(m_{H_0}). \tag{286}$$

Next comes the Goldstone boson Tadpole which reads



$$= i\mathcal{M}_H^3 \tag{287}$$

and its explicit form is

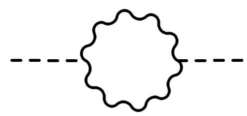
$$i\mathcal{M}_H^{3R_\xi} \quad = \quad 4i\mathcal{S}^3_{\mathcal{M}_H}\frac{\lambda_0}{m_{H_0}^2}\int \frac{d^4k}{(2\pi)^4}\frac{ik^2}{k^2 - m_{\chi_0}^2} \Leftrightarrow \tag{288}$$

$$\mathcal{M}_H^{3R_\xi} \quad = \quad -2\frac{\lambda_0}{m_{H_0}^2}m_{\chi_0}^2 \int \frac{d^4k}{(2\pi)^4}\frac{-i}{k^2 - m_{\chi_0}^2} \tag{289}$$

where here the symmetry factor reads $\mathcal{S}^3_{\mathcal{M}_H} = \frac{1}{2}$. Now in d-dimensions we get the following final result

$$(4\pi)^{d/2}\mathcal{M}_H^{3R_\xi} = -\frac{2\lambda_0}{m_{H_0}^2} m_{\chi_0}^2 \mu^{4-d} A_0(m_{\chi_0}). \tag{290}$$

So now we move on to more complicated contributions starting with the following



$$= i\mathcal{M}_H^{4R_\xi} \tag{291}$$

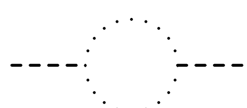
which is equal to

$$\begin{aligned} i\mathcal{M}_H^{4R_\xi} &= -18\mathcal{S}^4_{\mathcal{M}_H}\lambda_0 m_{H_0}^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{\left(k^2 - m_{H_0}^2\right)} \frac{i}{\left((k+p)^2 - m_{H_0}^2\right)} \Leftrightarrow \\ \mathcal{M}_H^{4R_\xi} &= 9\lambda_0 m_{H_0}^2 \int \frac{d^4k}{(2\pi)^4} \frac{-i}{\left(k^2 - m_{H_0}^2\right)\left((k+p)^2 - m_{H_0}^2\right)} . \end{aligned} \tag{292}$$

The symmetry factor here is $\mathcal{S}^4_{\mathcal{M}_H} = \frac{1}{2}$ and in d-dimensions, using the Veltman-Passarino reduction formula, $\mathcal{M}_H^{4R_\xi}$ reads

$$(4\pi)^{d/2}\mathcal{M}_H^{4R_\xi} = 9\lambda_0 m_{H_0}^2 \mu^{d-4} B_0(p, m_{H_0}, m_{H_0}) . \tag{293}$$

Next we have the Higgs-Goldstone self loop contribution which reads



$$= i\mathcal{M}_H^5 \tag{294}$$

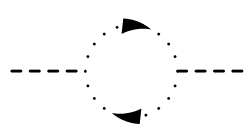
which is equal to

$$
\begin{aligned}
i\mathcal{M}_H^{5R_\xi} &= -8\mathcal{S}^5_{\mathcal{M}_H}\frac{\lambda_0}{m^2_{H_0}}\int\frac{d^4k}{(2\pi)^4}\frac{ik\cdot(k+p)}{\left(k^2-m^2_{\chi_0}\right)}\frac{ik\cdot(k+p)}{\left((k+p)^2-m^2_{\chi_0}\right)}\Leftrightarrow \\
\mathcal{M}_H^{5R_\xi} &= -4\mathcal{S}^5_{\mathcal{M}_H}\frac{\lambda_0}{m^2_{H_0}}\int\frac{d^4k}{(2\pi)^4}\frac{i[k^4+2k^2k\cdot p+\frac{1}{d}p^2k^2]}{\left(k^2-m^2_{\chi_0}\right)\left((k+p)^2-m^2_{\chi_0}\right)}
\end{aligned}
\qquad (295)
$$

where the symmetry factor here reads $\mathcal{S}^5_{\mathcal{M}_H}=\frac{1}{2}$.
From the above relation it is clear that Eq.(295) needs a specific treatment, since it belongs to the family of the U-integrals which we have defined in the case of Unitary gauge. Now, someone could say that this is quite unexpected since in R_ξ-gauge we do not see these specific integrals, but we should recall that we have used the Polar basis in order to express the scalar field as a function of the real Higgs field not the Cartesian. Thus our argument stays that if we have a theory in R_ξ-gauge and we consider the scalar field in Polar basis, then highly divergent integrals *i.e.* the U-integrals, should appear.
Now, using the Veltman-Passarino reduction formula and the $U_{\mathcal{M}_4}$'s results, in d-dimensions we obtain the following

$$
(4\pi)^{d/2}\mathcal{M}_H^{5R_\xi} = 4\frac{\lambda_0}{m^2_{H_0}}\mu^{d-4}\left\{[2m^2_{\chi_0}-\frac{3}{d}p^2]A_0(m_{\chi_0})+(m^4_{\chi_0}-\frac{3}{d}p^2m^2_{\chi_0})B_0(p,m_{\chi_0},m_{\chi_0})\right\}. \qquad (296)
$$

Next we consider the one-loop correction to the Higgs mass coming from the ghost fields, which have been inserted in the Lagrangian through the gauge fixing. Thus, we have that

 $= i\mathcal{M}_H^6$ (297)

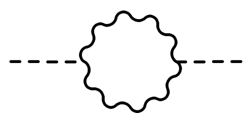
which reads

$$
\begin{aligned}
i\mathcal{M}_H^{6R_\xi} &= -2\mathcal{S}^6_{\mathcal{M}_H}\frac{\lambda_0\xi^2m^4_{Z_0}}{m^2_{H_0}}\int\frac{d^4k}{(2\pi)^4}\frac{i}{\left(k^2-m^2_{\chi_0}\right)}\frac{i}{\left((k+p)^2-m^2_{\chi_0}\right)}\Leftrightarrow \\
\mathcal{M}_H^{6R_\xi} &= 2\frac{\lambda_0\xi^2m^4_{Z_0}}{m^2_{H_0}}\int\frac{d^4k}{(2\pi)^4}\frac{-i}{\left(k^2-m^2_{\chi_0}\right)\left((k+p)^2-m^2_{\chi_0}\right)}
\end{aligned}
\qquad (298)
$$

with symmetry factor $\mathcal{S}^6_{\mathcal{M}_H} = 1$. Therefore using the Veltman-Passarino reduction formula in d-dimensions we obtain the following

$$(4\pi)^{d/2}\mathcal{M}_H^{6R_\xi} = 2\frac{\lambda_0\xi^2 m^4_{Z_0}}{m^2_{H_0}}\mu^{d-4}B_0(p, m_{\chi_0}, m_{\chi_0})\,. \tag{299}$$

Now we are left only with two more diagrams contributing to the corrections of Higgs mass. The first one exists also in the Unitary gauge prescription and reads



$$= i\mathcal{M}_H^{7R_\xi}\,. \tag{300}$$

Its explicit form is given by the following integral

$$\begin{aligned} i\mathcal{M}_H^{7R_\xi} &= -8\mathcal{S}^7_{\mathcal{M}_H} g^{\mu\nu}g^{\alpha\beta}\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{i\left(-g_{\mu\alpha}+\frac{(1-\xi)k_\mu k_\alpha}{k^2-\xi m^2_{Z_0}}\right)}{\left(k^2-m^2_{Z_0}\right)}\frac{i\left(-g_{\nu\beta}+\frac{(1-\xi)(k+p)_\nu(k+p)_\beta}{(k+p)^2-\xi m^2_{Z_0}}\right)}{\left((k+p)^2-m^2_{Z_0}\right)} \Leftrightarrow \\ \mathcal{M}_H^{7R_\xi} &= 4g^{\mu\nu}g^{\alpha\beta}\frac{m^4_{Z_0}}{m^2_{H_0}}\lambda_0\int\frac{d^4k}{(2\pi)^4}\frac{-i\left(-g_{\mu\alpha}+\frac{(1-\xi)k_\mu k_\alpha}{k^2-\xi m^2_{Z_0}}\right)}{\left(k^2-m^2_{Z_0}\right)}\frac{\left(-g_{\nu\beta}+\frac{(1-\xi)(k+p)_\nu(k+p)_\beta}{(k+p)^2-\xi m^2_{Z_0}}\right)}{\left((k+p)^2-m^2_{Z_0}\right)} \end{aligned} \tag{301}$$

where the symmetry factor here is $\mathcal{S}^7_{\mathcal{M}_H} = \frac{1}{2}$. Now, we perform the computation in the contracted numerator of $\mathcal{M}_H^{7R_\xi}$ from which we obtain the following

$$\begin{aligned} N &= g^{\mu\nu}g^{\alpha\beta}\left(-g_{\mu\alpha}+\frac{(1-\xi)k_\mu k_\alpha}{k^2-\xi m^2_{Z_0}}\right)\left(-g_{\nu\beta}+\frac{(1-\xi)\,(k+p)_\nu(k+p)_\beta}{(k+p)^2-\xi m^2_{Z_0}}\right) \Leftrightarrow \\ N &= (d+\varepsilon)-(1-\xi)\frac{d+\varepsilon}{d}\left\{\frac{k^2}{k^2-\xi m^2_{Z_0}}+\frac{k^2+2k\cdot p+p^2}{(k+p)^2-\xi m^2_{Z_0}}\right\} \\ &+ (1-\xi)^2\frac{d+\varepsilon}{d^2}\left\{\frac{k^4+2k^2k\cdot p+k^2p^2}{[k^2-\xi m^2_{Z_0}][(k+p)^2-\xi m^2_{Z_0}]}\right\}. \end{aligned} \tag{302}$$

Thus, now if we replace this expression in Eq.(301), performing the Veltamn-Passarino reduction, we will obtain the following form

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_H^{7R_\xi} &= 4\frac{m_{Z_0}^4}{m_{H_0}^2}\lambda_0\mu^{d-4}\Bigg\{(d+\varepsilon)B_0(p,m_{Z_0},m_{Z_0}) - (1-\xi)\frac{d+\varepsilon}{d}\Big\{m_{Z_0}^2 C_0^1(p,m_{Z_0},m_{Z_0},m_{\chi_0}) \\
&+ B_0^1(m_{Z_0},m_{\chi_0}) + (2m_{Z_0}^2 + f_1 + p^2)C_0^2(p,m_{Z_0},m_{Z_0},m_{\chi_0}) \\
&+ 2B_0^2(p,m_{Z_0},m_{\chi_0}) - B_0^2(p,m_{Z_0},m_{\chi_0})\Big\} \\
&+ (1-\xi)^2\frac{d+\varepsilon}{d^2}\Big\{B_0^3(p,m_{Z_0},m_{\chi_0}) + m_{Z_0}^2 C_0^a(p,m_{Z_0},m_{\chi_0},m_{\chi_0}) \\
&+ m_{Z_0}^2 g_{\mu\nu}D^{\mu\nu}(p,m_{Z_0},m_{Z_0},m_{\chi_0},m_{\chi_0})\Big\}\Bigg\}
\end{aligned}
\tag{303}
$$

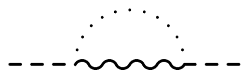
where the $i=1,2$ and $j=a,b$ indices correspond to the possible different combinations of the denominators in the $B_0's$ and $C_0's$ scalar integrals. To be more specific, for a given scalar integral B_0 we have that

$$
\begin{aligned}
B_0^1(m_{Z_0},m_{\chi_0}) &= \int\frac{d^4k}{(2\pi)^4}\frac{-i}{\left(k^2-m_{Z_0}^2\right)\left(k^2-m_{\chi_0}^2\right)} \\
B_0^2(p,m_{Z_0},m_{\chi_0}) &= \int\frac{d^4k}{(2\pi)^4}\frac{-i}{\left(k^2-m_{Z_0}^2\right)\left((k+p)^2-m_{\chi_0}^2\right)} \\
B_0^3(p,m_{Z_0},m_{\chi_0}) &= \int\frac{d^4k}{(2\pi)^4}\frac{-i}{\left(k^2-m_{Z_0}^2\right)\left((k+p)^2-m_{\chi_0}^2\right)}\,.
\end{aligned}
\tag{304}
$$

while for a C_0 we define the following relations

$$
\begin{aligned}
C_0^1(p,m_{Z_0},m_{Z_0},m_{\chi_0}) &= \int\frac{d^4k}{(2\pi)^4}\frac{-i}{\left(k^2-m_{Z_0}^2\right)\left((k+p)^2-m_{Z_0}^2\right)\left(k^2-m_{\chi_0}^2\right)} \\
C_0^2(p,m_{Z_0},m_{Z_0},m_{\chi_0}) &= \int\frac{d^4k}{(2\pi)^4}\frac{-i}{\left(k^2-m_{Z_0}^2\right)\left((k+p)^2-m_{Z_0}^2\right)\left((k+p)^2-m_{\chi_0}^2\right)} \\
C_0^a(p,m_{Z_0},m_{\chi_0},m_{\chi_0}) &= \int\frac{d^4k}{(2\pi)^4}\frac{-i}{\left(k^2-m_{Z_0}^2\right)\left(k^2-m_{\chi_0}^2\right)\left((k+p)^2-m_{\chi_0}^2\right)}\,.
\end{aligned}
\tag{305}
$$

Finally, the last contribution to the one-loop corrections of the Higgs mass comes from the following



$$= i\mathcal{M}_H^{8R_\xi} \tag{306}$$

and its explicit form reads

$$\begin{aligned}
i\mathcal{M}_H^{8R_\xi} &= -8\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} \mathcal{S}_{\mathcal{M}_H}^8 \int \frac{d^4k}{(2\pi)^4} \frac{ik^\mu k^\nu}{k^2 - m_{\chi_0}^2} \frac{i\left(-g_{\mu\nu} + \frac{(1-\xi)(k+p)_\mu (k+p)_\nu}{(k+p)^2 - \xi m_{Z_0}^2}\right)}{\left((k+p)^2 - m_{Z_0}^2\right)} \Leftrightarrow \\
\mathcal{M}_H^{8R_\xi} &= -8\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} \int \frac{d^4k}{(2\pi)^4} \frac{ik^\mu k^\nu}{k^2 - m_{\chi_0}^2} \frac{\left(-g_{\mu\nu} + \frac{(1-\xi)(k+p)_\mu (k+p)_\nu}{(k+p)^2 - \xi m_{Z_0}^2}\right)}{\left((k+p)^2 - m_{Z_0}^2\right)} \Leftrightarrow \\
\mathcal{M}_H^{8R_\xi} &= -8\frac{d+\varepsilon}{d}\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} g_{\mu\nu} B^{\mu\nu}(p, m_{\chi_0}, m_{Z_0}) \\
&+ 8(1-\xi)\frac{d+\varepsilon}{d^2}\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} \int \frac{d^4k}{(2\pi)^4} \frac{-i[k^4 + 2k^2 k\cdot p + k^2 p^2]}{[k^2 - m_{\chi_0}^2][(k+p)^2 - m_{Z_0}^2][(k+p)^2 - m_{\chi_0}^2]}
\end{aligned} \tag{307}$$

where its symmetry factor is $\mathcal{S}_{\mathcal{M}_H}^8 = 1$. So we see that $\mathcal{M}_H^{8R_\xi}$ can be split into two terms which we calculate separately. Therefore we have the following relations

$$\begin{aligned}
\mathcal{M}_H^{8R_\xi A} &= -8\frac{d+\varepsilon}{d}\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} g_{\mu\nu} B^{\mu\nu}(p, m_{\chi_0}, m_{Z_0}) \\
\mathcal{M}_H^{8R_\xi B} &= 8\frac{d+\varepsilon}{d^2}(1-\xi)\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} \int \frac{d^4k}{(2\pi)^4} \frac{-i[k^4 + 2k^2 k\cdot p + k^2 p^2]}{[k^2 - m_{\chi_0}^2][(k+p)^2 - m_{Z_0}^2][(k+p)^2 - m_{\chi_0}^2]} .
\end{aligned} \tag{308}$$

The first one in d-dimensions, using the Veltman-Passarino reduction formula and Eqs.(276) reads

$$(4\pi)^{d/2}\mathcal{M}_H^{8R_\xi A} = -8\frac{d+\varepsilon}{d}\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} \mu^{d-4} \left[m_{\chi_0}^2 B_0(p, m_{\chi_0}, m_{Z_0}) + A_0(m_{Z_0})\right] \tag{309}$$

while the second term, using the same arguments, obtains the following form

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_H^{8R_\xi B} &= 8\frac{d+\varepsilon}{d^2}(1-\xi)\lambda_0\frac{m_{Z_0}^2}{m_{H_0}^2}\mu^{d-4}\Bigg\{A_0(m_{\chi_0})+m_{\chi_0}^2B_0(p,m_{\chi_0},m_{\chi_0}) \\
&+ m_{Z_0}^2B_0(p,m_{Z_0},m_{\chi_0})+m_{\chi_0}^2m_{Z_0}^2C_0(p,m_{\chi_0},m_{Z_0},m_{\chi_0})\Bigg\}
\end{aligned}
\tag{310}
$$

therefore, in order to obtain the full contribution of $\mathcal{M}_H^{8R_\xi}$ we should add the two results calculated above.Therefore, we obtain the following two relation

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_H^{8R_\xi} &= 8\frac{d+\varepsilon}{d}\lambda_0\frac{m_{Z_0}^2}{m_{H_0}^2}\mu^{d-4}\Bigg\{-m_{\chi_0}^2B_0(p,m_{\chi_0},m_{Z_0})-A_0(m_{Z_0}) \\
&+ \frac{1}{d}(1-\xi)\Big\{A_0(m_{\chi_0})+m_{\chi_0}^2B_0(p,m_{\chi_0},m_{\chi_0}) \\
&+ m_{Z_0}^2B_0(p,m_{Z_0},m_{\chi_0})+m_{\chi_0}^2m_{Z_0}^2C_0(p,m_{\chi_0},m_{Z_0},m_{\chi_0})\Big\}\Bigg\}
\end{aligned}
\tag{311}
$$

Here we have a misleading point, since someone could say that we need to consider the $\mathcal{M}_H^{8R_\xi}$'s mirror diagram which occurs from the interchange of the Goldstone boson with the gauge boson inside the loop. Nevertheless, this is a step that should not be done here, since we have that the mirror contribution is exactly the same with that of $\mathcal{M}_H^{8R_\xi}$.

Until now we have calculated all of the necessary diagrams that contribute to the one-loop corrections of the Higgs mass. Thus, the final step is to evaluate their sum which will be the complete one-loop correction to the Higgs two point function. So this correction, which we define as $\mathcal{M}_H^{R_\xi}$ reads

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_H^{R_\xi}(p) &= \mu^{d-4}\Bigg\{3\lambda_0 A_0(m_{H_0}) + 6\frac{m_{Z_0}^2}{m_{H_0}^2}\lambda_0 A_0(m_{Z_0}) + 2\xi\frac{m_{Z_0}^2}{m_{H_0}^2}\lambda_0 A_0(m_{\chi_0}) \\
&+ 2(1-\xi)\frac{m_{Z_0}^4}{m_{H_0}^2}\lambda_0 - 2\frac{\lambda_0}{m_{H_0}^2}m_{\chi_0}^2 A_0(m_{\chi_0}) + 9\lambda_0 m_{H_0}^2 B_0(p, m_{H_0}, m_{H_0}) \\
&+ 4\frac{\lambda_0}{m_{H_0}^2}\left[2m_{\chi_0}^2 - \frac{3p^2}{d}\right]A_0(m_{\chi_0}) + 4\frac{\lambda_0}{m_{H_0}^2}\left(m_{\chi_0}^4 - \frac{3p^2}{d}p^2 m_{\chi_0}^2\right)B_0(p, m_{\chi_0}, m_{\chi_0}) \\
&+ 2\frac{\lambda_0\xi^2 m_{Z_0}^4}{m_{H_0}^2}B_0(p, m_{\chi_0}, m_{\chi_0}) + 4(d+\varepsilon)\frac{m_{Z_0}^4}{m_{H_0}^2}\lambda_0 B_0(p, m_{Z_0}, m_{Z_0}) \\
&- 8\frac{d+\varepsilon}{d}\lambda_0\frac{\xi^2 m_{Z_0}^4}{m_{H_0}^2}B_0(p, m_{\chi_0}, m_{Z_0}) - 8\frac{d+\varepsilon}{d}\lambda_0\frac{m_{Z_0}^2}{m_{H_0}^2}A_0(m_{Z_0}) \\
&+ \frac{m_{Z_0}^2}{m_{H_0}^2}\lambda_0(1-\xi)\Bigg\{-\frac{4m_{Z_0}^2(d+\varepsilon)}{d}(3m_{Z_0}^2 + f_1 + p^2)C_0^2(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \\
&- 4m_{Z_0}^2\frac{d+\varepsilon}{d}B_0^1(m_{Z_0}, m_{\chi_0}) - 8m_{Z_0}^2\frac{d+\varepsilon}{d}B_0^2(p, m_{Z_0}, m_{\chi_0}) + 4m_{Z_0}^2\frac{d+\varepsilon}{d}B_0^2(p, m_{Z_0}, m_{\chi_0}) \\
&+ 8\frac{d+\varepsilon}{d^2}A_0(m_{\chi_0}) + 16\frac{d+\varepsilon}{d^2}m_{\chi_0}^2 B_0(p, m_{\chi_0}, m_{\chi_0}) \\
&+ 8\frac{d+\varepsilon}{d^2}m_{Z_0}^2 B_0(p, m_{Z_0}, m_{\chi_0}) + 16\frac{d+\varepsilon}{d^2}m_{\chi_0}^2 m_{Z_0}^2 C_0(p, m_{\chi_0}, m_{Z_0}, m_{\chi_0})\Bigg\} \\
&+ 4\frac{m_{Z_0}^4}{m_{H_0}^2}\lambda_0(1-\xi)^2\frac{d+\varepsilon}{d^2}\Bigg\{B_0^3(p, m_{Z_0}, m_{\chi_0}) + m_{Z_0}^2 C_0^a(p, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) \\
&+ m_{Z_0}^2 g_{\mu\nu}D^{\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}, m_{\chi_0})\Bigg\}\Bigg\}.
\end{aligned}
\tag{312}
$$

where the $g_{\mu\nu}D^{\nu\mu}$ term gives only C_0 and D_0 contributions which are completely finite. Now, in order to have a complete description of what we have obtained with the above calculation, we present the corresponding one-loop correction coming from Unitary gauge. So, using the parametrization from Eq.(221) the correction to Higgs mass in this gauge reads

$$
\begin{aligned}
(4\pi)^{d/2}\mathcal{M}_H^U(p) &= \mu^{d-4}\Bigg\{6\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}A_0(m_{Z_0}) + \frac{3\lambda m_{Z_0}^4}{m_{H_0}^2} + 3\lambda_0 A_0(m_{H_0}) \\
&+ 9\lambda_0 m_{H_0}^2 B_0(p,m_{H_0},m_{H_0}) + 4(d+\varepsilon)\frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2}B_0(p,m_{Z_0},m_{Z_0}) \\
&- \frac{8(d+\varepsilon)}{d}\frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2}\left[B_0(p,m_{Z_0},m_{Z_0}) + \frac{1}{m_{Z_0}^2}A_0(m_{Z_0})\right] \\
&+ \frac{4(d+\varepsilon)}{d^2}\frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2}\left[B_0(p,m_{Z_0},m_{Z_0}) + \frac{2}{m_{Z_0}^2}A_0(m_{Z_0})\right]\Bigg\}.
\end{aligned} \tag{313}
$$

4 Renormalization of the Abelian Higgs Model

Until now we have calculated all the one-loop corrections of the two- and four-point functions in both the Unitary and the R_ξ gauge. Generally our final goal is to evaluate the LCP's, and thus we should evaluate the effective potential. This would be done by first adding the one-loop corrections and then renormalizing the potential. Thus, we need all of the above calculations in order to obtain the effective potential. Moreover, here we perform a comparison between the counterterms of the physical quantities in the two gauges, showing the procedure that makes them gauge-independent and clarifying some dark spots on this calculation. Finally, we show how we should treat the insertion of the Tadpoles in the effective potential, since in our case there is not any condition which absorbs them.

4.1 Renormalization in R_ξ gauge

Our Lagrangian reads

$$
\begin{aligned}
\mathcal{L}_{R_\xi} &= -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu\phi_0)(\partial^\mu\phi_0) + \frac{1}{2}(\partial_\mu\chi_0)(\partial^\mu\chi_0) - \frac{1}{2\xi}\left(\partial_\mu A_\mu^0\right)^2 + \frac{1}{2}m_{Z_0}^2 A_\mu^0 A^{0\mu} \\
&+ 2\frac{m_{Z_0}}{m_{H_0}}\sqrt{2\lambda_0}A_\mu^0\partial_\mu\chi_0\phi_0 + \frac{\sqrt{2\lambda_0}}{m_{H_0}}(\partial_\mu\chi_0)^2\phi_0 + \frac{2\lambda_0 m_{Z_0}}{m_{H_0}^2}A_\mu^0\partial_\mu\chi_0\phi_0^2 + \frac{\lambda_0}{m_{H_0}^2}(\partial_\mu\chi_0)^2\phi_0^2 \\
&+ g^{\mu\nu}\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2}A_\mu^0 A_\nu^0\phi_0^2 + g^{\mu\nu}\frac{m_{Z_0}^2}{m_{H_0}}\sqrt{2\lambda_0}\phi_0 A_\mu^0 A_\nu^0 - \frac{1}{2}\xi m_{Z_0}^2\chi_0^2 \\
&- \frac{1}{2}m_{H_0}^2\phi_0^2 - \frac{\lambda_0}{4}\phi_0^4 - \sqrt{\frac{\lambda_0}{2}}m_{H_0}\phi_0^3 + \mathcal{L}_{ghost} + const.
\end{aligned} \tag{314}
$$

We proceed with the renormalization of the Lagrangian (314). We define renormalized quantities only for the independent parameters which read

$$\begin{aligned} m^2_{H_0} &= m^2_H + \delta m_H \\ m^2_{Z_0} &= m^2_Z + \delta m_Z \\ \lambda_0 &= \lambda + \delta\lambda \end{aligned} \tag{315}$$

where the subscript 0 denotes the bare quantities. Substituting the above definitions into the classical Lagrangian, including all the one-loop corrections of the model we will be able to obtain the 1-loop effective Lagrangian. An important point here, according to the next section, is that there is a non-zero anomalous dimension only for the Higgs field and the un-physical Goldstone field. On the other hand, the Z-boson has zero anomalous dimension. Thus, we have to renormalize also these fields *i.e.*, we should consider counterterms for ϕ and χ fields. As a consequence, we obtain the following relations

$$\begin{aligned} \phi_0 &= \sqrt{Z_\phi}\phi \\ \chi_0 &= \sqrt{Z_\chi}\chi \end{aligned} \tag{316}$$

where $Z_j = 1 + \delta j$.
Therefore, considering all of the above arguments, the renormalized 1-loop effective Lagrangian reads

$$
\begin{aligned}
\mathcal{L}^{eff}_{R_\xi} &= -\frac{1}{4}F^2_{\mu\nu} + \frac{Z_\phi}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right) + \frac{Z_\chi}{2}\left(\partial_\mu\chi\right)\left(\partial^\mu\chi\right) - \frac{1}{2\xi}(\partial_\mu A_\mu)^2 \\
&+ \frac{1}{2}\left[m_Z^2 + \delta m_Z - \mathcal{M}_Z^{R_\xi} - d_i\mathcal{T}_H^{iR_\xi} + m_Z^2\delta A^{R_\xi}\right]A_\mu A^\mu \\
&+ \left[2\sqrt{Z_\phi Z_\chi}\frac{m_Z\sqrt{2\lambda}(1+\frac{\delta m_Z}{2m_Z^2}-\frac{\delta m_H}{2m_H^2}+\frac{\delta\lambda}{2\lambda})}{m_H} + \mathcal{K}^{R_\xi}_{Z\chi H}\right]A_\mu\partial_\mu\chi\phi \\
&+ \left[Z_\chi\sqrt{Z_\phi}\frac{\sqrt{2\lambda}(1+\frac{\delta\lambda}{2\lambda}-\frac{\delta m_H}{2m_H^2})}{m_H} + \mathcal{K}^{R_\xi}_{\chi\chi H}\right](\partial_\mu\chi)^2\phi \\
&+ \left[\frac{2Z_\phi\sqrt{Z_\chi}m_Z(\lambda+\lambda\frac{\delta m_Z}{2m_Z^2}-\lambda\frac{\delta m_H}{2m_H^2}+\delta\lambda)}{m_H} + \mathcal{B}^{R_\xi}_{\chi ZH}\right]A_\mu\partial_\mu\chi\phi^2 \\
&+ \left[Z_\phi Z_\chi\frac{\lambda+\delta\lambda-\lambda\frac{\delta m_H}{m_H^2}}{m_{H_0}^2} + \mathcal{B}^{R_\xi}_{\chi H}\right](\partial_\mu\chi)^2\phi^2 \\
&+ g^{\mu\nu}\left[Z_\phi\frac{(\lambda+\lambda\frac{\delta m_Z}{m_Z^2}-\lambda\frac{\delta m_H}{m_H^2}+\delta\lambda)m_{Z_0}^2}{m_{H_0}^2} + \mathcal{B}^{R_\xi}_{HZ}\right]A_\mu A_\nu\phi^2 \\
&+ g^{\mu\nu}\left[\sqrt{Z_\phi}\frac{m_Z^2\sqrt{2\lambda}(1-\frac{\delta m_H}{2m_H^2}+\frac{\delta m_Z}{m_Z^2}+\frac{\delta\lambda}{2\lambda})}{m_H} + \mathcal{K}^{R_\xi}_{HZ}\right]\phi A_\mu A_\nu \\
&- \frac{1}{2}\left[m_\chi^2 + \delta m_\chi - \mathcal{M}_\chi^{R_\xi} - \frac{\sqrt{2\lambda}}{m_H}b_i\mathcal{T}_H^{iR_\xi} + m_\chi^2\delta\chi\right]\chi^2 + \mathcal{L}^{eff}_{ghost} \\
&- V(\phi)
\end{aligned}
\tag{317}
$$

where the effective Lagrangian for the ghosts reads

$$
\mathcal{L}^{eff}_{ghost} = \partial_\mu\bar{c}\partial^\mu c - \left[m_\chi^2 + \delta m_\chi - \mathcal{M}_{gh}^{R_\xi}\right]\bar{c}c \tag{318}
$$

and the Higgs effective potential reads

$$
\begin{aligned}
V(\phi) &= \left[a_i\mathcal{T}_H^{iR_\xi} + \frac{m_H^3}{(2\lambda)^{3/2}}\frac{\delta\phi}{2}\right]\phi + \frac{1}{2}\left[m_H^2 + \delta m_H - \mathcal{M}_H^{R_\xi} - c_i\mathcal{T}_H^{iR_\xi} + m_H^2\delta\phi^{R_\xi}\right]\phi^2 \\
&+ \left[\sqrt{\frac{\lambda}{2}}m_H + \frac{3}{2}\sqrt{\frac{\lambda}{2}}m_H\delta\phi + \sqrt{\frac{\lambda}{2}}\frac{\delta m_H}{2m_H} + m_H\frac{\delta\lambda}{2\sqrt{2\lambda}} + \mathcal{K}_H^{R_\xi}\right]\phi^3 \\
&+ \frac{\left[\lambda + 2\lambda\delta\phi + \delta\lambda - \mathcal{B}_H^{R_\xi}\right]}{4}\phi^4
\end{aligned}
\tag{319}
$$

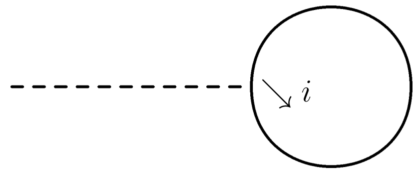
where, essentially, the Higgs wave function counterterm is just a shift of the re-minimized *vev*. As we can see from the above relations, we have inserted in both the Z- and the Higgs-quadratic terms the reducible and un-physical diagrams, *i.e.* the two-leg Tadpoles. Generally, this is a necessary step in order to obtain gauge independent physical quantities. Nevertheless, it is not exactly clear how and in what physical quantities we should use the two-leg Tadpoles.
In particular, we can recall from Eq.(314) that we have inserted a gauge fixing term which seems to break gauge invariance. Actually gauge invariance is steel there but is hidden and it can be revealed through the combination of Z- and the Goldstone boson. To be more specific, when we perform the resummation of the gauge boson propagator in tree and one-loop level, evaluating the quantum corrections to the Z-mass, we include both the physical and the un-physical degrees of freedom. Thus, since m_Z is protected through the gauge invariance, we would expect that the one-loop corrections, the counterterm and the β-function of the Z-mass should be ξ-independent without the use of the two-leg Tadpoles.
On the other hand, for the Higgs mass we do not have a specific symmetry that protects it from being ξ-dependent. All that we know, from the Nielsen Identities, is that the extrema of the effective potential should be gauge invariant and this includes only the m_H and not the quartic coupling. Unfortunately, the above statement does not indicate if and with what coefficients the reducible diagrams should be introduced to the Higgs-mass counterterm so as to get a gauge independent result. Therefore, in what follows we are going to clarify the above situation.

