Pole Inflation in Supergravity

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Based on:

- C.P., *J. Cosmol. Astropart. Phys.* **05**, *043 (2021)* [arXiv:2103.05534].
- C.P., *to appear*.

Outline

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Observational Status of Minimal Chaotic Inflation (CI)

- Motivation: The Power-Law Potentials, Employed in Modes of CI, of the form

$$V_{\rm I} = \lambda^2\phi^n \quad \text{or} \quad V_{\rm I} = \lambda^2(\phi^2 - M^2)^{n/2} \quad \text{For} \quad M \ll m_{\rm P} = 1.$$

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are Very Common in Physics and so It is Easy the **Identification** of the **Inflaton** ϕ With a Field Already Present in the Theory; E.g., Within Higgs Inflation (**HI**) the Inflaton Could Play, At The End Of Inflation, The Role Of A Higgs Field.

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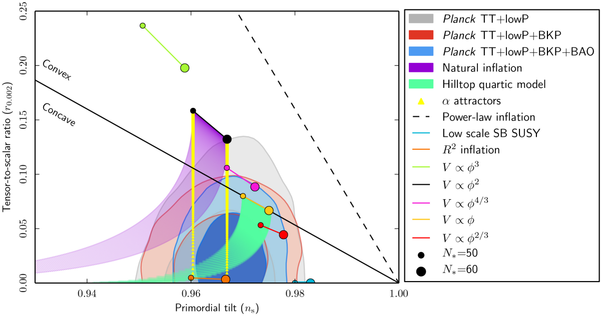
ARE VERY COMMON IN PHYSICS AND SO IT IS EASY THE **IDENTIFICATION** OF THE **INFLATON** ϕ WITH A FIELD ALREADY PRESENT IN THE THEORY; E.G., WITHIN HIGGS INFLATION (**HI**) THE INFLATON COULD PLAY, AT THE END OF INFLATION, THE ROLE OF A HIGGS FIELD.

- **HOWEVER**, FOR $n = 2, 4$ THE THEORETICALLY DERIVED VALUES FOR **SPECTRAL INDEX** $n_{\rm s}$ AND/OR **TENSOR-TO-SCALAR RATIO** r ARE NOT CONSISTENT WITH THE OBSERVATIONAL ONES.

- THE COMBINED BICEP2/*Keck Array* AND *Planck* RESULTS REQUIRE, FOR FITTED $A_{\rm s}$ AND $N_\star$ – SEE BELOW –,

$$n_{\rm s} = 0.968 \pm 0.009 \quad \text{and} \quad r = 0.028^{+0.026}_{-0.025} \quad \Rightarrow \quad 0.003 \lesssim r \lesssim 0.054 \quad \text{at 68\% c.l.} \quad \text{or} \quad r \lesssim 0.07 \quad \text{at 95\% c.l.}$$



Introducing A non-Minimal Kinetic Mixing In the Inflaton Sector

- Observational Requirements Indicate that We have to Invoke Some non-Minimality to (Or **Reconcile**) CI With Data – c.f. **Starobisky Model and α Attractors**
- Actually, There are **two Sources** of non-Minimality in Constructing Models of CI.
  - One due to **non-Minimal Coupling** of ϕ to the Ricci Scalar Curvature, $\mathcal{R}$, $f_{\mathcal{R}} \neq 1$. Here We take $f_{\mathcal{R}} = 1$
  - One due to the **non-Minimal Kinetic Mixing**, $f_{\rm K}(\phi) \neq 1$.

Under This Assumption, The Action Of the (initial real) inflaton ϕ is

$$\mathcal{S} = \int d^4x\sqrt{-\mathfrak{g}}\left(-\frac{1}{2}\mathcal{R} + \frac{f_{\rm K}(\phi)}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right),\quad \text{Where}$$

we set $m_{\rm P} = 1$ and $\mathfrak{g}$ is the Determinant Of The Background Metric $g^{\mu\nu}$.

[1] B.J. Broy et al. (2015); T. Terada (2016); T. Kobayashi et al. (2017).

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- If we Introduce the **Canonically Normalized Field**, $\widehat{\phi}$, Defined As Follows:

$$\left(\frac{d\widehat{\phi}}{d\phi}\right)^2 = J^2 = f_{\rm K} \;\;\Rightarrow\;\; \widehat{\phi} = \int d\phi J(\phi) \;\;\text{with}\;\; J = +\sqrt{f_{\rm K}}$$

the Action $\mathcal{S}$ in terms of $\widehat{\phi}$ Takes the Form

$$\mathcal{S} = \int d^4x\sqrt{-\mathfrak{g}}\left(-\frac{1}{2}\mathcal{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\widehat{\phi}\partial_\nu\widehat{\phi} - V_{\rm I}\left(\widehat{\phi}\right)\right) \;\;\text{With}\;\; V_{\rm I}(\widehat{\phi}) = V_{\rm I}\left(\widehat{\phi}(\phi)\right)\cdot$$

- We can Show that for a Suitable Choice of $f_{\rm K}$ Including A **Pole**[1] the Potential $V_{\rm I}(\widehat{\phi})$ Develops A **Plateau**, and so it Becomes Suitable to Drive Observationally Acceptable CI.
- The Analysis of CI Can Be Performed Exclusively in terms of $V_{\rm I}$ and $\widehat{\phi}$ Using The Standard Slow-Roll Approximation.

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Inflationary Observables and Requirements

- The **Number of e-foldings**, $N_\star$, that the Scale $k_\star = 0.05/\mathrm{Mpc}$ Underwent During CI has to be Sufficient to Resolve the Horizon and Flatness Problems of Standard Big Bang:

$$\widehat{N}_\star = \int_{\widehat{\phi}_\mathrm{f}}^{\widehat{\phi}_\star} d\widehat{\phi}\, \frac{V_\mathrm{I}}{V_{\mathrm{I},\widehat{\phi}}} = \int_{\phi_\mathrm{f}}^{\phi_\star} d\phi\, J^2 \frac{V_\mathrm{I}}{V_{\mathrm{I},\phi}} \simeq 52-56$$

Where $\phi_\star$ $[\widehat{\phi}_\star]$ is The Value of ϕ $[\widehat{\phi}]$ When $k_\star$ Crosses Outside The Inflationary Horizon;
ϕ_f $[\widehat{\phi}_\mathrm{f}]$ is the Value of ϕ $[\widehat{\phi}]$ at the end of HI Which Can Be Found From The Condition:

$$\max\{\widehat{\epsilon}(\phi_\mathrm{f}), |\widehat{\eta}(\phi_\mathrm{f})|\} = 1, \quad \text{With} \quad \widehat{\epsilon} = \frac{1}{2}\left(\frac{V_{\mathrm{I},\widehat{\phi}}}{V_\mathrm{I}}\right)^2 = \frac{1}{2J^2}\left(\frac{V_{\mathrm{I},\phi}}{V_\mathrm{I}}\right)^2 \quad \text{and} \quad \widehat{\eta} = \frac{V_{\mathrm{I},\widehat{\phi}\widehat{\phi}}}{V} = \frac{1}{J^2}\left(\frac{V_{\mathrm{I},\phi\phi}}{V_\mathrm{I}} - \frac{V_{\mathrm{I},\phi}}{V_\mathrm{I}}\frac{J_{,\phi}}{J}\right).$$

