The dynamics of Quark-Gluon Plasma and AdS/CFT

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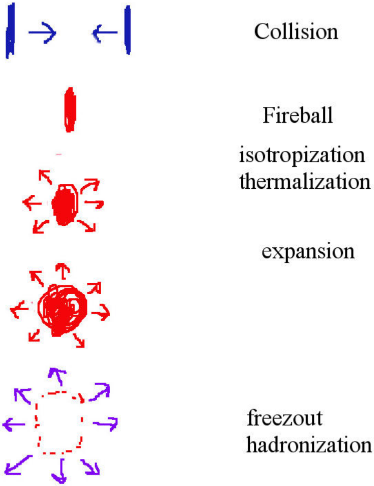
Based on work with R. Peschanski, M.P. Heller
For a review see M.P. Heller, RJ, R. Peschanski, 0811.3113
2^{nd} lecture G. Beuf, M.P. Heller, RJ, R. Peschanski, 0906.4423

Outline

Motivation

Aim: Use the AdS/CFT correspondence to study dynamical time-dependent processes for $\mathcal{N} = 4$ SYM plasma.

Point of reference: heavy-ion collision at RHIC:



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- Study properties of the expanding plasma system
- Initially focus on late stages of expansion
- Derive hydrodynamic expansion in its fully nonlinear regime
- Proceed to earlier times...
- Dissipative effects start to be important
- Consider far from equilibrium behaviour at very early times
- Understand early thermalization/isotropization

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- QCD plasma produced at RHIC is most probably a strongly coupled system
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- Conventional lattice QCD is inherently Euclidean

Study similar problems in $\mathcal{N} = 4$ SYM for which real-time nonperturbative methods exist — *the AdS/CFT correspondence*

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$\mathcal{N} = 4$ Super Yang-Mills theory	$\equiv$	Superstrings on $AdS_5 \times S^5$
strong coupling nonperturbative physics very difficult weak coupling 'easy'		(semi-)classical strings or supergravity 'easy' highly quantum regime very difficult

- New ways of looking at nonperturbative gauge theory physics...
- Intricate links with General Relativity...
- This is an equivalence! Any state/phenomenon on the gauge theory side should have its dual counterpart...
- **Caveat:** the dual counterpart does not neccessarily have to be in the well understood (super)gravity sector...

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Similarities:

- Deconfined phase
- Strongly coupled

Differences:

- No running coupling
- (Exactly) conformal equation of state
- No confinement/deconfinement phase transition

Consequently

- Plasma fireball cools indefinitely
- Even at very high energy densities the coupling remains strong

$\mathcal{N} = 4$ plasma versus QCD plasma

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- The applicability of using $\mathcal{N} = 4$ plasma to model real world phenomena depends on the questions asked..
- Use it as a toy model where we may compute from 'first principles'
- The natural language of the AdS/CFT correspondence appropriate to strongly coupled $\mathcal{N} = 4$ SYM is quite new w.r.t. conventional gauge theory methods
- Try to build some new physical intuitions within this new language
- In particular many gauge-theoretical problems are translated into quite geometrical General Relativity like questions
- Discover some universal properties? (like η/s)
- Use the results on strong coupling properties of $\mathcal{N} = 4$ plasma as a point of reference for analyzing/describing QCD plasma
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The AdS/CFT setup

- AdS_5 is the 5-dimensional spacetime

$$ds^2 = \frac{\eta_{\mu\nu} dx^\mu dx^\nu + dz^2}{z^2}$$

where $z \geq 0$

- $z = 0$ is the *boundary* of AdS_5
- $z > 0$ is the *'bulk'*
- Empty $AdS_5 \times S^5$ corresponds to the vacuum of $\mathcal{N} = 4$ SYM. In particular

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- We can excite gravitons in $AdS_5 \times S^5$ – this will correspond to some states in $\mathcal{N} = 4$ SYM with $\langle T_{\mu\nu} \rangle \neq 0$.
- When very many gravitons are excited it is better to interpret this as a change of the background

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z) dx^\mu dx^\nu + dz^2}{z^2}$$

- Seek to describe plasma in terms of the geometry $g_{\mu\nu}(x^\rho, z)$

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- Suppose our geometry is

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z)dx^\mu dx^\nu + dz^2}{z^2} \equiv g^{5D}_{\alpha\beta} dx^\alpha dx^\beta$$

- (I) What are the constraints imposed on $g_{\mu\nu}(x^\rho, z)$?
- (II) What is the corresponding energy-momentum profile $\langle T_{\mu\nu}(x^\rho)\rangle$?