Now, here we face a very interesting situation, since as we can see from Eq.(314) we do not have a linear in ϕ term in Polar basis. Nevertheless, inserting the one-loop contributions so as to obtain the effective potential, a liear term with Tadpoles and $\delta\phi$ appears. Moreover, these Tadpoles should come with a specific combination since the potential should be finite. Thus, we can perform the following steps:

Since any counterterm and Tadpole is a function of the scalar integrals we can write generally that

$$\delta a = \frac{\mu^{\varepsilon}}{(4\pi)^2}\left(\frac{C_\alpha}{\varepsilon} + \sum_k f^k_{A_0} \ln\frac{\mu^2}{m_k^2} + \sum_{k,i} f^k_{B_0}\int_0^1 dx \ln\left(\frac{\mu^2}{\Delta^i_k(m_k, m_i)}\right) + \sum_k f^k_{A_0}\right) \tag{320}$$

and having in mind that



(321)

where i indicates the kind of the field that is running the loop and the straight line indicates the Higgs field, we get that

$$\mathcal{T}_H^i = \frac{\mu^\varepsilon}{(4\pi)^2}\left(\frac{C_\mathcal{T}^i}{\varepsilon} + \sum_k F_{A_0}^{i,k}\ln\frac{\mu^2}{m_k^2} + \sum_k F_{B_0}^{i,k}\int_0^1 dx \ln\left(\frac{\mu^2}{\Delta_k^i(m_k,m_i)}\right) + \sum_k F_{A_0}^{i,k}\right) \tag{322}$$

where in our case, k corresponds to the subscript Z, H and χ, $\delta\alpha$ is an arbitrary counterterm and moreover $C_\alpha \equiv 2\sum_k \left[f_{A_0}^k + f_{B_0}^k\right]$ and $C_\mathcal{T} \equiv 2\sum_k \left[F_{A_0}^{i,k} + F_{B_0}^{i,k}\right]$.
So, since $\Delta_k^i \equiv \Delta_k^i(m_k,m_i)$ we can separate the last logarithm of the above relations and in each case we can write the following

$$\delta a = \frac{\mu^\varepsilon}{(4\pi)^2}\left(\frac{C_\alpha}{\varepsilon} + \sum_k \left[f_{A_0}^k + f_{B_0}^k\right]\ln\frac{\mu^2}{m_k^2} - \sum_{i,k} f_{B_0}^k \int_0^1 dx \ln \Delta_k'^i(m_k,m_i) + \sum_k f_{A_0}^k\right) \tag{323}$$

and

$$\mathcal{T}_H^i = \frac{\mu^\varepsilon}{(4\pi)^2}\left(\frac{C_\mathcal{T}^i}{\varepsilon} + \sum_k \left[F_{A_0}^{i,k} + F_{B_0}^{i,k}\right]\ln\frac{\mu^2}{m_k^2} - \sum_k F_{B_0}^{i,k} \int_0^1 dx \ln \Delta_k'^i(m_k,m_i) + \sum_k F_{A_0}^{i,k}\right) \tag{324}$$

where $\Delta_k^i(m_k,m_i) = -p^2 x(1-x) + m_i^2(1-x) + m_k^2 x$ and $\Delta_k'^i \equiv \frac{\Delta_k^i}{m_k}$.
Now, since we want a finite effective potential, we demand from Eq.(319) that

$$
\begin{aligned}
a_i C^i_{\mathcal{T}} + \frac{v_0^3}{2} C_\alpha &= 0 \Leftrightarrow \\
2\sum_k \left\{ a_i \left[F^{i,k}_{A_0} + F^{i,k}_{B_0} \right] + \frac{v_0^3}{2} \left[f^k_{A_0} + f^k_{B_0} \right] \right\} &= 0 \Leftrightarrow \\
\sum_k \left\{ a_i \left[F^{i,k}_{A_0} + F^{i,k}_{B_0} \right] + \frac{v_0^3}{2} \left[f^k_{A_0} + f^k_{B_0} \right] \right\} &= 0
\end{aligned}
\tag{325}
$$

obtaining a finite linear term

$$
\begin{aligned}
\mathcal{T}^{fR_\xi}_H &= \frac{\mu^\varepsilon}{(4\pi)^2} \left\{ \sum_k \left[a_i [F^{i,k}_{A_0} + F^{i,k}_{B_0}] + \frac{v_0^3}{2} [f^k_{A_0} + f^k_{B_0}] \right] \ln \frac{\mu^2}{m_k^2} \right. \\
&- \left. \sum_k \left[a_i F^{i,k}_{B_0} + f^k_{B_0} \right] \int_0^1 dx \ln \Delta'^i_k(m_k, m_i) + \sum_k [a_i F^{i,k}_{A_0} + \frac{v_0^3}{2} f^k_{A_0}] \right\}
\end{aligned}
\tag{326}
$$

which generally is not zero and it could be gauge-dependent. Here, the subscript f indicates the word *finite*.

Now, recalling that the Tadpoles in the above relation are fixed through Sec.3.1, while $\delta\phi$ is fixed through the anomalous dimension in Eq.(339), we can imply the above procedure in our case. To be more specific, we have that

$$
\begin{aligned}
a_i \mathcal{T}^{iR_\xi}_H &= 3a_1 \sqrt{\frac{\lambda}{2}} m_H \mu^\varepsilon A_0(m_H) + \frac{\sqrt{2\lambda} m_Z^2}{m_H} \mu^\varepsilon \left\{ a_2 (3A_0(m_Z) + \xi A_0(\sqrt{\xi} m_Z) + (1+\xi) m_Z^2) \right. \\
&- \left. a_3 \xi A_0(m_\chi) - a_4 \xi A_0(m_\chi) \right\}
\end{aligned}
\tag{327}
$$

while the Higgs field counterterm reads

$$
\delta\phi = \mu^\varepsilon \left\{ 3\frac{\lambda}{m_H^2} A_0(m_\chi) + 3\xi \frac{\lambda m_Z^2}{m_H^2} B_0(p, m_\chi, m_\chi) + \frac{4m_Z^4(1-\xi)}{m_H^2} C_0(p_1, p_2, m_Z, m_Z, m_\chi) \right\}
\tag{328}
$$

and from the above relations we can identify the following

$$
\begin{aligned}
f_{A_0}^{H} &= 0 \\
f_{A_0}^{Z} &= 0 \\
f_{A_0}^{\chi} &= 3\xi\frac{\lambda m_Z^2}{m_H^2} \\
f_{B_0}^{H} &= 0 \\
f_{B_0}^{Z} &= 0 \\
f_{B_0}^{\chi} &= 3\xi\frac{\lambda m_Z^2}{m_H^2} \\
f_{A_0}^{Z} &= 0 \\
a_i F_{A_0}^{i,H} &= 3\sqrt{\frac{\lambda}{2}} m_H^3 a_1 \\
a_i F_{A_0}^{i,Z} &= 3a_2\frac{\sqrt{2\lambda} m_Z^4}{m_H} \\
a_i F_{A_0}^{i,\chi} &= \xi^2\frac{\sqrt{2\lambda} m_Z^4}{m_H}\left\{a_2 - a_3 - a_4\right\} \\
a_i F_{B_0}^{i,H} &= 0 \\
a_i F_{B_0}^{i,Z} &= 0 \\
a_i F_{B_0}^{i,\chi} &= 0
\end{aligned}
\tag{329}
$$

Now, using Eq.(325) we obtain the following specific relation

$$
3\sqrt{\frac{\lambda}{2}} m_H^3 a_1 + \frac{\sqrt{2\lambda} m_Z^4}{m_H}\left\{(3+\xi^2)a_2 - \xi^2 a_3 - \xi^2 a_4\right\} + 6\frac{m_H m_Z^2}{2^{3/2}\sqrt{\lambda}}\xi \quad = \quad 0\,. \tag{330}
$$

So, now we can use Eq.(326) in order to define the remaining finite part, obtaining

$$
\begin{aligned}
\mathcal{T}_H^{fR_\xi} &= \mu^{\varepsilon}\left\{3\sqrt{\frac{\lambda}{2}} m_H^3 a_1 \ln\frac{\mu^2}{m_H^2} + 3\frac{\sqrt{2\lambda} m_Z^4}{m_H^2} a_2 \ln\frac{\mu^2}{m_Z^2}\right. \\
&+ \frac{\sqrt{2\lambda} m_Z^4}{m_H^2}\left\{\xi^2 a_2 - \xi^2 a_3 - \xi^2 a_4 + 6\frac{m_H m_Z^2}{2^{3/2}\sqrt{\lambda}}\xi\right\}\ln\frac{\mu^2}{m_\chi^2} - \xi\frac{3 m_H m_Z^2}{2^{3/2}\sqrt{\lambda}} b_0'(m_\chi, m_\chi) \\
&+ \left. 4(1-\xi)\frac{m_H m_Z^2}{2^{3/2}} c_0(m_Z, m_Z, m_\chi) + 3\frac{m_H m_Z^2}{2^{3/2}\sqrt{\lambda}}\xi + (1+\xi)\frac{\sqrt{2\lambda} m_Z^4}{m_H} a_2\right\}.
\end{aligned}
\tag{331}
$$

where we have defined that

$$b_0(m_1,m_2) \equiv \int_0^1 dx \ln \Delta_{B_0}(m_1,m_2)$$

$$b_0'(m_1,m_2) \equiv \int_0^1 dx \ln \frac{\Delta_{B_0}(m_1,m_2)}{m_1^2} \tag{332}$$

with $\Delta^{B_0}(m_1,m_2) = -p^2x(1-x) + m_2^2(1-x) + m_1^2 x$ and

$$c_0(m_1,m_2,m_3) \equiv -\int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{1}{\Delta_{C_0}(m_1,m_2,m_3)} \tag{333}$$

with $\Delta^{C_0}(m_1,m_2,m_3) = -p_1^2x(1-x) - 2p_2\cdot p_2 xz - p_2^2 z(1-z) + [m_1^2x + m_2^2y + m_3^2z]$.

Finally, we can use Eq.(325) in order to simplify the above result, but in order to do it we should first perform some calculations. To be more specific, we should add in Eq.(331) the terms $\pm 3a_1\frac{\lambda}{2}m_H^3 \ln m_\chi$ and $\pm 3a_2\frac{\sqrt{2\lambda}m_Z^4}{m_H^2}\ln\xi$, which help us to create the proper condition of Eq.(325) as a multiplicative coefficient of $\ln\frac{\mu^2}{m_\chi^2}$. Thus, following the above arguments we obtain a finite result for the linear ϕ term

$$\begin{aligned}\mathcal{T}_H^{fR_\xi} &= \mu^\varepsilon \Bigg\{ -\xi\frac{3m_Hm_Z^2}{2^{3/2}\sqrt{\lambda}}b_0(m_\chi,m_\chi) + 4(1-\xi)\frac{m_Hm_Z^2}{2^{3/2}}c_0(m_Z,m_Z,m_\chi) \\ &+ 3a_1\frac{\lambda}{2}m_H^3\ln\frac{m_\chi}{m_H} + 3a_2\frac{\sqrt{2\lambda}m_Z^4}{m_H^2}\ln\xi + 3\frac{m_Hm_Z^2}{2^{3/2}\sqrt{\lambda}}\xi + (1+\xi)\frac{\sqrt{2\lambda}m_Z^4}{m_H}a_2 \Bigg\}\end{aligned} \tag{334}$$

which is both non-zero and gauge dependent. Thus, in what follows we will see if and how the above result will affect the physical quantities.

For completeness, and before we move on, we should mention that the amplitudes $\mathcal{T}_H^{R_\xi}$, $\mathcal{M}_H^{R_\xi}$, $\mathcal{K}_H^{R_\xi}$ and $\mathcal{B}_H^{R_\xi}$ correspond to the one-loop corrections of the Higgs one-, two-, three- and four-point functions respectively. The quantity $\mathcal{M}_Z^{R_\xi}$ is an extraction of $\mathcal{M}_{Z\mu\nu}^{R_\xi}$ and corresponds to the one-loop corrections of the Z-boson two-point function. Moreover, $\mathcal{K}_{Z\chi H}^{R_\xi}$, $\mathcal{K}_{\chi\chi H}^{R_\xi}$, $\mathcal{K}_{\chi HH}^{R_\xi}$, $\mathcal{B}_{\chi H}^{R_\xi}$, $\mathcal{B}_{HZ}^{R_\xi}$ and $\mathcal{K}_{HZ}^{R_\xi}$ are amplitudes with both Higgs, Z and χ-boson external legs and will not be presented here since they will not contribute in the physical quantities under consideration.

Now, in order to continue with renormalization, we need to use specific conditions which will help us define the counterterms needed to adsorb the divergences from the one-loop corrections. Therefore, we choose to use a Physical prescription which is equivalent

to on-shell renormalization conditions.
To be more specific, and similarly with the Unitary gauge, our Physical prescription requires that:

- The physical Higgs mass is defined by

$$\delta m_H^{R_\xi} = \mathcal{M}_H^{R_\xi} + c_i \mathcal{T}_H^{iR_\xi} - m_H^2 \delta\phi^{R_\xi} + c_k \cdot \mathcal{T}_H^{fR_\xi} \tag{335}$$

  demanding that

$$V(v)'' = m_H^2 \tag{336}$$

  where, as we have shown, $\mathcal{T}_H^{fR_\xi}$ is non-zero and $m_H \equiv m_{H_{phy}}$, *i.e.* corresponds to the physical Higgs mass. Here all the c's have dimension of inverse mass.

- The Higgs quartic coupling is defined by

$$V(v)'''' = 6\lambda \tag{337}$$

- The physical Z-mass is

$$m_{Z_{phy}} \equiv m_Z \,. \tag{338}$$

So, using the above renormalization conditions we fix the needed counterterms, obtaining the 1-loop renormalized effective potential. To be more specific, from Eq.(317) we observe that there are six different counterterms involved into the renormalization procedure, from which, the wave function counterterms are determined through the following relations

$$\begin{aligned} \delta\phi^{R_\xi} &= - \left. \frac{d\mathcal{M}_H^{R_\xi}(p)}{dp^2} \right|_{p^2=m_H^2} \\ \delta\chi &= \delta\phi \,. \end{aligned} \tag{339}$$

Thus, we are left with four undetermined counterterms which will be fixed in the following. Actually, there are four conditions fixing δm_H, δm_Z, $\delta\lambda$ and δm_χ but in the last case we should be careful. The last counterterm comes from the renormalization of the χ-mass, and since Goldstone boson is un-physical there is not a uniq renormalization condition that could be used. Nevertheless, we are interested in the quantities that are determined through the Higgs potential and the Z-mass renormalization condition, thus, starting with Eq.(337) we get that

$$\delta\lambda^{R_\xi} = \mathcal{B}_H^{R_\xi} - 2\lambda\delta\phi^{R_\xi} \tag{340}$$

which is used to eliminate $\delta\lambda$ from the term in Eq.(319) proportional to ϕ^3. Next we consider the condition Eq.(338) from which we get that

$$\delta m_Z^{R_\xi} = \mathcal{M}_Z^{R_\xi} + \frac{1}{3} d_i \mathcal{T}_H^{iR_\xi} \tag{341}$$

which will shows us the necessity of the two-leg Tadpole insertion. Here, all the d's have dimension of inverse mass, and the condition Eq.(336) which fixes δm_H according to Eq.(335) .

Finally, we should notice that we have quantized a classical Lagrangian expanded around its true vacuum v_0, but after the one-loop corrections we end up wth an effective potential that needs to be re-minimized. This is maximally unconventional since we start from a "broken" action (its scalar mass and quartic terms have the same sign) and upon quantization we require that it generates an "unbroken" effective potential. Therefore, we consider the 1-loop corrected *vev* through the condition that minimises $V(\phi)$, which is given by

$$V(v)' = 0 \tag{342}$$

where, after renormalization, the one-loop effective potential reads

$$\begin{aligned} V(\phi) &= \mathcal{T}_H^{fR_\xi}\phi + \frac{1}{2}\left[m_H^2 - c_k\cdot\mathcal{T}_H^{fR_\xi}\right]\phi^2 \\ &+ \left[\sqrt{\frac{\lambda}{2}}m_H + \sqrt{\frac{\lambda}{2}}\frac{\mathcal{M}_H^{R_\xi}}{2m_H} + \sqrt{\frac{\lambda}{2}}\frac{c_i\mathcal{T}_H^{iR_\xi}}{2m_H} + \sqrt{\frac{\lambda}{2}}\frac{c_k\cdot\mathcal{T}_H^{fR_\xi}}{2m_H} - m_H\frac{\mathcal{B}_H^{R_\xi}}{2\sqrt{2\lambda}} + \mathcal{K}_H^{R_\xi}\right]\phi^3 + \frac{\lambda}{4}\phi^4 \end{aligned} \tag{343}$$

which generally could be gauge-dependent and hence un-physical. So, using the above renormalization conditions in order to fix the counterterms and then replacing them into

$V(\phi)$, we observe that $\delta\phi^{R_\xi}$ has been vanished from every term except the linear one. Therefore, it is valid to say that Higgs wave function counterterm plays just the role of a shift to the vacuum of the re-minimized one-loop effective potential.
Now, let us use our last condition, so as to calculate the 1-loop corrected *vev*. To be more specific, Eq.(381) gives the cubic equation

$$\lambda v^3 + 3\left[\sqrt{\frac{\lambda}{2}}m_H + \sqrt{\frac{\lambda}{2}}\frac{\mathcal{M}_H^{R_\xi}}{2m_H} + \sqrt{\frac{\lambda}{2}}\frac{c_i\mathcal{T}_H^{iR_\xi}}{2m_H} + \sqrt{\frac{\lambda}{2}}\frac{c_k\cdot\mathcal{T}_H^{fR_\xi}}{2m_H} - m_H\frac{\mathcal{B}_H^{R_\xi}}{2\sqrt{2\lambda}} + \mathcal{K}_H^{R_\xi}\right]v^2 + \left[m_H^2 - \frac{\sqrt{2\lambda}}{m_H}c_k\cdot\mathcal{T}_H^{fR_\xi}\right]v + \mathcal{T}_H^{fR_\xi} = 0 \tag{344}$$

with complex solutions in general.
In the present case, calculating the discriminant of the above cubic equation by expanding all the amplitudes to the first order we obtain that

$$\begin{aligned}\Delta_{cubic} &= \frac{m_H^6\lambda}{2} + \frac{m_H^4\lambda}{4}\Bigg\{18c_i\mathcal{T}_H^{iR_\xi} + 30c_k\cdot\mathcal{T}_H^{fR_\xi} \\ &+ 18\mathcal{M}_H^{R_\xi} + 36\frac{\sqrt{2}m_H}{\sqrt{\lambda}}\mathcal{K}_H^{R_\xi} - 18\frac{m_H^2}{\lambda}\mathcal{B}_H^{R_\xi}\Bigg\}\hbar + \mathcal{O}(\hbar^2)\end{aligned} \tag{345}$$

which is expected to be positive. Actually, writing the above relation as

$$\Delta_{cubic} = \Delta_0 + \Delta_1\hbar + \mathcal{O}(\hbar^2) \tag{346}$$

where

$$\begin{aligned}\Delta_0 &= \frac{m_H^6\lambda}{2} \\ \Delta_1 &= \frac{m_H^4\lambda}{4}\left\{18c_i\mathcal{T}_H^{iR_\xi} + 30c_k\cdot\mathcal{T}_H^{fR_\xi} + 18\mathcal{M}_H^{R_\xi} + 36\frac{\sqrt{2}m_H}{\sqrt{\lambda}}\mathcal{K}_H^{R_\xi} - 18\frac{m_H^2}{\lambda}\mathcal{B}_H^{R_\xi}\right\}\end{aligned} \tag{347}$$

we can see that there are additional constraints that should be fulfilled so as to have at least one real solution. To be more specific, since $\Delta_0 > 0$ always, then if

$$\Delta_1 > 0 \tag{348}$$

the discriminant stays positive and we have three real solutions one of which corresponds to the deepest minimum. An other case is if

$$\begin{aligned}\Delta_1 &< 0 \\ \Delta_0 &> \Delta_1 \hbar\end{aligned} \tag{349}$$

which corresponds to the previous case obtaining again three real solutions. Next we can consider that there is a high energy limit where

$$|\Delta_0| = |\Delta_1 \hbar| \tag{350}$$

and then if

$$\Delta_1 < 0 \tag{351}$$

we have that

$$\Delta_{cubic} = 0 \tag{352}$$

which has a multiple root and all of them are real corresponding to one minimum. Finally, if we suppose that there is a high energy limit where

$$|\Delta_0| < |\Delta_1 \hbar| \tag{353}$$

and

$$\Delta_1 < 0 \tag{354}$$

we get

$$\Delta_{cubic} < 0 \tag{355}$$

which means that the potential has three solutions, corresponding to one real and two complex conjugate roots. Thus, if there is a limit like that one should check only if the real solution corresponds to the deepest minimum.
Nevertheless, since the physical Higgs mass is of order

$$m_H^2 \gg \mathcal{O}(\hbar) \tag{356}$$

the Eq.(345) seems to correspond to the first case where the discriminant stays positive. Now, if we have a positive discriminant then there are three real independent solutions. Now, the next step is to determine the appropriate solution for the re-minimized *vev*, and a first constrain is that the correct solution should correspond to a global minimum. In particular, beginning with the last one we get that

$$v_3 = -\frac{m_H}{\sqrt{2\lambda}} + \left[-\frac{3\mathcal{M}_H^{R_\xi}}{2m_H\sqrt{2\lambda}} - \frac{3\mathcal{B}_H^{R_\xi} m_H}{(2\lambda)^{3/2}} + \frac{3\mathcal{K}_H^{R_\xi}}{\lambda} + \frac{3c_i \mathcal{T}_H^{iR_\xi}}{2m_H^2} - \frac{2\mathcal{T}_H^{fR_\xi}}{m_H^2} \right] \hbar + \mathcal{O}(\hbar^2) \tag{357}$$

therefore, replacing it to the second derivative of the potential Eq.(343), we get that

$$\begin{aligned} V''(v_3) = & -\frac{m_H^2}{2} + \left\{ -\frac{3\mathcal{M}_H^{R_\xi}}{2} + \frac{3\mathcal{B}_H^{R_\xi} m_H^2}{2\lambda} - \frac{3\sqrt{2}\mathcal{K}_H^{R_\xi} m_H}{2\sqrt{\lambda}} - \frac{3\sqrt{2\lambda} c_i \mathcal{T}_H^{iR_\xi}}{2m_H} \right. \\ & \left. - \frac{\sqrt{2\lambda}}{m_H} c_k \cdot \mathcal{T}_H^{fR_\xi} + 3\frac{\sqrt{2\lambda}}{m_H} \mathcal{T}_H^{fR_\xi} \right\} \hbar + \mathcal{O}(\hbar^2) \end{aligned} \tag{358}$$

where the above relation is negative and gives a local maximum for v_3. Thus we are left with the other two possible solutions which both correspond to a minimum. Actually, for the second solution we have that

$$v_2 = -2\frac{m_H}{\sqrt{2\lambda}} + \left[-\frac{3\mathcal{M}_H^{R_\xi}}{m_H\sqrt{2\lambda}} + \frac{3\mathcal{B}_H^{R_\xi} m_H}{\sqrt{2}(\lambda)^{3/2}} - \frac{6\mathcal{K}_H^{R_\xi}}{\lambda} - \frac{3c_i \mathcal{T}_H^{iR_\xi}}{m_H^2} - \frac{\mathcal{T}_H^{fR_\xi}}{m_H^2} \right] \hbar + \mathcal{O}(\hbar^2) \tag{359}$$

so if we replace it to the second derivative of the effective potential we get that

$$
\begin{aligned}
V''(v_2) &= m_H^2 + \Bigg\{ 6\mathcal{M}_H^{R_\xi} - \frac{6\mathcal{B}_H^{R_\xi} m_H^2}{\lambda} + \frac{12\sqrt{2}\mathcal{K}_H^{R_\xi} m_H}{\sqrt{\lambda}} + \frac{6\sqrt{2\lambda} c_i \mathcal{T}_H^{iR_\xi}}{m_H} \\
&- \frac{\sqrt{2\lambda}}{m_H} c_k \cdot \mathcal{T}_H^{fR_\xi} + 3\frac{\sqrt{2\lambda}}{m_H}\mathcal{T}_H^{fR_\xi} \Bigg\} \hbar + \mathcal{O}(\hbar^2)
\end{aligned}
\tag{360}
$$

which corresponds to a minimum since we have supposed that

$$
6\mathcal{M}_H^{R_\xi} - \frac{6\mathcal{B}_H^{R_\xi} m_H^2}{\lambda} + \frac{12\sqrt{2}\mathcal{K}_H^{R_\xi} m_H}{\sqrt{\lambda}} + \frac{6\sqrt{2\lambda} c_i \mathcal{T}_H^{iR_\xi}}{m_H} - \frac{\sqrt{2\lambda}}{m_H} c_k \cdot \mathcal{T}_H^{fR_\xi} + 3\frac{\sqrt{2\lambda}}{m_H}\mathcal{T}_H^{fR_\xi} < m_H^2\,. \tag{361}
$$

Finally we have the first solution which reads

$$
v_1 = -\frac{\mathcal{T}_H^{fR_\xi}}{2m_H^2}\hbar + \mathcal{O}(\hbar^2) \tag{362}
$$

and the second derivative of the effective potential with this solution gives

$$
V''(v_1) = m_H^2 - \left(3\frac{\sqrt{\lambda}}{\sqrt{2}m_H} + 2\cdot c_k\right)\mathcal{T}_H^{fR_\xi}\hbar + \mathcal{O}(\hbar^2)\,. \tag{363}
$$

Actually, if the inequality

$$
\begin{aligned}
-\left(3\frac{\sqrt{\lambda}}{\sqrt{2}m_H} + 2\cdot c_k\right)\mathcal{T}_H^{fR_\xi} &< 6\mathcal{M}_H^{R_\xi} - \frac{6\mathcal{B}_H^{R_\xi} m_H^2}{\lambda} + \frac{12\sqrt{2}\mathcal{K}_H^{R_\xi} m_H}{\sqrt{\lambda}} + \frac{6\sqrt{2\lambda} c_i \mathcal{T}_H^{iR_\xi}}{m_H} \\
&- \frac{\sqrt{2\lambda}}{m_H} c_k \cdot \mathcal{T}_H^{fR_\xi} + 3\frac{\sqrt{2\lambda}}{m_H}\mathcal{T}_H^{fR_\xi}
\end{aligned}
\tag{364}
$$

holds, then v_2 corresponds to a local minimum and v_1 to the global minimum. This seems to be the correct vacuum shift, since we should recall that we started by quantizing an "unbroken" Lagrangian and after the 1-loop correction of the potential we get a *vev* of order $\mathcal{O}(\hbar)$.