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- The **Amplitude A_s of the Power Spectrum** of the Curvature Perturbations is To Be Consistent with ***Planck*** Data:

$$A_\mathrm{s}^{1/2} = \frac{1}{2\sqrt{3}\,\pi}\,\frac{V_\mathrm{I}(\widehat{\phi}_\star)^{3/2}}{|V_{\mathrm{I},\widehat{\phi}}(\widehat{\phi}_\star)|} = \frac{|J(\phi_\star)|}{2\sqrt{3}\,\pi}\,\frac{V_\mathrm{I}(\phi_\star)^{3/2}}{|V_{\mathrm{I},\phi}(\phi_\star)|} = 4.588 \cdot 10^{-5}$$

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$$n_{\rm s} = 1 - 6\widehat{\epsilon}_\star + 2\widehat{\eta}_\star, \quad \alpha_{\rm s} = 2\left(4\widehat{\eta}_\star^2 - (n_{\rm s}-1)^2\right)/3 - 2\widehat{\xi}_\star \quad \text{and} \quad r = 16\widehat{\epsilon}_\star,$$

Where $\widehat{\xi} = V_{{\rm I},\widehat{\phi}} V_{{\rm I},\widehat{\phi}\widehat{\phi}\widehat{\phi}}/V_{\rm I}^2 = V_{{\rm I},\phi}\,\widehat{\eta}_{,\phi}/V_{\rm I}\,J^2 + 2\widehat{\eta}\widehat{\epsilon}$ And The Variables With Subscript $\star$ Are Evaluated at $\phi = \phi_\star$.

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- We Have To Check The Hierarchy Between **The Ultraviolet Cut-off** $\Lambda_{\rm UV} \sim m_{\rm P}$, of the Effective Theory And The Inflationary Scale. In Particular, The Validity Of The Effective Theory Implies:

$$\text{(a)}\ \ V_{\rm I}(\phi_*)^{1/4} \leq \Lambda_{\rm UV} \quad \text{for} \quad \text{(b)}\ \ \phi \leq \Lambda_{\rm UV}$$

Pole of Order Two (T-Model CI)

- Pole CI Is Most Usually Realized If We Introduce a **Pole of Order Two** in ${f_{\rm K}}$[2] i.e.,:

$$f_{\rm K} = 2N/f_{\rm 2P}^2 \ \text{ with } \ f_{\rm 2P} = 1-\phi^2 \ \text{ and } \ V_{\rm I} = V_{\rm HI} = \lambda^2\left(\phi^2 - M^2\right)^2/16 \ \text{ With } \ M \ll 1 \ \& \ N > 0\,.$$

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- **Canonically Normalizing** ϕ, we Obtain $\phi \sim \tanh\widehat{\phi}$ and hence the Name **T-Model (TM$_4$)** HI

$$\widehat{\phi} = \sqrt{N/2}\ln\left((1+\phi)/(1-\phi)\right) \quad \text{or} \quad \phi = \tanh\left(\widehat{\phi}/\sqrt{2N}\right)$$

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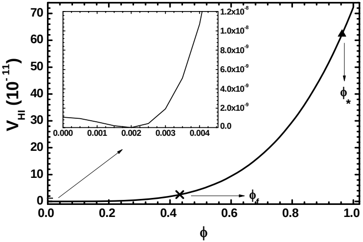
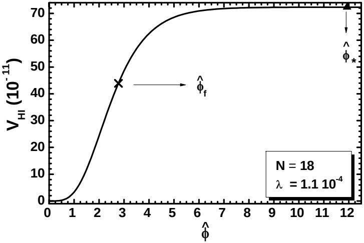
- $V_{\rm I}$ IN TERMS OF $\widehat\phi$ EXPERIENCES A **STRETCHING** FOR $\widehat\phi > 1$ WHICH RESULTS TO A PLATEAU, I.E., $V_{\rm I} = \lambda^2\tanh^4(\widehat\phi/\sqrt{2N})/16$.





HERE, $\epsilon \simeq 16 f_{\rm 2P}{}^2/N\phi^2$ AND $\eta \simeq 8 f_{\rm 2P}(3-5\phi^2)/N\phi^2$. THEREFORE, $N_\star \simeq N\phi_\star^2/f_{\rm 2p\star} \Rightarrow \phi_\star = \sqrt{4N_\star}/\sqrt{4N_\star+N} \sim 1 \gg \phi_{\rm f}$.

- THE **CONSTRAINT ON $A_{\rm s}$** YIELDS $A_{\rm s}^{1/2} \simeq \sqrt{2}\lambda N_\star/\sqrt{3N}\pi = 4.588\cdot 10^{-5} \Rightarrow \lambda \simeq 4\sqrt{6NA_{\rm s}}\pi/N_\star \Rightarrow \underline{\lambda \sim 10^{-5}}$ FOR $N_\star \simeq 55$
- THE **OTHER OBSERVABLES** ARE $n_{\rm s} \simeq 1-2/N_\star \simeq 0.965$, $\alpha_{\rm s} \simeq -2/N_\star^2 = 9.5\cdot 10^{-4}$ AND $r \simeq N/N_\star^2 \leq 0.07 \Rightarrow N \lesssim 211$.

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Pole of Order One

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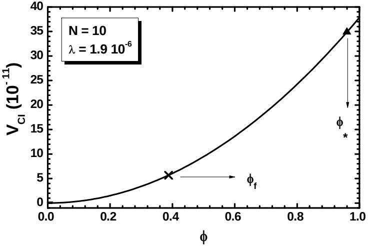
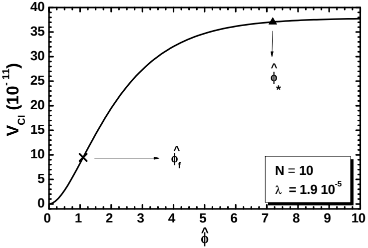
– E.G., FOR $n=2$ WE OBTAIN THE WELL-KNOWN **STAROBINSKY MODEL** AND THE PLOTS BELOW.





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- THE **OTHER OBSERVABLES** ARE $n_{\rm s}\simeq1-2/N_\star\simeq0.965$, $\alpha_{\rm s}\simeq-2/N_\star^2=9.5\cdot10^{-4}$ AND $r\simeq\underline{8N/N_\star^2\leq0.07} \Rightarrow \underline{N\lesssim26.5}$.

