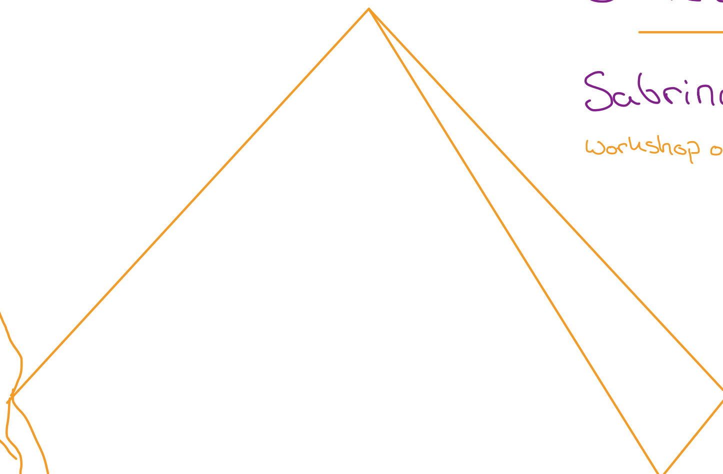
Celestial Pyramids

Sabrina Gonzalez Pasterski, PCTS

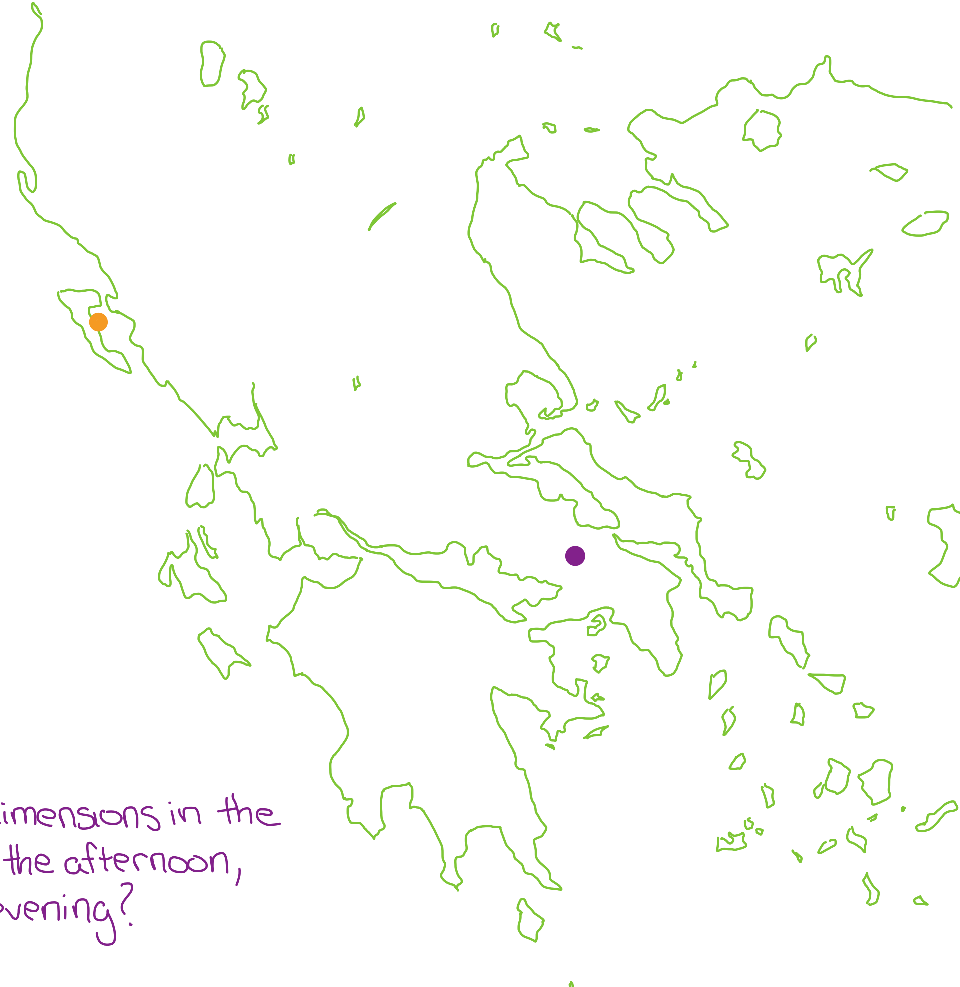
Workshop on Celestial Amplitudes & Flat Space Holography, Corfu



based on:

2012.15694, 2105.03516, 2105.09792, 2108.11422

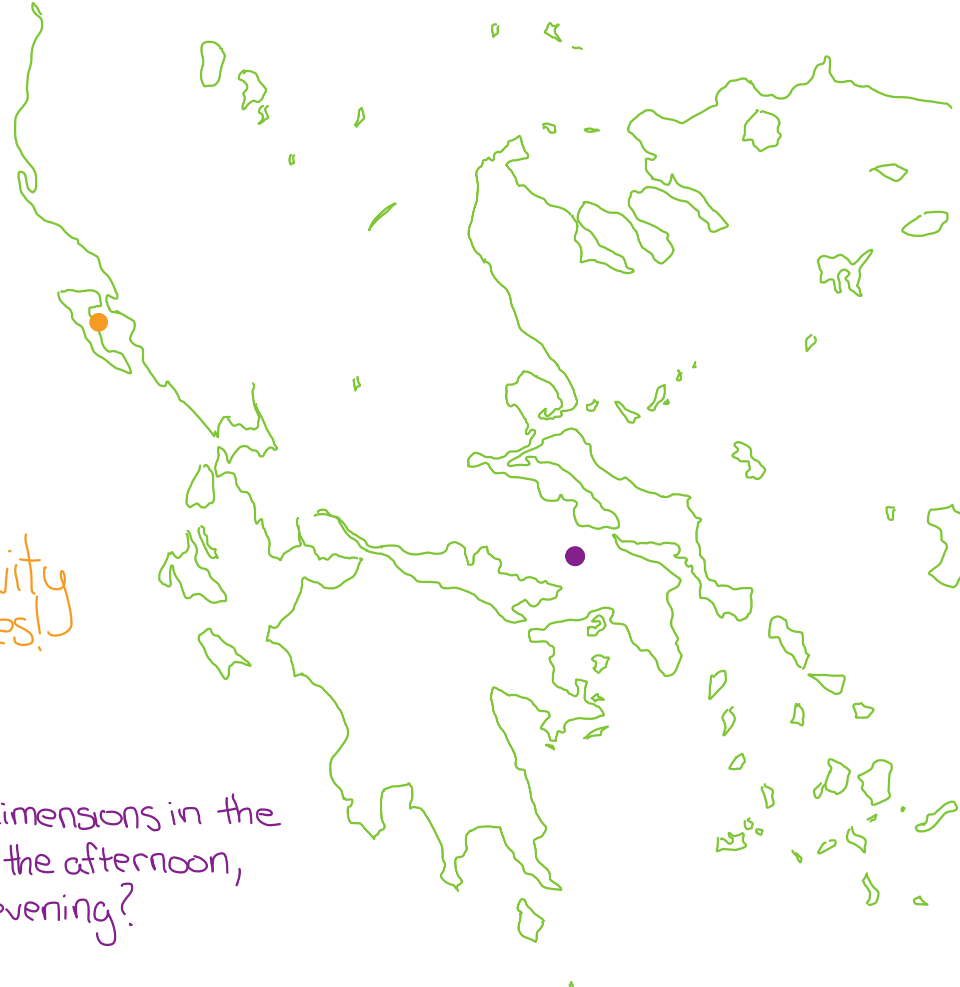
with Y. Pano, A. Puhm, & E. Trevisani



What theory lives in 4 dimensions in the morning, 2 dimensions in the afternoon, and 3 dimensions in the evening?

Quantum Gravity in ALF spacetimes!

What theory lives in 4 dimensions in the morning, 2 dimensions in the afternoon, and 3 dimensions in the evening?



Outline

→ Generalized Primaries

→ Celestial Operators via { inner product
extrapolate dict.

→ Celestial Diamonds { shadow reln's
soft charges
dressings

→ A Celestial Pyramid

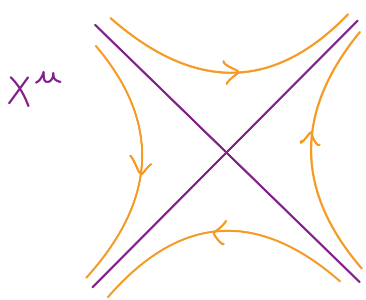
A systematic treatment of the conformal multiplets reveals surprising connections...

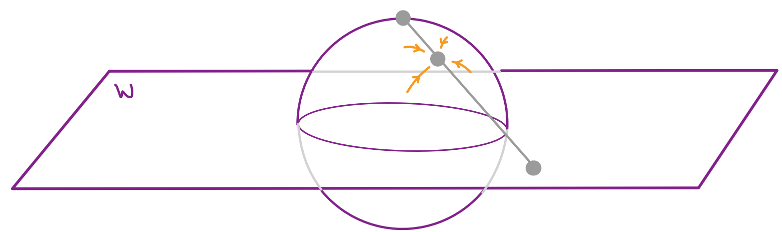
[SUSY OPEs EFTs Dressings
Null States BMS Fluxes Currents
Shadows Shockwaves]

A conformal primary wavefunction is a function of a bulk point X^μ and a reference point $w \in \mathbb{C}$ which transforms as follows

$$\Phi^{s}_{\Delta,J}\left(\Lambda^\mu{}_\nu X^\nu ; \frac{aw+b}{cw+d}, \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}\right) = (cw+d)^{\Delta+J}(\bar{c}\bar{w}+\bar{d})^{\Delta-J} D_s(\Lambda)\, \Phi^{s}_{\Delta,J}(X^\mu ; w, \bar{w})$$

Where D_s is the spin-s rep. of the Lorentz group. Radiative conformal primaries are $\Phi^{s}_{\Delta,J}$ with $s=|J|$ that solve the appropriate source free linearized eom.





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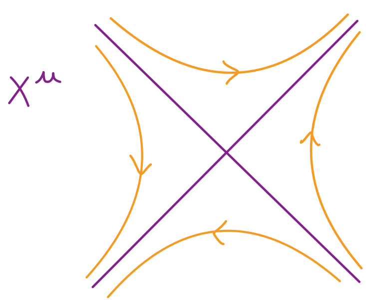
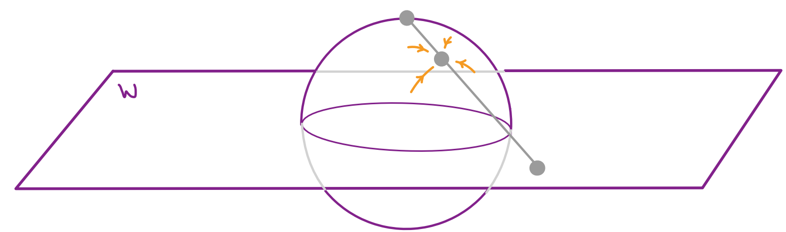
Where D_s is the spin-s rep. of the Lorentz group. For generalized conformal primaries we do not impose such additional restrictions.