Answers: see lectures by K. Skenderis

- $g_{\mu\nu}(x^\rho, z)$ has to satisfy (5D) Einstein's equations:

$$R_{\alpha\beta} - \frac{1}{2} g^{5D}_{\alpha\beta} R - 6\, g^{5D}_{\alpha\beta} = 0$$

- For a *physical state* the geometry should be nonsingular
- The profile of the energy momentum tensor can be extracted from the Taylor expansion of $g_{\mu\nu}(x^\rho, z)$ near the boundary

$$g_{\mu\nu}(x^\rho, z) = \eta_{\mu\nu} + z^4 g^{(4)}_{\mu\nu}(x^\rho) + \ldots$$

where

$$\langle T_{\mu\nu}(x^\rho)\rangle = \frac{N_c^2}{2\pi^2} \cdot g^{(4)}_{\mu\nu}(x^\rho)$$

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Answers: see lectures by K. Skenderis

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- One can turn this procedure around and *construct* for a given profile of the energy-momentum tensor $\langle T_{\mu\nu}(x^\rho) \rangle$ the dual geometry
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Example: Static uniform plasma

- Start from a constant diagonal energy momentum tensor (with $E = 3p$)

$$T_{\mu\nu} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

- Solve Einstein's equations with the above boundary condition for $g_{\mu\nu}(x^\rho, z)$...
- The result is a black hole geometry

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with z_0 expressed in terms of E $\left(E = \frac{3N_c^2}{2\pi^2 z_0^4}\right)$

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Interlude: Systems of coordinates

- Now one has to redfine the time coordinate t

$$t_{EF} = t - \frac{1}{4}\tilde{z}_0 \left(2 \arctan \frac{z_{std}}{\tilde{z}_0} + \log \frac{\tilde{z}_0 + z_{std}}{\tilde{z}_0 - z_{std}} \right)$$

- The metric becomes:

$$ds^2 = -\frac{1 - z_{std}^4/\tilde{z}_0^4}{z_{std}^2}\, dt_{EF}^2 + 2\frac{dt_{EF}\, dz_{std}}{z_{std}^2} + \frac{dx_i^2}{z_{std}^2}$$

- Now the metric is well defined at $z_{std} = \tilde{z}_0$. One can go inside the horizon...
- **Note:** The time coordinate gets an infinite shift near the horizon
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Strategy I

1. Pick some family of $\langle T_{\mu\nu}(x^\rho)\rangle$'s
2. Solve *5-dimensional* Einstein's equations to obtain the geometry

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z)dx^\mu dx^\nu + dz^2}{z^2}$$

3. Generically the above geometry will be singular.
   The $\langle T_{\mu\nu}(x^\rho)\rangle$ leading to a nonsingular geometry will be singled out as physical...

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1. Start from some (nonsingular!) initial data ($\equiv$ initial geometry in the bulk)
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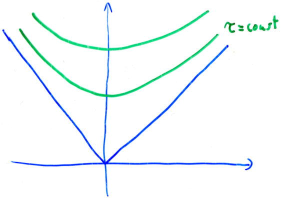
Bjorken '83

Assume a flow that is invariant under longitudinal boosts ($\equiv$ infinite energy) and does not depend on the transverse coordinates (*very* large nuclei), and has reflection symmetry.