SUGRA Scalar Potential

- How We Can Formulate **Pole-Inflation** Within SUGRA?
- The General **Action For The Scalar Fields** z^α Plus Gravity In Four Dimensional, $\mathcal{N}=1$ SUGRA is:

$$\mathcal{S}=\int d^4x\sqrt{-\mathfrak{g}}\left(-\frac{1}{2}\mathcal{R}+K_{\alpha\bar\beta}g^{\mu\nu}D_\mu z^\alpha D_\nu z^{*\bar\beta}-V\right)\ \text{Where}\ V=V_{\rm F}+V_{\rm D}\ \text{With}\ \begin{cases}V_{\rm D}=g^2{\rm D}_a^2/2\\ V_{\rm F}=e^K\left(K^{\alpha\bar\beta}{\rm F}_\alpha{\rm F}^*_{\bar\beta}-3|W|^2\right)\end{cases}$$

Also $K_{\alpha\bar\beta}=\dfrac{\partial^2K}{\partial z^\alpha\partial z^{*\bar\beta}}>0$ and $K^{\bar\beta\alpha}K_{\alpha\bar\gamma}=\delta^{\bar\beta}_{\bar\gamma}$; $D_\mu z^\alpha=\partial_\mu z^\alpha+igA^a_\mu T^a_{\alpha\beta}z^\beta$, ${\rm F}_\alpha=W_{,z^\alpha}+K_{,z^\alpha}W$ and ${\rm D}_a=z_\alpha\left(T_a\right)^\alpha_\beta K_{,z^\beta}$

A^a_μ is The Vector Gauge Fields, g is the Gauge Coupling and T_a are the Generators of the Gauge Transformations Of z^α.

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A^a_μ is The Vector Gauge Fields, g is the Gauge Coupling and T_a are the Generators of the Gauge Transformations Of z^α.

- The Kinetic Mixing is Controlled by The **Kähler Potential** K Which Affects **Also** V. This Consists a **Complication** With Respect the non-SUSY case And We Show Below How We Arrange it in **two** Ways. V Depends on **Superpotential** W Too.
- We Concentrate on **CI Driven by $V_{\rm F}$** – As we show Below We Can Easily Assure $V_{\rm D}=0$ During CI.

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SUGRA Scalar Potential

- How We Can Formulate **Pole-Inflation** Within SUGRA?
- The General **Action For The Scalar Fields z^α** Plus Gravity In Four Dimensional, $\mathcal{N}=1$ SUGRA is:

$$\mathcal{S}=\int d^4x\sqrt{-\mathfrak{g}}\left(-\frac{1}{2}\mathcal{R}+K_{\alpha\bar\beta}g^{\mu\nu}D_\mu z^\alpha D_\nu z^{*\bar\beta}-V\right)\ \text{Where}\ V=V_{\rm F}+V_{\rm D}\ \text{With}\ \begin{cases}V_{\rm D}=g^2{\rm D}_a^2/2\\ V_{\rm F}=e^K\left(K^{\alpha\bar\beta}{\rm F}_\alpha{\rm F}^*_{\bar\beta}-3|W|^2\right)\end{cases}$$

Also $K_{\alpha\bar\beta}=\dfrac{\partial^2K}{\partial z^\alpha\partial z^{*\bar\beta}}>0$ and $K^{\bar\beta\alpha}K_{\alpha\bar\gamma}=\delta^{\bar\beta}_{\bar\gamma};\ \ D_\mu z^\alpha=\partial_\mu z^\alpha+igA^a_\mu T^a_{\alpha\beta}z^\beta,\ \ {\rm F}_\alpha=W_{,z^\alpha}+K_{,z^\alpha}W$ and ${\rm D}_a=z_\alpha\left(T_a\right)^\alpha_\beta K_{,z^\beta}$

A^a_μ is The Vector Gauge Fields, g is the Gauge Coupling and T_a are the Generators of the Gauge Transformations Of z^α.

- The Kinetic Mixing is Controlled by The **Kähler Potential** K Which Affects **Also** V. This Consists a **Complication** With Respect the non-SUSY case And We Show Below How We Arrange it in **two** Ways. V Depends on **Superpotential** W Too.
- We Concentrate on **CI Driven by $V_{\rm F}$** – As we show Below We Can Easily Assure $V_{\rm D}=0$ During CI.

Introduction of the Stabilizer Field

- Pole CI Can be Systematically Formulated in SUGRA If We Introduce A Gauge Singlet Superfield $z^1=S$ called **Stabilizer or Goldstino**. Its Introduction is Necessary For the Following Reasons:

[3] *C.P. and N. Toumbas (2016).*

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- Pole CI Can be Systematically Formulated in SUGRA If We Introduce A Gauge Singlet Superfield $z^1=S$ called **Stabilizer or Goldstino**. Its Introduction is Necessary For the Following Reasons:
  - It Generates the **non-SUSY Potential** From the term $|W_{,S}|^2$ for $S=0$. E.g., For $W=\lambda S\Phi^{\mathbf{n/2}}$ We Obtain
  $$V_{\rm F}=e^KK^{SS^*}|W_{,S}|^2\in V_{\rm non-SUSY}=\lambda^2\phi^n\quad\text{with}\quad\phi={\rm Re}(\Phi)\ \text{the (initial) inflaton.}$$
  - It Assures the **Boundedness of $V_{\rm F}$:** If We set $S=0$ During Inflation, the Terms $K_{,z^\alpha}W$, $\alpha\neq1$, and $-3|W|^2$ Vanish. The 2nd one May Render $V_{\rm F}$ Unbounded From Below.
  - It can be **Stabilized** at $S=0$ Without Invoking Higher Order Terms, if we Select [3]:
  $K_2=N_S\ln\left(1+|S|^2/N_S\right)\ \Rightarrow\ K_2^{SS^*}=1$ With $0<N_S<6$ Which Parameterizes the Compact Manifold $SU(2)/U(1)$.

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Gauge Singlet Inflaton

- **Generation of Inflaton Kinetic Mixing:** If we Adopt

$$K_{1s} = -N\ln\left(1-(\Phi+\Phi^*)/2\right),$$

We Obtain A Pole Of order 1 in the Kinetic Terms

I.e., $\mathcal{S} = \int d^4x\sqrt{-\mathfrak{g}}\left(K_{\Phi\Phi^*}\dot{\Phi}\dot{\Phi}^* + \cdots\right)$ Where $K_{\Phi\Phi^*} = \partial_\Phi\partial_{\Phi^*}K = \frac{N}{4}\frac{1}{(1-(\Phi+\Phi^*)/2)^2} = \frac{N}{4}\frac{1}{(1-\phi)^2}$ for $\Phi=\Phi^*=\phi$

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- **Cancellation of Denominator:** Due to Factor e^K, the Adopted K Results to a **Disturbing** Denominator, I.e.,

$$V_{\rm F}=e^KK^{SS^*}|W_{,S}|=\frac{V_{\rm non-SUSY}}{(1-(\Phi+\Phi^*)/2)^N}=\frac{\lambda^2\phi^n}{(1-\phi)^N}\quad\text{with}\quad K=K_2+K_{1s}$$

The Avoidance of this Denominator Is Obtained Using One of the Following Two Methods:

- **Tuning** the Form of W So that the Denominator Is Cancelled. E.g., If $W=\lambda S(\Phi-\Phi^2)$ and $N=2$, then

$$V_{\rm F}=e^KK^{SS^*}|W_{,S}|=\frac{\lambda^2(\phi-\phi^2)^2}{(1-\phi)^2}=\frac{\lambda^2\phi^2(1-\phi)^2}{(1-\phi)^2}=\lambda^2\phi^2$$

- **Modifying** K So that the new ("tilded") one,

$$\widetilde{K}_{1s}(\Phi,\Phi^*)=K_{1s}(\Phi,\Phi^*)-K_{\rm H}(\Phi)-K_{\rm A}(\Phi^*)=K_{1s}+N\ln(1-\Phi)/2+N\ln(1-\Phi^*)/2,$$

yields the Desired Kinetic Mixing But not Denominator, I.e., $\partial_\Phi\partial_{\Phi^*}\tilde{K}_{1s}=\partial_\Phi\partial_{\Phi^*}K_{1s}$ But $e^K=1$ for $\Phi=\Phi^*$.