Taking an inner product of such a wavefunction with the field operator gives a (quasi-)primary operator with 2D conformal dimension Δ and spin J.

$$\mathcal{O}^{s,\pm}_{\Delta,J}(w,\bar{w}) = i\left(\hat{\mathcal{O}}^{s}(X^{\mu}),\ \Phi^{s}_{\Delta^*,-J}(X^{\mu}_{\mp}; w,\bar{w})\right)_{\Sigma}$$

By taking a wavefunction-based approach we can

- ▷ Illuminate what relations hold at the level of the kinematics.
- ▷ Give a bulk interpretation of the CCFT states we are creating

The building blocks for such wavefunctions are:

- A generalized scalar primary wavefunction $\varphi_\Delta^{gen} = f(X^2)\frac{1}{(-q\cdot X)^\Delta}$ + distributional options
- A tetrad $\{l^\mu, n^\mu, m^\mu, \bar{m}^\mu\}$ & spin frame $\{o, \bar{o}, \iota, \bar{\iota}\}$ with definite $SL(2,\mathbb{C})$ weights.

	$l^\mu = \frac{q^\mu}{-q\cdot X}$	$n^\mu = X^\mu + \frac{X^2}{2} l^\mu$	$m^\mu = \varepsilon_+^\mu + \varepsilon_+ \cdot X l^\mu$	$\bar{m}^\mu = \varepsilon_-^\mu + \varepsilon_- \cdot X l^\mu$
Δ:	0	0	0	0
J:	0	0	+1	−1

obey the standard orthonormality conditions

$$l\cdot n = -1 \qquad m\cdot\bar{m} = 1 \qquad l^2 = n^2 = m^2 = l\, m = n\cdot m = 0 \qquad \bar{m} = (m)^*$$

and further decompose into a spin frame

$$l = o\bar{o} \qquad n = \iota\bar{\iota} \qquad m = o\bar{\iota} \qquad \bar{m} = \iota\bar{o}$$

where

	$o = \sqrt{\frac{2}{q\cdot X}}\begin{pmatrix}\bar{w}\\ -1\end{pmatrix}$	$\iota = \frac{1}{\sqrt{2}} X^\mu \sigma_\mu \bar{o}$
Δ:	0	0
J:	+1/2	−1/2

For a given s we can construct wavefunctions with $|J| \leq s$ by taking tensor products

$$\left\{ \begin{matrix} l, n, m, \bar{m} \\ o, \bar{o}, \iota, \bar{\iota} \end{matrix} \right\} \times \frac{1}{(-q \cdot X)^{\Delta}} \times f(X^2)$$

Looking at the case $s=1$ as an example

$$A^{\mu}_{\Delta, J=+1} = m^{\mu} f(X^2) \frac{1}{(-q \cdot X)^{\Delta}} \qquad A^{\mu}_{\Delta, J=0} = \left[l^{\mu} f_1(X^2) + n^{\mu} f_2(X^2) \right] \frac{1}{(-q \cdot X)^{\Delta}}$$

	$X^{\mu} A_{\mu}$	$\nabla^{\mu} A_{\mu}$	$\Box A_{\mu}$
$A^{gen}_{\Delta,+1}$	0	0	$4[(2-\Delta) f' + X^2 f''] m_{\mu} \varphi^{\Delta}$
$A^{gen}_{\Delta,0}$	$[-f_1 + \frac{X^2}{2} f_2] \varphi^{\Delta}$	$[-2 f_1' + (3-\Delta) f_2 + X^2 f_2'] \varphi^{\Delta}$	$\Big\{ 4[(1-\Delta) f_1' + X^2 f_1'' + \frac{1}{2} f_2] l_{\mu}$ $+4[(3-\Delta) f_2' + X^2 f_2''] n_{\mu} \Big\} \varphi^{\Delta}$

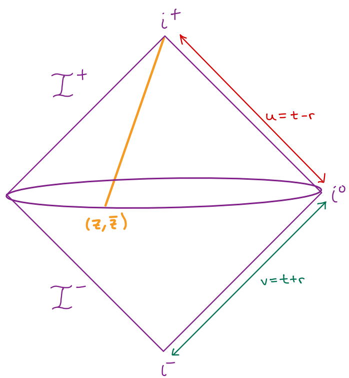
We see that $|J| = s$ + primary amounts to a gauge-fixing. As expected, the radiative solutions are $f(X^2) = c_1 + c_2 (-X^2)^{\Delta - 1}$ corresponding to $\simeq$ Mellin & shadow modes.

The question of the operator spectrum is two-fold

▷ $|J| = s$ $\quad \Delta = 1 + i\lambda$ vs $\Delta \in \mathbb{C}$

▷ $|J| < s$ $\quad$ play what role?

Let's consider the radiative solutions first. For $\mathrm{Re}\,\Delta = 1$ the spacetime fall-offs are standard. The saddlepoint approximation gives us an extrapolate-style dictionary.



$$h_{\mu\nu} = \sum_{\alpha=\pm} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k^0} \left[\varepsilon^{\alpha *}_{\mu\nu} a_\alpha e^{ik\cdot x} + \varepsilon^{\alpha}_{\mu\nu} a^\dagger_\alpha e^{-iq\cdot x} \right]$$

$$\Big\downarrow \quad \Sigma \to \mathcal{I}^\pm$$

$$\lim_{r\to\infty} \frac{1}{r} h_{\bar{z}\bar{z}} = \frac{-i}{4\pi^2} \frac{2}{(1+z\bar{z})^2} \int_0^\infty dk^0 \left[a_-(k^0\hat{x}) e^{-ik^0 u} - a^\dagger_+(k^0\hat{x}) e^{ik^0 u} \right]$$