- Pass to proper-time/spacetime rapidity coordinates $(\tau, y, x_1.x_2)$

$$t = \tau \cosh y \qquad\qquad x_3 = \tau \sinh y$$

- The only non-vanishing components of the energy-momentum tensor are $T_{\tau\tau}$, T_{yy} and $T_{xx} \equiv T_{x_1x_1} = T_{x_2x_2}$
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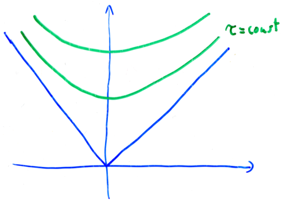

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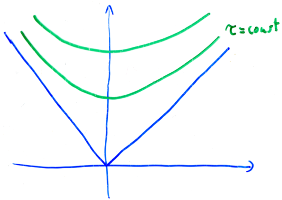

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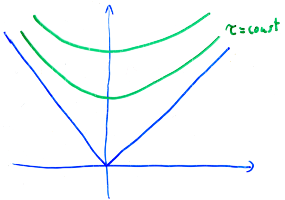

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Boost-invariant flow

- Impose tracelessness $T^\mu_\mu = 0$ and conservation of energy momentum $T^{\mu\nu}_{;\nu} = 0$
  In these coordinates they take the form

$$-T_{\tau\tau} + \frac{1}{\tau^2} T_{yy} + 2T_{xx} = 0$$

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- These equations determine $T_{\mu\nu}$ uniquely in terms of a single function $\varepsilon(\tau)$

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon(\tau) & 0 & 0 & 0 \\ 0 & -\tau^3 \frac{d}{d\tau}\varepsilon(\tau) - \tau^2 \varepsilon(\tau) & 0 & 0 \\ 0 & 0 & \varepsilon(\tau) + \frac{1}{2}\tau \frac{d}{d\tau}\varepsilon(\tau) & 0 \\ 0 & 0 & 0 & \varepsilon(\tau) + \frac{1}{2}\tau \frac{d}{d\tau}\varepsilon(\tau) \end{pmatrix}$$

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- The above decomposition was purely 'kinematical' – valid in *any* conformal 4D theory
- The determination of $\varepsilon(\tau)$ will be an issue of understanding the dynamics of the theory of interest — here $\mathcal{N} = 4$ SYM
- E.g. suppose that the system of interest behaves as a perfect fluid... Then we have
$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p\eta^{\mu\nu}$$
with $\varepsilon = 3p$
- By our symmetry assumptions $u^\mu = (1, 0, 0, 0)$ and we get in particular
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- The determination of $\varepsilon(\tau)$ will be an issue of understanding the dynamics of the theory of interest — here $\mathcal{N} = 4$ SYM
- E.g. suppose that the system of interest behaves as a perfect fluid... Then we have

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p\eta^{\mu\nu}$$

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which gives a differential equation for $\varepsilon(\tau)$

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Boost-invariant flow

- There is rich dynamical information contained in $\varepsilon(\tau)$
- We would like *not* to assume hydrodynamics but just use the AdS/CFT correspondence to determine $\varepsilon(\tau)$ for the $\mathcal{N} = 4$ SYM plasma system at strong coupling
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Examples of $\varepsilon(\tau)$

- Weak coupling (e.g. Color Glass Condensate)– free streaming

$$\varepsilon(\tau) = \frac{1}{\tau}$$

- Perfect fluid assumption

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- Fluid with viscosity $\eta = \frac{\eta_0}{\tau}$

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- Follow **'Strategy I'** discussed before...
- Consider some $\varepsilon(\tau)$ RJ,Peschanski
- Construct the dual geometry

$$\boxed{\varepsilon(\tau)} \longrightarrow \boxed{ds^2 = \frac{g_{\mu\nu}(z,\tau)dx^\mu dx^\nu + dz^2}{z^2}}$$

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$$\varepsilon(\tau) = 1/\tau^s + \ldots$$

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$$T_{\mu\nu} t^\mu t^\nu \geq 0$$

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- Construct dual geometry with the same symmetries

$$ds^2 = \frac{1}{z^2}\left(-e^{a(z,\tau)}d\tau^2 + e^{b(z,\tau)}\tau^2 dy^2 + e^{c(z,\tau)}dx_\perp^2\right) + \frac{dz^2}{z^2}$$

- Impose the boundary conditions

$$a(z,\tau) = -z^4\varepsilon(\tau) + z^6 a_6(\tau) + z^8 a_8(\tau) + \ldots$$

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$$R_{\alpha\beta} - \frac{1}{2}g^{5D}_{\alpha\beta}R - 6\, g^{5D}_{\alpha\beta} = 0$$