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Working Models

- **Superpotential** $W=\lambda S(\Phi-M^2)-\lambda' S\Phi^2$.

It is the Most General W Consistent With an ***R* Symmetry** Under Which $R(S)=R(W)$.

- Model 1 **(CI1)**: $K=K_{1\rm s}=K_2+K_{1\rm s}$ with $N=2$ and $M\ll 1$ and $\lambda'\simeq\lambda(1+\delta_\lambda)$ with $\delta_\lambda=\mathcal{O}(10^{-5})$.

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$$K_{1s} = -N\ln\left(1-(\Phi+\Phi^*)/2\right),$$

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I.e., $\mathcal{S} = \int d^4x\sqrt{-\mathfrak{g}}\left(K_{\Phi\Phi^*}\dot\Phi\dot\Phi^*+\cdots\right)$ Where $K_{\Phi\Phi^*} = \partial_\Phi\partial_{\Phi^*}K = \frac{N}{4}\frac{1}{(1-(\Phi+\Phi^*)/2)^2} = \frac{N}{4}\frac{1}{(1-\phi)^2}$ for $\Phi=\Phi^*=\phi$

• **Cancellation of Denominator:** Due to Factor e^K, the Adopted K Results to a **Disturbing** Denominator, I.e.,

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- Model 2 **(CI2)**: $K=\widetilde{K}_{1s}=K_2+\widetilde{K}_{1s}$ with Free N and $M\ll 1$ in W (λ'/λ is Free).

Gauge non-Singlet Inflaton

- We Use 2 Superfields $z^2 = \Phi$, $z^3 = \bar\Phi$, **Charged** Under a Local Symmetry, e.g. $U(1)_{B-L}$, and the **"Stabilizer"** $z^1 = S$.
  - **Superpotential** $W = \lambda S\left(\bar\Phi\Phi - M^2/2\right)/2 - \lambda' S(\bar\Phi\Phi)^2$
  - W Is Uniquely Determined Using $U(1)_{B-L}$ and an R Symmetry and Leads to a **Grand Unified Theory (GUT)** Phase Transition

Charge Assignments

Superfields:	S	Φ	$\bar\Phi$
$U(1)_R$	1	0	0
$U(1)_{B-L}$	0	1	−1

At The SUSY Vacuum $\langle S\rangle = 0, |\langle\Phi\rangle| = |\langle\bar\Phi\rangle| \sim M/\sqrt{2}$

Since in The SUSY Limit, After HI, We Expect $V_{\rm eff} \simeq \frac{1}{2}\lambda^2\left|\bar\Phi\Phi + 2\frac{\lambda'}{\lambda}(\bar\Phi\Phi)^2 - \frac{1}{2}M^2\right|^2 + |S|^2(\cdots) + \text{D-terms}$

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- To Assure The **Presence Of The Pole** in $K_{\alpha\bar\beta}$ we Select One Of The Following Kähler Potentials

$$K_{21}=-N\ln\left(1-|\Phi|^2-|\bar\Phi|^2\right)\ \text{Or}\ \widetilde{K}_{21}=K_{21}+N\ln\left(1-2\bar\Phi\Phi\right)/2+N\ln\left(1-2\bar\Phi^*\Phi^*\right)/2$$

- We Observe that $\partial_\alpha\partial_{\bar\beta}\widetilde{K}_{21}=\partial_\alpha\partial_{\bar\beta}K_{21}$ Since $\partial_{\bar\beta}K_{\rm H}=\partial_\alpha K_{\rm A}=0$ where $K_{\rm H}$ and $K_{\rm A}$ Are defined as follows

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- For Both K's, the D term Due to $B-L$ Symmetry is $\mathrm{D}_{BL}=N\left(|\Phi|^2-|\bar\Phi|^2\right)/\left(1-|\Phi|^2-|\bar\Phi|^2\right)\ \Rightarrow\ V_{\rm D}=0$ If $|\Phi|=|\bar\Phi|$

I.e., D-term Can Be Eliminated During HI, if we identify inflaton With The **Radial Parts** Of Φ and $\bar\Phi$.

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- To Assure The **Presence Of The Pole** in $K_{\alpha\bar{\beta}}$ we Select One Of The Following Kähler Potentials

$$K_{21} = -N\ln\left(1 - |\Phi|^2 - |\bar{\Phi}|^2\right) \text{ Or } \widetilde{K}_{21} = K_{21} + N\ln(1 - 2\bar{\Phi}\Phi)/2 + N\ln(1 - 2\bar{\Phi}^*\Phi^*)/2$$

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- For Both K's, the D term Due to $B-L$ Symmetry is ${\rm D}_{BL} = N\left(|\Phi|^2 - |\bar{\Phi}|^2\right)/\left(1 - |\Phi|^2 - |\bar{\Phi}|^2\right) \Rightarrow V_{\rm D} = 0$ If $|\Phi| = |\bar{\Phi}|$

I.e., D-term Can Be Eliminated During HI, if we identify inflaton With The **Radial Parts** Of Φ and $\bar{\Phi}$.

- The **Difference Between K_{21} and $\widetilde{K}_{21}$** Arises from e^K in $V_{\rm HI}$. Along the Inflationary Path, $|\Phi| = |\bar{\Phi}|$,
  - $K = K_{21}$ Yields a Denominator in $V_{\rm F}$ Which Can be Almost Cancelled Out by **Tuning λ'/λ** in W
  - $K = \widetilde{K}_{21}$ does not Lead to a Denominator and so We can Use $\lambda' = 0$.

GAUGE NON-SINGLET INFLATON

- WE USE 2 SUPERFIELDS $z^2 = \Phi$, $z^3 = \bar\Phi$, **CHARGED** UNDER A LOCAL SYMMETRY, E.G. $U(1)_{B-L}$, AND THE **"STABILIZER"** $z^1 = S$.
  - **SUPERPOTENTIAL** $W = \lambda S\left(\bar\Phi\Phi - M^2/2\right)/2 - \lambda' S(\bar\Phi\Phi)^2$
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SINCE IN THE SUSY LIMIT, AFTER HI, WE EXPECT $V_{\rm eff} \simeq \frac{1}{2}\lambda^2\left|\bar\Phi\Phi + 2\frac{\lambda'}{\lambda}(\bar\Phi\Phi)^2 - \frac{1}{2}M^2\right|^2 + |S|^2(\cdots) + $ D-TERMS

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- WE OBSERVE THAT $\partial_\alpha\partial_{\bar\beta}\widetilde{K}_{21} = \partial_\alpha\partial_{\bar\beta}K_{21}$ SINCE $\partial_{\bar\beta}K_{\rm H} = \partial_\alpha K_{\rm A} = 0$ WHERE $K_{\rm H}$ AND $K_{\rm A}$ ARE DEFINED AS FOLLOWS

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I.E., D-TERM CAN BE ELIMINATED DURING HI, IF WE IDENTIFY INFLATON WITH THE **RADIAL PARTS** OF Φ AND $\bar\Phi$.