Finally, the renormalization condition Eq.(336) with the chosen *vev* reads

$$V''(v) = m_H^2 - \left(3\frac{\sqrt{\lambda}}{\sqrt{2}m_H} + 2 \cdot c_k\right) \mathcal{T}_H^{fR_\xi} \tag{365}$$

and since $\mathcal{T}_H^{fR_\xi}$ is non-zero and our renormalization condition defines the physical Higgs mass as the second derivative of the potential calculated on the *vev*, we get that

$$c_k = -\frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H} \tag{366}$$

and if we express the above relation as a function of the *vev*, then we get that

$$c_k \sim \frac{1}{2v_0} \,. \tag{367}$$

Thus, with the above definition we have that

$$V''(v) = m_H^2 \tag{368}$$

and that the physical Higgs mass and the corresponding counterterm would be given by

$$m_H^2 = m_{H_0}^2 - \mathcal{M}_H^{R_\xi} - c_i \mathcal{T}_H^{iR_\xi} - \frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H}\mathcal{T}_H^{fR_\xi} + m_H^2 \delta\phi^{R_\xi} \tag{369}$$

and

$$\delta m_H^{R_\xi} = \mathcal{M}_H^{R_\xi} + c_i \mathcal{T}_H^{iR_\xi} - m_H^2 \delta\phi^{R_\xi} - \frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H}\mathcal{T}_H^{fR_\xi} \tag{370}$$

respectively. Here, notice that even if $\mathcal{T}_H^{fR_\xi}$ is finite will affect the physical Higgs mass through its relation with the bare Higgs mass, which we will use for the *LCP*'s. We will see how to handle this in following sections.

Finally, using all the above arguments, the renormalized one-loop effective potential reads

$$\begin{aligned} V^{R_\xi}(\phi) &= \mathcal{T}_H^{fR_\xi}\phi + \frac{1}{2}\left[m_H^2 + \frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H}\mathcal{T}_H^{fR_\xi}\right]\phi^2 \\ &+ \left[\sqrt{\frac{\lambda}{2}}m_H + \sqrt{\frac{\lambda}{2}}\frac{\mathcal{M}_H^{R_\xi}}{2m_H} + \frac{\sqrt{\lambda}}{2\sqrt{2}m_H}c_i\mathcal{T}_H^{iR_\xi} - \frac{3\sqrt{\lambda}}{4\sqrt{2}m_H}\mathcal{T}_H^{fR_\xi} - m_H\frac{\mathcal{B}_H^{R_\xi}}{2\sqrt{2\lambda}} + \mathcal{K}_H^{R_\xi}\right]\phi^3 + \frac{\lambda}{4}\phi^4 \,. \end{aligned} \tag{371}$$

At this point and before we move on to the next section, we would like to present some comments concerning the choice of the Polar instead of the Cartesian Basis and how this could affect our results. To be more specific, we have inserted the physical Higgs field through the Polar Basis and all the calculations and the arguments that have been developed here are based on this choice. Now, if we have used the Cartesian Basis, then the Feynman rules would be quite different preventing us from seeing the unexpected appearance of the highly divergent integral, $U_{\mathcal{M}_4}$, in R_ξ-gauge. Of course this would not affect the results of the physical quantities that we have.
On the other hand, it is common for R_ξ-Lagrangian in Cartesian Basis to be renormalized before the *SSB*. Thus, after the *SSB*, adding the one-loop corrections, the effective potential would contain a linear ϕ term multiplied by a counterterm combination. In that case we could define a condition which would absorb all the Tadpoles from the linear ϕ term making it finite and this would affect the *vev*. This procedure seems helpful with the Tadpoles, but recall that the above counterterm combination would be proportional to the counterterm of the Higgs-mass and that indicates that the contributions of these tadpoles would affect the Higgs two-point functions. Thus, with the above procedure essentially the Tadpoles disappear from one quantity and then they appear in an other.

4.2 Renormalization in Unitary gauge

In the previous section we have presented the way of renormalizing the Lagrangian Eq.(314) in the R_ξ gauge. Actually, we have shown that we can have a renormalized potential which is finite and generally ξ-dependent, by implying specific conditions for the counterterms. Therefore, in order to have a complete picture of the renormalization procedure we should do the same calculation in the Unitary gauge where actually the result is quite unexpected. To be more specific, both the effective potential and the *vev* in Unitary gauge could be infinite, without that result affecting the physical quantities.
Now recall that the Lagrangian in Unitary gauge is

$$
\begin{aligned}
\mathcal{L}_{AH}^{U} =& -\frac{1}{4}F_{\mu\nu}^{2}+\frac{1}{2}\left(\partial_{\mu}\phi_{0}\right)\left(\partial^{\mu}\phi_{0}\right)+\frac{1}{2}m_{Z_0}^{2}A_{\mu}^{0}A^{0\mu}+g^{\mu\nu}\frac{\lambda_0 m_{Z_0}^{2}}{m_{H_0}^{2}}\phi_0^2 A_\mu^0 A_\nu^0 \\
&+g^{\mu\nu}\frac{\sqrt{2\lambda_0}m_{Z_0}^{2}}{m_{H_0}}\phi A_\mu^0 A_\nu^0-\frac{1}{2}m_{H_0}^{2}\phi_0^2-\sqrt{\frac{\lambda_0}{2}}m_{H_0}\phi_0^3-\frac{\lambda_0}{4}\phi_0^4+const.
\end{aligned}
\tag{372}
$$

Notice here that there is not any contribution from the Goldstone and ghost fields, since we are in the physical gauge and thus all the unphysical degrees of freedom have been decoupled.

Now, generally the potential is gauge-dependent and thus unphysical, but nevertheless, the physical quantities are gauge-independent and so there should be a matching between Unitary and R_ξ gauge. Thus, we start with the Lagrangian (372) defining the same independent renormalized quantities with Eq.(315) and again here we should consider wave function renormalization for both the gauge and the Higgs boson at 1-loop. Substituting the above definitions into the classical Lagrangian, we obtain the 1-loop effective Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{AH}}^{\mathrm{effU}} &= -\frac{Z_A}{4}F_{\mu\nu}^{2}+\frac{Z_\phi}{2}\left(\partial_{\mu}\phi\right)\left(\partial^{\mu}\phi\right)+\frac{1}{2}\left[m_Z^2+\delta m_Z-\mathcal{M}_Z^U-d_i'\mathcal{T}_H^{iU}+m_Z^2\delta A\right]A_\mu A^\mu \\
&+ g^{\mu\nu}\left[Z_A Z_\phi\frac{(\lambda+\lambda\frac{\delta m_Z}{m_Z^2}-\lambda\frac{\delta m_H}{m_H^2}+\delta\lambda)m_{Z_0}^2}{m_{H_0}^2}+\mathcal{B}_{HZ}^U\right]A_\mu A_\nu\phi^2 \\
&+ g^{\mu\nu}\left[Z_A\sqrt{Z_\phi}\frac{m_Z^2\sqrt{2\lambda}(1-\frac{\delta m_H}{2m_H^2}+\frac{\delta m_Z}{m_Z^2}+\frac{\delta\lambda}{2\lambda})}{m_H}+\mathcal{K}_{HZ}^U\right]\phi A_\mu A_\nu \\
&- V(\phi)
\end{aligned}
\tag{373}
$$

where

$$
\begin{aligned}
V(\phi) &= \left[a_i'\mathcal{T}_H^{iU}+\frac{m_H^3}{(2\lambda)^{3/2}}\frac{\delta\phi^U}{2}\right]\phi+\frac{1}{2}\left[m_H^2+\delta m_H-\mathcal{M}_H^U-c_i'\mathcal{T}_H^{iU}+m_H^2\delta\phi^U\right]\phi^2 \\
&+ \left[\sqrt{\frac{\lambda}{2}}m_H+\frac{3}{2}\sqrt{\frac{\lambda}{2}}m_H\delta\phi^U+\sqrt{\frac{\lambda}{2}}\frac{\delta m_H}{2m_H}+m_H\frac{\delta\lambda}{2\sqrt{2\lambda}}+\mathcal{K}_H^U\right]\phi^3 \\
&+ \frac{\left[\lambda+2\lambda\delta\phi^U+\delta\lambda-\mathcal{B}_H^U\right]}{4}\phi^4
\end{aligned}
\tag{374}
$$

is the effective Higgs potential. Again here, somebody would expect that we should define a relation between the Tadpoles and $\delta\phi$ in such a way that we would get a finite result like the following

$$a_i'\mathcal{T}_H^{iU} + \frac{m_H^3}{(2\lambda)^{3/2}}\frac{\delta\phi^U}{2} = \mathcal{T}_H^{Uf} \,. \tag{375}$$

Unfortunately, in Unitary gauge we have that

$$\delta\phi^U = -\frac{d\mathcal{M}_H^U}{dp^2} = 0 \tag{376}$$

and form the above equation we are left only with

$$a_i'\mathcal{T}_H^{iU} = a_i'\mathcal{T}_H^{iU}(\varepsilon) + a_i'\mathcal{T}_H^{Uf} \,. \tag{377}$$

Thus, here we have something very interesting and seemingly disastrous, since Eq.(377) will get in our calculations. To be more specific, this infinite contribution will get in the first derivative of the effective potential, affecting the *vev*, and moreover it will be inserted into the definition of the Higgs-mass counterterm. Thus, in the following we show how we should treat this situation.

Now, before we move on and similarly with the previous section, we should mention that the amplitudes $\mathcal{T}_H^U$, $\mathcal{M}_H^U$, $\mathcal{K}_H^U$ and $\mathcal{B}_H^U$ which are computed in Sec.2. The quantity $\mathcal{M}_Z$ is an extraction of $\mathcal{M}_{Z,\mu\nu}^U$ which was determined in Sect. 2.2. The $\mathcal{K}_{HZ}^U$ and $\mathcal{B}_{HZ}^U$ are amplitudes with both Higgs and Z-boson external legs and will not be presented here because they will not play a role in what follows.

In Unitary gauge our Physical prescription requires:

- That the physical Higgs mass and quartic coupling are defined by

$$\begin{aligned} \delta m_H &= \mathcal{M}_H^U + c_i'\mathcal{T}_H^{iU} + c_k'\left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right] - m_H^2\delta\phi^U && (378)\\ V(v)'''' &= 6\lambda \Leftrightarrow \\ \delta\lambda^U &= \mathcal{B}_H^U - 2\lambda\delta\phi^U && (379) \end{aligned}$$

  with c_i' and c_k' two yet undetermined constants with inverse mass dimension.

- That the physical Z-mass is

$$m_Z = m_{Z_{phy}} \,. \tag{380}$$

Finally, the 1-loop corrected *vev* v is the one that minimises $V(\phi)$

$$V'(v) = 0 \tag{381}$$

where $V(\phi)$ is the one-loop, renormalized effective potential

$$\begin{aligned} V(\phi) &= \left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right]\phi + \frac{1}{2}\left\{m_H^2 - c_k' \cdot \left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right]\right\}\phi^2 \\ &+ \left\{\sqrt{\frac{\lambda}{2}}m_H + \sqrt{\frac{\lambda}{2}}\frac{\mathcal{M}_H^U}{2m_H} + \frac{\sqrt{\lambda}}{2\sqrt{2}m_H}c_i'\mathcal{T}_H^{Ui} \right. \\ &+ \left. \frac{\sqrt{\lambda}}{2\sqrt{2}m_H}c_k' \cdot \left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right] - m_H\frac{\mathcal{B}_H^U}{2\sqrt{2\lambda}} + \mathcal{K}_H^U\right\}\phi^3 + \frac{\lambda}{4}\phi^4 \end{aligned} \tag{382}$$

which has the same form with that of Eq.(343). Thus using the same arguments and performing the same calculations with the previous section we obtain

- that the correct vacuum shift is

$$v^U = -\frac{\mathcal{T}_H^U}{2m_H^2}\hbar + \mathcal{O}(\hbar^2) \equiv -\frac{\left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right]}{2m_H^2}\hbar + \mathcal{O}(\hbar^2) \tag{383}$$

  which shows clearly that in Unitary gauge the *vev* is divergent and the divergence is proportional to the physical Tadpoles of the theory.

- that the second derivative of the potential is

$$V''(v^U) = m_H^2 \tag{384}$$

  which, recalling the similar situation in R_ξ-gauge, gives that

$$V''(v^U) = m_H^2 - \left(3\frac{\sqrt{\lambda}}{\sqrt{2}m_H} + 2c_k'\right)\left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right] \tag{385}$$

  and demanding the above condition to hold, we get that $c_k' = -\frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H}$, giving the following relation

$$m_H \equiv m_{H_{phy}} = m_{H_0} - \mathcal{M}_H^U - c_i' \mathcal{T}_H^{iU} + \frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H}\left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right] + m_H^2 \delta\phi^U \tag{386}$$

- that the counterterms of the physical Higgs and Z-mass reads

$$\delta m_H^U = \mathcal{M}_H^U + c_i' \mathcal{T}_H^{iU} - m_H^2 \delta\phi^U - \frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H}\left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right] \tag{387}$$

and

$$\delta m_Z^U = \mathcal{M}_Z^U + \frac{1}{3} d_i' \mathcal{T}_H^{iU} - m_Z^2 \delta A^U \tag{388}$$

respectively. Here, the d''s are dimensionfull with dimension of inverse mass,

Finally we get that the one-loop renormalized effective potential in Unitary gauge reads

$$\begin{aligned} V^U(\phi) &= \left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right]\phi + \frac{1}{2}\left\{m_H^2 + \frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H}\left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right]\right\}\phi^2 \\ &+ \left\{\sqrt{\frac{\lambda}{2}} m_H + \sqrt{\frac{\lambda}{2}}\frac{\mathcal{M}_H^U}{2m_H} + \frac{\sqrt{\lambda}}{2\sqrt{2}m_H} c_i' \mathcal{T}_H^{Ui} \right. \\ &- \left. \frac{3\sqrt{\lambda}}{4\sqrt{2}m_H}\left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right] - m_H \frac{\mathcal{B}_H^U}{2\sqrt{2\lambda}} + \mathcal{K}_H^U\right\}\phi^3 + \frac{\lambda}{4}\phi^4 \,. \end{aligned} \tag{389}$$

which, generally, is infinite.

4.3 Evaluation of the Counterterms

In this part of the present document we calculate the counterterms associated with the renormalized Higgs and gauge boson mass. Moreover, we give a qualitative relation between the renormalized quartic coupling and its counterterm. According to the previous section, we should recall that we perform the renormalization using only the independent parameters of the Lagrangian through the relation Eq.(315).

Thus, using these arguments along with the Physical renormalization conditions, we have developed in Sec.4.1, specific relations for our counterterms. But before we move on, let us present the divergent part of the Tadpoles calculated in Sec.3.1 . This is very important since we will use it to investigate if there would be a ξ-cancelation from the physical quantities. Actually, for completeness we present the divergent part of the Tadpoles in both R_ξ and Unitary gauge, so

$$\mathcal{T}_H^{R_\xi} \;=\; \frac{\mu^\varepsilon}{16\pi^2}\left\{3\sqrt{\frac{\lambda_0}{2}}m_{H_0}A_0(m_H)+3\frac{\sqrt{2\lambda_0}m_{Z_0}^2}{m_{H_0}}A_0(m_Z)+\xi\frac{\sqrt{2\lambda_0}m_{Z_0}^2}{m_{H_0}}A_0(m_\chi)\right\} \tag{390}$$

from which we can extract the divergent part reading

$$\mathcal{T}_H^{R_\xi}(\varepsilon) \;=\; \frac{\mu^\varepsilon}{16\pi^2}\left\{6\sqrt{\frac{\lambda}{2}}m_H^3+6\frac{\sqrt{2\lambda}m_Z^4}{m_H}+2\xi^2\frac{\sqrt{2\lambda}m_Z^4}{m_H}\right\}\frac{1}{\varepsilon}\,. \tag{391}$$

On the other hand, concerning the Unitary gauge, we have the following relations

$$\mathcal{T}_H^{U} \;=\; \frac{\mu^\varepsilon}{16\pi^2}\left\{3\sqrt{\frac{\lambda_0}{2}}m_{H_0}A_0(m_H)+3\frac{\sqrt{2\lambda_0}m_{Z_0}^2}{m_{H_0}}A_0(m_Z)\right\} \tag{392}$$

which gives

$$\mathcal{T}_H^{U}(\varepsilon) \;=\; \frac{\mu^\varepsilon}{16\pi^2}\left\{6\sqrt{\frac{\lambda}{2}}m_H^3+6\frac{\sqrt{2\lambda}m_Z^4}{m_H}\right\}\frac{1}{\varepsilon}\,. \tag{393}$$

Now, we move on to the calculation of the counterterm of the Higgs mass. To be more specific, according to the Physical renormalization conditions that we applied on the Higgs effective potential, the Higgs-mass counterterm is given by Eq.(335)

$$\delta m_H^{R_\xi} = \mathcal{M}_H^{R_\xi} + c_i\mathcal{T}_H^{IR_\xi} - m_H^2\delta\phi^{R_\xi} - \frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H}\mathcal{T}_H^{fR_\xi} \tag{394}$$

for $c_k = -\frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H}$ and $i = 1, 2, 3, 4$ in R_ξ gauge.
Here, the term $c_i \mathcal{T}_H^{iR_\xi}$ includes all the possible terms corresponding to the two-leg Tadpoles. This is a very important notation since, considering the contribution of these reducible diagrams to the Higgs and Z two-point functions we will investigate if our results will be gauge dependent, *i.e.* unphysical, or not.
Moreover, here we should notice that the term $-\frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H}\mathcal{T}_H^{fR_\xi}$ is finite and thus, the counterterm is not just the divergent part of the corresponding amplitudes. This argument is not so strange if we recall that the renormalization conditions that we have used in order to fix this counterterm was a generalization of the On-shell renormalization scheme. Therefore, our δm_H is not a $\overline{MS}$ counterterm so it can include finite parts.

Nevertheless, for the following analysis it i enough to consider only the divergent part of the one-loop corrections in order to obtain the Higgs mass counterterm and to verify the previous argument. The reason is that β-functions do not understand the convention that we make in order to fix the counterterms, they need only the divergent part of them. Therefore at On-shell, namely at $p^2 = m_H^2$, using Eq.(391) and Eq.(339) we obtain for $\delta m_H^{R_\xi}$ that

$$\begin{aligned}\delta m_H^{R_\xi} &= \frac{\mu^\varepsilon}{16\pi^2}\Bigg\{\left(24\lambda m_H^2 + 6\sqrt{\frac{\lambda}{2}}m_H^3 c_1\right) \\ &+ \left(-24\lambda m_Z^2 + 20\frac{\lambda m_Z^4}{m_H^2} + 2\frac{\lambda m_Z^4}{m_H^2}\xi + [c_2 - c_3 - c_4]\xi\frac{\sqrt{2\lambda}m_Z^4}{m_H}\right)\xi \\ &+ \left(18\frac{\lambda m_Z^4}{m_H^2} + 6c_2\frac{\sqrt{2\lambda}m_Z^4}{m_H}\right)\Bigg\}\frac{1}{\varepsilon}\end{aligned} \tag{395}$$

where $\delta\phi^{R_\xi} = 12\xi\frac{\lambda m_Z^2}{m_H^2}$. Therefore, demanding the counterterm, and as a consequence the physical Higgs mass, to be gauge independent we get the following

$$\begin{aligned}c_2 - c_3 - c_4 &= 24\frac{\sqrt{\lambda}m_H}{\xi\sqrt{2}m_Z^2} - 20\frac{\sqrt{\lambda}}{\xi\sqrt{2}m_H} - 2\frac{\sqrt{\lambda}}{\sqrt{2}m_H} \Leftrightarrow \\ c_2 - c_3 - c_4 &= \left(12\frac{m_H^2}{\xi m_Z^2} - \frac{10}{\xi} - 1\right)\frac{1}{v_0}\end{aligned} \tag{396}$$

while c_1 and c_2 stay yet undetermined. Now, replacing the above relation in Eq.(395) we get that the Higgs mass counterterm reads

$$\delta m_H^{R_\xi} = \frac{\mu^\varepsilon}{16\pi^2}\left\{\left(24\lambda m_H^2 + 6\sqrt{\frac{\lambda}{2}}m_H^3 c_1\right) + \left(18\frac{\lambda m_Z^4}{m_H^2} + 6c_2\frac{\sqrt{2\lambda}m_Z^4}{m_H}\right)\right\}\frac{1}{\varepsilon}\,. \tag{397}$$

On the other hand, we should perform the same calculation for the Unitary gauge in order to compare the corresponding counterterms. But, here we should recall that the expression that gives mass counterterm in Unitary gauge, is enhanced by the quantity $-\frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H}\left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right]$.

In particular, we have that

$$\delta m_H^U = \mathcal{M}_H^U + c_i'\mathcal{T}_H^{Ui} - \frac{3}{2}\frac{\sqrt{\lambda}}{\sqrt{2}m_H}\left[\mathcal{T}_H^U(\varepsilon) + \mathcal{T}_H^{Uf}\right] - m_H^2\delta\phi^U \tag{398}$$

and thus, concerning here only for the divergent parts, the above relation gives

$$\begin{aligned}
\delta m_H^U &= \frac{\mu^\varepsilon}{16\pi^2}\left\{24\lambda m_H^2 + 18\frac{\lambda m_Z^4}{m_H^2} + 6c_1'\sqrt{\frac{\lambda}{2}}m_H^3 + 6c_2'\frac{\sqrt{2\lambda}m_Z^4}{m_H} - 9\lambda m_H^2 - 18\frac{\lambda m_Z^4}{m_H^2}\right\}\frac{1}{\varepsilon} \Leftrightarrow \\
\delta m_H^U &= \frac{\mu^\varepsilon}{16\pi^2}\left\{\left(24\lambda m_H^2 + 6c_1'\sqrt{\frac{\lambda}{2}}m_H^3 - 9\lambda m_H^2\right) + \left(18\frac{\lambda m_Z^4}{m_H^2} + 6c_2'\frac{\sqrt{2\lambda}m_Z^4}{m_H} - 18\frac{\lambda m_Z^4}{m_H^2}\right)\right\}\frac{1}{\varepsilon}
\end{aligned} \tag{399}$$

where $\delta\phi^U = 0$.
So, since $\delta m_H^{R_\xi}$ has been left only with its ξ-independent parts, in order to perform a matching between the R_ξ and Unitary gauge counterterms, the following relation should be fulfilled

$$\begin{aligned}
c_1 &= c_1' - \frac{3}{2}\sqrt{\frac{\lambda}{2}}\frac{1}{m_H} \equiv c_1' - \frac{3}{2v_0} \\
c_2 &= c_2' - \frac{3}{4}\sqrt{\frac{\lambda}{2}}\frac{1}{m_H} \equiv c_2' - \frac{3}{4v_0}\,.
\end{aligned} \tag{400}$$

Finally, as we can see from the above relation, there is an extra freedom in the choice of c_1' and c_2'. Nevertheless, we know that the Tadpoles should not affect the physical quantities and generally there are arguments which say that they cancel from them.

Thus, for our case this possibility comes from the choice $c'_1 = \frac{3}{2v_0}$ and $c'_2 = \frac{3}{4v_0}$ which set $c_1 = c_2 = 0$. Thus, now in both the Unitary and the R_ξ gauge we are left with two identical relations, which read

$$\begin{aligned} \delta m_H^{R_\xi} &= \frac{\mu^\varepsilon}{16\pi^2}\left\{24\lambda m_H^2 + 18\frac{\lambda m_Z^4}{m_H^2}\right\}\frac{1}{\varepsilon} \\ \delta m_H^{U} &= \frac{\mu^\varepsilon}{16\pi^2}\left\{24\lambda m_H^2 + 18\frac{\lambda m_Z^4}{m_H^2}\right\}\frac{1}{\varepsilon} \end{aligned} \tag{401}$$

respectively.

Here, before we move on to obtain the counterterm for the Z-mass, let us present a very important comment concerning the introduction of the reducible two-leg Tadpoles. In particular, it is well known that in R_ξ gauges in order to have ξ-independent counterterms for the masses, we should consider also the two-leg Tadpoles. But here, we face a contradiction. To be more specific, the general idea is that we add the two-leg Tadpoles as corrections of the propagator, and thus, performing the resumation we obtain their contribution to the masses. But this procedure says something very constrained, since if we follow it, then the two-leg Tadpoles should have coefficient unity, there should not be any mixing and we should add all of them despite of their ξ-dependence.

Now, our analysis shows something different, since recalling Eq.(400) we see that the physical two-leg Tadpoles can be and should be absent from the physical quantities, which means that we need only the un-physical Tadpole contributions in order to make our result ξ-independent. In addition, from Eq.(396) for $c_2 = 0$, we observe that there is not solution for $c_3 = c_4 = \frac{1}{v_0}$ and moreover, in order to make $\delta m_H^{R_\xi}$ gauge independent we should consider mixing between the two-leg Tadpoles.

Now, lets move on to the calculation of the Z-boson mass counterterm. Again from the On-shell renormalization condition that we apply in the effective Lagrangian for the Z-boson pole mass we get that

$$\delta m_Z^{R_\xi} = \mathcal{M}_Z^{R_\xi} + \frac{1}{3} d_i \mathcal{T}_H^{iR_\xi} - m_Z^2 \delta A^{R_\xi} \tag{402}$$

where $i = 1, 2, \cdots, n$. Now, according to our calculations we have that

$$\mathcal{M}_Z^{R_\xi} = \Pi^T(m_Z^2) = \frac{-1}{3}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\mathcal{M}_{ZR_\xi}^{\mu\nu}(p^2 = m_Z^2)\,. \tag{403}$$

which could be split into two parts and we will calculate them separately. Thus, form Eq.(277) the first part reads

$$
\begin{aligned}
g_{\mu\nu}\mathcal{M}^{R_\xi}_{Z,\mu\nu} &= (d+\varepsilon)\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\frac{\mu^\varepsilon}{16\pi^2}\Bigg\{-2A_0(m_{H_0}) - 8B_0(p,m_{Z_0},m_{H_0}) \\
&+ 8(1-\xi)\frac{1}{d}\left[m^2_{Z_0}C_0(m_{Z_0},\sqrt{\xi}m_{Z_0},m_{H_0}) + B_0(p,m_{\chi_0},m_{H_0})\right] \\
&+ 8\frac{1}{d}m^2_{\chi_0}B_0(p,m_{\chi_0},m_{H_0}) + 8\frac{1}{d}A_0(m_{H_0})\Bigg\}.
\end{aligned} \tag{404}
$$

As we showed previously, the second part of $\mathcal{M}_{ZR_\xi}$ corresponds to the contraction of Eq.(273) with the term $\frac{p^\mu p^\nu}{p^2}$, so performing this calculation we get the following relation

$$
\begin{aligned}
\frac{p^\mu p^\nu}{p^2}\mathcal{M}^{\mu\nu}_{ZR_\xi}(p) &= \frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\frac{\mu^\varepsilon}{16\pi^2}\Bigg\{-2A_0(m_{H_0}) - 8B_0(p,m_{Z_0},m_{H_0}) \\
&+ 8(1-\xi)\frac{1}{d}\left[m^2_{Z_0}C_0(m_{Z_0},\sqrt{\xi}m_{Z_0},m_{H_0}) + B_0(p,m_{\chi_0},m_{H_0})\right] \\
&+ 8\frac{1}{d}m^2_{\chi_0}B_0(p,m_{\chi_0},m_{H_0}) + 8\frac{1}{d}A_0(m_{H_0})\Bigg\}.
\end{aligned} \tag{405}
$$

Finally, as we mentioned previously, the term that we need to evaluate is given by Eq.(403), so following this relation we get that

$$
\begin{aligned}
\mathcal{M}_{ZR_\xi}(p) &= \frac{-1}{3}\frac{m^2_{Z_0}}{m^2_{H_0}}\lambda_0\frac{\mu^\varepsilon}{16\pi^2}\Bigg\{(d+\varepsilon)\Big\{-2A_0(m_{H_0}) - 8B_0(p,m_{Z_0},m_{H_0}) \\
&+ 8(1-\xi)\frac{1}{d}\left[m^2_{Z_0}C_0(m_{Z_0},\sqrt{\xi}m_{Z_0},m_{H_0}) + B_0(p,m_{\chi_0},m_{H_0})\right] \\
&+ 8\frac{1}{d}m^2_{\chi_0}B_0(p,m_{\chi_0},m_{H_0}) + 8\frac{1}{d}A_0(m_{H_0})\Big\} \\
&- \Big\{-2A_0(m_{H_0}) - 8B_0(p,m_{Z_0},m_{H_0}) \\
&+ 8(1-\xi)\frac{1}{d}\left[m^2_{Z_0}C_0(m_{Z_0},\sqrt{\xi}m_{Z_0},m_{H_0}) + B_0(p,m_{\chi_0},m_{H_0})\right] \\
&+ 8\frac{1}{d}m^2_{\chi_0}B_0(p,m_{\chi_0},m_{H_0}) + 8\frac{1}{d}A_0(m_{H_0})\Big\}\Bigg\}.
\end{aligned} \tag{406}
$$

Thus according to Eq.(341), with $\delta A^{R_\xi} = 0$, we get at On-shell that

$$\delta m_Z^{R_\xi} = \frac{\mu^\varepsilon}{(4\pi)^2}\left\{12\frac{\lambda m_Z^4}{m_H^2} + 6d_1\sqrt{\frac{\lambda}{2}}m_H^3 + 6d_2\frac{\sqrt{2\lambda}m_Z^4}{m_H}\right\}\frac{1}{\varepsilon} \tag{407}$$

and as we can see it is ξ-independent without using the two-leg Tadpoles as we have expected through the gauge invariance argumnets that we have developed previously. So a non-trivial check for our calculation and a physical expectation, is to find exactly the same counterterm in Unitary gauge where there are only the physical degrees of freedom and is gauge independent from the beginning.