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$$-z^4\ \tau^{-s} + z^6\ \left(\frac{1}{6}\tau^{-s-2}s - \frac{1}{12}\tau^{-s-2}s^2\right) +$$
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- Analyzing higher orders we will see that the dominant terms in $a_n(\tau)$ for large τ will be of the form

$$z^n a_n(\tau) \sim \frac{z^n}{\tau^{\frac{ns}{4}}} = \left(\frac{z}{\tau^{\frac{s}{4}}}\right)^n \qquad \text{for large } \tau$$

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- The appearance of the scaling variable at late times is a dynamical consequence of the structure of Einstein's equations...
- At early times there does not seem to be a place for a scaling variable (see 2nd lecture)
- The separation of dynamics into a scaling variable and an expansion in inverse powers of τ corresponds to a gradient expansion recall lecture by V. Hubeny
- The appearance of a scaling variable reduces equations to ordinary differential equations!

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Asymptotic solution at large proper time

- The metric coefficients become

$$a(v) = A(v) - 2m(v)$$
$$b(v) = A(v) + (2s - 2)m(v)$$
$$c(v) = A(v) + (2 - s)m(v)$$

where

$$A(v) = \frac{1}{2}\left(\log(1 + \Delta(s)\, v^4) + \log(1 - \Delta(s)\, v^4)\right)$$
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$$\Delta(s) = \sqrt{\frac{3s^2 - 8s + 8}{24}}$$

- Now we can check the singularity of this geometry...

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Curvature singularity

- We check for curvature singularity in the limit

$$\tau \longrightarrow \infty \qquad z \longrightarrow \infty \qquad \text{with } v = \frac{z}{\tau^{\frac{s}{4}}} \text{ fixed}$$

- **Caution:** This is a subtle point to which we will return later!
- We calculate $\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}$ in the scaling limit

$$\mathfrak{R}^2 = \frac{4}{\left(1-\Delta(s)^2 v^8\right)^4} \cdot \Big[10\,\Delta(s)^8 v^{32} - 88\,\Delta(s)^6 v^{24} + 42\,v^{24} s^2 \Delta(s)^4 + \\ +112\,v^{24}\Delta(s)^4 - 112\,v^{24}\Delta(s)^4 s + 36\,v^{20} s^3 \Delta(s)^2 - 72\,v^{20} s^2 \Delta(s)^2 + \\ +828\,\Delta(s)^4 v^{16} + 288\,v^{16}\Delta(s)^2 s - 288\,v^{16}\Delta(s)^2 - 108\,v^{16} s^2 \Delta(s)^2 + \\ -136\,v^{16} s^3 + 27\,v^{16} s^4 - 320\,v^{16} s + 160\,v^{16} + 296\,v^{16} s^2 + 36\,v^{12} s^3 + \\ -72\,v^{12} s^2 - 88\,\Delta(s)^2 v^8 + 42\,v^8 s^2 + 112\,v^8 - 112\,v^8 s + 10 \Big] + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right)$$

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- The above expression is finite *only* for

$$s = \frac{4}{3}$$

- In this way we obtained that hydrodynamic evolution is the only possible behaviour of boost invariant strongly coupled plasma in $\mathcal{N} = 4$ SYM at asymptotically large proper times...

How does this geometry look like explicitly?

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The perfect fluid geometry

- The late time geometry for $s = \frac{4}{3}$ is

$$ds^2 = \frac{1}{z^2}\left[-\frac{\left(1-\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}\right)^2}{1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}}d\tau^2 + \left(1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}\right)(\tau^2 dy^2 + dx_\perp^2)\right] + \frac{dz^2}{z^2}$$

- Compare with the black hole geometry...

$$ds^2 = \frac{1}{z^2}\left[-\frac{\left(1-\frac{z^4}{z_0^4}\right)^2}{1+\frac{z^4}{z_0^4}}\,dt^2 + (1+\frac{z^4}{z_0^4})dx_i^2\right] + \frac{dz^2}{z^2}$$

- The perfect fluid geometry looks like a black hole with the position of the horizon *changing* with proper time as $z_0 = \sqrt[4]{\frac{3}{e_0}} \cdot \tau^{\frac{1}{3}}$
- Naively generalizing static formulas this corresponds to cooling of the plasma as in Bjorken expansion

$$T = \frac{\sqrt{2}}{\pi z_0} = \frac{2^{\frac{1}{2}} e_0^{\frac{1}{4}}}{\pi 3^{\frac{1}{4}}}\tau^{-\frac{1}{3}}$$

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Is this an exact perfect fluid?