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WORKING MODELS

- MODEL 1 **(HI1)**: $K = K_{221} = K_2 + K_{21}$ WITH $N = 2$ AND $\lambda' = \lambda(1 + \delta_\lambda)$ IN W WITH $\delta_\lambda = O(10^{-5})$;

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Working Models

- Model 1 **(HI1)**: $K = K_{221} = K_2 + K_{21}$ with $N = 2$ and $\lambda' = \lambda(1+\delta_\lambda)$ in W with $\delta_\lambda = O(10^{-5})$;
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Geometry of $K_{1\rm s}$ and $\widetilde{K}_{1\rm s}$

- The Geometry of $K_{1\rm s}$ and $\widetilde{K}_{1\rm s}$ is Determined by **Riemannian Metric And The Scalar Curvature**, $\mathcal{R}_K$, calculated by

$$ds_K^2 = K_{\Phi\Phi^*}\,d\Phi d\Phi^* \quad \text{and} \quad \mathcal{R}_K = -K^{\Phi\Phi^*}\partial_\Phi\partial_{\Phi^*}\ln\left(K_{\Phi\Phi^*}\right).$$

- For $K = K_{1\rm s}$ and $\widetilde{K}_{1\rm s}$, We Obtain the Line Element and the Scalar Curvature

$$ds_{1\rm s}^2 = \frac{N}{4}\frac{d\Phi d\Phi^*}{(1-(\Phi+\Phi^*)/2)^2} \quad \text{and} \quad \mathcal{R}_{1\rm s} = -\frac{2}{N}.$$

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$$ds_{1\rm s}^2 = \frac{N}{4}\,\frac{d\Phi d\Phi^*}{(1-(\Phi+\Phi^*)/2)^2} \quad \text{and} \quad \mathcal{R}_{1\rm s} = -\frac{2}{N}.$$

- $ds_{1\rm s}^2$ Remains **Invariant** under the Transformations

$$\frac{\Phi}{2} \to \frac{a\Phi/2+b}{c\Phi/2+d} \quad \text{Represented By} \quad \mathcal{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad \text{Provided that } b=0,\ c=2a,\ d=-1/a^* \text{ and } |a|^2=1. \quad (T_1)$$

- The Matrix $\mathcal{M}$ Has the Following Forms and Properties Respectively

$$\mathcal{M} = \begin{pmatrix} a & 0 \\ 2a & -1/a^* \end{pmatrix} \quad \text{and} \quad \mathcal{M}^\dagger\Omega\mathcal{M} = -\Omega \quad \text{with} \quad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Therefore, the Matrices $\mathcal{M}$ Are not Elements of a Subgroup of $GL(2,\mathbb{C})$.

- The **Iwasawa Decomposition** of $\mathcal{M}$ is

$$\mathcal{M} = KAN \quad \text{with} \quad K = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} \sqrt{5}a & 0 \\ 0 & -a/\sqrt{5} \end{pmatrix} \quad \text{and} \quad N = \begin{pmatrix} 1 & -2/5 \\ 0 & 1 \end{pmatrix}.$$

Where the Matrices K, A and N Parametrize The Compact, Abelian And Nilpotent Transformations of the Möbius Group.

Geometry of K_{1s} and $\widetilde{K}_{1s}$

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The Kähler Manifold Corresponding to K_{21} and $\widetilde{K}_{21}$

- The Geometry of the Kähler Manifolds is Determined by **Riemannian Metric And The Scalar Curvature**, $\mathcal{R}_K$, calculated by

$$ds_K^2 = K_{\alpha\bar{\beta}} dz^\alpha dz^{*\bar{\beta}} \quad \text{and} \quad \mathcal{R}_K = -K^{\alpha\bar{\beta}}\partial_\alpha\partial_{\bar{\beta}} \ln\left(\det M_{\bar{\Phi}\Phi}\right), \quad z^{\alpha,\beta} = \Phi, \bar{\Phi}$$

Here $M_{\Phi\bar{\Phi}}$ Expresses The Kinetic Mixing In The Inflationary Sector.

- For $K = K_{21}$ and $\widetilde{K}_{21}$, We Obtain the **Bergmann Metric**, Which Parameterize the $SU(2,1)/(SU(2)\times U(1))$ Manifold. I.e.,

$$ds_{21}^2 = N\left(\frac{|d\Phi|^2 + |d\bar{\Phi}|^2}{1-|\Phi|^2-|\Phi|^2} + \frac{|\Phi^* d\Phi + \bar{\Phi}^* d\bar{\Phi}|^2}{\left(1-|\Phi|^2-|\bar{\Phi}|^2\right)^2}\right) \quad \text{and} \quad \mathcal{R}_{21} = -\frac{6}{N}.$$

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$$U = \mathcal{U}\mathcal{P} \quad \text{with} \quad \mathcal{U} = \underbrace{\begin{pmatrix} 1/N_a & 0 & a \\ N_a b a^* & N_a\gamma & b \\ N_a\gamma a^* & N_a b^* & \gamma\end{pmatrix}}_{\in SU(2,1)/(SU(2)\times U(1))} \quad \text{and} \quad \mathcal{P} = e^{i\vartheta}\underbrace{\begin{pmatrix} d & f & 0 \\ -f^* & d^* & 0 \\ 0 & 0 & e^{-3i\vartheta}\end{pmatrix}}_{\in SU(2)\times U(1)}, \quad \text{where} \quad \begin{cases} N_a = 1/\sqrt{1+|a|^2} \\ |a|^2+|b|^2-\gamma^2 = -1 \\ |d|^2+|f|^2 = 1.\end{cases}$$

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Whereas $\widetilde{K}_{21}$ does not Enjoy such an Invariance.

Gauge Singlet Inflaton

- Expanding Φ and S as Follows:

$$\Phi = \phi e^{i\theta} \quad \text{and} \quad S = (s_1 + is_2)/\sqrt{2},$$

We Can Introduce The Canonically Normalized Fields,

$$d\widehat{\phi}/d\phi = J \simeq \sqrt{N/2}/f_{1P}, \quad \widehat{\theta} \simeq J\phi\theta \quad \text{and} \quad \widehat{s_i} = s_i \quad \text{with} \quad i = 1, 2 \quad (\text{Recall} \quad f_{1P} = 1 - \phi)$$

Where We observe that We Established the Correct **Non-Minimal Kinetic Mixing**.