Thus, according to Eq.(388), again with $\delta A^U = 0$ we find that

$$\delta m_Z^U = \frac{\mu^\varepsilon}{(4\pi)^2}\left\{12\frac{\lambda m_Z^4}{m_H^2} + 6d_1'\sqrt{\frac{\lambda}{2}}m_H^3 + 6d_2'\frac{\sqrt{2\lambda}m_Z^4}{m_H}\right\}\frac{1}{\varepsilon} \tag{408}$$

which is identical with the $\delta m_Z^{R_\xi}$ relation. Again we should consider that $d_1 = d_2 = d_1' = d_2' = 0$ getting that

$$\delta m_Z^{R_\xi} = \delta m_Z^U = \frac{\mu^\varepsilon}{(4\pi)^2}\frac{12\lambda m_Z^4}{m_H^2}\frac{1}{\varepsilon}\,. \tag{409}$$

Thus, we have ended up with some very interesting conclusions. To be more specific, we saw that we indeed need the reducible two-leg Tadpoles in order to make the Higgs mass counterterm gauge independent, but for that purpose we should considered specific coefficients, different to unity, and mixing between the Tadpoles. On the other hand we saw that in the case of the Z-boson mass counterterm, the result was ξ-independent from the very beginning and we did not worry about the insertion of the two-leg Tadpoles.

5 Physical quantities and the β-functions

5.1 General Framework

Let us start here with a theoretical framework for the evaluation of the β-functions so as to have a complete picture of the derivation that follows in the next sections. Therefore, we begin by denoting a generic bare coupling by α_0, the corresponding renormalized coupling and counter-term by α and $\delta\alpha$ respectively and its beta function by β_α. Quantum corrections introduce a μ-dependence of the counterterms that induces a μ-dependence of the renormalized coupling so that the bare coupling

$$\alpha_0 = \alpha(\mu) + \delta\alpha(\mu)\,, \tag{410}$$

is μ-independent. Now, $\alpha(\mu)$ will denote the renormalized running coupling and the counterterm $\delta\alpha(\mu)$ is considered to be a function of the renormalized couplings, through their dependence on μ. In addition, we define the value of the renormalized running coupling at some renormalization scale $\mu = m_{phys.}$ to be

$$\alpha(\mu = m_{phy.}) \equiv \alpha\,. \tag{411}$$

Now, we can express any counterterm as

$$\delta a \;=\; \frac{\mu^\varepsilon}{(4\pi)^2}\left(\frac{C_\alpha}{\varepsilon} + \sum_k f^k_{A_0}\ln\frac{\mu^2}{m_k^2} + \sum_{k,i} f^k_{B_0}\int\limits_0^1 dx\ln\left(\frac{\mu^2}{\Delta^i_k(m_k,m_i)}\right) + \sum_k f^k_{A_0}\right) \tag{412}$$

where we have defined that the summation on index i corresponds to all the possible different fields that contribute to the quantum corrections. In addition, we should notice that the relation $\varepsilon = 4 - d$, indicates that the μ-dependence is only explicit.
Now, the RG flow equation in d-dimensions, where for dimensional reasons all the renormalized couplings, must be rescaled by μ^ε which is pulled out in front of the counterterm, and thus, defining the following relations

$$\begin{aligned}\delta_\alpha &\equiv \frac{\delta\alpha(\mu)}{a(\mu)}\\ \beta_\alpha &= \mu\frac{d}{d\mu}\alpha(\mu)\\ \tilde{\beta}_\alpha &\equiv \frac{\beta_\alpha}{\alpha}\end{aligned} \tag{413}$$

we have for a general coupling that

$$
\begin{aligned}
0 &= \mu\frac{d}{d\mu}\alpha_0 = \mu\frac{d}{d\mu}\Big\{\mu^{\varepsilon}\alpha(\mu)(1+\delta_\alpha)\Big\} = \mu\frac{d}{d\mu}\Big\{\mu^{\varepsilon}\alpha(\mu) + \mu^{\varepsilon}\delta\alpha(\mu)\Big\} \Leftrightarrow \\
0 &= \mu\left\{\varepsilon\alpha(1+\delta_\alpha) + (1+\delta_\alpha)\mu\frac{\partial\alpha}{\partial\mu} + \alpha\mu\frac{\partial\delta_\alpha}{\partial\mu}\right\} \Leftrightarrow \\
\beta_\alpha(1+\delta_\alpha) &= -\varepsilon\alpha(1+\delta_\alpha) - \alpha\mu\frac{\partial\delta_\alpha}{\partial\mu} \Leftrightarrow \\
\beta_\alpha &= -\varepsilon\alpha - \alpha\mu\frac{\partial\delta_\alpha}{\partial\mu}(1+\delta_\alpha)^{-1} \Leftrightarrow \\
\beta_\alpha &= -\varepsilon\alpha - \alpha\mu\frac{\partial\delta_\alpha}{\partial\mu} \Leftrightarrow
\end{aligned}
\tag{414}
$$

where, since $\delta_\alpha \equiv \delta_\alpha\hbar$, we have performed an expansion in $\hbar$ in order to get rid of terms of $\mathcal{O}(\hbar^2)$ like $\delta_\alpha \cdot \frac{\partial\delta_\alpha}{\partial\mu}$. Now, since we have more than one couplings, the above relation should become as follows

$$
\beta_\alpha = -\varepsilon\alpha - \alpha\left\{\beta_\lambda\frac{\partial\delta_\alpha}{\partial\lambda} + \beta_{m_H^2}\frac{\partial\delta_\alpha}{\partial m_H^2} + \beta_{m_Z^2}\frac{\partial\delta_\alpha}{\partial m_Z^2}\right\}.
\tag{415}
$$

Now, in our case where the only couplings are λ, m_H and m_Z we get that

$$
\begin{aligned}
\beta_\lambda &= -\varepsilon\lambda - \lambda\left\{\beta_\lambda\frac{\partial\delta_\lambda}{\partial\lambda} + \beta_{m_H^2}\frac{\partial\delta_\lambda}{\partial m_H^2} + \beta_{m_Z^2}\frac{\partial\delta_\lambda}{\partial m_Z^2}\right\} \\
\beta_{m_H^2} &= -\varepsilon m_H^2 - m_H^2\left\{\beta_\lambda\frac{\partial\delta_{m_H}}{\partial\lambda} + \beta_{m_H^2}\frac{\partial\delta_{m_H}}{\partial m_H^2} + \beta_{m_Z^2}\frac{\partial\delta_{m_H}}{\partial m_Z^2}\right\} \\
\beta_{m_Z^2} &= -\varepsilon m_Z^2 - m_Z^2\left\{\beta_\lambda\frac{\partial\delta_{m_Z}}{\partial\lambda} + \beta_{m_H^2}\frac{\partial\delta_{m_Z}}{\partial m_H^2} + \beta_{m_Z^2}\frac{\partial\delta_{m_Z}}{\partial m_Z^2}\right\}
\end{aligned}
\tag{416}
$$

which, collecting the equal β-functions in each case, reads

$$
\begin{aligned}
\beta_\lambda(1+\frac{\partial\delta_\lambda}{\partial\lambda})+\lambda\left\{+\beta_{m_H^2}\frac{\partial\delta_\lambda}{\partial m_H^2}+\beta_{m_Z^2}\frac{\partial\delta_\lambda}{\partial m_Z^2}\right\} &= -\varepsilon\lambda \\
\beta_{m_H^2}(1+\frac{\partial\delta_{m_H}}{\partial m_H^2})+m_H^2\left\{\beta_\lambda\frac{\partial\delta_{m_H}}{\partial\lambda}+\beta_{m_Z^2}\frac{\partial\delta_{m_H}}{\partial m_Z^2}\right\} &= -\varepsilon m_H^2 \\
\beta_{m_Z^2}(1+\frac{\partial\delta_{m_Z}}{\partial m_Z^2})+m_Z^2\left\{\beta_\lambda\frac{\partial\delta_{m_Z}}{\partial\lambda}+\beta_{m_H^2}\frac{\partial\delta_{m_Z}}{\partial m_H^2}\right\} &= -\varepsilon m_Z^2
\end{aligned}
\tag{417}
$$

which could be written in the following matrix form

$$
\begin{pmatrix}
1+\frac{\partial\delta_\lambda}{\partial\lambda} & \lambda\frac{\partial\delta_\lambda}{\partial m_H^2} & \lambda\frac{\partial\delta_\lambda}{\partial m_Z^2} \\
m_H^2\frac{\partial\delta_{m_H}}{\partial\lambda} & 1+\frac{\partial\delta_{m_H}}{\partial m_H^2} & m_H^2\frac{\partial\delta_{m_H}}{\partial m_Z^2} \\
m_Z^2\frac{\partial\delta_{m_Z}}{\partial\lambda} & m_Z^2\frac{\partial\delta_{m_Z}}{\partial m_H^2} & 1+\frac{\partial\delta_{m_Z}}{\partial m_Z^2}
\end{pmatrix}
\cdot
\begin{pmatrix}
\beta_\lambda \\ \beta_{m_H^2} \\ \beta_{m_Z^2}
\end{pmatrix}
= -\varepsilon
\begin{pmatrix}
\lambda \\ m_H^2 \\ m_Z^2
\end{pmatrix}
\tag{418}
$$

thus, inverting the matrix in our case we get that

$$
\begin{aligned}
\beta_\lambda &= \lambda^2\frac{\partial\frac{C_\lambda}{\lambda}}{\partial\lambda}+\lambda m_H^2\frac{\partial\frac{C_\lambda}{\lambda}}{\partial m_H}+\lambda\frac{\partial\frac{C_\lambda}{\lambda}}{\partial m_Z} \\
\beta_{m_H^2} &= \lambda m_H^2\frac{\partial\frac{C_{m_H^2}}{m_H^2}}{\partial\lambda}+\frac{1}{2}m_H^2\frac{\partial\frac{C_{m_H^2}}{m_H^2}}{\partial m_H}+\frac{1}{2}m_H^2\frac{\partial\frac{C_{m_H^2}}{m_H^2}}{\partial m_Z} \\
\beta_{m_Z^2} &= \lambda m_Z^2\frac{\partial\frac{C_{m_Z^2}}{m_Z^2}}{\partial\lambda}+\frac{1}{2}m_Z^2\frac{\partial\frac{C_{m_Z^2}}{m_Z^2}}{\partial m_H}+\frac{1}{2}m_Z^2\frac{\partial\frac{C_{m_Z^2}}{m_Z^2}}{\partial m_Z}\,.
\end{aligned}
\tag{419}
$$

Thus, all that we need to do in order to obtain the various β-functions, is to identify from the explicit form of the counterterms the quantities C_α and f_α defined in Eq.(412) and build the β-function, according to Eq.(419). Moreover, the solution of the differential equation of the second term in (413) gives the following relation

$$
\begin{aligned}
\alpha &= \alpha(\mu)+\beta_\alpha\,\ln\left(\frac{m_{phy.}}{\mu}\right)\Leftrightarrow \\
\alpha &= \frac{\alpha(\mu)}{1-\tilde{\beta}_\alpha\,\ln\left(\frac{m_{phy.}}{\mu}\right)}
\end{aligned}
\tag{420}
$$

for the RG evolution of the coupling. We can write it also as

$$\alpha(\mu) = \frac{\alpha}{1+\tilde{\beta}_\alpha \ln\left(\frac{m_{phy.}}{\mu}\right)} \tag{421}$$

where the latter determines the Landau pole associated with the coupling α to be

$$\begin{aligned} \mu^{\alpha}_{Lp} &= m\, e^{\frac{1}{\tilde{\beta}_\alpha}} \Leftrightarrow \\ \mu^{\alpha}_{Lp} &= m\, e^{\frac{\alpha}{\beta_\alpha}} . \end{aligned} \tag{422}$$

5.2 Evaluation of the β-functions

In the previous section we have developed the full procedure that someone should follow in order to calculate the β-functions of a model. Thus, we are ready to imply all the above arguments on the counterterms that we have extracted in Sec.4.3, obtaining the corresponding beta functions and as a consequence the "running" of the independent couplings that we have defined previously.
Let us start with the β-function of the Higgs mass β_{m_H}, thus comparing Eq.(394) with the general form Eq.(412) for the counterterms we can see that

$$C_{m_H^2} = 24\lambda m_H^2 + 18\frac{\lambda m_Z^4}{m_H^2} . \tag{423}$$

Now, using the definition of the β-function Eq.(419) we obtain the following

$$\beta_{m_H} = \frac{1}{(4\pi)^2}\left\{24\lambda m_H^2 + 18\frac{\lambda m_Z^4}{m_H^2}\right\} \tag{424}$$

and from Eq.(413)

$$\tilde{\beta}_{m_H} = \frac{1}{(4\pi)^2}\left\{24\lambda + 18\frac{\lambda m_Z^4}{m_H^4}\right\} . \tag{425}$$

Therefore, according to the definition in Eq.(421) we get that the "running" Higgs mass reads

$$m_H^2(\mu) = \frac{m_H^2}{1 + \tilde{\beta}_{m_H} \ln\left(\frac{m^2_{H_{phy.}}}{\mu^2}\right)} . \tag{426}$$

Now, we can perform the same calculation for the Z-boson mass beta function β_{m_Z}. To be more specific, following the same arguments with the previous derivation we get that

$$C_{m_Z} = 12\frac{\lambda m_Z^4}{m_H^2} . \tag{427}$$

So, using again Eq.(419) we obtain the following

$$\beta_{m_Z} = \frac{1}{(4\pi)^2} 12 \frac{\lambda m_Z^4}{m_H^2} \tag{428}$$

and

$$\tilde{\beta}_{m_Z} = \frac{1}{(4\pi)^2} 12 \frac{\lambda m_Z^2}{m_H^2} . \tag{429}$$

Thus, here the "running" Z-mass reads

$$m_Z^2(\mu) = \frac{m_Z^2}{1 + \tilde{\beta}_{m_Z} \ln\left(\frac{m^2_{Z_{phy.}}}{\mu^2}\right)} . \tag{430}$$

Finally, in order to investigate further the results that we obtained in the above relations, we present here the *Renormalization Group Equation*'s flow for the physical Higgs- and Z-mass in Fig.1 and Fig.2 respectively. This, would be very helpful for the next section where we evaluate the *LCP*'s since, in order to have a complete picture about the physical results, we should compare the *RGE*'s flow with them.

Here, we should notice that both the physical Higgs- and Z-mass are getting bigger for a vast variety of energy scales. Nevertheless, this is not generally the case since, looking carefully the above Figures, we can observe that there is a energy scale where both masses go to infinity. This energy scale is the known *Landau Pole*, which was obtained through

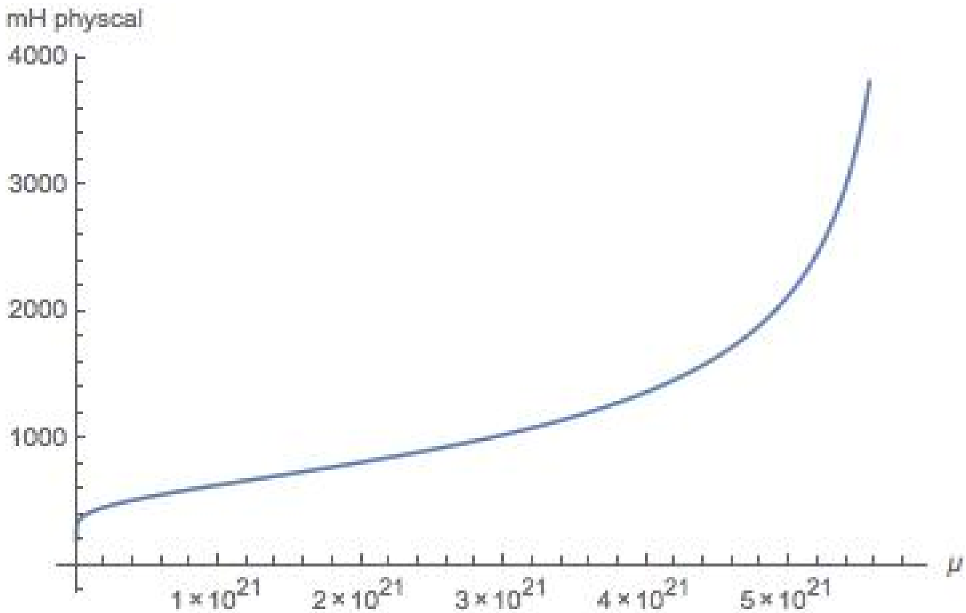


Figure 1: The "running" of the physical Higgs-mass as a function of the renormalization scale μ

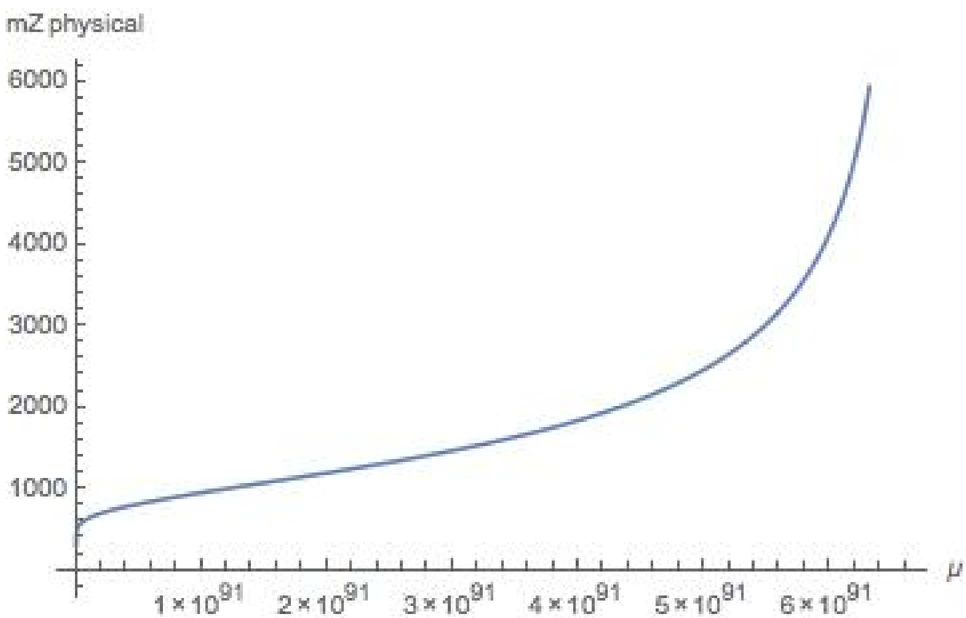


Figure 2: The "running" of the physical Z-mass as a function of the renormalization scale μ

Eq.(422) for each mass. Generally, the *Landau Pole* is different for each coupling and here reads for the Higgs- and the Z-mass

$$\begin{aligned} \mu_{Lp}^{m_H} &= 5 \times 10^{21} GeV \\ \mu_{Lp}^{m_Z} &= 6 \times 10^{91} GeV \end{aligned} \tag{431}$$

respectively.

5.3 Lines of Constant Physics

In this last subsection, as we have already mentioned, we are dealing with the evaluation of the *Lines of Constant Physics*. Therefore, here we should make an important notation here concerning the physical quantities. To be more specific, we know that in order to obtain the RGE's, Eq.(426) and Eq.(430) are what we need. Nevertheless, in what follows we will evaluate the LCP's, and thus, we will need a relation between the bare and the physical independent couplings. Thus, for the purpose of this specific calculation we should use the general relation of the renormalized quantities which reads

$$\alpha = \alpha_0 - \delta\alpha \tag{432}$$

where α is an arbitrary coupling. Thus, preparing our selves for the calculation of the LCP's we perform a mini comparison between the physical quantities coming form the β-functions analysis and the physical quantities coming from the above relation.
Therefore, we will use Eq.(320) in order to see the exact expanded form of the counterterms, but now including also their finite parts and excluding the divergent ones. This is a legitimate step since we have renormalized our Lagrangian. Then, with the help of Eq.(432) we will find the physical quantities in order to see if and how it differs from that of the previous section.
Let us start with the Higgs mass where, implying all the previous arguments and adding the appropriate terms so as to create everywhere $\ln \frac{\mu^2}{m_H^2}$, we get that

$$
\begin{aligned}
m_H^2 &= m_{H_0}^2 - \left\{12\lambda m_H^2 + 9\frac{\lambda m_Z^4}{m_H^2} - 12\lambda m_Z^2\xi + 10\frac{\lambda m_Z^4}{m_H^2}\xi + \frac{\lambda m_Z^4}{m_H^2}\xi^2 + [-c_3 - c_4]\xi\frac{\sqrt{2\lambda} m_Z^4}{2m_H}\right\}\ln\frac{\mu^2}{m_H^2} \\
&+ 9\lambda m_H^2 b_0'(m_H, m_H) + 16\frac{\lambda m_Z^4}{m_H^2}b_0'(m_Z, m_Z) - 16\frac{\lambda m_Z^4}{m_H^2}\ln\frac{m_H^2}{m_Z^2} \\
&- \frac{\lambda m_Z^4}{m_H^2}\left\{-6 + 6\xi\right\}[-b_0'(m_Z, m_\chi) + \ln\frac{m_H^2}{m_Z^2}] + 8\frac{\lambda m_Z^4}{m_H^2}[-b_0'(m_\chi, m_Z) + \ln\frac{m_H^2}{m_Z^2}] \\
&+ \frac{\lambda m_Z^4}{m_H^2}\left\{1 - \frac{m_H^2}{m_Z^2} + 2\xi + 3\xi^2\right\}[-b_0'(m_\chi, m_\chi) + \ln\frac{m_H^2}{m_\chi^2}] \\
&+ \frac{\lambda m_Z^6}{m_H^2}\left\{2 - 2\xi^2\right\}c_0(m_Z, m_Z, m_\chi) - \frac{\lambda m_Z^8}{m_H^2}\left\{1 - 2\xi + \xi^2\right\}d_0(m_Z, m_Z, m_\chi, m_\chi) \\
&- 2\frac{\lambda m_Z^4}{m_H^2}\ln\frac{m_H^2}{m_Z^2} - \frac{\lambda m_Z^4}{m_H^2}\left\{-3\frac{m_H^2}{m_Z^2} + 2\xi + 6\xi^2 - \xi^2 c_3\frac{m_H}{\sqrt{2\lambda}} - \xi^2 c_4\frac{m_H}{\sqrt{2\lambda}}\right\}\ln\frac{m_H^2}{m_\chi^2} + \xi^2[c_3 + c_4]\frac{\sqrt{2\lambda} m_Z^4}{m_H} \\
&- 3\lambda m_H^2 - 3\frac{\lambda m_Z^4}{2m_H^2} + 3\lambda m_Z^2\xi - 9\frac{\lambda m_Z^4}{m_H^2}\xi - \frac{\lambda m_Z^4}{2m_H^2}\xi^2 \qquad (433)
\end{aligned}
$$

which generally seems ξ-dependent. Here, as in the previous section, we have define that

$$
\begin{aligned}
b_0(m_1, m_2) &\equiv \int_0^1 dx \ln\Delta_{B_0}(m_1, m_2) \\
b_0'(m_1, m_2) &\equiv \int_0^1 dx \ln\frac{\Delta_{B_0}(m_1, m_2)}{m_1^2} \\
c_0(m_1, m_2, m_3) &\equiv -\int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{1}{\Delta_{C_0}(m_1, m_2, m_3)} \\
d_0(m_1, m_2, m_3, m_4) &\equiv \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dw \frac{1}{\Delta_{D_0}^2(m_1, m_2, m_3)}\,. \qquad (434)
\end{aligned}
$$

Nevertheless, we should not forget that the relation between the bare and the renormalized couplings is quite arbitrary from the very beginning. To be more specific, we know that generally a bare quantity can take any value that it wants and it could be infinite, so we always can write that

$$
\begin{aligned}
a_0 &= a_R + \delta a \Leftrightarrow \\
a_0(\varepsilon) + a_0^f(\xi) + a_0^f &= a_R + \delta a(\varepsilon) + \delta^f(\xi) + \delta^f a\,. \qquad (435)
\end{aligned}
$$

Now, recall that here we want to evaluate the *LCP*'s coming form a perturbative regime and then to compare them with *LCP*'s coming from a non-perturbative regime from *Lattice*. In order to do so, we have to consider a specific action in *Lattice* which would correspond to a specific prescription choice in the perturbation theory. For our case, this choice corresponds to the absorption of $\delta a(\varepsilon)$ and $\delta^f(\xi)$ from $a_0(\varepsilon)$ and $a_0^f(\xi)$. Thus, for the *LCP*'s, we are left only with the following relation

$$m_H^2 = m_{H_0}^2 - \left\{ 12\lambda m_H^2 + 9\frac{\lambda m_Z^4}{m_H^2} \right\} \ln \frac{\mu^2}{m_{H_{phy}}^2} + \delta m_H^f \tag{436}$$

where using the above arguments, we define that

$$\begin{aligned} \delta m_H^f &= 9\lambda m_H^2 b_0'(m_H, m_H) + 16\frac{\lambda m_Z^4}{m_H^2} b_0'(m_Z, m_Z) \\ &- 4\frac{\lambda m_Z^4}{m_H^2} \ln \frac{m_H^2}{m_Z^2} - 3\lambda m_H^2 - 3\frac{\lambda m_Z^4}{2m_H^2} \,. \end{aligned} \tag{437}$$

which is completely ξ-independent and finite. Now, from Eq.(426) we have that

$$m_H^2 = m_H^2(\mu) + \beta_{m_H} \ln \frac{m_{H_{phy}}^2}{\mu^2} \equiv m_H^2(\mu) - \left\{ 24\lambda m_H^2 + 18\frac{\lambda m_Z^4}{m_H^2} \right\} \ln \frac{\mu^2}{m_{H_{phy}}^2} \tag{438}$$

which gives a very interesting result since, comparing the above relation with Eq.(436) we see that it lacks a finite part but nevertheless, the two results have the same relative signs in the coefficient of $\ln \frac{\mu^2}{m_{H_{phy}}^2}$. This result is crucial since any difference here could affect the physical Higgs mass and as a consequence the *LCP*'s. On the other hand, the term δm_H^f could be an indication of the Hierarchy problem, which is hidden in dimensional regularization, since it contains terms proportional to the square of the masses. The validity of this statement will be checked when we will construct the *LCP*'s in the next section.

Following the same arguments with the Higgs-mass, we perform the same calculation for the m_Z obtaining the following relations

$$m_Z^2 = m_{Z_0}^2 - 12\frac{\lambda m_Z^4}{m_H^2} \ln \frac{\mu^2}{m_{Z_{phy}}^2} + \delta m_Z^f \tag{439}$$

where we have defined that

$$\delta m_Z^f = -\frac{8\lambda m_Z^4}{m_H^2} b_0'(m_Z, m_H) + \frac{4\lambda m_Z^4}{m_H^2} \tag{440}$$

and from Eq.(430) we get that

$$m_Z^2 = m_Z^2(\mu) + \beta_{m_Z} \ln \frac{m_{Z_{phy}}^2}{\mu^2} \equiv m_Z^2(\mu) - 12\frac{\lambda m_Z^4}{m_H^2} \ln \frac{\mu^2}{m_{Z_{phy}}^2} \tag{441}$$

Thus, again here, we have found that the second relation lacks a finite part but the relative signs of the $\ln \frac{\mu^2}{m_{Z_{phy}}^2}$ coefficient are the same.