Is $\varepsilon(\tau) = 1/\tau^{\frac{4}{3}}$ exact?

- Recall that we computed just the leading part of the metric corresponding to $\varepsilon(\tau) = 1/\tau^{\frac{4}{3}}$
- One can compute the subleading corrections appearing at order

$$a(z,\tau) = a_0(v) + \frac{1}{\tau^{\frac{4}{3}}} a_2(v) + \ldots$$

- At subleading order we find 4^{th} order pole singularities in the curvature

$$\mathfrak{R}^2 = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = \underbrace{R_0(v)}_{nonsingular} + \frac{1}{\tau^{\frac{4}{3}}} \underbrace{R_2(v)}_{singular!} + \ldots$$

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$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}}$$

- Keep the correction very generic and set

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- Subsequent work showed that conventional Israel-Stewart theory is incomplete! [Baier, Romatschke, Son, Starinets, Stephanov] [Bhattacharyya, Hubeny, Minwalla, Rangamani]
- There are in total four new transport coefficients in (conformal) second order viscous hydrodynamics. λ_1 is related to boost-invariant expansion.

Pitfalls with Fefferman-Graham

- It turns out that at 3^{rd} order in the late proper time expansion in *Fefferman-Graham* coordinates there is a logarithmic singularity in $\mathfrak{R}^2$
- This is a sign of the pathologies of Fefferman-Graham coordinates

- Fefferman-Graham coordinates do not allow us to pass through the horizon, in contrast to Eddington-Finkelstein
- *If* we would have an exact solution then its properties could be analyzed completely equivalently in both coordinate systems
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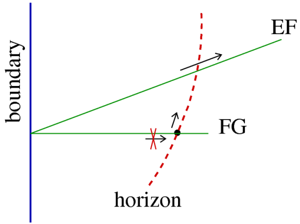

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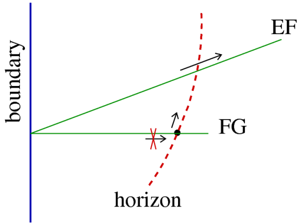

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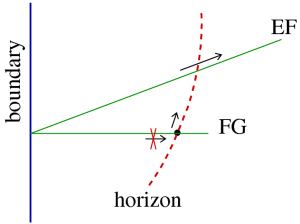

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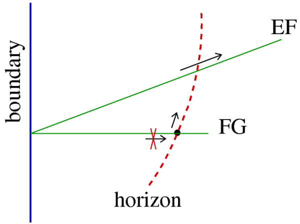

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- We picked boost-invariant setup with full transverse symmetry
- Energy-momentum tensor completely expressed in terms of $\varepsilon(\tau)$

AdS/CFT computation

- Construct dual geometry – solve Einstein's equations
- Fix $\varepsilon(\tau)$ from nonsingularity

Link with hydrodynamics

- Take $\varepsilon(\tau)$ from AdS/CFT
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Question

- Can one lift the symmetry assumptions?
- Is it possible to see hydrodynamic equations more directly?

The approach of [Bhattacharyya,Hubeny,Minwalla,Rangamani]

- Start from a static black hole with fixed temperature T which describes a fluid at rest, $u^\mu = (1,0,0,0)$ with constant energy density
- Perform a boost to obtain a uniform fluid moving with constant velocity u^μ
- The resulting metric (in Eddington-Finkelstein coordinates) is

$$ds^2 = -2u_\mu dx^\mu dr - r^2 \left(1 - \frac{T^4}{\pi^4 r^4}\right) u_\mu u_\nu dx^\mu dx^\nu + r^2(\eta_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu$$

where $r = \infty$ corresponds to the boundary, $r = T/\pi$ is the horizon while $r = 0$ is the position of the singularity.