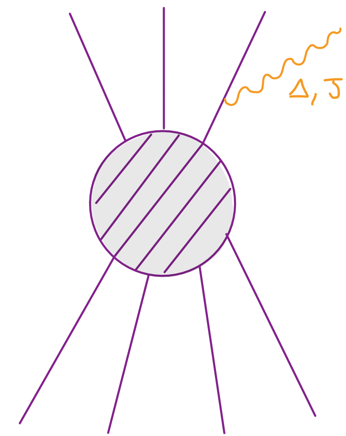
$$\Big\downarrow \quad \int_0^\infty d\omega\, \omega^{\Delta-1} \circ \int du\, e^{i\omega u}$$

$$\bar{\mathcal{O}}^-_{\Delta,-2} \propto \lim_{r\to\infty} \frac{1}{r} \int du\, u_+^{-\Delta} h_{\bar{z}\bar{z}}(u_+, r, z, \bar{z})$$

However we want to be able to explore Δ analytically continued off the principal series. Indeed, many interesting currents lie at special values of $\Delta \notin 1+i\lambda$.

$$\mathcal{O}_{\Delta,J} = i\left(\hat{\mathcal{O}}, \Phi^{G}_{\Delta^*,-J}\right)_{\Sigma}$$

↖ pure gauge



$\lvert J \rvert$	Δ	Soft Thm.	Current	Asym. Sym.
1	1	ω^{-1}	J	large U(1)
3/2	1/2	$\omega^{-1/2}$	S	large SUSY
2	1	ω^{-1}	P	supertranslations
	0	ω^{0}	$\tilde{T}$	superrotations / Diff(S^2)

While some of these Δ can be identified by demanding that the radiative primaries be pure gauge, there are more sub-leading soft theorems with a less obvious ASG interpretation.

These are all nicely captured by Celestial Diamonds.

A primary state: $L_1|h,\bar{h}\rangle = \bar{L}_1|h,\bar{h}\rangle = 0$ $h=\frac{1}{2}(\Delta+J)$ $\bar{h}=\frac{1}{2}(\Delta-J)$

will have a primary descendant at level-k when: $L_1(L_{-1})^k|h,\bar{h}\rangle = -k\overbrace{(2h+k-1)}^{0}(L_{-1})^{k-1}|h,\bar{h}\rangle = 0$

Similarly for $\bar{L}_{-1}$. When both conditions are met we get nested primaries:

$$\begin{tikzcd}
 & \left|\frac{1-k}{2},\frac{1-\bar{k}}{2}\right\rangle \arrow[dl, "(\bar{L}_{-1})^{\bar{k}}"'] \arrow[dr, "(L_{-1})^{k}"] & \\
\left|\frac{1-k}{2},\frac{1+\bar{k}}{2}\right\rangle \arrow[dr] & & \left|\frac{1+k}{2},\frac{1-\bar{k}}{2}\right\rangle \arrow[dl] \\
 & \left|\frac{1+k}{2},\frac{1+\bar{k}}{2}\right\rangle &
\end{tikzcd}$$

$k,\bar{k} \in \mathbb{Z}_{>}$

Because the hermiticity conditions of the 4D boosts differ from standard radial quantization, we should take care not to conflate primary descendants and null states.

radial quantization	CCFT
$L_i^\dagger = L_{-i}$	$L_i^\dagger = -\bar{L}_i$
$\langle h_1, \bar{h}_1 \vert (L_{-1})^k \vert h_2, \bar{h}_2 \rangle =$	$\langle h_1, \bar{h}_1 \vert (L_{-1})^k \vert h_2, \bar{h}_2 \rangle =$
$(\langle h_1, \bar{h}_1 \vert L_1^\dagger)(L_{-1})^{k-1} \vert h_2, \bar{h}_2 \rangle = 0$	$(-\langle h_1, \bar{h}_1 \vert \bar{L}_{-1}^\dagger)(L_{-1})^{k-1} \vert h_2, \bar{h}_2 \rangle \overset{?}{\neq} 0$

Understanding the out-states and the 2D state operator correspondence is an active topic. Here we will just need

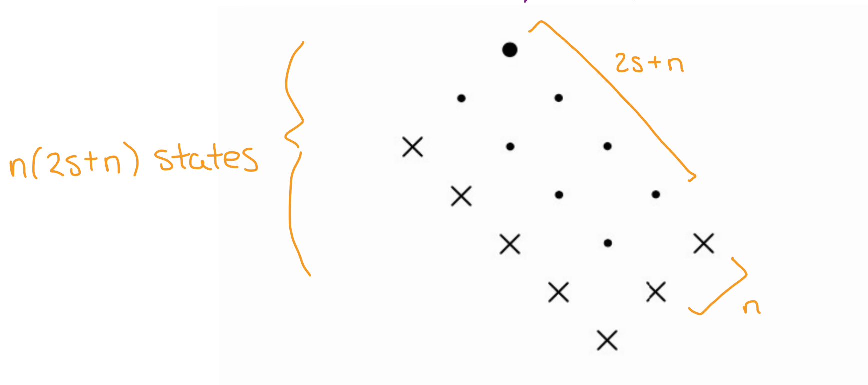
$$|h, \bar{h}\rangle \leftrightarrow \mathcal{O}_{\Delta, J} \implies L_{-1} \leftrightarrow \partial_w$$

Now there are an infinite tower of radiative primaries for which the primary descendants vanish identically and the $SL(2,\mathbb{C})$ multiplets are finite dimensional.