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- For $S = 0$ and $\theta = 0$, the only Surviving term of $V_{\rm F}$ is

$$V_{\rm CI} = e^K K^{SS^*} |W_{,S}|^2 = \lambda^2 \cdot \begin{cases} \left(\phi - (1+\delta_\lambda)\phi^2 - M^2\right)^2 / f_{1\rm P}^N & \simeq \lambda^2\phi^2 \quad \text{for } N = 2 \text{ and } M = \delta_\lambda = 0 \text{ (CI1)}, \\ \left(\phi - \lambda'\phi^2/\lambda - M^2\right)^2 & \simeq \lambda^2\phi^2 \quad \text{for } \lambda' = M = 0 \text{ (CI2)} \end{cases}$$

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Scalar **Mass-Squared Spectrum** for $K = K_{1\mathrm{S}}$ and $\widetilde{K}_{1\mathrm{S}}$ Along The Inflationary Trajectory

Fields	Eigen-states	Masses Squared		
			$K = K_{1\mathrm{S}}$	$K = \widetilde{K}_{1\mathrm{S}}$
1 real scalar	$\widehat{\theta}$	$\widehat{m}^2_\theta$	$6H^2_\mathrm{CI}$	
2 real scalars	$\widehat{s}_1, \widehat{s}_2$	$\widehat{m}^2_s$	$6H^2_\mathrm{CI}/N_S$	
2 Weyl spinors	$\widehat{\psi}_\pm = \frac{\widehat{\psi}_\Phi \pm \widehat{\psi}_S}{\sqrt{2}}$	$\widehat{m}^2_{\psi\pm}$	$\sqrt{6}(1-\phi)H^2_\mathrm{CI}/\sqrt{N}\phi$	

We Observe the Following:

- All mass$^2 > 0$. Especially $m^2_{\widehat{S}} > 0 \;\Leftrightarrow\; N_S < 6$;
- All mass$^2 > H^2_\mathrm{CI}$ and So Any Inflationary Perturbations Of The Fields Other Than The Inflaton Are Safely Eliminated.
- The One-Loop Radiative Corrections Have No Significant Effect.

Gauge Non-Singlet Inflaton

- If We Use The **Parametrizations**: $\Phi = \phi e^{i\theta}\cos\theta_\Phi$ and $\bar\Phi = \phi e^{i\bar\theta}\sin\theta_\Phi$ with $0 \leq \theta_\Phi \leq \pi/2$ and $S = (s + i\bar s)/\sqrt{2}$

We Can Show That A **D-Flat Direction** Is $\theta = \bar\theta = 0,\ \theta_\Phi = \pi/4$ and $S = 0$ (: P) Which Is Qualified as **Inflationary Path**.

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- The only Surviving term of $V_{\rm F}$ Along the Path in Eq. (P) is

$$V_{\rm HI} = e^K K^{SS^*}\,|W_{,S}|^2 = \frac{\lambda^2}{16}\cdot\begin{cases}\left(\phi^2-(1+\delta_\lambda)\phi^4-M^2\right)^2/f_{2\rm P}^N & \simeq \lambda^2\phi^4/16 \ \text{ for } N=2 \text{ and } M=\delta_\lambda=0 \text{ (HI1)},\\ \left(\phi^2-M^2\right)^2 & \simeq \lambda^2\phi^4/16 \ \text{ for } \lambda'=M=0 \text{ (HI2)}\end{cases}$$

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- To Obtain TM_4, We Have to Establish the Correct **Non-Minimal Kinetic Mixing**.
- To This End We Compute The **Kähler Metric $K_{\alpha\bar\beta}$** Along the Path in Eq. (P) Which Takes The Form

$$\left(K_{\alpha\bar\beta}\right) = \mathrm{diag}\left(M_{\Phi\bar\Phi}, K_{SS^*}\right) \text{ with } M_{\Phi\bar\Phi} = \frac{\kappa\phi^2}{2}\begin{pmatrix} 2/\phi^2 - 1 & 1\\ 1 & 2/\phi^2 - 1\end{pmatrix},\ \kappa = N/f_{2\rm P}^2 \text{ and } K_{SS^*} = 1.$$

And **Diagonalize** $M_{\Phi\bar\Phi}$ Via A Similarity Transformation As Follows:

$$U_{\Phi\bar\Phi} M_{\Phi\bar\Phi} U_{\Phi\bar\Phi}^{\mathsf T} = \mathrm{diag}\left(\kappa_+, \kappa_-\right), \text{ where } U_{\Phi\bar\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1\\ -1 & 1\end{pmatrix} \text{ and } \kappa_+ = \kappa,\ \kappa_- = \kappa f_{2\rm P}$$

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$$\frac{d\widehat\phi}{d\phi} = J = \sqrt{2\kappa_+} \ \Rightarrow\ \phi = \tanh\frac{\widehat\phi}{2\sqrt N},\ \widehat\theta_+ = \sqrt{\kappa_+}\phi\theta_+,\ \widehat\theta_- = \sqrt{\kappa_-}\phi\theta_-,\ \text{ and } \ \widehat\theta_\Phi = \phi\sqrt{2\kappa_-}\left(\theta_\Phi-\pi/4\right),\ \left(\widehat s,\widehat{\bar s}\right) = (s,\bar s)\cdot$$

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And **Diagonalize** $M_{\Phi\bar\Phi}$ Via A Similarity Transformation As Follows:

$$U_{\Phi\bar\Phi}M_{\Phi\bar\Phi}U_{\Phi\bar\Phi}^{\mathsf T} = \mathrm{diag}\left(\kappa_+,\kappa_-\right), \text{ WHERE } U_{\Phi\bar\Phi} = \frac{1}{\sqrt2}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix} \text{ AND } \kappa_+ = \kappa,\ \kappa_- = \kappa f_{2\rm P}$$

- The EF **Canonically Normalized Fields**, Which Are Denoted By Hat, Can Be Obtained As Follows:

$$\frac{d\widehat\phi}{d\phi} = J = \sqrt{2\kappa_+} \Rightarrow \phi = \tanh\frac{\widehat\phi}{2\sqrt{N}},\ \widehat\theta_+ = \sqrt{\kappa_+}\phi\theta_+,\ \widehat\theta_- = \sqrt{\kappa_-}\phi\theta_-, \text{ AND } \widehat\theta_\Phi = \phi\sqrt{2\kappa_-}\left(\theta_\Phi-\pi/4\right),\ \left(\widehat s,\widehat{\bar s}\right) = (s,\bar s)\cdot$$

- We Have, Also, to Check the **Stability** of the Trajectory in Eq. (P) w.r.t the Fluctuations Of The Various Fields, i.e.

$$\left.\frac{\partial V}{\partial\widehat z^\alpha}\right|_{\rm Eq.\ (P)} = 0 \text{ AND } \widehat m^2_{z^\alpha} > 0 \text{ Where } \widehat m^2_{z^\alpha} = \mathrm{Egv}\left[\widehat M^2_{\alpha\beta}\right] \text{ With } \widehat M^2_{\alpha\beta} = \left.\frac{\partial^2 V}{\partial\widehat z^\alpha\partial\widehat z^\beta}\right|_{\rm Eq.\ (P)} \text{ AND } z^\alpha = \theta_-,\theta_+,\theta_\Phi,s,\bar s.$$

Here Egv are the Eigenvalues of the Matrix $\widehat M^2_{\alpha\beta}$.