Promote T and u^μ to (slowly-varying) functions of x^μ

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- Perform an expansion of the Einstein equations in gradients of spacetime fields.
- Find corrections to the metric at first and second order
- Require nonsingularity to fix integration constants
- Read off the resulting energy-momentum tensor $T_{\mu\nu}$
- $T_{\mu\nu}$ is expressed in terms u^μ and T and their derivatives

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$
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Going beyond hydrodynamics

Some very interesting (and very difficult) open problems are beyond the reach of hydrodynamics.

Example: isotropisation of uniform anisotropic plasma

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p_{\parallel}(t) & 0 & 0 \\ 0 & 0 & p_{\perp}(t) & 0 \\ 0 & 0 & 0 & p_{\perp}(t) \end{pmatrix}$$

$\Longrightarrow$ Cannot be treated within (even dissipative) hydrodynamics

Key questions:

- why is thermalization/isotropisation so fast?
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- Initial conditions should play an important role
- Recent studies:

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Proceed to smaller times:

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- We will need to deal with full Einstein's equations — from the point of view of hydrodynamics treated as gradient expansion these encompass all orders of viscous hydrodynamics together with infinite set of higher transport coefficients + "higher QNM modes"
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- Follow **'Strategy II'** discussed before — solve Einstein's equations starting from some initial data..
- We will use Fefferman-Graham coordinates...
- Why???
- We will solve Einstein's equations exactly so it should not matter

- We want to put initial data on a spacelike hypersurface ($\tau = \tau_0 \to 0^+$)
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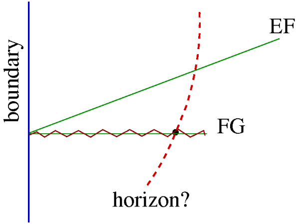

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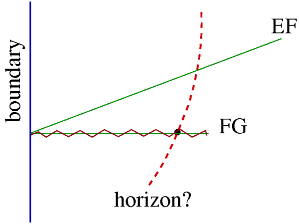

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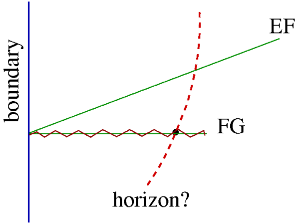

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- Construct dual geometry with the same symmetries

$$ds^2 = \frac{1}{z^2}\left(-e^{a(z,\tau)}d\tau^2 + e^{b(z,\tau)}\tau^2 dy^2 + e^{c(z,\tau)}dx_\perp^2\right) + \frac{dz^2}{z^2}$$

- Impose the boundary conditions

$$a(z,\tau) = -z^4\varepsilon(\tau) + z^6 a_6(\tau) + z^8 a_8(\tau) + \ldots$$

- Integrate Einstein's equations

$$R_{\alpha\beta} - \frac{1}{2}g^{5D}_{\alpha\beta}R - 6\,g^{5D}_{\alpha\beta} = 0$$

- The first few terms give...

$$\begin{aligned} a(\tau,z) \;=\; & -\varepsilon(\tau)\cdot z^4 + \left\{-\frac{\varepsilon'(\tau)}{4\tau} - \frac{\varepsilon''(\tau)}{12}\right\}\cdot z^6 + \left\{\frac{1}{6}\varepsilon(\tau)^2 + \frac{1}{6}\tau\varepsilon'(\tau)\varepsilon(\tau) + \right. \\ & \left. + \frac{1}{16}\tau^2\varepsilon'(\tau)^2 + \frac{\varepsilon'(\tau)}{128\tau^3} - \frac{\varepsilon''(\tau)}{128\tau^2} - \frac{\varepsilon^{(3)}(\tau)}{64\tau} - \frac{1}{384}\varepsilon^{(4)}(\tau)\right\}\cdot z^8 + \cdots \end{aligned}$$

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Early time dynamics — Interlude: scaling variable??