$$|J|=s \quad \Delta = 1-s-n \quad n\in\mathbb{Z}_{>} \quad \Longrightarrow \quad \partial_{\bar{w}}^{n}\,\bar{\Phi}^{s}_{1-s-n,-s} = \partial_{w}^{2s+n}\,\bar{\Phi}^{s}_{1-s-n,-s}=0$$

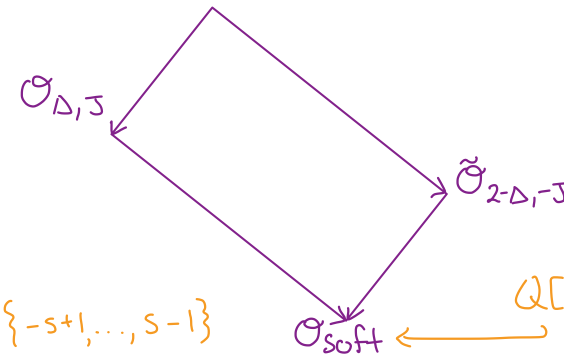
These radiative modes appear at the top of their corresponding diamonds.

$|\Delta=1-s-n, J=-s\rangle$



Meanwhile, the soft modes from our earlier table of $\Phi^{G}_{\Delta,J}$ appear at the left & right corners of their respective diamonds.

$\Delta = 1+s-n$



$Q[Y] = \int d^2z\, \mathcal{O}_{soft} \cdot Y$

$\Delta' = 1+s \qquad J' = \{-s+1, \dots, s-1\}$

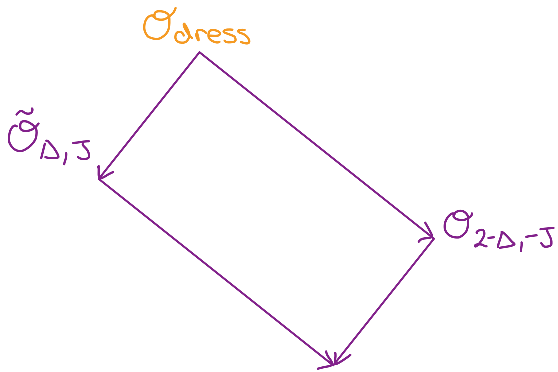
For a given diamond the left & right corners are each other's shadows

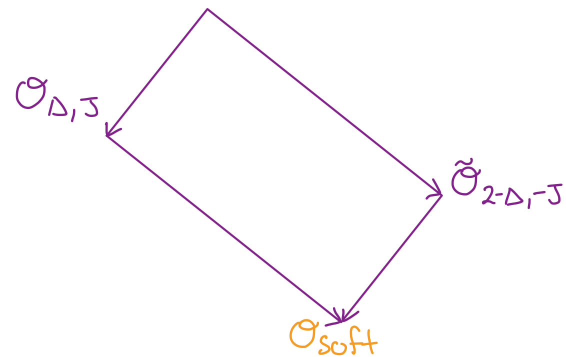
$$\partial_{\omega}^{k} \mathcal{O}_{\frac{1-k}{2},\frac{1-\bar{k}}{2}} = \frac{\Gamma(k+1)}{2\pi\Gamma(\bar{k})} \int d^2\omega' \, \frac{\partial_{\bar{\omega}'}^{\bar{k}} \mathcal{O}_{\frac{1-k}{2},\frac{1-\bar{k}}{2}}(\omega',\bar{\omega}')}{(\omega'-\omega)^{1+k}(\bar{\omega}'-\bar{\omega})^{1+\bar{k}}}$$

They descend to operators generating asymptotic symmetry transformations.

The modes at the bottom corner are generalized primaries with $|J|<s$.

By adding generalized primaries with $\Delta = 1-s$ at the top corner, we complete the nested submodule structure.





For each conformally soft theorem there are paired Goldstone & Memory diamonds which capture the spontaneous symmetry breaking dynamics of CCFT.

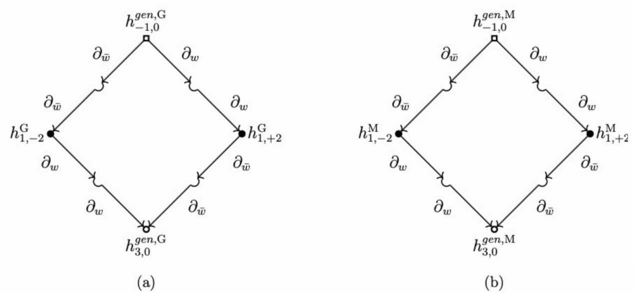
The most subleading soft theorems in momentum space correspond to degenerate diamonds. The radiative primaries at $\Delta = 1-s$ descend to their own shadows.

$$\Delta = 1+s \qquad \mathcal{O}_{\Delta,J} \xrightarrow{\;\partial_w^{2s}\;} \widetilde{\mathcal{O}}_{2-\Delta,-J}$$