Stability of The Inflationary Direction

Scalar **Mass-Squared Spectrum** for $K = K_{221}$ and $\widetilde{K}_{221}$ Along The Inflationary Trajectory

Fields	Eigen-states	Masses Squared		
			$K = K_{221}$	$K = \widetilde{K}_{221}$
2 real scalars	$\widehat{\theta}_+$	$m^2_{\widehat{\theta}+}$	$3H^2_{\rm HI}$	
	$\widehat{\theta}_\Phi$	$\widehat{m}^2_{\theta_\Phi}$	$M^2_{BL} + 6H^2_{\rm HI}(1 + 1/N - 1/N\phi^2)$	
1 complex scalar	$s, \bar{s}$	$\widehat{m}^2_s$	$6H^2_{\rm HI}(1/N_S - 4(1-\phi^2)/N + N\phi^2 + 2(1-2\phi^2) + 4\phi^2/N)$	$6H^2_{\rm HI}(1/N_S - 2/N + 1/N\phi^2 + \phi^2/N)$
1 gauge boson	A_{BL}	M^2_{BL}	$4Ng^2\phi^2/f^2_{2\rm P}$	
4 Weyl spinors	$\widehat{\psi}_\pm$	$\widehat{m}^2_{\psi\pm}$	$3f^2_{2\rm P}H^2_{\rm HI}/N^2\phi^2$	
	$\lambda_{BL}, \widehat{\psi}_{\Phi-}$	M^2_{BL}	$4Ng^2\phi^2/f^2_{2\rm P}$	

- We can Obtain $\forall\,\alpha,\ \widehat{m}^2_{\chi^\alpha} > 0$. Especially $\widehat{m}^2_s > 0 \ \Leftrightarrow\ N_S < 6$.

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Stability of The Inflationary Direction

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- We Determine M **Demanding** That The Unification Scale $M_{\rm GUT}\simeq 2/2.433\times10^{-2}$ is Identified with M_{BL} at the Vacuum, I.e.,

$$\langle M_{BL}\rangle=\sqrt{2N}gM/\langle f_{\rm 2P}\rangle=M_{\rm GUT}\ \Rightarrow\ M\simeq M_{\rm GUT}/g\sqrt{2N}\ \text{ with }\ g\simeq0.7\ \text{(GUT Gauge Coupling)}.$$

Stability of The Inflationary Direction

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- The One-Loop **Radiative Corrections** à la Coleman-Weinberg to $V_{\rm I}$ Can Be Kept Under Control.

Testing Against The Inflationary Data

- Enforcing $N_\star \simeq 52 - 56$ and $\sqrt{A_s} = 4.588 \cdot 10^{-5}$, we Obtain the Allowed Curves for Our Models In the $n_s - r_{0.002}$ Plane
- In **Both Models** $\phi_\star \sim 1$ and the Relevant **Tuning** can be Qualified by Computing $\Delta_\star = (1 - \phi_\star)/1$.

TESTING AGAINST THE INFLATIONARY DATA

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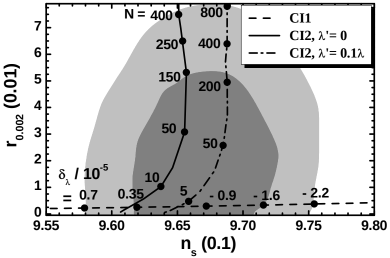
- IN BOTH MODELS $\phi_\star \sim 1$ AND THE RELEVANT TUNING CAN BE QUALIFIED BY COMPUTING $\Delta_\star = (1-\phi_\star)/1$.

GAUGE SINGLET INFLATON

- OUR INFLATIONARY SCENARIA DEPEND ON THE PARAMETERS: M, λ AND δ_λ FOR CI1, AND N FOR CI2. WE TAKE $M \leq 0.01$.



MODEL:	CI1	CI2 ($\lambda' = 0$)
δ_λ	$-9\cdot 10^{-6}$	-1
N	2	10
$\phi_\star/0.1$	9.9106	9.598
$\Delta_\star(\%)$	0.89	4
$\phi_f/0.1$	5.9	3.9
$\lambda/10^{-5}$	1	1.95
$n_s/0.1$	9.67	9.64
$-\alpha_s/10^{-4}$	8.37	6.8
$r/10^{-2}$	0.3	1.1

- FOR CI1 THE WHOLE OBSERVATIONALLY FAVORED RANGE CAN BE COVERED FOR δ_λ'S CLOSE TO 10^{-5} AND r REMAINING BELOW 0.01.

$$-22 \lesssim \frac{\delta_\lambda}{10^{-6}} \lesssim 7,\quad 8.5 \gtrsim \frac{\Delta_\star}{10^{-3}} \gtrsim 9.5,\quad 7.7 \gtrsim \frac{-\alpha_s}{10^{-4}} \gtrsim 5.3 \ \text{AND}\ 3.9 \gtrsim \frac{r}{10^{-3}} \gtrsim 2.5\,.$$

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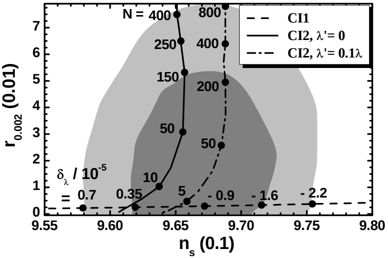

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For $n_s = 0.967$ We find $\delta_\lambda = -9 \cdot 10^{-6}$ And $r = 0.003$.

- For CI2 and Using $0 \leq \lambda'/\lambda \leq 0.1$ we see that $r \lesssim 0.07$ Increases With N and $\Delta_\star$ Yielding Upper Bounds

I.e., $0.96 \lesssim n_s \lesssim 0.968, \quad 0.5 \lesssim N \lesssim 800, \quad 0.24 \gtrsim \Delta_\star/10^{-2} \gtrsim 52$ and $0.00076 \lesssim r \lesssim 0.07$.

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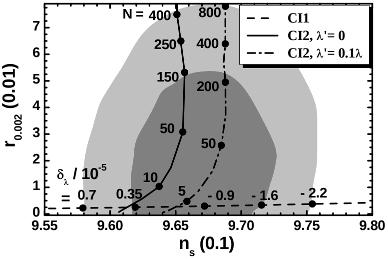

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Gauge non-Singlet Inflaton

- Our Inflationary Scenaria Depends On The Parameters: M, λ and δ_λ for HI1, or N for HI2.
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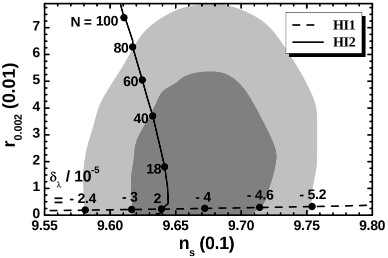

Model:	HI1	HI2
$\delta_\lambda/10^{-5}$	-4	–
N	2	18
$\phi_\star/0.1$	9.9564	9.6322
$\Delta_\star(\%)$	0.44	3.7
$\phi_{\rm f}/0.1$	7.1	4.3
$\lambda/10^{-5}$	3.8	10.7
$M/10^{-3}$	5.87	1.96
$n_{\rm s}/0.1$	9.67	9.64
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$r/10^{-2}$	0.28	2

- For **HI1** the Whole Observationally Favored Range **Can Be Covered** For δ_λ's close to 10^{-5} and r Remaining Below 0.01.

$$\text{I.e.,}\quad 2.4 \lesssim \frac{-\delta_\lambda}{10^{-5}} \lesssim 5.2,\quad 4.6 \gtrsim \frac{\Delta_\star}{10^{-3}} \gtrsim 4.1,\quad 5.4 \lesssim \frac{-\alpha_{\rm s}}{10^{-4}} \lesssim 8.6 \quad\text{and}\quad 2.1 \lesssim \frac{r}{10^{-3}} \lesssim 3.4\,.$$

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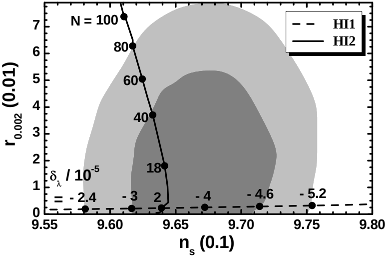

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For $n_{\rm s} = 0.967$ We find $\delta_\lambda = -4\cdot 10^{-5}$ And $r = 0.0028$.