Appendices

A Feynman Rules

As we have mentioned in the first section of this document, here we study the Abelian Higgs model in the Polar basis, and its Lagrangian in the unitary gauge has the form

$$\mathcal{L}_{AH} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) + \frac{1}{2}m_Z^2 A_\mu A^\mu + \frac{g^{\mu\nu}}{2} g^2 \phi^2 A_\mu A_\nu$$
$$+ g^{\mu\nu} g m_Z \phi A_\mu A_\nu - \frac{1}{2}m_H^2 \phi^2 - \frac{\lambda}{4}\phi^4 - \lambda v_0 \phi^3 + const.$$

The loop Feynman diagrams that we encounter in section 2 are based on the specific Feynman rules of this Lagrangian which read:

Gauge boson propagator

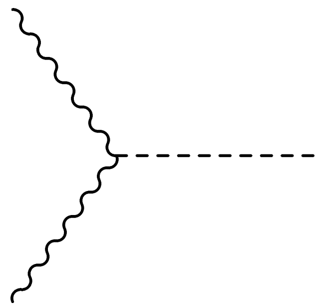
$$= \frac{i\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{m_Z^2}\right)}{k^2 - m_Z^2 + i\varepsilon} \tag{442}$$

Higgs boson propagator

$$= \frac{i}{k^2 - m_H^2 + i\varepsilon} \tag{443}$$

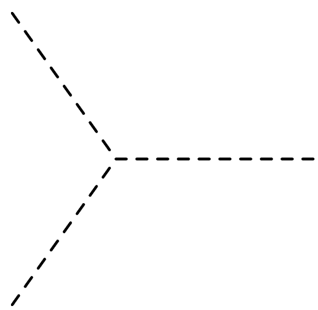
and now we go on to present the Feynman rules for the trilinear vertices:

Higgs-Z-Z vertex



$$= 2ig^{\mu\nu} g m_Z \tag{444}$$

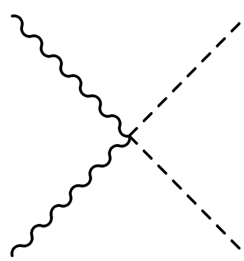
Three-Higgs vertex



$$= -6i\lambda v_0 \tag{445}$$

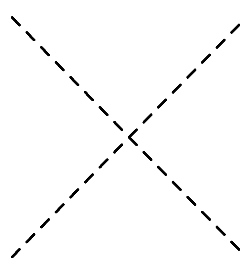
next we have the Feynman rules of the quadrilinear vertices:

Higgs-Higgs-Z-Z vertex



$$= 2ig^2 g^{\mu\nu} \tag{446}$$

Four-Higgs vertex



$$= -6i\lambda \tag{447}$$

B Veltman-Passarino Reduction Formula

Here we demonstrate the full mathematical formulation that we have used in the section 2 following [1] . This has been done since we would like to simplify our results coming from the calculation of the one loop Feynman diagrams. In contrast the traditional approach that someone uses along with the dimensional regularization(DR), in order to calculate one-loop diagrams, gives quite messy results. To be more specific, when we have to calculate one-loop diagrams in any possible model, then we have deal with integrals of the following form

$$I_N \sim \int \frac{d^4k}{(4\pi)^2} \frac{\mathcal{N}(k)}{\left((k+q_0)^2 - m_1^2\right)\left((k+q_1)^2 - m_2^2\right)\cdots\left((k+q_{N-1})^2 - m_N^2\right)} \tag{448}$$

where N is the number of the external particles, $q_j = \sum\limits_{k=1}^{j} p_k$ and $p_1+p_2+\cdots+p_N = 0$ due to the momentum conservation at the loop vertexes. Generally, the $\mathcal{N}(k)$ is a polynomial function of the loop momentum k , the external momenta p_i , the external polarization vectors, the spinors etc. Moreover there is a special case where $\mathcal{N}(k) = 1$ which is referred to as the scalar integral. Now, if we have to calculate an integral like (448) , then scalar integrals make our life easier since, as we will show, in them we can compactify all the bits of information encoded in the original integral. Thus, since we would like to obtain an efficient way to perform this calculations without doing it explicitly with the traditional way, the scalar integrals played a crucial role in our computation.

Generally it turns out that in the limit $D \to 4$ any integral I_N can be written as a linear combination of the one-loop scalar integrals which include one-, two-, three- and four-point functions and a remnant of the dimensional regularization that is called rational part $\mathcal{R}$. The form of that specific combination is

$$I_N = c_{4;j} I_{4;j} + c_{3;j} I_{3;j} + c_{2;j} I_{2;j} + c_{1;j} I_{1;j} + \mathcal{R} + \mathcal{O}\left(d-4\right) \tag{449}$$

where the coefficients $c_{N;j}$ with $(N = 1, ...4)$ are evaluated in $d = 4$, namely they do not have any dependence on ε. In addition $I_{L;j}$ stands for an L-point one-loop scalar integral of type j, specifying the combination of the external momenta p_i which built up the q_i. This kind of decomposition has its origin on simple Lorentz invariance which allows us to decompose a tensor integral to invariant form factors and on the four dimensional nature of space time which allows scalar higher point integrals to be reduced to sums of boxes.

Specifically the possible scalar integrals that may appear in (449) are tadpoles, bubbles, triangles and boxes. Their form is

$$I_1\left(m_1^2\right) = \frac{\mu^{d-4}}{i\pi^{d/2}r_\Gamma}\int\frac{d^dk}{d_1} \tag{450}$$

$$I_2\left(p_1^2;m_1^2,m_2^2\right) = \frac{\mu^{d-4}}{i\pi^{d/2}r_\Gamma}\int\frac{d^dk}{d_1d_2} \tag{451}$$

$$I_3\left(p_1^2,p_2^2,p_3^2;m_1^2,m_2^2,m_3^2\right) = \frac{\mu^{d-4}}{i\pi^{d/2}r_\Gamma}\int\frac{d^dk}{d_1d_2d_3} \tag{452}$$

$$I_4\left(p_1^2,p_2^2,p_3^2,p_4^2;s_{12},s_{23};m_1^2,m_2^2,m_3^2,m_4^2\right) = \frac{\mu^{d-4}}{i\pi^{d/2}r_\Gamma}\int\frac{d^dk}{d_1d_2d_3d_4} \tag{453}$$

where $d_i = (k+p_{i-1})^2 - m_i + i\varepsilon$, $q_n = \sum_1^n p_i$, $s_{ij} = (p_i+p_j)^2$ and $r_\Gamma = \Gamma^2(1-\varepsilon)\,\Gamma(1+\varepsilon)\,/\Gamma(1-2\varepsilon)$. Now, here we calculate one-loop Feynman diagrams which give rise to tensor integrals containing powers of the loop momentum in the numerator, as we can see in section 2. The calculation of these integrals is simple but quite messy, thus we should reduce them to the scalar integrals that we mentioned previously. Following [1] we define the scalars integrals A_0, B_0, C_0, D_0 which correspond to scalar tadpole, bubble, triangle and box integrals respectively. The exact form of these integrals reads

$$A_0(m_1) = \frac{1}{i\pi^{d/2}}\int d^dk\frac{1}{d_1} \tag{454}$$

$$B_0;B^\mu;B^{\mu\nu}(p_1,m_1,m_2) = \frac{1}{i\pi^{d/2}}\int d^dk\frac{1;k^\mu;k^\mu k^\nu}{d_1d_2} \tag{455}$$

$$C_0;C^\mu;C^{\mu\nu};C^{\mu\nu\alpha}(p_1,p_2,m_1,m_2,m_3) = \frac{1}{i\pi^{d/2}}\int d^dk\frac{1;k^\mu;k^\mu k^\nu;k^\mu k^\nu k^\alpha}{d_1d_2d_3} \tag{456}$$

$$D_0;D^\mu;D^{\mu\nu};D^{\mu\nu\alpha};D^{\mu\nu\alpha\beta}(p_1,p_2,p_3,m_1,m_2,m_3,m_4) = \frac{1}{i\pi^{d/2}}\int d^dk\frac{1;k^\mu;k^\mu k^\nu;k^\mu k^\nu k^\alpha;k^\mu k^\nu k^\alpha k^\beta}{d_1d_2d_3d_4} \tag{457}$$

where the denominators are given by $d_i = \left(k+\sum_{k=1}^{i-1}p_k\right)^2 - m_i + i\varepsilon$ with $, i = 1,..,4$. Now, the next step that we should follow in order to make this reduction to scalar integrals procedure easier, is that we should present the pattern which we use.

B.1 B_0 scalar integrals

A non-trivial example that we can give is that of the reduction of the rank-one and rank-two tensor bubble functions to scalar integrals. Thus form Lorentz invariance we have that

$$\begin{aligned} B^\mu &= p^\mu B_1 \\ B^{\mu\nu} &= g^{\mu\nu}B_{00} + p^\mu p^\nu B_{11} \end{aligned} \tag{458}$$

where we refer to the coefficients B_i, B_{00}, B_{11} as form factors. The exact dependence of these form factors on the appropriate Lorentz invariants, namely p^2, m_i^2, have been dropped for simplicity.

In order to express these form factors as scalars integrals we have first to perform two steps. The first one is to contract B^μ and $B^{\mu\nu}$ with respect to $p_{1\mu}$ and the second one is to contract $B^{\mu\nu}$ with the metric $g_{\mu\nu}$. Thus contracting with $p_{1\mu}$ we obtain that

$$p_\mu B^\mu = p \cdot p B_1. \tag{459}$$

No we have to calculate the lefthand side so we use the relation

$$k \cdot p_1 = \frac{1}{2}\left(f_1 + d_2 - d_1\right) \tag{460}$$

where we should define here the following useful relations

$$\begin{aligned} f_1 &= m_2^2 - m_1^2 - p_1^2 \\ \left\langle \frac{k^\mu}{d_1 d_2 d_3 \cdots} \right\rangle &= \frac{1}{i\pi^{d/2}} \int d^d k \frac{k^\mu}{d_1 d_2 d_3 \cdots} \end{aligned} \tag{461}$$

and we get that

$$\begin{aligned} p_\mu B^\mu &= \frac{1}{i\pi^{d/2}} \int d^d k \frac{p_\mu k^\mu}{d_1 d_2} = \frac{1}{i\pi^{d/2}} \int d^d k \frac{\frac{1}{2}\left(f_1 + d_2 - d_1\right)}{d_1 d_2} \\ &= \frac{1}{2}\left(f_1 B_0\left(1,2\right) + A_0\left(1\right) - A_0\left(2\right)\right) \end{aligned} \tag{462}$$

thus replacing this to equation (459) it reads

$$\begin{aligned} p \cdot p B_1 &= \frac{1}{2}\left(f_1 B_0\left(1,2\right) + A_0\left(1\right) - A_0\left(2\right)\right) \Leftrightarrow \\ B_1(p, m_1, m_2) &= \frac{1}{2p^2}\left(f_1 B_0\left(1,2\right) + A_0\left(1\right) - A_0\left(2\right)\right) \end{aligned} \tag{463}$$

Since we have fund the form factor B_1 now we contract with p_μ the $B^{\mu\nu}$ getting

$$p_\mu B^{\mu\nu} = p^\nu B_{00} + p^\nu p \cdot p B_{11} \tag{464}$$

and therefore the left hand side becomes

$$\begin{aligned} p_\mu B^{\mu\nu} &= \frac{1}{i\pi^{d/2}} \int d^d k \frac{p_\mu k^\mu k^\nu}{d_1 d_2} = \frac{1}{i\pi^{d/2}} \int d^d k \frac{\frac{1}{2}\left(f_1 + d_2 - d_1\right) k^\nu}{d_1 d_2} \\ &= \frac{p^\nu}{2}\left(f_1 B_1\left(1,2\right) + A_0\left(2\right)\right). \end{aligned} \tag{465}$$

Finally, if we put this relation into equation (464), then we will obtain that

$$\begin{aligned} p^{\nu}(p\cdot pB_{11}+B_{00}) &= \tfrac{p^{\nu}}{2}\left(f_1 B_1\left(1,2\right)+A_0\left(2\right)\right)\Leftrightarrow \\ B_{11}(p,m_1,m_2) &= \tfrac{1}{2p^2}\left(f_1 B_1\left(1,2\right)+A_0\left(2\right)-2B_{00}\right). \end{aligned} \tag{466}$$

Now, the second step that we should make, as we have already mentioned, is to contract with $g_{\mu\nu}$ the $B^{\mu\nu}$, which will give the following

$$\begin{aligned} g_{\mu\nu}B^{\mu\nu} &= \left\langle k^2\right\rangle = dB_{00}+p^2B_{11}\Leftrightarrow \\ \left\langle k^2-m_1^2\right\rangle &= dB_{00}+p^2B_{11}-m_1^2B_0(1,2)\Leftrightarrow \\ m_1^2B_0(1,2) &= dB_{00}+\frac{1}{2}\left(f_1 B_1\left(1,2\right)-A_0\left(2\right)\right)-B_{00}\Leftrightarrow \\ B_{00}(p,m_1,m_2) &= \frac{1}{2(d-1)}\Big(2m_1^2B_0(1,2)-f_1B_1\left(1,2\right)+A_0\left(2\right)\Big) \end{aligned} \tag{467}$$

where according to (454) and (455) we have that

$$\begin{aligned} A_0(2) &= \frac{1}{i\pi^{d/2}}\int d^dk\frac{1}{k^2-m_2^2} \\ g_{\mu\nu}B^{\mu\nu} &= \frac{1}{i\pi^{d/2}}\int d^dk\frac{g_{\mu\nu}k^{\mu}k^{\nu}}{d_1d_2} = \frac{1}{i\pi^{d/2}}\int d^dk\frac{k^2}{d_1d_2} \end{aligned}$$

respectively. Thus, until now we have presented the exact formula that we follow in order to reduce B^{μ} and $B^{\mu\nu}$ into scalar integrals.

Nevertheless, before we move on in this section the last step that is missing and it is very useful for our calculation, is to obtain an explicit general form of $B^{\mu\nu}$ and its contraction $g_{\mu\nu}B^{\mu\nu}$ as functions of the B_0's and A_0's. To do so, we expand $B^{\mu\nu}$ as in (458) and we use the relations (466) and (467) along with the relation $f=m_H^2-m_Z^2-p^2$ in order to obtain the following

$$
\begin{aligned}
B^{\mu\nu} &= g^{\mu\nu}B_{00} + p^{\mu}p^{\nu}B_{11} \Leftrightarrow \\
B^{\mu\nu}(p,m_Z,m_H) &= \left\{ -\frac{g^{\mu\nu}}{p^2}\left[m_H^4 + \left(m_Z^2 - p^2\right)^2 - 2m_H^2\left(m_Z^2 + p^2\right)\right]\right. \\
&+ \left.\frac{p^{\mu}p^{\nu}}{p^4}\left[-4m_Z^2p^2 + d\left(-m_H^2 + m_Z^2 + p^2\right)^2\right]\right\}\frac{B_0(p,m_Z,m_H)}{4(d-1)} \\
&+ \left\{-\frac{g^{\mu\nu}}{p^2}\left(m_Z^2 - m_H^2 - p^2\right) + \frac{p^{\mu}p^{\nu}}{p^4}\left[-4p^2 + d\left(3p^2 + m_Z^2 - m_H^2\right)\right]\right\}\frac{A_0\left(m_H\right)}{4(d-1)} \\
&+ \left\{-\frac{g^{\mu\nu}}{p^2}\left(m_H^2 - m_Z^2 - p^2\right) + \frac{p^{\mu}p^{\nu}}{p^4}d\left(m_H^2 - m_Z^2 - p^2\right)\right\}\frac{A_0\left(m_Z\right)}{4(d-1)} \qquad (468)
\end{aligned}
$$

and moreover, after contracting this relation with respect to $g_{\mu\nu}$ we get that

$$
g_{\mu\nu}B^{\mu\nu}(p,m_Z,m_H) = m_Z^2 B_0(p,m_Z,m_H) + A_0(m_H). \qquad (469)
$$

and if we do this with $p_\mu p_\nu$ we get that

$$
\begin{aligned}
p_\mu p_\nu B^{\mu\nu}(p,m_Z,m_H) &= \frac{m_H^2 - m_Z^2 - p^2}{4}A_0(m_Z) + \frac{m_Z^2 - m_H^2 + 3p^2}{4}A_0(m_H) \\
&+ \left(\frac{m_Z^4 + m_H^4 - m_Z^2m_H^2}{4} + \frac{p^2(m_Z^2 - m_H^2 + p^2)}{4}\right)B_0(p,m_Z,m_H)
\end{aligned} \qquad (470)
$$

which for equal masses $m_Z = m_H = m_a$, it reads

$$
p_\mu p_\nu B^{\mu\nu}(p,m_a,m_a) = \frac{p^4}{4}B_0(p,m_a,m_a) + \frac{p^2}{2}A_0(m_a)\,. \qquad (471)
$$

Moreover, if we had the relation $B_0(p,m_a,m_a)$ we just have to replace each mass with m_a obtaining

$$
\begin{aligned}
B^{\mu\nu}(p,m_a,m_a) &= \left\{-\frac{g^{\mu\nu}}{p^2}\left[p^4 - 4m_a^2p^2\right] + \frac{p^{\mu}p^{\nu}}{p^4}\left[-4m_a^2p^2 + dp^4\right]\right\}\frac{B_0(p^2,m_a,m_a)}{4(d-1)} \\
&+ \left\{g^{\mu\nu} + \frac{p^{\mu}p^{\nu}}{p^4}\left[-4p^2 + 3dp^2\right]\right\}\frac{A_0\left(m_a\right)}{4(d-1)} \\
&+ \left\{g^{\mu\nu} - \frac{dp^{\mu}p^{\nu}}{p^2}\right\}\frac{A_0\left(m_a\right)}{4(d-1)}
\end{aligned} \qquad (472)
$$

and thus, performing the contraction with the metric, we will obtain the following relation

$$g_{\mu\nu}B^{\mu\nu}(p, m_a, m_a) = m_a^2 B_0(p, m_a, m_a) + A_0(m_a). \tag{473}$$

B.2 C_0 scalar integrals

Now we move on to another useful example of this formula is the reduction of the triangle functions to scalar integrals. This will be sufficient to illustrate the general pattern of the reduction method that we follow through out this work, in order to simplify our results. Thus from Lorentz invariance we have that

$$C^\mu = p_1^\mu C_1 + p_2^\mu C_2 \tag{474}$$

$$C^{\mu\nu} = g^{\mu\nu}C_{00} + \sum_{i,j=1}^{2} p_i^\mu p_j^\nu C_{ij} \tag{475}$$

where $C_{12} = C_{21}$ and we define the coefficients C_i, C_{00}, C_{ij}, $i, j = 1, 2$ as form factors. Of course these coefficients have an appropriate dependance on specific Lorentz invariants,namely p_1^2, p_2^2, $(p_1 + p_2)^2$, m_1^2, m_2^2 and m_3^2 which for now we have suppressed. Therefore, contracting (474) with p_1 and p_2 we get that

$$p_{1\mu}C^\mu = p_1 \cdot p_1 C_1 + p_1 \cdot p_2 C_2 \tag{476}$$

$$p_{2\mu}C^\mu = p_2 \cdot p_1 C_1 + p_2 \cdot p_2 C_2. \tag{477}$$

The numerator in the left hand side of the previous equation can be expressed according to the following relations

$$k \cdot p_1 = \frac{1}{2}(f_1 + d_2 - d_1) \tag{478}$$

$$k \cdot p_2 = \frac{1}{2}(f_2 + d_3 - d_2) \tag{479}$$

where $f_1 = m_2^2 - m_1^2 - p_1^2$ and $f_2 = m_3^2 - m_2^2 - p_2^2 - 2p_1 \cdot p_2$. With that in mind and using the second case of the terms (456) the left hand side of (476) and (477) obtain a specific form. Particularly the first term reads

$$p_{1\mu}C^{\mu} = \frac{1}{i\pi^{d/2}}\int d^dk\frac{p_{1\mu}k^{\mu}}{d_1d_2d_3} = \frac{1}{i\pi^{d/2}}\int d^dk\frac{\frac{1}{2}\left(f_1+d_2-d_1\right)}{d_1d_2d_3} \tag{480}$$

$$= \frac{1}{2}\left(f_1C_0\left(1,2,3\right)+B_0\left(1,3\right)-B_0\left(2,3\right)\right) \tag{481}$$

and the second

$$p_{2\mu}C^{\mu} = \frac{1}{i\pi^{d/2}}\int d^dk\frac{p_{2\mu}k^{\mu}}{d_1d_2d_3} = \frac{1}{i\pi^{d/2}}\int d^dk\frac{\frac{1}{2}\left(f_2+d_3-d_2\right)}{d_1d_2d_3} \tag{482}$$

$$= \frac{1}{2}\left(f_2C_0\left(1,2,3\right)+B_0\left(1,2\right)-B_0\left(1,3\right)\right). \tag{483}$$

Thus, we can make a system of equations for the coefficients C_1, C_2 of the form

$$G_2\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} \langle k\cdot p_1\rangle \\ \langle k\cdot p_2\rangle \end{pmatrix} = \begin{pmatrix} R_1^{[c]} \\ R_2^{[c]} \end{pmatrix} \tag{484}$$

where the G_2 is a 2×2 Gram matrix which reads

$$G_2 = \begin{pmatrix} p_1\cdot p_1 & p_1\cdot p_2 \\ p_1\cdot p_2 & p_2\cdot p_2 \end{pmatrix}$$

and we have used the notation

$$\langle k\cdot p_i\rangle = \frac{1}{i\pi^{d/2}}\int d^dk\frac{p_i\cdot k}{d_1d_2d_3}$$

for $i=1,2$. Thus, from (484) we can define $R_{1,2}^{[c]}$ as follows

$$R_1^{[c]} = \frac{1}{2}\left(f_1C_0\left(1,2,3\right)+B_0\left(1,3\right)-B_0\left(2,3\right)\right) \tag{485}$$

$$R_2^{[c]} = \frac{1}{2}\left(f_2C_0\left(1,2,3\right)+B_0\left(1,2\right)-B_0\left(1,3\right)\right) \tag{486}$$

where we have used a compact notation which labels the form factors according to the denominators that they have, namely we get that

$$B_0\left(2,3\right)\equiv B_0\left(p_1,p_2,m_2,m_3\right) = \int\frac{d^dk}{i\pi^{d/2}}\frac{1}{\left(k^2-m_2^2\right)\left(\left(k+p_2\right)^2-m_3^2\right)}. \tag{487}$$

where the loop momentum k has been shifted with respect to the defining equation for the triangle integrals since d_1 has been cancelled.

Finally, in order to solve the system of the equations that we have obtained, we should invert G_2 which reads

$$G_2^{-1} = \frac{1}{\det G_2} \begin{pmatrix} p_2 \cdot p_2 & -p_1 \cdot p_2 \\ -p_1 \cdot p_2 & p_1 \cdot p_1 \end{pmatrix}$$

so we get that our system becomes

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = G_2^{-1} \begin{pmatrix} R_1^{[c]} \\ R_2^{[c]} \end{pmatrix} \tag{488}$$

which gives the coefficients C_1 and C_2. For this example we get that

$$C_1(p_1, p_2, m_1, m_2, m_3) = \frac{1}{\det G_2} \left(p_2 \cdot p_2 R_1^{[c]} - p_1 \cdot p_2 R_2^{[c]} \right) \tag{489}$$

$$C_2(p_1, p_2, m_1, m_2, m_3) = \frac{1}{\det G_2} \left(-p_1 \cdot p_2 R_1^{[c]} + p_1 \cdot p_1 R_2^{[c]} \right) \tag{490}$$

where, as we have already mentioned, $R_1^{[c]}$ and $R_2^{[c]}$ are given from equation (485) and (486) respectively.

Now, we move on to the next case which refers to the equation (475). Here we contract this relation with p_1 and p_2 in order to define C_{11} and C_{12}. Moreover we want to find the exact form of the C_{00} so we should multiply from left with the metric $g_{\mu\nu}$, which in our case reads $g_{\mu\nu} = diag[1, -1, -1, -1]$. Thus, the first step in this calculation gives

$$p_{1\mu} C^{\mu\nu} = p_1^\nu \left(p_1 \cdot p_1 C_{11} + p_1 \cdot p_2 C_{12} + C_{00} \right) + p_2^\nu \left(p_1 \cdot p_1 C_{12} + p_1 \cdot p_2 C_{22} \right)$$

$$p_{2\mu} C^{\mu\nu} = p_1^\nu \left(p_1 \cdot p_2 C_{11} + p_2 \cdot p_2 C_{12} \right) + p_2^\nu \left(p_1 \cdot p_2 C_{12} + p_2 \cdot p_2 C_{22} + C_{00} \right). \tag{491}$$

As we can see from the above equation along with the obtained relations of $k \cdot p_1$ and $k \cdot p_2$, here we have to solve two systems independently, namely we get that

$$G_2 \begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} = \begin{pmatrix} R_1^{[c_1]} \\ R_2^{[c_1]} \end{pmatrix} \qquad G_2 \begin{pmatrix} C_{12} \\ C_{22} \end{pmatrix} = \begin{pmatrix} R_1^{[c_2]} \\ R_2^{[c_2]} \end{pmatrix}. \tag{492}$$

Now we have to define the terms $R_1^{[c_1]}$, $R_2^{[c_1]}$, $R_1^{[c_2]}$ and $R_2^{[c_2]}$, which could be done by calculating the lefthand side of the (491). Thus concerning the first case we have that

$$\begin{aligned} p_{1\mu}C^{\mu\nu} &= \frac{1}{i\pi^{d/2}}\int d^dk\frac{p_{1\mu}k^\mu k^\nu}{d_1d_2d_3} = \frac{1}{2}\left(f_1+d_2-d_1\right)C^\nu \\ &= \frac{p_1^\nu}{2}\left(f_1C_1\left(1,2,3\right)+B_1\left(1,3\right)-B_1\left(2,3\right)-2C_{00}\left(1,2,3\right)\right) \\ &+ \frac{p_2^\nu}{2}\left(f_1C_2\left(1,2,3\right)+B_1\left(1,3\right)-B_1\left(2,3\right)\right) \end{aligned} \tag{493}$$

and for the second case we obtain

$$\begin{aligned} p_{2\mu}C^{\mu\nu}(p_1,p_2,m_1,m_2,m_3) &= \frac{1}{i\pi^{d/2}}\int d^dk\frac{p_{2\mu}k^\mu k^\nu}{d_1d_2d_3} = \frac{1}{2}\left(f_2+d_3-d_2\right)C^\nu \\ &= \frac{p_1^\nu}{2}\left(f_2C_1\left(1,2,3\right)+B_1\left(1,2\right)-B_1\left(1,3\right)\right) \\ &+ \frac{p_2^\nu}{2}\left(f_2C_2\left(1,2,3\right)+B_1\left(1,2\right)-B_1\left(1,3\right)-2C_{00}\left(1,2,3\right)\right) \end{aligned} \tag{494}$$

thus we get the desired relations

$$R_1^{[c_1]} = \frac{1}{2}\left(f_1C_1\left(1,2,3\right)+B_1\left(1,3\right)+B_0\left(2,3\right)-2C_{00}\left(1,2,3\right)\right) \tag{495}$$

$$R_2^{[c_1]} = \frac{1}{2}\left(f_2C_1\left(1,2,3\right)+B_1\left(1,2\right)-B_1\left(1,3\right)\right) \tag{496}$$

and

$$R_1^{[c_2]} = \frac{1}{2}\left(f_1C_2\left(1,2,3\right)+B_1\left(1,3\right)-B_1\left(2,3\right)\right) \tag{497}$$

$$R_2^{[c_2]} = \frac{1}{2}\left(f_2C_2\left(1,2,3\right)-B_1\left(1,3\right)-2C_{00}\left(1,2,3\right)\right). \tag{498}$$

Finally, contracting (475) with the metric we obtain that in one hand

$$\begin{aligned} g_{\mu\nu}C^{\mu\nu} &= \left\langle k^2\right\rangle \Leftrightarrow \\ g_{\mu\nu}C^{\mu\nu} &= \left\langle k^2-m_1^2\right\rangle+\left\langle m_1^2\right\rangle \Leftrightarrow \\ g_{\mu\nu}C^{\mu\nu} &= m_1^2C_0(1,2,3)+B_0\left(2,3\right) \end{aligned} \tag{499}$$

where we have used the first case of (456) and the equation (487), and in the other hand that

$$g_{\mu\nu}C^{\mu\nu} = dC_{00} + \frac{1}{2}\left(f_1 C_1\left(1,2,3\right) + B_1\left(1,2\right) + B_0\left(2,3\right)\right) - C_{00} + \frac{1}{2}\left(f_2 C_2\left(1,2,3\right) - B_1\left(1,2\right)\right) - C_{00} - m_1^2 C_0 \tag{500}$$

thus combining this two relations we get that

$$C_{00}(p_1, p_2, m_1, m_2, m_3) = \frac{1}{2\left(d-2\right)}\Big(2m_1^2 C_0 - f_2 C_2\left(1,2,3\right) - f_1 C_1\left(1,2,3\right) + B_0\left(2,3\right)\Big). \tag{501}$$

Thus, as follows from the analysis that has been done here, we note that the coefficients which we face through the diagram calculation, follow a special pattern which reads

$$\begin{aligned} C_{ij} &\rightarrow C_{00}, C_i, B_i, (B_0) \\ C_{00} &\rightarrow C_i, (C_0, B_0) \\ C_i &\rightarrow (C_0, B_0)\,. \end{aligned}$$

Similar patterns with the previous one govern the B, as we have already seen, and the D integrals, making our life easier and demanding from us to calculate only the corresponding scalar integrals B_0 , C_0 and D_0.

