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Celestial Free Fields

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Top-down model?

- In recent years, we have seen a lot of exciting developments on celestial amplitudes and flat space holography.
- Many general properties of the celestial amplitudes were discovered and understood, such as those related to the soft theorems.
- However, as Andy Strominger and many others like to emphasize, there isn't a concrete top-down model for the celestial holography.
- It would be desirable to have an example analogous to IIB SUGRA and 4d N=4 SYM.
- In this 10-min presentation I will share some thoughts on arguably the simplest model for celestial holography.

Celestial dual of a free field

- Consider a free, massive complex bulk scalar field Φ in the $d+2$ dimensional Minkowski spacetime $\mathbb{R}^{1,d+1}$:

$$\mathcal{L} = \frac{1}{2}\left|\partial_\mu \Phi\right|^2 - \frac{1}{2} m^2 |\Phi|^2$$

- The four-point amplitude is disconnected:

$$\mathcal{A}\left(p_i^\mu\right) \sim \delta^{(d+1)}(\hat{p}_1, \hat{p}_3)\delta^{(d+1)}(\hat{p}_2, \hat{p}_4) + \delta^{(d+1)}(\hat{p}_1, \hat{p}_4)\delta^{(d+1)}(\hat{p}_2, \hat{p}_3)$$

where $\delta^{(d+1)}(\hat{p}, \hat{p}')$ is an $SO(1, d+1)$ invariant delta function for unit time-like vectors $\hat{p} \in \mathbb{R}^{1,d+1}$.

Celestial dual of a free field

- Next, we transform the free, disconnected four-point amplitude in $\mathbb{R}^{1,d+1}$ to a celestial amplitude on $\mathbb{R}^d$. We will take the conformal dimensions to be $\Delta_i = \frac{d}{2} + i\nu_i$ with $\nu_1 = \nu_2 = \nu > 0$ and $\nu_3 = \nu_4 = \nu' > 0$ to simplify the answer (no contact terms).
- The final celestial amplitude is

$$\tilde{\mathcal{A}}(\nu, \nu', \vec{w}_i) = 2\pi m^{-2d} C(\nu)^2 \delta(\nu - \nu')^2 \frac{1}{|\vec{w}_{12}|^{2(\frac{d}{2}+i\nu)} |\vec{w}_{34}|^{2(\frac{d}{2}+i\nu)}}$$

$$\times \left[(z\bar{z})^{\frac{d}{2}+i\nu} + \left(\frac{z\bar{z}}{(1-z)(1-\bar{z})} \right)^{\frac{d}{2}+i\nu} \right]$$

Here $\vec{w}_i \in \mathbb{R}^d$ and $z\bar{z} = w_{12}^2 w_{34}^2 / w_{13}^2 w_{24}^2$ and $(1-z)(1-\bar{z}) = w_{14}^2 w_{23}^2 / w_{13}^2 w_{24}^2$. $C(\nu) = \frac{\pi^{\frac{d}{2}} \Gamma(i\nu)}{\Gamma\left(\frac{d}{2}+i\nu\right)}$.

Generalized free field

$$\tilde{\mathcal{A}}(\nu,\nu',\vec{w_i}) = 2\pi m^{-2d} C(\nu)^2 \delta(\nu-\nu')^2 \frac{1}{|\vec{w}_{12}|^{2(\frac{d}{2}+i\nu)} |\vec{w}_{34}|^{2(\frac{d}{2}+i\nu)}}$$

$$\times \left[(z\bar{z})^{\frac{d}{2}+i\nu} + \left(\frac{z\bar{z}}{(1-z)(1-\bar{z})} \right)^{\frac{d}{2}+i\nu} \right]$$

- Up to the prefactor, this is the correlation function of a complex d-dimensional generalized free field with scaling dimension $\Delta = \frac{d}{2} + i\nu$.

Free scalar in $\mathbb{R}^{1,d+1}$ $\xleftrightarrow[\textit{celestial dual?}]{}$ Generalized free scalar in $\mathbb{R}^d$

Generalized free field

- The generalized free field theory is labeled by a scaling dimension Δ_ϕ.
- It is a conformal theory whose primary operator spectrum includes a scalar field ϕ with scaling dimension Δ_ϕ:

$$\langle \bar{\phi}(\vec{w}_1)\phi(\vec{w}_2)\rangle = \frac{1}{|\vec{w}_{12}|^{2\Delta_\phi}}$$

- The other primary operators are the double-trace operators:

$$\bar{\phi}\partial_{\mu_1} \dots \partial_{\mu_\ell}\partial^{2n}\phi$$

Generalized free field

- All correlation functions are given by products of the two-point functions:

$$\langle\phi(w_1)\phi(w_2)\bar{\phi}(w_3)\bar{\phi}(w_4)\rangle$$

$$= \frac{1}{|\vec{w}_{12}|^{2\Delta_\phi}|\vec{w}_{34}|^{2\Delta_\phi}}\left[(z\bar{z})^{\Delta_\phi} + \left(\frac{z\bar{z}}{(1-z)(1-\bar{z})}\right)^{\Delta_\phi}\right]$$

- Correlation functions admit unitary conformal block and partial wave decompositions.
- See, for example, [Karateev, Kravchuk, Simmons-Duffin 2018] for the OPE functions and the OPE coefficients.

Generalized free field in AdS/CFT

- In AdS_{d+1}/CFT_d, the generalized free field is the boundary dual of a massive free bulk scalar field in AdS with mass given by $\Delta_\phi(\Delta_\phi + d) = m^2R^2$.
- It generally does not have a stress tensor, so is not a conformal *field* theory. Relatedly, there is no graviton in the bulk AdS.
- The generalized free field can be thought of as the large N limit of a scalar operator in a holographic CFT. The stress tensor only appears in the subleading order in 1/N [..., Heemskerk, Penedones, Polchinski, Sully 2009,...].
- When $\Delta_\phi = \frac{d-2}{2}$, it becomes the literal free scalar *field* theory. ($d = 2$ is more subtle.)

Conclusion

- AdS/CFT:

Free scalar in $AdS_{d+1} \leftrightarrow$ Generalized free scalar in $\mathbb{R}^d$

- Celestial holography:

Free scalar in $\mathbb{R}^{1,d+1} \underset{?}{\leftrightarrow}$ Generalized free scalar in $\mathbb{R}^d$ with $\Delta_\phi \in \frac{d}{2} + i\mathbb{R}$

- What does this celestial free theory mean?
- How do we understand the contact terms for more general scaling dimensions ν_i?