$$\Delta' = 1+s \qquad J' = -J$$

g wavefunctions		m wavefunctions	
$\partial_{\bar w}\psi_{\frac12,+\frac12} = \widetilde{\psi}_{\frac32,-\frac12}$	$\partial_w\overline{\psi}_{\frac12,-\frac12} = \widetilde{\overline{\psi}}_{\frac32,+\frac12}$	$\partial_{\bar w}\widetilde{\overline{\psi}}_{\frac12,+\frac12} = -\psi_{\frac32,-\frac12}$	$\partial_w\widetilde{\psi}_{\frac12,-\frac12} = -\psi_{\frac32,+\frac12}$
$\frac{1}{2!}\partial_{\bar w}^2 A_{0,+1} = -\widetilde{A}_{2,-1}$	$\frac{1}{2!}\partial_w^2 A_{0,-1} = -\widetilde{A}_{2,+1}$	$\frac{1}{2!}\partial_{\bar w}^2 \widetilde{A}_{0,+1} = -A_{2,-1}$	$\frac{1}{2!}\partial_w^2 \widetilde{A}_{0,-1} = -A_{2,+1}$
$\frac{1}{3!}\partial_{\bar w}^3 \chi_{-\frac12,+\frac32} = -\widetilde{\chi}_{\frac52,-\frac32}$	$\frac{1}{3!}\partial_w^3 \overline{\chi}_{-\frac12,-\frac32} = -\widetilde{\overline{\chi}}_{\frac52,+\frac32}$	$\frac{1}{3!}\partial_{\bar w}^3 \widetilde{\chi}_{-\frac12,+\frac32} = \chi_{\frac52,-\frac32}$	$\frac{1}{3!}\partial_w^3 \widetilde{\chi}_{-\frac12,-\frac32} = \chi_{\frac52,+\frac32}$
$\frac{1}{4!}\partial_{\bar w}^4 h_{-1,+2} = \widetilde{h}_{3,-2}$	$\frac{1}{4!}\partial_w^4 h_{-1,-2} = \widetilde{h}_{3,+2}$	$\frac{1}{4!}\partial_{\bar w}^4 \widetilde{h}_{-1,+2} = h_{3,-2}$	$\frac{1}{4!}\partial_w^4 \widetilde{h}_{-1,-2} = h_{3,+2}$

Leading Conformally Soft Graviton



Goldstone (a) and memory (b) diamonds for the leading soft graviton theorem.

Corner	Δ	J	$i(\hat h, h^{\mathrm{G}}_{\Delta,-J})$	$h^{\log}_{\Delta,J}$
Top	-1	0	$\mathcal{N}$	$\frac{1}{2}l_\mu l_\nu \log(X^2)\varphi^{-1}$
Left	1	-2	$\frac{1}{2!}\partial_{\bar w}^2\mathcal{N}$	$\bar m_\mu \bar m_\nu \log(X^2)\varphi^1$
Right	1	2	$\frac{1}{2!}\partial_w^2\mathcal{N}$	$m_\mu m_\nu \log(X^2)\varphi^1$
Bottom	3	0	$\frac{1}{(2!)^2}\partial_w^2\partial_{\bar w}^2\mathcal{N}$	$\Big[\Big(\frac{X^2}{2}\Big)^2 l_\mu l_\nu + n_\mu n_\nu + \frac{X^2}{2}\eta_{\mu\nu} + X^2(l_\mu n_\nu + n_\mu l_\nu)\Big]\log(X^2)\varphi^3$

Elements of the celestial diamond corresponding to gravitational memory.

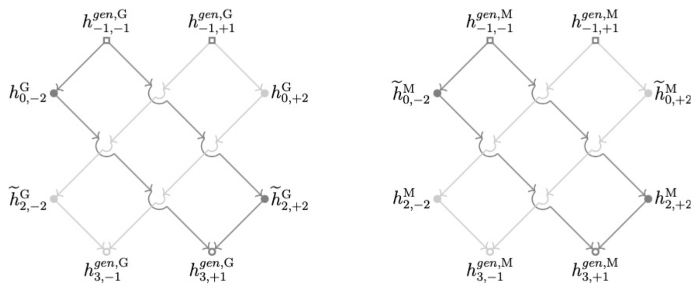
→ self-shadow & helicity redundancy tied to descendancy relations.

→ iε variant of parent is the Aichelburg Sexl geometry.

Corner	Δ	J	$i(\hat h, h^{\mathrm{M}}_{\Delta,-J})$	$h^{\mathrm{G}}_{\mu\nu;\Delta,J}$	$\xi_{\Delta,J}$	$\Lambda_{\Delta,J}$
Top	-1	0	$\mathscr{C}$	$\frac{1}{2}l_\mu l_\nu\varphi^{-1}$	$-\frac{1}{2}\varphi^{-1}\log\varphi^{-1}l^\mu$	$\frac{1}{2}\varphi^{-1}\log\varphi^{-1}$
Left	1	-2	$\frac{1}{2!}\partial_{\bar w}^2\mathscr{C}$	$\bar m_\mu \bar m_\nu\varphi^1$	$\frac{1}{2!}\partial_w^2\xi^\mu_{-1,0}$	$\frac{1}{2!}\partial_{\bar w}^2\Lambda_{-1,0}$
Right	1	2	$\frac{1}{2!}\partial_w^2\mathscr{C}$	$m_\mu m_\nu\varphi^1$	$\frac{1}{2!}\partial_{\bar w}^2\xi^\mu_{-1,0}$	$\frac{1}{2!}\partial_w^2\Lambda_{-1,0}$
Bottom	3	0	$\frac{1}{(2!)^2}\partial_w^2\partial_{\bar w}^2\mathscr{C}$	$\Big[\Big(\frac{X^2}{2}\Big)^2 l_\mu l_\nu + n_\mu n_\nu + \frac{X^2}{2}\eta_{\mu\nu} + X^2(l_\mu n_\nu + n_\mu l_\nu)\Big]\varphi^3$	$\frac{1}{(2!)^2}\partial_w^2\partial_{\bar w}^2\xi^\mu_{-1,0}$	$\frac{1}{(2!)^2}\partial_w^2\partial_{\bar w}^2\Lambda_{-1,0}$

Elements of the celestial diamond corresponding to supertranslation symmetry.

Subleading Conformally Soft Graviton



(a) (b)

Goldstone (a) and memory (b) diamonds for the subleading soft graviton theorem.