B.3 D_0 scalar integrals

Until now, we have presented specific examples explaining the procedure that we follow through the loop-integrals calculation referring to the bubble and triangular integral cases. Nevertheless, in our calculation we face also box integrals where we follow the same reasoning as we described here. Thus in the following we present the needed relations that we deal with in 2.

So, we start here by presenting the coefficients that we deal with in the case of the box integrals, specifically here we have that

$$D^{\mu} = p_1^{\mu} D_1 + p_2^{\mu} D_2 + p_3^{\mu} D_3 \tag{502}$$

$$D^{\mu\nu} = g^{\mu\nu} D_{00} + \sum_{i,j=1}^{3} p_i^{\mu} p_j^{\nu} D_{ij} \tag{503}$$

$$D^{\mu\nu\alpha} = \sum_{i=1}^{3} g^{\{\mu\nu} p_i^{\alpha\}} D_{00i} + \sum_{i,j,k=1}^{3} p_i^{\mu} p_j^{\nu} p_k^{\alpha} D_{ijk} \tag{504}$$

$$D^{\mu\nu\alpha\beta} = g^{\{\mu\nu} g^{\alpha\beta\}} D_{0000} + \sum_{i,j=1}^{3} g^{\{\mu\nu} p_i^{\alpha} p_j^{\beta\}} D_{00ij} + \sum_{i,j,k,l=1}^{3} p_i^{\mu} p_j^{\nu} p_k^{\alpha} p_l^{\beta} D_{ijkl} \tag{505}$$

where the curly braces denote fully symmetrization of the indices. The main result that we demonstrated before and we use here in order to reduce the box integrals to scalar ones is that many of mentioned coefficients satisfy equations like

$$\begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} = G_3^{-1} \begin{pmatrix} R_1^{[d]} \\ R_2^{[d]} \\ R_3^{[d]} \end{pmatrix} \tag{506}$$

where G_3 is the 3×3 Gram matrix whose definition reads

$$G_3 = \begin{pmatrix} p_1 \cdot p_1 & p_1 \cdot p_2 & p_1 \cdot p_3 \\ p_2 \cdot p_1 & p_2 \cdot p_2 & p_2 \cdot p_3 \\ p_3 \cdot p_1 & p_3 \cdot p_2 & p_3 \cdot p_3 \end{pmatrix}. \tag{507}$$

The interesting feature of this procedure is that all the form factor triplets which satisfy equations of the form of equation (506) can be defined through the reasoning that we have followed till now corresponding to the bubble and triangle integrals. Now we demonstrate a short list with the needed relations that govern the form factors of the box integrals. Therefore we give the Table which contains these relations, which reads

Form Factors			
			RHS
D_1	D_2	D_3	R^d
D_{11}	D_{12}	D_{13}	R^{d1}
D_{21}	D_{22}	D_{23}	R^{d2}
D_{31}	D_{23}	D_{33}	R^{d3}
D_{001}	D_{002}	D_{003}	R^{d00}
D_{112}	D_{122}	D_{123}	R^{d12}
D_{113}	D_{123}	D_{133}	R^{d13}
D_{123}	D_{223}	D_{233}	R^{d23}
D_{111}	D_{112}	D_{113}	R^{d11}
D_{122}	D_{222}	D_{223}	R^{d22}
D_{133}	D_{233}	D_{333}	R^{d33}
D_{0011}	D_{0012}	D_{0013}	R^{d001}
D_{0012}	D_{0022}	D_{0023}	R^{d002}
D_{0013}	D_{0023}	D_{0033}	R^{d003}
D_{1111}	D_{1112}	D_{1113}	R^{d111}
D_{1222}	D_{2222}	D_{2223}	R^{d222}
D_{1333}	D_{2333}	D_{3333}	R^{d333}
D_{1112}	D_{1122}	D_{1123}	R^{d112}
D_{1113}	D_{1123}	D_{1133}	R^{d113}
D_{1122}	D_{1222}	D_{1223}	R^{d122}
D_{1133}	D_{1233}	D_{1333}	R^{d133}
D_{1223}	D_{2223}	D_{2233}	R^{d223}
D_{1233}	D_{2233}	D_{2333}	R^{d233}
D_{1123}	D_{1223}	D_{1233}	R^{d123}

C Integrals in d-Dimensions

In the present work we adopted the formalism explained in the previous section, coming from [1], in order to calculate the emergent integrals coming from the loop Feynman diagrams. However, we face situations where these integrals do not correspond to any relation familiar with what we have presented until now. Thus these cases, which we refer to as the U_i integrals, are treated separately. To be more specific, when we deal with them we use the traditional calculation procedure as could be seen from section 2. Therefore, here we present the basic tools that we use in order to perform that kind of calculation.

C.1 Feynman Parameterization

The first step is to demonstrate the way that the products of the denominators can be rewritten according to the Feynman parameterization, namely the general case is

$$\frac{1}{A_1 A_2 \cdots A_n} = (n-1)! \int_0^1 dx_1 \int_0^1 dx_2 \cdots \int_0^1 dx_n \delta\left(1 - \sum_n x_n\right) \times \frac{1}{[A_1 x_1 + A_2 x_2 + \cdots + A_n x_n]^n}. \tag{508}$$

In the case that we have $n = 2$ and the denominators are in first power then we take

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{[A + (B - A)\, x]^2} \tag{509}$$

but if the denominators are raised to a general power in the $n = 2$ case then we obtain

$$\frac{1}{A^a B^b} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 dx \frac{x^{\alpha-1}(1-x)^{\beta-1}}{[xA + (1-x)\, B]^{\alpha+\beta}}. \tag{510}$$

As a finall example we give the $n = 3$ case which reads

$$\frac{1}{ABC} = 2 \int_0^1 dx \int_0^1 dy \frac{y}{[xyA + (1-x)\, yB + (1-y)\, C]^3} \tag{511}$$

C.2 d-Dimensional Integrals with Gamma Functions

Now that we have demonstrated the Feynman parameterization we move on to the next step that we follow when we perform the traditional loop diagram calculation. Specifically, we refer to d-dimensional integrals which occur from the transormation of the 4-dimensional ones when we perform a specific variable changing. Generally each one of the resulting integrals has its own mass dimension, but here we would like to obtain dimensonless relations thus we multiply them with the factor $(m_a^2)^x$. The resulting integrals read

$$J_0(n, \Delta) = (m_a^2)^{n-2} \int \frac{d^d k}{(2\pi)^d} \frac{-i}{(k^2 - \Delta)^n} = i(m_a^2)^{n-2} \frac{(-1)^n}{(4\pi)^{d/2}} \frac{\Gamma\left(n - \frac{d}{2}\right)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-d/2} \tag{512}$$

$$g_{\mu\nu}U^{\mu\nu}_{\mathcal{T}}(n,\Delta)=g_{\mu\nu}J^{\mu\nu}=(m_a^2)^{n-3}\int\frac{d^dk}{(2\pi)^d}\frac{-ik^2}{(k^2-\Delta)^n}=-i\frac{d}{2}(m_a^2)^{n-3}\frac{(-1)^n}{(4\pi)^{d/2}}\frac{\Gamma\left(n-1-\frac{d}{2}\right)}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-1-d/2} \tag{513}$$

$$J^{\mu\nu}(n,\Delta)=U^{\mu\nu}_{\mathcal{T}}(n,\Delta)=(m_a^2)^{n-3}\int\frac{d^dk}{(2\pi)^d}\frac{-ik^\mu k^\nu}{(k^2-\Delta)^n}=-\frac{i}{2}(m_a^2)^{n-3}g^{\mu\nu}\frac{(-1)^n}{(4\pi)^{d/2}}\frac{\Gamma\left(n-1-\frac{d}{2}\right)}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-1-d/2} \tag{514}$$

$$g_{\mu\nu}g_{\rho\sigma}J^{\mu\nu\rho\sigma}(n,\Delta)=(m_a^2)^{n-4}\int\frac{d^dk}{(2\pi)^d}\frac{-i\left(k^2\right)^2}{(k^2-\Delta)^n}=i(m_a^2)^{n-4}\frac{d\left(d+2\right)}{4}\frac{(-1)^n}{(4\pi)^{d/2}}\frac{\Gamma\left(n-2-\frac{d}{2}\right)}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-2-d/2} \tag{515}$$

$$\begin{aligned}J^{\mu\nu\rho\sigma}(n,\Delta)=(m_a^2)^{n-4}\int\frac{d^dk}{(2\pi)^d}\frac{-ik^\mu k^\nu k^\rho k^\sigma}{(k^2-\Delta)^n}=&i(m_a^2)^{n-4}\frac{(-1)^n}{(4\pi)^{d/2}}\frac{\Gamma\left(n-2-\frac{d}{2}\right)}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-2-d/2}\\&\times\frac{1}{4}\left(g^{\mu\nu}g^{\rho\sigma}+g^{\mu\rho}g^{\sigma\nu}+g^{\mu\sigma}g^{\rho\nu}\right)\end{aligned} \tag{516}$$

where we have used the demand that

$$\begin{aligned}4-2n+2x=0\Leftrightarrow x=n-2\\4+2-2n+2x=0\Leftrightarrow x=n-3\\4+4-2n+2x=0\Leftrightarrow x=n-4\end{aligned}$$

for equations (512), (513) and (515) respectively. Thus the final result of these d-dimensional integrals remains dimensionless.
Next for symmetry reasons we demand that the odd powers of the k^μ in the numerator should vanish, namely

$$\int\frac{d^dk}{(2\pi)^d}k^\mu f(k^2)=0. \tag{517}$$

Before we move on to the final part of this subsection and since we have defined $B^{\mu\nu}$, B_0 and A_0 we would like to obtain a relation between the above integrals and these quantities because it would be very interesting for our calculation. Therefore starting from equation (513) for $n=1$ and $\Delta=m$, and defining that

$$\left\langle\frac{k^2}{k^2-m^2}\right\rangle=\int\frac{d^dk}{i\left(2\pi\right)^d}\frac{k^2}{k^2-m^2} \tag{518}$$

we can write the following relations

$$
\begin{aligned}
U_{\mathcal{T}}^{\mu\nu}(1,m) &= \frac{1}{m^4}\left\langle \frac{k^\mu k^\nu}{k^2-m^2}\right\rangle = \frac{1}{m^4}\frac{g_{\mu\nu}}{d}\left\langle \frac{k^2}{k^2-m^2}\right\rangle \Leftrightarrow \\
g_{\mu\nu}U_{\mathcal{T}}^{\mu\nu}(1,m) &= \frac{d+\varepsilon}{m^4 d}\left\langle \frac{k^2}{k^2-m^2}\right\rangle \Leftrightarrow \\
g_{\mu\nu}U_{\mathcal{T}}^{\mu\nu}(1,m) &= \frac{d+\varepsilon}{m^2 d}\left\langle \frac{1}{k^2-m^2}\right\rangle + \frac{d+\varepsilon}{m^4 d}\left\langle \frac{k^2-m^2}{k^2-m^2}\right\rangle
\end{aligned} \tag{519}
$$

and now, since we know that the volume integral in dimensional regularization is zero and the scalar integral A_0 is written as

$$
A_0(m) = m^2\left[\frac{2}{\varepsilon} + \ln\frac{\mu^2}{m^2} + 1\right]
$$

we obtain that Eq.(519) reads

$$
g_{\mu\nu}U_{\mathcal{T}}^{\mu\nu}(1,m) = \frac{1}{m^2}A_0(m) + \frac{1}{2} \tag{520}
$$

An other relation occurs if we start from (455) which gives

$$
\begin{aligned}
B^{\mu\nu} &= -i(2\sqrt{\pi})^d \int \frac{d^d k}{(2\pi)^d}\frac{k^\mu k^\nu}{d_1 d_2} \Leftrightarrow \\
g_{\mu\nu}B^{\mu\nu} &= -i(2\sqrt{\pi})^d \int \frac{d^d k}{(2\pi)^d}\frac{k^2}{d_1 d_2}.
\end{aligned}
$$

Now according to the traditional calculation of these integrals, in order to calculate them we should find the Feynman parameters. Thus for the $n = 2$ case we get from Appendix D that our relation reads

$$
g_{\mu\nu}B^{\mu\nu} = -i(2\sqrt{\pi})^d \int_0^1 dx \int \frac{d^d k}{(2\pi)^d}\frac{k^2 + p^2(x-1)^2}{(k^2-\Delta)^2}
$$

where we have defined that $\Delta = -p^2 x(1-x) + m_a^2$ and we have made the shift $k \to k + p(x-1)$. Then using the relations from (512) and (513) we get that

$$
\begin{aligned}
g_{\mu\nu}B^{\mu\nu} &= -i(2\sqrt{\pi})^d \int_0^1 dx \int \frac{d^d k}{(2\pi)^d}\frac{k^2 + p^2(x-1)^2}{(k^2-\Delta)^2} \Leftrightarrow \\
g_{\mu\nu}B^{\mu\nu} &= (2\sqrt{\pi})^d \int_0^1 dx \left(-m_a^2 g_{\mu\nu}J^{\mu\nu}(2,\Delta) + p^2(x-1)^2 J_0(2,\Delta)\right).
\end{aligned} \tag{521}
$$

Now, as we know from equation (458), we have that

$$g_{\mu\nu} B^{\mu\nu} = g_{\mu\nu} \left(g^{\mu\nu} B_{00} + p^\mu p^\nu B_{11} \right)$$

so replacing this relation into (521) we obtain that

$$\begin{aligned} g_{\mu\nu} \left(g^{\mu\nu} B_{00} + p^\mu p^\nu B_{11} \right) &= -g_{\mu\nu} (2\sqrt{\pi})^d m_a^2 \int_0^1 dx J^{\mu\nu}(2,\Delta) + g_{\mu\nu} (2\sqrt{\pi})^d \int_0^1 dx p^\mu p^\nu (x-1)^2 J_0(2,\Delta) \Leftrightarrow \\ g^{\mu\nu} B_{00} + p^\mu p^\nu B_{11} &= -(2\sqrt{\pi})^d m_a^2 \int_0^1 dx J^{\mu\nu}(2,\Delta) + p^\mu p^\nu (2\sqrt{\pi})^d \int_0^1 dx (x-1)^2 J_0(2,\Delta) \Leftrightarrow \end{aligned} \tag{522}$$

and therefore comparing the lefthand side with righthand side we get that

$$\begin{aligned} (2\sqrt{\pi})^d \int_0^1 dx (x-1)^2 J_0(2,\Delta) &= B_{11} \Leftrightarrow \\ \int_0^1 dx (x-1)^2 J_0(2,\Delta) &= \frac{1}{(2\sqrt{\pi})^d} B_{11} \Leftrightarrow \\ \int_0^1 dx (x-1)^2 J_0(2,\Delta) &= \frac{1}{2(2\sqrt{\pi})^d p^2} \left[\left(\frac{dp^2 - 4m_a^2}{2(d-1)} \right) B_0\left(p^2, m_a, m_a\right) + \frac{d}{d-1} A_0\left(m_a^2\right) \right] \end{aligned} \tag{523}$$

and

$$\begin{aligned} -(2\sqrt{\pi})^d m_a^2 \int_0^1 dx J^{\mu\nu}(2,\Delta) &= g^{\mu\nu} B_{00} \Leftrightarrow \\ \int_0^1 dx J^{\mu\nu}(2,\Delta) &= -\frac{1}{m_a^2 (2\sqrt{\pi})^d} g^{\mu\nu} B_{00} \Leftrightarrow \\ \int_0^1 dx J^{\mu\nu}(2,\Delta) &= -\frac{g^{\mu\nu}}{2m_a^2 (2\sqrt{\pi})^d (d-1)} \left[\left(2m_a^2 - \frac{p^2}{2} \right) B_0\left(p^2, m_a, m_a\right) - A_0\left(m_a^2\right) \right] \end{aligned} \tag{524}$$

where we have expanded B_{11} and B_{00} according to the equations (466) and (467) respectively. Thus we have obtained a specific relation between the $J^{\mu\nu}$ integrals and the scalar integrals B_0 and A_0.
An other relation that can be produced is that between the $J^{\mu\nu\rho\sigma}$ and $B^{\mu\nu}$ when we have the $n = 2$ case, which could be useful for our calculation. In particular we have from (516) that

$$
\begin{aligned}
iJ^{\mu\nu\rho\sigma}(2,\Delta) &= \frac{1}{m_a^4}\int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu k^\rho k^\sigma}{(k^2-\Delta)^2} \Leftrightarrow \\
ip_\rho p_\sigma J^{\mu\nu\rho\sigma}(2,\Delta) &= \frac{1}{m_a^4}\int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu (p\cdot k)(p\cdot k)}{(k^2-\Delta)^2} \Leftrightarrow \\
ip_\rho p_\sigma J^{\mu\nu\rho\sigma}(2,\Delta) &= \frac{1}{m_a^4}\int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu \frac{1}{4}(f_1+d_2-d_1)^2}{(k^2-\Delta)^2} \Leftrightarrow \\
ip_\rho p_\sigma J^{\mu\nu\rho\sigma}(2,\Delta) &= \frac{1}{4m_a^4}\int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu [f_1^2+f_1(d_2-d_1)+(d_2-d_1)^2]}{(k^2-\Delta)^2}
\end{aligned} \tag{525}
$$

where we have used the relation (460). Now, from the relation (455) we get that

$$
\begin{aligned}
B^{\mu\nu} &= -i(2\sqrt{\pi})^d \int_0^1 dx \int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu + p^\mu p^\nu (x-1)^2}{(k^2-\Delta)^2} \Leftrightarrow \\
B^{\mu\nu} &= -i(2\sqrt{\pi})^d \int_0^1 dx \int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu}{(k^2-\Delta)^2} - i(2\sqrt{\pi})^d \int_0^1 dx p^\mu p^\nu (x-1)^2 J_0(2,\Delta)
\end{aligned} \tag{526}
$$

so integrating (525) with respect to x and combining it with (526) we get that

$$
\begin{aligned}
\int_0^1 dx p_\rho p_\sigma J^{\mu\nu\rho\sigma}(2,\Delta) &= \frac{1}{4(2\sqrt{\pi})^d m_a^4}B^{\mu\nu} - \frac{1}{4m_a^4}\int_0^1 dx p^\mu p^\nu (x-1)^2 J_0(2,\Delta) \\
&- i\frac{1}{4m_a^4}\int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu [f_1(d_2-d_1)+(d_2-d_1)^2]}{(k^2-\Delta)^2} \Leftrightarrow \\
\int_0^1 dx p_\rho p_\sigma J^{\mu\nu\rho\sigma}(2,\Delta) &= \frac{1}{4(2\sqrt{\pi})^d m_a^4}B^{\mu\nu}(p^2,m_a,m_b) \\
&- \frac{1}{8(2\sqrt{\pi})^d m_a^6 p^2}\left[\left(\frac{dp^2-4m_a^2}{2\,(d-1)}\right)B_0\left(p^2,m_a,m_b\right) + \frac{d}{d-1}A_0\left(m_a^2\right)\right] \\
&- i\frac{1}{4m_a^4}\int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu [f_1(d_2-d_1)+(d_2-d_1)^2]}{(k^2-\Delta)^2}
\end{aligned} \tag{527}
$$

which gives a specific relation between the quantities $J^{\mu\nu\rho\sigma}$ and $B^{\mu\nu}$.

Now lets forget about the above calculations and move on to the next subject of this Appendix. In particular, we present the necessary steps that should be followed, concerning the dimensional expansion, in order to use properly the DR procedure. Therefore, the terms $\Delta^{-\varepsilon}$, μ^ε and $(4\pi)^\varepsilon$ would be expanded as follows

$$\begin{aligned}
\Delta^{-\varepsilon} &= 1-\varepsilon\ln\Delta+\mathcal{O}\left(\varepsilon^2\right)\\
\mu^{\varepsilon} &= 1+\varepsilon\ln\mu+\mathcal{O}\left(\varepsilon^2\right)\\
(4\pi)^{\varepsilon} &= 1+\varepsilon\ln 4\pi+\mathcal{O}\left(\varepsilon^2\right)
\end{aligned} \tag{528}$$

where we have used the general form

$$\left(\frac{1}{\Delta}\right)^{n-d/2}=\left(\frac{1}{\Delta}\right)^{n-2}\cdot\left[1-\left(2-\frac{d}{2}\right)\ln\Delta+\mathcal{O}\left(\left(2-\frac{d}{2}\right)^2\right)\right].$$

Moreover we should deal with the Gamma function $\Gamma(x)$. Here we deal with two cases where the first one is when Gamma function has integer arguments, which gives $\Gamma(1)=1$, $\Gamma(2)=1$, $\Gamma(3)=2$, $\Gamma(x)=(x-1)!$. The second one occurs when the argument of the Gamma function is zero or negative integer, where it has a pole[1]. So if we expand the argument around the two kind of poles we get that

$$\begin{aligned}
\Gamma(\varepsilon) &= \frac{1}{\varepsilon}-\gamma_E+\mathcal{O}(\varepsilon)+\cdots\\
\Gamma\left(-n+\frac{\varepsilon}{2}\right) &= \frac{(-1)^n}{n!}\left[\frac{2}{\varepsilon}+\psi(n+1)+\mathcal{O}(\varepsilon)\right]
\end{aligned}$$

respectively, where $\psi(z+1)=\psi(z)+\frac{1}{z}$, $\psi(1)=-\gamma_E$ and γ_E is the Euler-Mascheroni constant defined as $\gamma_E\approx 0.577$.

D Calculation of the U-integrals

In this part of the Appendices we present two different methods in order to calculate several integrals that we face through out our calculation. Generally this integrals could be either identified with the cases contained in Appendix B or not. In particular, here we deal with a specific form of highly divergent integrals, namely the U_i's, which will evaluated by using the reduction formulas from Appendix B and explicitly. Of course the straightforward calculation of these integrals is the explicit one, but here, we use the scalar integrals coming from the Veltman - Passarino reduction formula to compactify our results. It is very interesting to see if these two different procedures are essentially equivalent, containing the same physical information and leading to the same physical results.

[1] A short notation that we have to mention is that when we perform the expansion around $d=2-\varepsilon$ we do not have negative integers as Gamma function pole.

D.1 U_i's as scalar integrals

Lets start the calculation of the U-integrals with the first case that we mentioned before, namely using the scalar integrals and their relations. Thus we start with the dimensionless integral that we face for the first time at the calculation of the one-loop two-point functions of the Higgs boson in section 2, which reads

$$\begin{aligned}
U_{\mathcal{M}4}(p, m_a, m_a) &= \int \frac{d^4k}{(2\pi)^4} \frac{ik^4}{m_a^4\,(k^2 - m_a^2)\left((k+p)^2 - m_a^2\right)} \Leftrightarrow \\
U_{\mathcal{M}4}(p, m_a, m_a) &= \left\langle \frac{k^4}{m_a^4\,(k^2 - m_a^2)\left((k+p)^2 - m_a^2\right)} \right\rangle
\end{aligned} \tag{529}$$

where we used the second case of relation (461). Generally, the evaluation of integrals like this one or like the upcoming ones which are more and more divergent, could be done using several mathematical tricks. A common one, which is mainly used, refers to specific shifts of the integrating momentum. In particular, someone could perform shifts of the integrating constant momenta contained in the denominator of the integrals under consideration. This can reduce the order of divergence but this is done in an inappropriate way, since it adds extra infinite integrals which give wrong physical quantities.
Therefore, the way that we choose to consider so as to lower the divergence of our integrals, namely of the U-integrals, is quite different. Specifically we use the terms in the integral's numerator in order to construct the corresponding denominators. This reduces the divergence without inserting extra infinities and seems to work correctly. Nevertheless, for completeness, after evaluating $U_{\mathcal{M}4}$ we present the same calculation using the shift-procedure in order to highlight the differences that appear even in the simplest case of the U's. Thus we start the calculation-procedure as follows

$$\begin{aligned}
U_{\mathcal{M}4}(p, m_a, m_a) &= \left\langle \frac{k^4}{m_a^4\,(k^2 - m_a^2)\left((k+p)^2 - m_a^2\right)} \right\rangle \\
&= \left\langle \frac{(k^2 - m_a^2)k^2}{m_a^4\,(k^2 - m_a^2)\left((k+p)^2 - m_a^2\right)} \right\rangle + \left\langle \frac{k^2 m_a^2}{m_a^4\,(k^2 - m_a^2)\left((k+p)^2 - m_a^2\right)} \right\rangle \Leftrightarrow \\
U_{\mathcal{M}4}(p, m_a, m_a) &= \left\langle \frac{k^2}{m_a^4\left((k+p)^2 - m_a^2\right)} \right\rangle + \frac{1}{m_a^2} g_{\mu\nu} B^{\mu\nu}(p, m_a, m_a)\,.
\end{aligned} \tag{530}$$

Now lets deal with the first term since the others are already known, therefore we have that

$$\begin{aligned}
\left\langle \frac{k^2}{m_a^4\left((k+p)^2-m_a^2\right)}\right\rangle &= \left\langle \frac{(k+p)^2-m_a^2}{m_a^4\left((k+p)^2-m_a^2\right)}\right\rangle - \left\langle \frac{2p\cdot k}{m_a^4\left[(k+p)^2-m_a^2\right]}\right\rangle \\
&- \left\langle \frac{p^2}{m_a^4\left[(k+p)^2-m_a^2\right]}\right\rangle + \frac{1}{m_a^2}A_0\left(m_a\right) \\
&= \frac{1}{2} - \frac{p^2}{m_a^4}A_0\left(m_a\right) + \frac{1}{m_a^2}A_0\left(m_a\right)
\end{aligned} \tag{531}$$

where we have canceled the $2k\cdot p$ term because it is odd under $k\to -k$ and we have used the relation (454). Therefore adding (530) and (531) and using the equation (469), we get the final form

$$U_{\mathcal{M}4}(p,m_a,m_a) = \frac{1}{2} + \left(\frac{2m_a^2-p^2}{m_a^4}\right)A_0(m_a) + B_0(p,m_a,m_a)\,. \tag{532}$$

Now that we have finished with this calculation we present the results coming from the evaluation of $U_{\mathcal{M}4}$ using the shift procedure. In order to do so we start again with the first part of the equation (530) where now we complete the square obtaining the following

$$\begin{aligned}
U^{shift}_{\mathcal{M}4}(p,m_a,m_a) &= \left\langle \frac{(k^2-m_a^2)^2}{m_a^4\left(k^2-m_a^2\right)\left((k+p)^2-m_a^2\right)}\right\rangle + \left\langle \frac{2k^2m_a^2}{m_a^4\left(k^2-m_a^2\right)\left((k+p)^2-m_a^2\right)}\right\rangle \\
&- \left\langle \frac{m_a^4}{m_a^4\left(k^2-m_a^2\right)\left((k+p)^2-m_a^2\right)}\right\rangle \Leftrightarrow \\
U^{shift}_{\mathcal{M}4}(p,m_a,m_a) &= \left\langle \frac{k^2-m_a^2}{m_a^4\left((k+p)^2-m_a^2\right)}\right\rangle + \frac{2}{m_a^2}g_{\mu\nu}B^{\mu\nu}(p,m_a,m_a) - B_0(p,m_a,m_a)\,.
\end{aligned} \tag{533}$$

Now lets deal with the first term since the others are already known, therefore doing a shift of the form $k\to k-p$ we have that

$$\begin{aligned}
\left\langle \frac{k^2-m_a^2}{m_a^4\left((k+p)^2-m_a^2\right)}\right\rangle &= \left\langle \frac{k^2-2k\cdot p+p^2}{m_a^4\left(k^2-m_a^2\right)}\right\rangle - \frac{1}{m_a^2}A_0(m_a^2) \Leftrightarrow \\
\left\langle \frac{k^2-m_a^2}{m_a^4\left((k+p)^2-m_a^2\right)}\right\rangle &= g_{\mu\nu}J^{\mu\nu}(1,m_a) + \frac{p^2}{m_a^4}A_0(m_a^2) - \frac{1}{m_a^2}A_0(m_a^2)
\end{aligned} \tag{534}$$

where we have canceled the $2k \cdot p$ term because it is odd under $k \to -k$ and we have used the relation (454) . Therefore adding (533) and (534) we get that

$$U^{shift}_{\mathcal{M}4}(p, m_a, m_a) = g_{\mu\nu} J^{\mu\nu}(1, m_a) + \left(\frac{p^2}{m_a^4} - \frac{1}{m_a^2}\right) A_0(m_a) + \frac{2}{m_a^2} g_{\mu\nu} B^{\mu\nu}(p, m_a, m_a) - B_0(p, m_a, m_a) \tag{535}$$

so using the Eqs. (473) and (520) the final form of $U_{\mathcal{M}4}$ integral reads

$$U^{shift}_{\mathcal{M}4}(p, m_a, m_a) = \frac{1}{2} + \left(\frac{p^2 + 2m_a^2}{m_a^4}\right) A_0(m_a) + B_0(p, m_a, m_a)\,. \tag{536}$$

Therefore if we compare Eqs. (532) and (536) we will see that there is only one difference. In particular, p^2 in the coefficient of $A_0(m_a)$ has opposite sign in the two cases. This is a tiny but very important difference since we can see that $U_{\mathcal{M}4}$ calculated using shifts, and calculated using Veltman-Passarino reduction formula, has a difference analog to a Tadpole contribution. And this is exactly the problem with the insertion of the U-integrals in Unitary gauge. In particular, we should be very careful when we use these integrals, since they are highly divergent and using shifts, in order to reduce them, seems that is not allowed.
Now, even in this case which is quite easy to handle there is an inconsistence between the two procedures. So, it is clear that things getting worst as we move to more complicated integrals and we should be very careful about the correct usage of the shifts when we apply them on highly divergent integrals.