- For **HI2** $n_{\rm s}$ is Concentrated A Little Lower Than Its Central Value And $r \lesssim 0.07$ Increases With $N \lesssim 80$ and $\Delta_\star$

$$0.962 \lesssim n_{\rm s} \lesssim 0.964,\quad 1 \lesssim N \lesssim 80,\quad 0.45 \gtrsim \Delta_\star/10^{-2} \gtrsim 13.6 \quad\text{and}\quad 0.0025 \lesssim r \lesssim 0.07\,.$$

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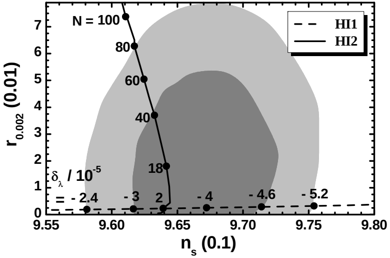

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- In the case of **Gauge Singlet Inflaton**, We obtain **Less Tuning** Regarding $\Delta_\star$

Conclusions

- We Proposed Two **Types** of Models Implementing **Pole Inflation** Within SUGRA:
  - One where K has **one** Logarithm and the Pole Appears not only in the Inflationary Kinetic term but also in $V_{\rm I}$. **Mildly Tuning** two W terms We can Almost Eliminate the Pole from $V_{\rm I}$.
  - One where K has **three** logarithmic terms and the Pole Appears Only in the Inflationary Kinetic term.

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  - One where K has **three** logarithmic terms and the Pole Appears Only in the Inflationary Kinetic term.
- **Both Types** of Models work for Both Gauge Singlet and non-Singlet Inflatons.

Comparison of the The Proposed Inflationary Models

Model	Quantity	Inflaton-Type: **Gauge Singlet** ($M \ll 1$)	Inflaton-Type: **Gauge non-Singlet** ($\langle M_{BL}\rangle = M_{\rm GUT}$)
$1\ (N=2)$	$K = K_2(S)+$	$K_{1\rm s} = -N\ln(1-(\Phi+\Phi^*)/2)$	$K_{21} = -N\ln\left(1-\lvert\bar\Phi\rvert^2-\lvert\Phi\rvert^2\right)$
	$W =$	$\lambda S\left(\Phi-(1+\delta_\lambda)\Phi^2-M^2\right)$	$\lambda S\left(\bar\Phi\Phi-2(1+\delta_\lambda)(\bar\Phi\Phi)^2-M^2/2\right)/2$
	$n_{\rm s} =$	$0.96-0.974$	
	$\delta_\lambda/10^{-5} \simeq$	$((-22)-7)\cdot10^{-1}$	$(-5.2)-(-2.4)$
	$r/10^{-3} \simeq$	$2.2-3.9$	$2.1-3.4$
2	$K = K_2(S)+$	$\widetilde K_{1\rm s} = K_{1\rm s} + (N\ln(1-\Phi)/2 + {\rm c.c.})$	$\widetilde K_{21} = K_{21} + \left(N\ln\left(1-\bar\Phi\Phi\right)/2+{\rm c.c.}\right)$
	$W =$	$\lambda S\left(\Phi-\lambda'\Phi^2/\lambda-M^2\right)$	$\lambda S\left(\bar\Phi\Phi-M^2/2\right)/2$
	$r =$	$0.00076-0.07$	
	$N \leq$	$800,\ (0\leq\lambda'/\lambda\leq0.1)$	80
	$n_{\rm s} \simeq$	$0.96-0.968$	$0.961-0.963$

Thank You!

Supplementary Material: CI With Pole of Order 2

- CI can be also Realized with Pole of Order 2[4], Using One of The Following K's for the Inflaton Sector:

$$K_{11} = -N\ln\left(1-|\Phi|^2\right) \ \text{or}\ \widetilde{K}_{11} = K_{11} + N\ln\left(1-\Phi^2\right)/2 + N\ln\left(1-\Phi^{*2}\right)/2$$

Which lead to the Kinetic Mixing $J = \sqrt{2N}/f_{2P}$ with $f_{2P} = 1-\phi^2$ and $\Phi = \phi e^{i\theta}$ for $\theta = 0$.

- For $K = K_{11}$ and $\widetilde{K}_{11}$, We Obtain $ds^2_{11} = N|d\Phi|^2/\left(1-|\Phi|^2\right)^2$ and $\mathcal{R}_{11} = -2/N$.
- ds^2_{11} Remains Invariant under the Transformations

$$\Phi \to \frac{\alpha\Phi + b}{b^*\Phi + \alpha} \ \text{Represented By}\ U = \begin{pmatrix}\alpha & b\\ b^* & \alpha\end{pmatrix}, \ \text{Provided that}\ \alpha^2 - |b|^2 = 1. \quad (T_2)$$

Therefore, U Provides Representation Of The $SU(1,1)/U(1)$ Kähler Manifold, since $U^\dagger\sigma_3 U = \sigma_3$ with $\sigma_3 = \text{diag}(1,-1)$.

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$$K \to K + \Lambda + \Lambda^* \quad \text{and} \quad W \to We^{-\Lambda} \quad \text{with} \quad \Lambda = N\ln(b^*\Phi + \alpha).$$

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- CI can be Implemented by the Following Combinations (K, W)
  - $K = K_2 + K_{11}$ and $W_2 = \lambda S(\Phi - \lambda'\Phi^3/\lambda - M^2)$ or $W_4 = \lambda S(\Phi^2 - \lambda'\Phi^4/\lambda - M^2)$.
    If we use $N = 2$ and $\lambda' \simeq \lambda(1+\delta_\lambda) \neq 0$ with $\delta_\lambda \simeq 0$ We Obtain

$$V_{\rm I} = e^K K^{SS^*}|W_{n,S}|^2 \simeq \lambda^2\phi^n\frac{(1-\phi^2)^2}{(1-\phi^2)^2} \simeq \lambda^2\phi^n, \quad n = 2, 4$$

    The Form of W may be Motivated from the Breaking of the Conformal Symmetry[4].
  - $K = K_2 + \widetilde{K}_{11}$ and $W_n = \lambda S\Phi^n$. We obtain

$$V_{\rm I} = e^K K^{SS^*}|W_{,S}|^2 = \lambda^2\phi^{2n}$$

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