- Suppose that we would like nevertheless to introduce a scaling variable for

$$\varepsilon(\tau) \sim \frac{1}{\tau^s} \qquad \text{for } \tau \to 0$$

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Early time dynamics

- We have to solve the General Relativity constraint equations for the initial data — for $\tau = 0$ this can be done analytically...
- We get for $a_0(z) \equiv a(\tau = 0, z)$ etc.

$$a_0(z) = b_0(z) \qquad \dot{a}_0 = \dot{b}_0 = \dot{c}_0 = 0$$

- And we are left with a single nonlinear equation

$$a_0'' + c_0'' + \frac{1}{2}(a_0')^2 + \frac{1}{2}(c_0')^2 - \frac{1}{z}(a_0' + c_0') = 0$$

- Introduce $v(z^2) = \frac{1}{4z} a_0'(z)$ and similarly $w(z^2)$ for c_0. Then

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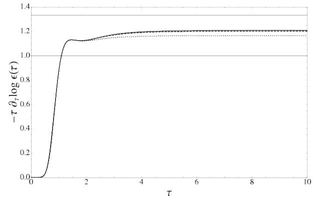

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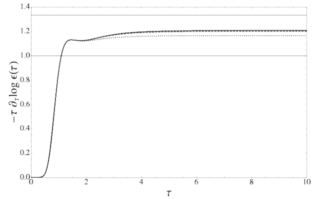

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Early time dynamics

- Assume the correct late time asymptotics

$$\varepsilon(\tau) \sim \frac{const}{\tau^{\frac{4}{3}}}$$

for the function $\varepsilon(\tau)$ and use a resummation procedure with this asymptotics...

- We get a range of energy profiles for various initial conditions

- More detailed study needed... (currently developing numerical code with P. Witaszczyk, A. Rostworowski)

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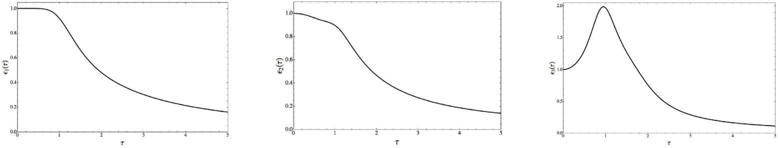

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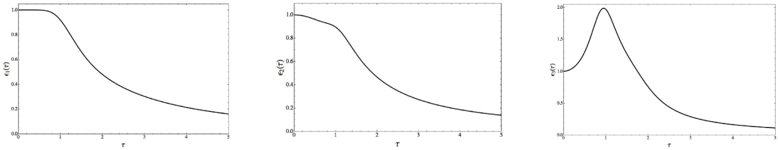

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Final remarks:

- In the boost invariant setting, late time asymptotics was determined by a single dimensionful constant

$$\varepsilon(\tau) \sim \frac{constant}{\tau^{\frac{4}{3}}} \qquad \text{for } \tau \to \infty$$

- At early times, the evolution depends on infinitely many scales ($\equiv$ shape of the initial data $a_0(z)$)
- This is very natural as we expect dissipation to wash out the initial details
- Many open questions remain to be studied — need numerics!
- Related work [Chesler,Yaffe] analyzes numerically a situation where the gauge theory metric is perturbed..

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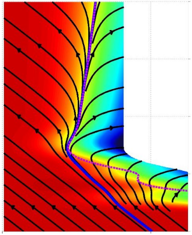
Final remarks:

- In the boost invariant setting, late time asymptotics was determined by a single dimensionful constant

$$\varepsilon(\tau) \sim \frac{constant}{\tau^{\frac{4}{3}}} \qquad \text{for } \tau \to \infty$$

- At early times, the evolution depends on infinitely many scales ($\equiv$ shape of the initial data $a_0(z)$)
- This is very natural as we expect dissipation to wash out the initial details
- Many open questions remain to be studied — need numerics!
- Related work [Chesler,Yaffe] analyzes numerically a situation where the gauge theory metric is perturbed..



Summary

- AdS/CFT predicts hydrodynamic behaviour of $\mathcal{N} = 4$ plasma
- This extends to the nonlinear regime!
- Very general framework for studying time-dependent dynamical processes or out-of-equilibrium configuration
- Investigate early time dynamics
- Get insight into thermalization...
- Can one make sense of an entropy here? Apparent/dynamical horizons very far from equilibrium??
- Eventually consider a collision process c.f. lecture by A. Yarom
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