Now, we move on to an other dimensionless integral, which is similar with the previous one, and comes from the triangle one-loop diagrams. To be more specific we have that

$$\begin{aligned} U_{\mathcal{K}4}(p_1, p_2, m_a, m_a, m_a) &= \int \frac{d^4k}{(2\pi)^4} \frac{ik^4}{m_a^2\,(k^2 - m_a^2)\left((k+p_1)^2 - m_a^2\right)\left((k+p_1+p_2)^2 - m_a^2\right)} \Leftrightarrow \\ U_{\mathcal{K}4}(p_1, p_2, m_a, m_a, m_a) &= \left\langle \frac{k^4}{m_a^2\,(k^2 - m_a^2)\left((k+p_1)^2 - m_a^2\right)\left((k+p_1+p_2)^2 - m_a^2\right)} \right\rangle \end{aligned} \tag{537}$$

so, again here we try to construct all the possible denominators until we get a reduced divergent integral, which we know how to calculate. Therefore we get that

$$
\begin{aligned}
U_{\mathcal{K}4}(p_1,p_2,m_a,m_a,m_a) &= \left\langle \frac{(k^2-m_a^2)k^2}{m_a^2\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)}\right\rangle \\
&+ \left\langle \frac{k^2 m_a^2}{m_a^2\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)}\right\rangle \Leftrightarrow \\
U_{\mathcal{K}4}(p_1,p_2,m_a,m_a,m_a) &= \frac{1}{m_a^2} g_{\mu\nu} B^{\mu\nu}_{\{2,3\}}(p_1,m_a,m_a) + g_{\mu\nu} C^{\mu\nu}_{\{1,2,3\}}(p_1,p_2,m_a,m_a,m_a)
\end{aligned}
\tag{538}
$$

and again here appears a dilemma between the usage of shifts and the usage of the general Veltman-Passarino reduction formula. To be more specific the first term of the above relation forgetting $\frac{1}{m_a^2}$, reads

$$
\begin{aligned}
A &= \left\langle \frac{k^2}{\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)}\right\rangle = \left\langle \frac{k^2}{d_2 d_3}\right\rangle \Leftrightarrow \\
A &= A_0(m_a) - \left\langle \frac{2k\cdot p}{d_2 d_3}\right\rangle + \left(m_a^2 - p_1^2\right) B_0(m_a,m_a)
\end{aligned}
\tag{539}
$$

and thus in the second term there is not d_1 in the denominator as usual. Therefore, we have two chooses. To be more specific, we can perform a shift of the form $k \to k - p_1$ and then to use the Veltman-Passarino formula obtaining

$$
U_{\mathcal{K}4}^{2shifts} = p_1^2 B_0(m_a,m_a,0,p_2) \tag{540}
$$

or we can use directly the Veltman-Passarino reduction formula getting

$$
U_{\mathcal{K}4}^{2V-P} = -p_1^2 B_0(m_a,m_a,p_1,p_1+p_2)\,. \tag{541}
$$

Therefore, there is a difference in B_0's arguments which does not play any role, and also we can observe that, for equal masses, the two results are equal and opposite. This is a very important difference which could affect drastically our results and thus, we should choose correctly the way that we will calculate these integrals. Thus, since in these highly divergent integrals the usage of shifts is ambiguous, here and in what follows we will evaluate the U-integrals performing directly the Veltman-Passarino reduction formula.

Now, let us move on to the next case, where as we can see from equation (117), except from the $U_{\mathcal{K}4}$ we face in addition two more dimensionless integrals, namely $U_{\mathcal{K}5}$ and $U_{\mathcal{K}6}$. Starting with the first one we get that

$$U_{\mathcal{K}5}(p_1,p_2,m_a,m_a,m_a) = \int \frac{d^4k}{(2\pi)^4} \frac{ik^4k^\mu}{m_a^3\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \Leftrightarrow$$

$$U_{\mathcal{K}5}(p_1,p_2,m_a,m_a,m_a) = \left\langle \frac{k^4k^\mu}{m_a^3\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle \tag{542}$$

so following the same reasoning with the previous cases we get that

$$\begin{aligned} U_{\mathcal{K}5}(p_1,p_2,m_a,m_a,m_a) &= \left\langle \frac{(k^2-m_a^2)k^2k^\mu}{m_a^3\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle \\ &+ \left\langle \frac{k^2m_a^2k^\mu}{m_a^3\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle \Leftrightarrow \\ U_{\mathcal{K}5}(p_1,p_2,m_a,m_a,m_a) &= \left\langle \frac{k^2k^\mu}{m_a^3\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle \\ &+ \frac{g_{\mu\nu}}{m_a} C^{\mu\nu\rho}(p_1,p_2,m_a,m_a,m_a)\,. \end{aligned} \tag{543}$$

So now we deal with the first term which becomes

$$\begin{aligned} \left\langle \frac{k^2k^\mu}{m_a^3\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle &= \left\langle \frac{[(k+p_1)^2-m_a^2]k^\mu}{m_a^3\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle \\ &- \frac{2p_{1\nu}}{m_a^3} B^{\mu\nu}_{\{2,3\}}(p_1,m_a,m_a) - \frac{p_1^2}{m_a^3} B^{\mu}_{\{2,3\}}(p_1,m_a,m_a) \\ &+ \frac{1}{m_a} B^{\mu}_{\{2,3\}}(p_1,m_a,m_a) \Leftrightarrow \\ \left\langle \frac{k^2k^\mu}{m_a^3\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle &= -\frac{2p_{1\nu}}{m_a^3} B^{\mu\nu}_{\{2,3\}}(p_1,m_a,m_a) - \frac{p_1^2}{m_a^3} B^{\mu}_{\{2,3\}}(p_1,m_a,m_a) \\ &+ \frac{1}{m_a} B^{\mu}_{\{2,3\}}(p_1,m_a,m_a) \end{aligned} \tag{544}$$

where again here we have canceled the k-odd term. Therefore combining (544) with the second term that we obtained from the $U_{\mathcal{K}5}$ we take the form

$$U_{\mathcal{K}5} = -\frac{2p_{1\nu}}{m_a^3} B^{\mu\nu}_{\{2,3\}}(p_1,m_a,m_a) + \left(\frac{1}{m_a} - \frac{p_1^2}{m_a^3}\right) B^{\mu}_{\{2,3\}}(p_1,m_a,m_a) + \frac{g_{\mu\nu}}{m_a} C^{\mu\nu\rho}(p_1,p_2,m_a,m_a,m_a)\,. \tag{545}$$

Finally we move on to the second integrals that we mentioned before, namely the $U_{\mathcal{K}6}$. Therefore, following the same procedure as before this integral reads

$$\begin{aligned}
U_{\mathcal{K}6}(p_1,p_2,m_a,m_a,m_a) &= \int \frac{d^4k}{(2\pi)^4} \frac{ik^6}{m_a^4\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \Leftrightarrow \\
U_{\mathcal{K}6}(p_1,p_2,m_a,m_a,m_a) &= \left\langle \frac{k^6}{m_a^4\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle
\end{aligned} \tag{546}$$

again here we have that

$$\begin{aligned}
U_{\mathcal{K}6}(p_1,p_2,m_a,m_a,m_a) &= \left\langle \frac{(k^2-m_a^2)k^4}{m_a^4\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle \\
&+ \left\langle \frac{m_a^2k^4}{m_a^2\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle \Leftrightarrow \\
U_{\mathcal{K}6}(p_1,p_2,m_a,m_a,m_a) &= \left\langle \frac{k^4}{m_a^4\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle \\
&+ \left\langle \frac{m_a^2(k^2-m_a^2)k^2}{m_a^4\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle \Leftrightarrow \\
U_{\mathcal{K}6}(p_1,p_2,m_a,m_a,m_a) &= \left\langle \frac{k^4}{m_a^4\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right\rangle \\
&+ \frac{g_{\mu\nu}}{m_a^2} B^{\mu\nu}_{\{2,3\}}(p_1,m_a,m_a) + g_{\mu\nu} C^{\mu\nu}_{\{1,2,3\}}(p_1,p_2,m_a,m_a,m_a)\,.
\end{aligned} \tag{547}$$

Now we should evaluate the first term of the above relation therefore using the same reasoning with the previous calculations we get that

$$
\begin{aligned}
\left\langle\frac{k^4}{m_a^4\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)}\right\rangle &= \left\langle\frac{k^2\left[(k+p_1)-m_a^2\right]}{m_a^4\left[(k+p_1)^2-m_a^2\right]\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle \\
&- \left\langle\frac{2k^2 p_1\cdot k}{m_a^4\left[(k+p_1)^2-m_a^2\right]\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle \\
&- \left\langle\frac{p_1^2 k^2}{m_a^4\left[(k+p_1)^2-m_a^2\right]\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle \\
&+ \left\langle\frac{k^2}{m_a^2\left[(k+p_1)^2-m_a^2\right]\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle \Leftrightarrow \\
\left\langle\frac{k^4}{m_a^4\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)}\right\rangle &= \left\langle\frac{k^2}{m_a^4\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle \\
&- \left\langle\frac{2p_{1\mu}k^2k^\mu}{m_a^4\left[(k+p_1)^2-m_a^2\right]\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle \\
&+ \left(\frac{1}{m_a^2}-\frac{p_1^2}{m_a^4}\right) g_{\mu\nu}B^{\mu\nu}_{\{2,3\}}(p_1,m_a,m_a) \qquad (548)
\end{aligned}
$$

so now we have to deal with the first and the second term of (548). Starting with the first one we we get

$$
\begin{aligned}
\left\langle\frac{k^2}{m_a^4\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle &= \left\langle\frac{(k+P_1)^2-m_a^2}{m_a^4\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle \\
&- \left\langle\frac{2P_1\cdot k}{m_a^4\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle \\
&- \left\langle\frac{P_1^2}{m_a^4\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle + \frac{1}{m_a^2}A_0\left(m_a\right) \Leftrightarrow \\
\left\langle\frac{k^2}{m_a^4\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle &= \left\langle\frac{1}{m_a^4}\right\rangle + \left[\frac{1}{m_a^2}-\frac{P_1^2}{m_a^4}\right]A_0\left(m_a\right)\ . \qquad (549)
\end{aligned}
$$

where we have defined that $P_1 = p_1 + p_2$. Now we move on to the second term which reads

$$
\begin{aligned}
A &= -\frac{2p_{1\mu}}{m_a^4}\left\langle\frac{k^2k^\mu}{m_a^4\left[(k+p_1)^2-m_a^2\right]\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle \Leftrightarrow \\
A &= -\frac{2p_{1\mu}}{m_a^4}\left\{\left\langle\frac{k^\mu\left[(k+p_1)^2-m_a^2\right]}{\left[(k+p_1)^2-m_a^2\right]\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle\right. \\
&- \left\langle\frac{2k^\mu p_1\cdot k}{\left[(k+p_1)^2-m_a^2\right]\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle \\
&- \left\langle\frac{k^\mu p_1^2}{\left[(k+p_1)^2-m_a^2\right]\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle \\
&+ m_a^2\left\langle\frac{k^\mu}{\left[(k+p_1)^2-m_a^2\right]\left[(k+p_1+p_2)^2-m_a^2\right]}\right\rangle \Leftrightarrow \\
A &= \frac{4p_{1\mu}p_{1\nu}}{m_a^4}B^{\mu\nu}_{\{2,3\}}(p_1,m_a,m_a) \\
&+ \frac{2p_1^2}{m_a^4}p_{1\mu}B^{\mu}_{\{2,3\}}(p_1,m_a,m_a) \\
&- \frac{2p_{1\mu}}{m_a^2}B^{\mu}_{\{2,3\}}(p_1,m_a,m_a)
\end{aligned} \tag{550}
$$

therefore combining all these together we obtain the final form of $U_{\mathcal{K}6}$ which reads

$$
\begin{aligned}
U_{\mathcal{K}6} &= \frac{1}{2}+\left(\frac{1}{m_a^2}-\frac{P_1^2}{m_a^4}\right)A_0(m_a)+\left(\frac{2p_1^2}{m_a^4}p_{1\mu}-\frac{2p_{1\mu}}{m_a^2}\right)B^{\mu}_{\{2,3\}}(p_1,m_a,m_a) \\
&+ \left(\frac{2g_{\mu\nu}}{m_a^2}-\frac{g_{\mu\nu}p_1^2}{m_a^4}+\frac{4p_{1\mu}p_{1\nu}}{m_a^4}\right)B^{\mu\nu}_{\{2,3\}}(p_1,m_a,m_a)+g_{\mu\nu}C^{\mu\nu}_{\{1,2,3\}}(p_1,p_2,m_a,m_a,m_a)\,.
\end{aligned} \tag{551}
$$

Since now we have seen all the U_i's that occur in the case of the scalar reduction of the Triangle-integrals. Thus it is time to move on to the U_i integrals that occur in the calculation of the Box diagrams which we have defined in subsection 2.4 . Generally, the method that we use in order to reduce these integrals is exactly the same with that we have followed throughout the calculation of the Triangle U_i's. To be more specific, again we are trying to construct the appropriate denominator in the numerator of the corresponding integrals, and then we manage to reduce its divergence in a proper way. Therefore it is straightforward to write the exact form of these integrals since the calculation is already known. So we start with the dimensionless integral $U_{\mathcal{B}4}$ which reads

$$
\begin{aligned}
U_{\mathcal{B}4}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \int \frac{d^4k}{(2\pi)^4}\Bigg\{\frac{-ik^4}{(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \\
&\times \frac{1}{((k+p_1+p_2+p_3)^2-m_a^2)}\Bigg\} \Leftrightarrow \\
U_{\mathcal{B}4}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \left\langle \frac{k^4}{[k]\,[k+p_1][k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle
\end{aligned} \tag{552}
$$

where we have defined that $[k] \equiv (k^2-m_a^2)$, $[k+p_1] \equiv \left((k+p_1)^2-m_a^2\right)$, $[k+p_1+p_2] \equiv \left((k+p_1+p_2)^2-m_a^2\right)$ and $[k+p_1+p_2+p_3] \equiv \left((k+p_1+p_2+p_3)^2-m_a^2\right)$ in order to have a more compact notation which we will follow in the Box-U_i integrals. Thus we can write here the following relation

$$
\begin{aligned}
U_{\mathcal{B}4}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \left\langle \frac{k^2\,[k]}{[k]\,[k+p_1]\,[k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle \\
&+ m_a^2 \left\langle \frac{k^2}{[k]\,[k+p_1]\,[k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle \Leftrightarrow \\
U_{\mathcal{B}4}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= g_{\mu\nu}C^{\mu\nu}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) + m_a^2 g_{\mu\nu}D^{\mu\nu}(a,a,a,a)\,.
\end{aligned} \tag{553}
$$

Next, following the same reasoning we present the reduced form of the $U_{\mathcal{B}5}$ integral which reads

$$
\begin{aligned}
U_{\mathcal{B}5}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \int \frac{d^4k}{(2\pi)^4}\Bigg\{\frac{-ik^4k^\mu}{m_a\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \\
&\times \frac{1}{((k+p_1+p_2+p_3)^2-m_a^2)}\Bigg\} \Leftrightarrow \\
U_{\mathcal{B}5}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \left\langle \frac{k^4k^\mu}{m_a\,[k]\,[k+p_1][k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle
\end{aligned} \tag{554}
$$

so after using the relations that we have obtained previously we have that

$$\begin{aligned}
U_{\mathcal{B}5}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \left\langle \frac{k^\mu k^2\,[k]}{m_a\,[k]\,[k+p_1]\,[k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle \\
&+ m_a \left\langle \frac{k^\mu k^2}{[k]\,[k+p_1]\,[k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle \Leftrightarrow \\
U_{\mathcal{B}5}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \frac{g_{\mu\nu}}{m_a} C^{\mu\nu\rho}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) + m_a g_{\mu\nu} D^{\mu\nu\rho}(a,a,a,a)\,.
\end{aligned} \tag{555}$$

Now using the same arguments for the $g_{\mu\nu}D^{\mu\nu\rho}(a,a,a,a)$ we get the final form of $U_{\mathcal{B}5}$ which reads

$$\begin{aligned}
U_{\mathcal{B}5}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \frac{g_{\mu\nu}}{m_a} C^{\mu\nu\rho}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) + m_a C^{\mu}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) \\
&+ m_a^3 D^{\mu}(a,a,a,a)
\end{aligned} \tag{556}$$

where we have left intact the terms with indices since they are contracted with the momenta or the metric when we consider their full calculation. Now we move on to the next dimensionless Box integral which is

$$\begin{aligned}
U_{\mathcal{B}6}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \int \frac{d^4k}{(2\pi)^4} \Bigg\{ \frac{-ik^6}{m_a^2\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \\
&\times \frac{1}{((k+p_1+p_2+p_3)^2-m_a^2)} \Bigg\} \Leftrightarrow \\
U_{\mathcal{B}6}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \left\langle \frac{k^2k^4}{m_a^2\,[k]\,[k+p_1][k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle .
\end{aligned} \tag{557}$$

This integral could be seen as the $U_{\mathcal{B}4}$ integral with an extra k^2 term in the nominator and divided by m_a^2. Therefore we get that the reduced form reads

$$\begin{aligned}
U_{\mathcal{B}6}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \left\langle \frac{k^4}{m_a^2\,[k+p_1]\,[k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle \\
&+ g_{\mu\nu} C^{\mu\nu}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) + m_a^2 g_{\mu\nu} D^{\mu\nu}(a,a,a,a) \Leftrightarrow \\
U_{\mathcal{B}6}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \left\langle \frac{k^4}{m_a^2\,[k+p_1]\,[k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle + U_{\mathcal{B}4}\,.
\end{aligned} \tag{558}$$

Now we should deal with the first term of the above equation which we define as B and it reads

$$\begin{aligned}
B &= \left\langle \frac{k^4}{m_a^2\,[k+p_1]\,[k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle \Leftrightarrow \\
B &= \left\langle \frac{k^2\,[k+p_1]}{m_a^2\,[k+p_1]\,[k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle \\
&- \left\langle \frac{2k^2 k\cdot p_1}{m_a^2\,[k+p_1]\,[k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle \\
&- \left\langle \frac{k^2 p_1^2}{m_a^2\,[k+p_1]\,[k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle \\
&+ \left\langle \frac{k^2}{[k+p_1]\,[k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle \Leftrightarrow \\
B &= \frac{g_{\mu\nu}}{m_a^2} B^{\mu\nu}_{\{3,4\}}(p_1,m_a,m_a) - \frac{2p_{1\mu}}{m_a^2} g_{\nu\rho} C^{\mu\nu\rho}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) \\
&+ \left(1-\frac{p_1^2}{m_a^2}\right) g_{\mu\nu} C^{\mu\nu}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a)
\end{aligned} \tag{559}$$

thus we obtain the final form of the $U_{\mathcal{B}6}$ which reads

$$\begin{aligned}
U_{\mathcal{B}6}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \frac{g_{\mu\nu}}{m_a^2} B^{\mu\nu}_{\{3,4\}}(p_1,m_a,m_a) - \frac{2p_{1\mu}}{m_a^2} g_{\nu\rho} C^{\mu\nu\rho}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) \\
&+ \left(1-\frac{p_1^2}{m_a^2}\right) g_{\mu\nu} C^{\mu\nu}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) + U_{\mathcal{B}4}\,.
\end{aligned} \tag{560}$$

Here we have an other dimensionless integral of the same numerator dimension, namely $U^{\mu\nu}_{\mathcal{B}6}$ which is exactly as the $U_{\mathcal{B}6}$ but without being contracted with either the metric or the momentum. Therefore we can write straightforward that

$$\begin{aligned}
U^{\mu\nu}_{\mathcal{B}6}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{-ik^4 k^\mu k^\nu}{m_a^2\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \right. \\
&\times \left. \frac{1}{((k+p_1+p_2+p_3)^2-m_a^2)} \right\} \Leftrightarrow \\
U^{\mu\nu}_{\mathcal{B}6}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \left\langle \frac{k^4 k^\mu k^\nu}{m_a^2\,[k]\,[k+p_1][k+p_1+p_2]\,[k+p_1+p_2+p_3]} \right\rangle
\end{aligned} \tag{561}$$

so we have that

$$
\begin{aligned}
U^{\mu\nu}_{\mathcal{B}6}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \frac{1}{m_a^2}B^{\mu\nu}_{\{3,4\}}(p_1,m_a,m_a) - \frac{2p_{1\rho}}{m_a^2}C^{\mu\nu\rho}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) \\
&+ \left(2-\frac{p_1^2}{m_a^2}\right)C^{\mu\nu}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) + m_a^2 D^{\mu\nu}(a,a,a,a)\,.
\end{aligned}
\tag{562}
$$

Now we move on to the next dimensionless integral that we face in the Box calculation which is $U_{\mathcal{B}7}$ and has the following form

$$
\begin{aligned}
U_{\mathcal{B}7}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \int\frac{d^4k}{(2\pi)^4}\left\{\frac{-ik^6k^\mu}{m_a^3\,(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)}\right. \\
&\times \left.\frac{1}{((k+p_1+p_2+p_3)^2-m_a^2)}\right\}\Leftrightarrow \\
U_{\mathcal{B}7}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \left\langle\frac{k^6k^\mu}{m_a^3\,[k]\,[k+p_1][k+p_1+p_2]\,[k+p_1+p_2+p_3]}\right\rangle
\end{aligned}
\tag{563}
$$

Since now we have seen in a very analytical way the method that we use in order to evaluate the U-integrals. Thus, the remaining two highly divergent integrals could be written without the intermediate steps, since we have already seen them. So we get that

$$
\begin{aligned}
U_{\mathcal{B}7}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \left(\frac{1}{m_a}-\frac{P_1^2}{m_a^3}\right)B^{\mu}_{\{3,4\}}(p_1,m_a,m_a) + \left(-\frac{2p_{1\nu}}{m_a^3}-\frac{2P_{1\nu}}{m_a^3}\right)B^{\mu\nu}_{\{3,4\}}(p_1,m_a,m_a) \\
&+ \left(\frac{2p_1^2}{m_a^3}-\frac{2}{m_a}\right)p_{1\nu}C^{\mu\nu}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) \\
&+ \left(\frac{4p_{1\rho}p_{1\nu}}{m_a^3}-\frac{p_1^2}{m_a^3}g_{\nu\rho}+\frac{g_{\nu\rho}}{m_a}\right)p_{1\nu}C^{\mu\nu\rho}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) + U_{\mathcal{B}5}\,.
\end{aligned}
\tag{564}
$$

Finally, we have the last dimensionless Box integral which is $U_{\mathcal{B}8}$ which has the following form

$$
\begin{aligned}
U_{\mathcal{B}8}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \int \frac{d^4k}{(2\pi)^4}\Bigg\{\frac{-ik^8}{m_a^4\left(k^2-m_a^2\right)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)} \\
&\times \frac{1}{((k+p_1+p_2+p_3)^2-m_a^2)}\Bigg\} \Leftrightarrow \\
U_{\mathcal{B}8}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \left\langle \frac{k^8}{m_a^4\,[k]\,[k+p_1][k+p_1+p_2]\,[k+p_1+p_2+p_3]}\right\rangle
\end{aligned} \tag{565}
$$

therefore its reduced form reads

$$
\begin{aligned}
U_{\mathcal{B}8}(p_1,p_2,p_3,m_a,m_a,m_a,m_a) &= \frac{1}{2}+\left(\frac{1}{m_a^2}-\frac{P_2^2}{m_a^4}\right)A_0(m_a)+\left(\frac{4P_1^2}{m_a^2}-\frac{4}{m_a^4}\right)p_{1\mu}B^{\mu}_{\{3,4\}}(p_1,m_a,m_a) \\
&+ \left(\frac{2g_{\mu\nu}}{m_a^2}-\frac{P_1^2}{m_a^4}g_{\mu\nu}-\frac{p_1^2}{m_a^4}g_{\mu\nu}+\frac{8p_{1\mu}P_{1\nu}}{m_a^4}+\frac{4p_{1\mu}p_{1\nu}}{m_a^4}\right)B^{\mu\nu}_{\{3,4\}}(p_1,m_a,m_a) \\
&+ \Bigg\{g_{\mu\nu}-2\frac{p_1^2}{m_a^2}g_{\mu\nu}+\frac{p_1^4}{m_a^4}g_{\mu\nu} \\
&+ \frac{4}{m_a^2}p_{1\mu}p_{1\nu}-\frac{4p_1^4}{m_a^4}p_{1\mu}p_{1\nu}\Bigg\}C^{\mu\nu}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a) \\
&+ \Bigg\{-\frac{2p_{1\mu}}{m_a^2}g_{\nu\rho}+\frac{2p_1^2p_{1\mu}}{m_a^4}g_{\nu\rho} \\
&+ \left(\frac{2p_1^2}{m_a^4}-\frac{2}{m_a^2}\right)g_{\mu\nu}p_{1\rho}-\frac{8}{m_a^4}p_{1\mu}p_{1\nu}p_{1\rho}\Bigg\}C^{\mu\nu\rho}_{\{2,3,4\}}(p_1,p_2,m_a,m_a,m_a)\,.
\end{aligned} \tag{566}
$$

XXX

D.2 Explicit calculation of the U-integrals

The first case that we have is the following

$$
U_1(p,m_a,m_a)=\int\frac{d^4k}{(2\pi)^4}\frac{i}{(k^2-m_a^2)\left((k+p)^2-m_a^2\right)} \tag{567}
$$

here we can see that the Feynman parameters are that of the equation (509), thus we have that the denominator takes the form

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{\left[A + (B - A)\,x\right]^2} \tag{568}$$

where $A = k^2 - m_a^2$ and $B = (k+p)^2 - m_a^2$. This gives

$$\begin{aligned} \Pi &= \left[A + (B - A)\,x\right] = \left[k^2 - m_a^2 + \left((k+p)^2 - m_a^2 - k^2 + m_a^2\right) x\right] \\ &= \left[k^2 + p^2 x + 2kpx - m_a^2\right] \end{aligned} \tag{569}$$

then we complete the square adding the term $\pm p^2 (1-x)^2$ and we get

$$\begin{aligned} \Pi &= \left[(k + px)^2 + p^2 x\,(1-x) - m_a^2\right] \\ &= k^2 - \Delta \end{aligned} \tag{570}$$

where we have performed the shift $k \to k - px$ and we have defined that $\Delta = -p^2 x\,(1-x) + m_a^2$. Thus, now U_1 becomes

$$U_1(p, m_a, m_a) = \int_0^1 dx \int \frac{d^4 k}{(2\pi)^4} \frac{i}{(k^2 - \Delta)^2}.$$

Now in d-dimensions and using equation (512) this integral reads

$$\begin{aligned} U_1(p, m_a, m_a) &= \int_0^1 dx \int \frac{d^d k}{(2\pi)^d} \frac{i}{(k^2 - \Delta)^2} \Leftrightarrow \\ U_1(p, m_a, m_a) &= \frac{1}{(4\pi)^{d/2}} \int_0^1 dx \Gamma\left(2 - \frac{d}{2}\right)\left(\frac{1}{\Delta}\right)^{2-d/2}. \end{aligned} \tag{571}$$

and using the expansion relations of the Gamma function and expanding around $d = 4 - \varepsilon$ the $U_1(p, m_a, m_a)$ becomes

$$U_1(p, m_a, m_a) = \frac{1}{(4\pi)^2}\left(\frac{2}{\varepsilon} + \int_0^1 dx \ln \frac{4\pi e^{-\gamma_E}}{\Delta}\right). \tag{572}$$