Corner	Δ	J	$i(\hat{h}, h^{G}_{\Delta,-J})$	$h^{M}_{\Delta,J}$
Top	-1	-1	$\mathscr{E}^w$	$\frac{1}{2\sqrt{2}X^2}(l_\mu \bar{m}_\nu + \bar{m}_\mu l_\nu)\varphi^{-1}$
Left	0	-2	$\partial_{\bar{w}}\mathscr{E}^w$	$\frac{1}{X^2}\bar{m}_\mu \bar{m}_\nu$
Right	2	2	$\frac{1}{3!}\partial_w^3\mathscr{E}^w$	$m_\mu m_\nu \varphi^2$
Bottom	3	1	$\frac{1}{3!}\partial_w^3\partial_{\bar{w}}\mathscr{E}^w$	$\left[\frac{X^2}{2}(l_\mu m_\nu + m_\mu l_\nu) + (n_\mu m_\nu + m_\mu n_\nu)\right]\varphi^3$

Elements of the celestial diamond corresponding to spin memory.

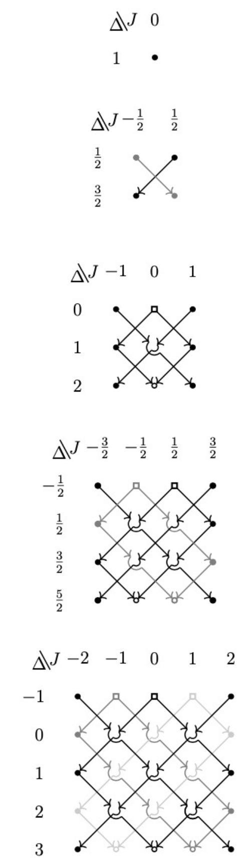
→ $\Delta = 0$ soft limit & stress tensor appear in the same diamond.

→ Dual stress tensor appears in the symplectically paired diamond.

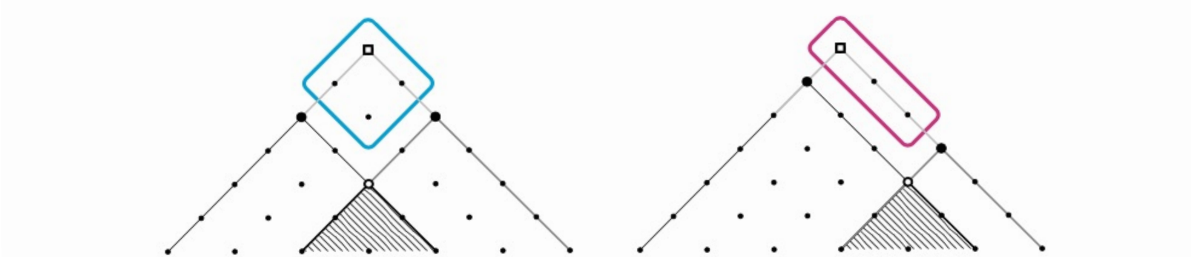
Corner	Δ	J	$i(\hat{h}, h^{M}_{\Delta,-J})$	$h^{G}_{\Delta,J}$	$\xi_{\Delta,J}$
Top	-1	-1	$\mathscr{F}^w$	$\frac{1}{2\sqrt{2}}(l_\mu \bar{m}_\nu + \bar{m}_\mu l_\nu)\varphi^{-1}$	$-\frac{1}{2\sqrt{2}}\varphi^{-1}\log\varphi^{-1}\bar{m}^\mu$
Left	0	-2	$\partial_{\bar{w}}\mathscr{F}^w$	$\bar{m}_\mu \bar{m}_\nu$	$\frac{1}{3!}\partial_w^3\xi^\mu_{-1,-1}$
Right	2	2	$\frac{1}{3!}\partial_w^3\mathscr{F}^w$	$X^2 m_\mu m_\nu \varphi^2$	$\partial_{\bar{w}}\xi^\mu_{-1,-1}$
Bottom	3	1	$\frac{1}{3!}\partial_w^3\partial_{\bar{w}}\mathscr{F}^w$	$X^2\left[\frac{X^2}{2}(l_\mu m_\nu + m_\mu l_\nu) + (n_\mu m_\nu + m_\mu n_\nu)\right]\varphi^3$	$\frac{1}{3!}\partial_w^3\partial_{\bar{w}}\xi^\mu_{-1,-1}$

Elements of the celestial diamond corresponding to superrotation symmetry.

This structure easily generalizes to arbitrary m=0 fields ½ integer spin.



While completing the celestial diamonds leads us to the conformally soft dressings.



FK dressings can be adapted to the conformal basis. For E&M

$$W_j = \exp\left[-eQ_j \int \frac{d^3k}{(2\pi)^3} \underbrace{\frac{1}{2k^0}}_{\Delta=1} \frac{p_j^{\mu}}{p_j \cdot k} \left(\varepsilon^{*}_{\alpha\mu} a - \varepsilon_{\alpha\mu} a^{+} \right)\right] = e^{iQ_j \underbrace{\Phi(z_j,\bar{z}_j)}_{SL(2,\mathbb{C})\ \text{ascendant}}}$$

and we see that the dressings take the form of a vertex operator.

The celestial amplitudes then factorize

$$\mathcal{A} = \mathcal{A}_{soft}\,\mathcal{A}_{hard}$$

where

$$\mathcal{A}_{soft} = \left\langle e^{iQ_1\Phi(z_1,\bar{z}_1)} \cdots e^{iQ_n\Phi(z_n,\bar{z}_n)} \right\rangle$$

while $\mathcal{A}_{hard}$ equals the amplitude for dressed operators.