The next integral that we deal here is a little bit more complicated but we use the same reasoning in order to calculate it. In particular we have the following

$$U_2(p, m_a, m_a) = \int \frac{d^4 k}{(2\pi)^4} \frac{i k^2}{(k^2 - m_a^2)\left((k+p)^2 - m_a^2\right)} \tag{573}$$

where we have again the same $n = 2$ case concerning the Feynman parameters and thus we obtain that the shift is $k \to k - p\,(x-1)$ and $\Delta = -p^2 x\,(1-x) + m_a^2$.
The only difference with the previous calculation occurs in the numerator where here it has to be shifted, namely

$$\begin{aligned} N &= k^2 = (k - px)^2 \\ &= k^2 + p^2 x^2 \end{aligned}$$

where we have omitted terms which are odd to the k. Therefore we obtain that

$$U_2(p, m_a, m_a) = \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{i(k^2 + p^2x^2)}{(k^2 - \Delta)^2}$$

so in d-dimensions and using equations (512) and (513) this integral reads

$$\begin{aligned} U_2(p, m_a, m_a) &= \int_0^1 dx \int \frac{d^dk}{(2\pi)^d} \frac{i(k^2 + p^2x^2)}{(k^2 - \Delta)^2} \Leftrightarrow \\ U_2(p, m_a, m_a) &= -\frac{1}{(4\pi)^{d/2}} \int_0^1 dx \Gamma\left(1 - \frac{d}{2}\right)\left(\frac{1}{\Delta}\right)^{1-d/2} + \frac{1}{(4\pi)^{d/2}} \int_0^1 dx p^2x^2 \Gamma\left(2 - \frac{d}{2}\right)\left(\frac{1}{\Delta}\right)^{2-d/2} \end{aligned} \tag{574}$$

and using the expansion relations of the Gamma function and expanding around $d = 4 - \varepsilon$ the $U_2(p, m_a, m_a)$ becomes

$$\begin{aligned} U_2(p, m_a, m_a) &= \frac{1}{(4\pi)^2}\left(-\frac{p^2}{3\varepsilon} + \frac{m_a^2}{\varepsilon} + \int_0^1 dx \Delta \ln\frac{4\pi e^{-\gamma_E}}{\Delta} - \frac{p^2}{6} + m_a^2\right) \\ &+ \frac{1}{(4\pi)^2}\left(\frac{2p^2}{3\varepsilon} + \int_0^1 dx p^2 x^2 \ln\frac{4\pi e^{-\gamma_E}}{\Delta}\right) \Leftrightarrow \\ U_2(p, m_a, m_a) &= \frac{1}{(4\pi)^2}\left(\frac{p^2}{3\varepsilon} - \frac{m_a^2}{\varepsilon} + \int_0^1 dx p^2 x\,(2x-1) \ln\frac{4\pi e^{-\gamma_E}}{\Delta} - \frac{p^2}{6} + m_a^2\right) \end{aligned} \tag{575}$$

Now, lets deal with the explicit calculation of the dimensionless integral $U_{\mathcal{M}4}$ where we already know that the Feynman parameters are in the $n = 2$ case. In addition the corresponding shift and Δ are the same with the previous two integrals. Moreover the numerator after the shift becomes

$$\begin{aligned} N &= k^4 = (k + px)^4 \\ &= k^4 + 6k^2p^2x^2 + p^4x^4 \end{aligned}$$

therefore we obtain that

$$U_{\mathcal{M}4}(p,m_a,m_a)=\int\limits_0^1 dx\int\frac{d^4k}{(2\pi)^4}\frac{i(k^4+6k^2p^2x^2+p^4x^4)}{m_a^4\left(k^2-\Delta\right)^2}$$

so in d-dimensions and using equations (512) , (513) and (515) this integral reads

$$\begin{aligned}
U_{\mathcal{M}4}(p,m_a,m_a) &= \int\limits_0^1 dx\int\frac{d^dk}{(2\pi)^d}\frac{i(k^4+6k^2p^2x^2+p^4x^4)}{m_a^4\left(k^2-\Delta\right)^2}\Leftrightarrow\\
U_{\mathcal{M}4}(p,m_a,m_a) &= -\frac{1}{(4\pi)^{d/2}}\int\limits_0^1 dx g_{\mu\nu}g_{\rho\sigma}J^{\mu\nu\rho\sigma}(2,\Delta)+\frac{6}{(4\pi)^{d/2}m_a^2}\int\limits_0^1 dx p^2x^2g_{\mu\nu}J^{\mu\nu}(2,\Delta)\\
&-\frac{1}{(4\pi)^{d/2}m_a^4}\int\limits_0^1 dx p^4x^4J_0(2,\Delta)\Leftrightarrow\\
U_{\mathcal{M}4}(p,m_a,m_a) &= \frac{1}{(4\pi)^{d/2}}\Bigg\{-\int\limits_0^1 dx g_{\mu\nu}g_{\rho\sigma}J^{\mu\nu\rho\sigma}(2,\Delta)+\frac{6}{m_a^2}\int\limits_0^1 dx p^2x^2g_{\mu\nu}J^{\mu\nu}(2,\Delta)\\
&-\frac{1}{m_a^4}\int\limits_0^1 dx p^4x^4J_0(2,\Delta)\Bigg\}
\end{aligned}\tag{576}$$

so after expanding around $d=4-\varepsilon$ the $U_{\mathcal{M}4}$ becomes

$$\begin{aligned}
U_{\mathcal{M}4}(p,m_a,m_a) &= \frac{1}{(4\pi)^2}\left(\left(6-\frac{2p^2}{m_a^2}+\frac{p^4}{5m_a^4}\right)\frac{1}{\varepsilon}+\int\limits_0^1 dx6\frac{\Delta^2}{m_a^4}\ln\frac{4\pi e^{-\gamma_E}}{\Delta}+\frac{9}{2}+\frac{3}{2}\frac{p^2}{m_a^2}+\frac{3}{20}\frac{p^4}{m_a^4}\right)\\
&+ \frac{1}{(4\pi)^2}\left(\left(\frac{8p^2}{m_a^2}-\frac{6p^4}{5m_a^4}\right)\frac{1}{\varepsilon}+\int\limits_0^1 dx12\frac{\Delta p^2x^2}{m_a^4}\ln\frac{4\pi e^{-\gamma_E}}{\Delta}+\frac{p^2}{m_a^2}-\frac{3p^4}{5m_a^4}\right)\\
&+ \frac{1}{(4\pi)^2}\left(\frac{2p^4}{5m_a^4\varepsilon}+\int\limits_0^1 dx\frac{p^4x^4}{m_a^4}\ln\frac{4\pi e^{-\gamma_E}}{\Delta}\right)\Leftrightarrow\\
U_{\mathcal{M}4}(p,m_a,m_a) &= \Bigg\{\left(6+\frac{6p^2}{m_a^2}-\frac{3p^4}{5m_a^4}\right)\frac{1}{\varepsilon}+\int\limits_0^1 dx\Big\{3+\frac{3p^2}{m_a^2}\left(-2x+6x^2+\frac{p^2x}{m_a^2}-\frac{6p^2x^4}{m_a^2}\right)\\
&+ \frac{16p^4x^4}{m_a^4}\Big\}\ln\frac{4\pi e^{-\gamma_E}}{\Delta}+\frac{9}{2}+\frac{5}{2}\frac{p^2}{m_a^2}-\frac{9}{20}\frac{p^4}{m_a^4}\Bigg\}
\end{aligned}\tag{577}$$

Finally a common case that we face in that kind of calculations is that of

$$U'_{\mathcal{M}4}(p,m_a,m_a)=\mu^{4-d}U_{\mathcal{M}4}(p,m_a,m_a)$$

so if we combine equation (577) with $U'_{\mathcal{M}4}$ and do the expansion around $d = 4 - \varepsilon$, then we will obtain the following form

$$
\begin{aligned}
(4\pi)^2 U'_{\mathcal{M}4}(p, m_a, m_a) &= \Bigg\{\left(6 + \frac{6p^2}{m_a^2} - \frac{3p^4}{5m_a^4}\right)\frac{1}{\varepsilon} + \int_0^1 dx \Bigg\{3 + \frac{3p^2}{m_a^2}\left(-2x + 6x^2 + \frac{p^2 x}{m_a^2} - \frac{6p^2 x^4}{m_a^2}\right) \\
&+ \frac{16p^4 x^4}{m_a^4}\Bigg\} \ln\frac{\mu^2}{\Delta} + \frac{9}{2} + \frac{5}{2}\frac{p^2}{m_a^2} - \frac{9}{20}\frac{p^4}{m_a^4}\Bigg\}. \qquad (578)
\end{aligned}
$$

Next integral that corresponds to interesting features of our work is the dimensionless $U_{\mathcal{K}4}$. This case is between the most common integrals that we face during the calculation of the one-loop three-point functions in section 2. So we have that

$$
U_{\mathcal{K}4} = \int \frac{d^4k}{(2\pi)^4} \frac{-ik^4}{m_a^2\left(k^2 - m_a^2\right)\left((k+p_1)^2 - m_a^2\right)\left((k+p_1+p_2)^2 - m_a^2\right)} \qquad (579)
$$

where its Feynman parameters correspond to the $n = 3$ case. In particular we notice that this integral is exactly the same with (597) concerning the denominators and the Feynman parametrization thus using the relation (511) for $A = k^2 - m_a^2$, $B = (k+p_1)^2 - m_a^2$ and $C = (k+p_1+p_2)^2 - m_a^2$ the denominator reads

$$
\begin{aligned}
\Pi &= [Ax + By + Cz] \Leftrightarrow \\
\Pi &= \left[xk^2 - xm_a^2 + yk^2 + yp_1^2 + 2ykp_1 - ym_a^2 + zk^2 + z(p_1+p_2)^2 + 2zk(p_1+p_2) - zm_a^2\right] \\
&= \left[k^2 + 2k\left(p_1 y + (p_1+p_2)z\right) + yp_1^2 + z(p_1+p_2) - m_a^2\right] \\
&= \left[k^2 + 2k\left(p_1 y + (p_1+p_2)z\right) \pm (yp_1 + z(p_1+p_2))^2 + yp_1^2 + z(p_1+p_2) - m_a^2\right] \Leftrightarrow \\
\Pi &= k^2 - \Delta \qquad (580)
\end{aligned}
$$

where we have completed the square adding the term $\pm(yp_1 + z(p_1+p_2))^2$, we have used the relation $x + y + z = 1$ and we have done the shift $k \to k - p_1 y - (p_1+p_2)z$. Finally considering everything mentioned above, we have defined that $\Delta = -p_1^2 x(1-x) - 2p_1p_2xz - p_2^2 z(1-z) + m_a^2$. Now we should do the shift in the numerator which becomes

$$
\begin{aligned}
N &= k^4 \to (k - p_1 y - (p_1+p_2)z)^4 \Leftrightarrow \\
N &= k^4 - 4(p_1 y + (p_1+p_2)z)k^3 + 6(p_1 y + (p_1+p_2)z)^2 k^2 - 4(p_1 y + (p_1+p_2)z)^3 k + (p_1 y + (p_1+p_2)z)^4 \qquad (581)
\end{aligned}
$$

and if we putt it in (579) it will read

$$\begin{aligned}
U_{\mathcal{K}4}(p_1,p_2,m_a) &= 2\int\limits_0^1 dx\int\limits_0^1 dy\int\limits_0^1 dz\delta\left(1-\sum_{i=1}^3 x_i\right)\int\frac{d^4k}{(2\pi)^4}\frac{-ik^4}{m_a^2\left(k^2-\Delta\right)^3}\\
&- 2\int\limits_0^1 dx\int\limits_0^1 dy\int\limits_0^1 dz\delta\left(1-\sum_{i=1}^3 x_i\right)\int\frac{d^4k}{(2\pi)^4}\frac{-i4(p_1y+(p_1+p_2)z)k^3}{m_a^2\left(k^2-\Delta\right)^3}\\
&+ 2\int\limits_0^1 dx\int\limits_0^1 dy\int\limits_0^1 dz\delta\left(1-\sum_{i=1}^3 x_i\right)\int\frac{d^4k}{(2\pi)^4}\frac{-i6(p_1y+(p_1+p_2)z)^2k^2}{m_a^2\left(k^2-\Delta\right)^3}\\
&- 2\int\limits_0^1 dx\int\limits_0^1 dy\int\limits_0^1 dz\delta\left(1-\sum_{i=1}^3 x_i\right)\int\frac{d^4k}{(2\pi)^4}\frac{-i4(p_1y+(p_1+p_2)z)^3k}{m_a^2\left(k^2-\Delta\right)^3}\\
&+ 2\int\limits_0^1 dx\int\limits_0^1 dy\int\limits_0^1 dz\delta\left(1-\sum_{i=1}^3 x_i\right)\int\frac{d^4k}{(2\pi)^4}\frac{-i(p_1y+(p_1+p_2)z)^4}{m_a^2\left(k^2-\Delta\right)^3}. \quad (582)
\end{aligned}$$

Similarly with the case of the Triangular integrals here we face integrals that have numerator which is different from one. One case like that is the dimensionless integral

$$U_{\mathcal{B}4}=\int\frac{d^4k}{(2\pi)^4}\frac{-ik^4}{(k^2-m_a^2)\left((k+p_1)^2-m_a^2\right)\left((k+p_1+p_2)^2-m_a^2\right)\left((k+p_1+p_2+p_3)^2-m_a^2\right)} \quad (583)$$

and this integral has the same Feynman parameterization with that of (602), thus using the $n=4$ case for the Feynman parameters we get that the denominator gives

$$\begin{aligned}
\Pi &= [Ax+By+Cz+Dw]\Leftrightarrow\\
\Pi &= \Big\{xk^2-xm_a^2+yk^2+yp_1^2+2ykp_1-ym_a^2+zk^2+z(p_1+p_2)^2+2zk(p_1+p_2)-zm_a^2\\
&+ wk^2+w(p_1+p_2+p_3)^2+2wk(p_1+p_2+p_3)-wm_a^2\Big\}\\
&= \Big\{k^2+2k\left(p_1y+(p_1+p_2)z+w(p_1+p_2+p_3)\right)+yp_1^2+z(p_1+p_2)^2+w(p_1+p_2+p_3)^2-m_a^2\Big\}\\
&= \Big\{k^2+2k\left(p_1y+(p_1+p_2)z+w(p_1+p_2+p_3)\right)\pm\left(yp_1+z(p_1+p_2)+w(p_1+p_2+p_3)\right)^2\\
&+ yp_1^2+z(p_1+p_2)^2+w(p_1+p_2+p_3)^2-m_a^2\Big\}\Leftrightarrow\\
\Pi &= k^2-\Delta \quad (584)
\end{aligned}$$

where we have completed the square adding the term $\pm(yp_1+z(p_1+p_2)+w(p_1+p_2+p_3))^2$, we have used the relation $x+y+z+w=1$ and we have done the shift

$k \to k - p_1 y - (p_1+p_2)z - w(p_1+p_2+p_3)$. Finally we have defined that $\Delta = -yp_1^2 - z(p_1+p_2)^2 - w(p_1+p_2+p_3)^2 + (yp_1 + z(p_1+p_2) + w(p_1+p_2+p_3))^2 + m_a^2$.
Now performing the shift that we have mentioned we get that the numerator reads

$$\begin{aligned} N &= k^4 \to (k - p_1 y - (p_1+p_2)z - w(p_1+p_2+p_3))^4 \Leftrightarrow \\ N &= k^4 - 4(p_1 y + (p_1+p_2)z + w(p_1+p_2+p_3))k^3 + 6(p_1 y + (p_1+p_2)z + w(p_1+p_2+p_3))^2 k^2 \\ &- 4(p_1 y + (p_1+p_2)z + w(p_1+p_2+p_3))^3 k + (p_1 y + (p_1+p_2)z + w(p_1+p_2+p_3))^4 \end{aligned} \tag{585}$$

and if we putt it in (583) it will read

$$\begin{aligned} U_{\mathcal{B}4}(p_i, m_a) &= 6\int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dw \delta\left(1 - \sum_{i=1}^4 x_i\right) \int \frac{d^4k}{(2\pi)^4} \frac{-ik^4}{(k^2-\Delta)^4} \\ &- 6\int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dw \delta\left(1 - \sum_{i=1}^4 x_i\right) \int \frac{d^4k}{(2\pi)^4} \frac{-i4(p_1(1-x) + p_2(z+w) + wp_3)k^3}{(k^2-\Delta)^4} \\ &+ 6\int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dw \delta\left(1 - \sum_{i=1}^4 x_i\right) \int \frac{d^4k}{(2\pi)^4} \frac{-i6(p_1(1-x) + p_2(z+w) + wp_3)^2 k^2}{(k^2-\Delta)^4} \\ &- 6\int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dw \delta\left(1 - \sum_{i=1}^4 x_i\right) \int \frac{d^4k}{(2\pi)^4} \frac{-i4(p_1(1-x) + p_2(z+w) + wp_3)^3 k}{(k^2-\Delta)^4} \\ &+ 6\int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dw \delta\left(1 - \sum_{i=1}^4 x_i\right) \int \frac{d^4k}{(2\pi)^4} \frac{-i(p_1(1-x) + p_2(z+w) + wp_3)^4}{(k^2-\Delta)^4}. \end{aligned} \tag{586}$$

where i takes the values $i = 1, 2, 3$.

XXX

E Calculation of the A_0, B_0, C_0 and D_0 Integrals

In the present section of the Appendices we demonstrate the analytic calculation of the scalar integrals that we have used throughout this work. This is very important since every reduced result that we have obtain in section 2 is according to these scalar integrals, therefore in order to use them to calculate properly the β-functions we will need their analytic form. So, we start with the $A_0(m_a)$ which reads

$$A_0(m_a) = \mu^{d-4}\int \frac{d^4k}{(2\pi)^4}\frac{-i}{k^2-m_a^2} \tag{587}$$

and with the help of equation (512) for the $n=1$ case in d-dimensions we get that

$$\begin{aligned} A_0(m_a) &= \mu^{d-4}m_a^2 J_0(1,m_a) \Leftrightarrow \\ A_0(m_a) &= \frac{-\mu^{d-4}}{(4\pi)^{d/2}}\Gamma\left(1-\frac{d}{2}\right)\frac{1}{m_a^{1-d/2}}\,. \end{aligned} \tag{588}$$

Now, we should perform the expansion around the $d=4-\varepsilon$ which will give us the following relation

$$A_0(m_a) = \frac{m_a^2}{\varepsilon} + m_a^2\ln\frac{\mu^2}{m_a^2} + m_a^2\,. \tag{589}$$

Next we move on to the $B_0(p,m_1,m_2)$ case which reads

$$B_0(p,m_1,m_2) = \int \frac{d^4k}{(2\pi)^4}\frac{-i}{(k^2-m_1^2)\left((k+p)^2-m_2^2\right)} \tag{590}$$

and as we can see, the Feynman parameters are that of the equation (509), thus we get that the denominator takes the form

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{\left[A+(B-A)\,x\right]^2} \tag{591}$$

where $A=(k+p)^2-m_2^2$ and $B=k^2-m_1^2$. This gives

$$\begin{aligned} \Pi = [A+(B-A)\,x] &= \left[(k+p)^2-m_2^2+\left(k^2-m_1^2-(k+p)^2+m_2^2\right)x\right] \\ &= \left[k^2+p^2\,(1-x)+2kp\,(1-x)-m_2^2(1-x)-m_1^2x\right] \end{aligned} \tag{592}$$

then we complete the square adding the term $\pm p^2\,(1-x)^2$ and we get

$$\begin{aligned} \Pi &= \left[(k+p\,(1-x))^2+p^2x\,(1-x)-m_2^2(1-x)-m_1^2x\right] \\ &= k^2-\Delta \end{aligned} \tag{593}$$

where we have performed the shift $k \to k + p\,(x-1)$ and we have defined that $\Delta = -p^2 x\,(1-x) + m_2^2(1-x) + m_1^2 x$. Thus, now $B_0(p^2, m_1, m_2)$ becomes

$$B_0(p, m_1, m_2) = \int\limits_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{-i}{\left(k^2 - \Delta\right)^2}.$$

Now in d-dimensions, using equation (512), this integral reads

$$\begin{aligned} B_0(p, m_1, m_2) &= \int\limits_0^1 dx \int \frac{d^dk}{(2\pi)^d} \frac{-i}{\left(k^2 - \Delta\right)^2} \Leftrightarrow \\ B_0(p, m_1, m_2) &= \frac{1}{(4\pi)^{d/2}} \int\limits_0^1 dx \Gamma\left(2 - \frac{d}{2}\right)\left(\frac{1}{\Delta}\right)^{2-d/2} \end{aligned} \tag{594}$$

thus, using the expansion relations of the Gamma function and expanding around $d = 4 - \varepsilon$ we obtain

$$B_0(p, m_1, m_2) = \frac{1}{(4\pi)^2}\left(\frac{2}{\varepsilon} + \int\limits_0^1 dx \ln \frac{4\pi e^{-\gamma_E}}{\Delta}\right) \tag{595}$$

or we can have the following useful relation for our calculation which reads

$$\begin{aligned} B_0(p, m_1, m_2) &= \mu^{d-4} B_0'(p, m_1, m_2) \Leftrightarrow \\ B_0(p, m_1, m_2) &= \frac{1}{(4\pi)^2}\left(\frac{2}{\varepsilon} + \int\limits_0^1 dx \ln \frac{\mu^2}{\Delta_{B_0}(x)}\right) \end{aligned} \tag{596}$$

Following the same procedure we can see that the integral $B_0(p, m_2, m_1)$ has the same ε-expansion with the one that we have just calculated, namely with $B_0(p, m_1, m_2)$, interchanging the two masses m_1^2 and m_2^2 in the Δ expression.
Now lets move on to consider a different case from what we have seen till now, namely we will explicitly calculate the integral corresponding to the $C_0(p_1, p_2, m_a, m_a, m_a)$ case of the Appendix B. To be more specific, integrals like $C_0(p_1, p_2, m_a, m_a, m_a)$ has $1/m_a^2$ mass dimensions, therefore here

$$C_0(1,2,3) = \int \frac{d^4k}{(2\pi)^4} \frac{-i}{(k^2 - m_a^2)\left((k+p_1)^2 - m_a^2\right)\left((k+p_1+p_2)^2 - m_a^2\right)} \tag{597}$$

and as we can see its Feynman parameters correspond to the $n = 3$ case. Therefore using the relation (511) for $A = k^2 - m_a^2$, $B = (k + p_1)^2 - m_a^2$ and $C = (k + p_1 + p_2)^2 - m_a^2$ the denominator reads

$$\begin{aligned}
\Pi &= [Ax + By + Cz] \Leftrightarrow \\
\Pi &= \left[xk^2 - xm_a^2 + yk^2 + yp_1^2 + 2ykp_1 - ym_a^2 + zk^2 + z(p_1+p_2)^2 + 2zk(p_1+p_2) - zm_a^2\right] \\
&= \left[k^2 + 2k\left(p_1 y + (p_1+p_2)z\right) + yp_1^2 + z(p_1+p_2)^2 - m_a^2\right] \\
&= \left[k^2 + 2k\left(p_1 y + (p_1+p_2)z\right) \pm \left(yp_1 + z(p_1+p_2)\right)^2 + yp_1^2 + z(p_1+p_2)^2 - m_a^2\right] \Leftrightarrow \\
\Pi &= k^2 - \Delta \qquad (598)
\end{aligned}$$

where we have completed the square adding the term $\pm(yp_1 + z(p_1+p_2))^2$, we have used the relation $x + y + z = 1$ and we have done the shift $k \to k - p_1 y - (p_1+p_2)z$. Finally considering everything mentioned above, we have defined that $\Delta = -p_1^2 x(1-x) - 2p_1p_2xz - p_2^2 z(1-z) + m_a^2$. Therefore equation (597) becomes

$$C_0(1,2,3) = 2\int_0^1 dx \int_0^1 dy \int_0^1 dz \delta\left(1 - \sum_{i=1}^{3} x_i\right) \int \frac{d^4k}{(2\pi)^4} \frac{-i}{(k^2-\Delta)^3} \qquad (599)$$

and this specific form corresponds to the equation (512) for $n = 3$, thus the above integral in d-dimensions reads

$$\begin{aligned}
C_0(1,2,3) &= 2\int_0^1 dx \int_0^1 dy \int_0^1 dz \delta\left(1 - \sum_{i=1}^{3} x_i\right) \int \frac{d^dk}{(2\pi)^d} \frac{-i}{(k^2-\Delta)^3} \Leftrightarrow \\
C_0(1,2,3) &= 2\int_0^1 dx \int_0^1 dy \int_0^1 dz \delta\left(1 - \sum_{i=1}^{3} x_i\right) \frac{J_0(3,\Delta)}{m_a^2}. \qquad (600)
\end{aligned}$$

Now, in order to compute explicitly the $C_0(1,2,3)$ we should do the expansion $d = 4-\varepsilon$, thus using the expansion forms that we have presented in Appendix C.2 we obtain the final form

$$\begin{aligned}
C_0(p_1,p_2,m_a,m_a,m_a) &= \frac{-1}{(4\pi)^{d/2}} \int_0^1 dx \int_0^1 dy \int_0^1 dz \delta\left(1 - \sum_{i=1}^{3} x_i\right) \Gamma\left(3 - \frac{d}{2}\right) \left(\frac{1}{\Delta_{C_0}(x,y,z)}\right)^{3-d/2} \Leftrightarrow \\
C_0(p_1,p_2,m_a,m_a,m_a) &= -\frac{1}{(4\pi)^2} \int_0^1 dx \int_0^1 dy \int_0^1 dz \delta\left(1 - \sum_{i=1}^{3} x_i\right) \frac{1}{\Delta_{C_0}(x,y,z)} \qquad (601)
\end{aligned}$$

where we have defined the notation $x_1 \equiv x$, $x_2 \equiv y$, $x_3 \equiv z$. As we could have seen from the beginning, the above result is completely finite.

Now we move on to another interesting case that occurs in section 2 and in particular, it refers to the Box diagrams. To be more specific, here we deal with the case of the $D_0(p_1, p_2, p_3, m_a, m_a, m_a, m_a)$ which has mass-dimension $1/m_a^4$. Thus we have the following

$$D_0(1,2,3,4) = \int \frac{d^4k}{(2\pi)^4} \frac{-i}{(k^2 - m_a^2)\left((k+p_1)^2 - m_a^2\right)\left((k+p_1+p_2)^2 - m_a^2\right)\left((k+p_1+p_2+p_3)^2 - m_a^2\right)} \tag{602}$$

and as we can see its Feynman parameters correspond to the $n = 4$ case. Therefore using the relation (511) for $A = k^2 - m_a^2$, $B = (k+p_1)^2 - m_a^2$, $C = (k+p_1+p_2)^2 - m_a^2$ and $D = (k+p_1+p_2+p_3)^2 - m_a^2$ the denominator reads

$$\begin{aligned}
\Pi &= [Ax + By + Cz + Dw] \Leftrightarrow \\
\Pi &= \Big\{xk^2 - xm_a^2 + yk^2 + yp_1^2 + 2ykp_1 - ym_a^2 + zk^2 + z(p_1+p_2)^2 + 2zk(p_1+p_2) - zm_a^2 \\
&+ wk^2 + w(p_1+p_2+p_3)^2 + 2wk(p_1+p_2+p_3) - wm_a^2\Big\} \\
&= \Big\{k^2 + 2k\left(p_1 y + (p_1+p_2)z + w(p_1+p_2+p_3)\right) + yp_1^2 + z(p_1+p_2)^2 + w(p_1+p_2+p_3)^2 - m_a^2\Big\} \\
&= \Big\{k^2 + 2k\left(p_1 y + (p_1+p_2)z + w(p_1+p_2+p_3)\right) \pm \left(yp_1 + z(p_1+p_2) + w(p_1+p_2+p_3)\right)^2 \\
&+ yp_1^2 + z(p_1+p_2)^2 + w(p_1+p_2+p_3)^2 - m_a^2\Big\} \Leftrightarrow \\
\Pi &= k^2 - \Delta
\end{aligned} \tag{603}$$

where we have completed the square adding the term $\pm(yp_1 + z(p_1+p_2) + w(p_1 + p_2 + p_3))^2$, we have used the relation $x + y + z + w = 1$ and we have done the shift $k \to k - p_1 y - (p_1+p_2)z - w(p_1+p_2+p_3)$. Finally considering everything mentioned above, we have defined that $\Delta = -yp_1^2 - z(p_1+p_2)^2 - w(p_1+p_2+p_3)^2 + (yp_1 + z(p_1 + p_2) + w(p_1+p_2+p_3))^2 + m_a^2$. Therefore equation (602) becomes

$$D_0(1,2,3,4) = 6\int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dw\, \delta\left(1 - \sum_{i=1}^4 x_i\right) \int \frac{d^4k}{(2\pi)^4} \frac{-i}{(k^2 - \Delta)^4} \tag{604}$$

and in d-dimensions using equation (512) for $n = 4$ it reads

$$D_0(1,2,3,4) = 6\int_0^1 dx\int_0^1 dy\int_0^1 dz\int_0^1 dw\delta\left(1-\sum_{i=1}^{4}x_i\right)\int\frac{d^dk}{(2\pi)^d}\frac{-i}{(k^2-\Delta)^4} \Leftrightarrow$$
$$D_0(1,2,3,4) = 6\int_0^1 dx\int_0^1 dy\int_0^1 dz\int_0^1 dw\delta\left(1-\sum_{i=1}^{4}x_i\right)\frac{J_0(4,\Delta)}{m_a^4}. \quad (605)$$

Now in order to have an explicit calculation of this integral we use the exact form of the equation (512) and therefore we obtain the following

$$D_0(p_1,p_2,p_3,m_a,m_a,m_a,m_a) = \frac{1}{(4\pi)^{d/2}}\int_0^1 dx\int_0^1 dy\int_0^1 dz\int_0^1 dw\delta\left(1-\sum_{i=1}^{4}x_i\right)\frac{\Gamma\left(4-\frac{d}{2}\right)}{(\Delta_{D_0}(x,y,z,w))^{4-d/2}}$$
$$D_0(p_1,p_2,p_3,m_a,m_a,m_a,m_a) = \frac{1}{(4\pi)^2}\int_0^1 dx\int_0^1 dy\int_0^1 dz\int_0^1 dw\delta\left(1-\sum_{i=1}^{4}x_i\right)\frac{1}{\Delta^2_{D_0}(x,y,z,w)}. \quad (606)$$

where here we have defined that $x_1 \equiv x$, $x_2 \equiv y$, $x_3 \equiv z$ and $x_4 \equiv w$

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