The spontaneous symmetry breaking dynamics for 4D asymptotic symmetries is captured by simple 2D models

$$\text{Large-}U(1) \iff \text{free boson}$$

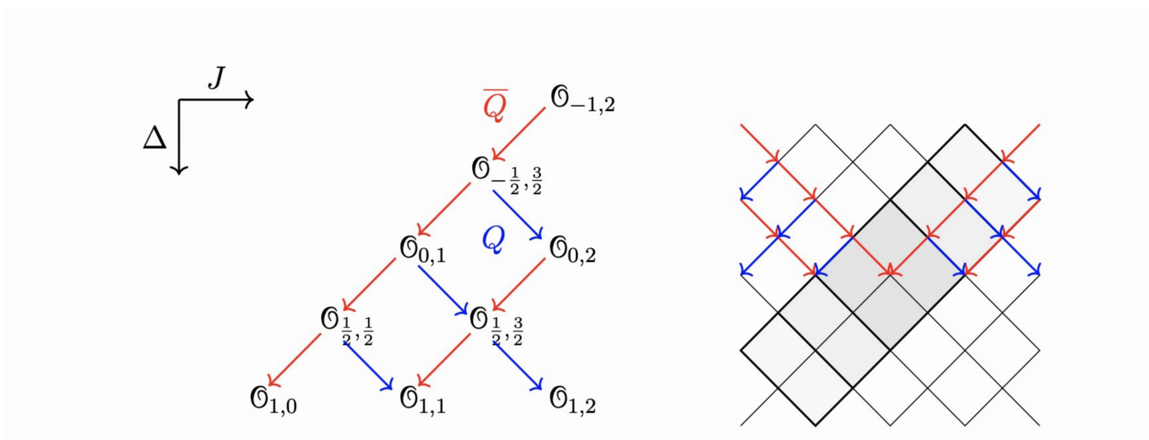
with the important observation that the levels of the 2D current algebra are set by cusp anomalous dimensions in 4D.

$$\langle \Phi(z,\bar{z})\,\Phi(w,\bar{w})\rangle = \frac{e^2}{4\pi^2} \ln\Lambda_{IR} \ln|z-w|^2$$

When we include supersymmetry, conformally soft theorems of different spin are related by the supercharges

$$Q = \partial_\theta |q\rangle e^{\partial_\Delta/2}, \qquad \overline{Q} = \theta |q] e^{\partial_\Delta/2}.$$

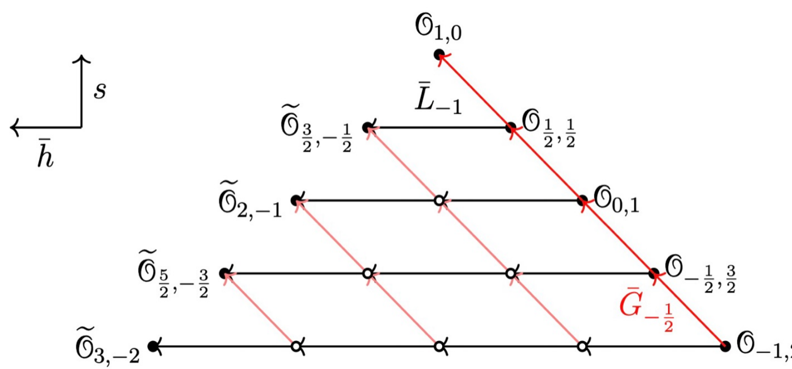
These spin-shifting symmetries tie together the celestial diamonds of different s, which stack together into a celestial pyramid.



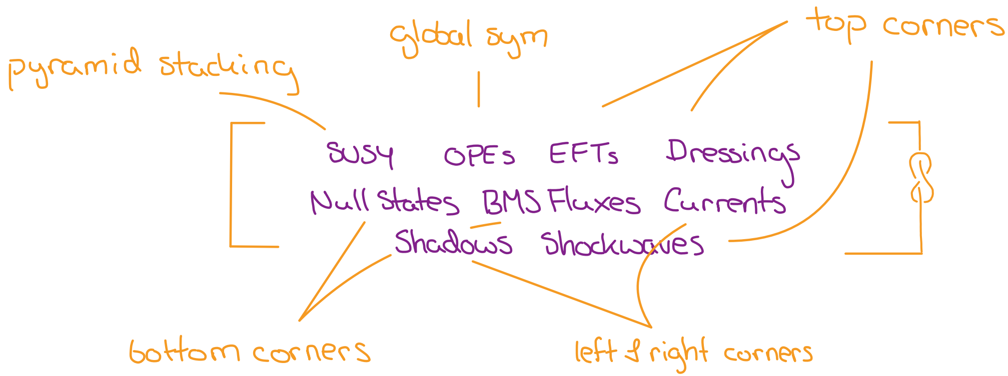
Indeed, a chiral subalgebra of the global super-poincaré multiplet structure

$$[\bar{G}_{-1/2}, \bar{L}_{-1}] = 0 \quad + \quad \begin{cases} \bar{G}_{1/2}\mathcal{O}^s_{\Delta,s} = \bar{L}_1 \mathcal{O}^s_{\Delta,s} = 0 \\ \bar{G}_{-1/2}\mathcal{O}^s_{\Delta,s} = \mathcal{O}^{s-1/2}_{\Delta+1/2,s-1/2} \end{cases} \quad + \quad \mathcal{O}^s_{soft} = \frac{1}{\bar{h}!}\partial^{\bar{h}}_{\bar{\omega}}\mathcal{O}^s_{\Delta,s}$$

explains why the parameters for susy-related soft thm ≅ ASG identities match.



We have shown how a systematic treatment of global primary descendants connects the stories of . . .



Thank You!