Signature change of the emergent space-time in the IKKT matrix model

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Ref.) J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077 [arXiv:1904.05919 [hep-th]]
Anagnostopoulos-Azuma-Hatakeyama-Hirasawa-Ito-J.N.-Tsuchiya-Papadoudis,
work in progress

IKKT matrix model

Ishibashi, Kawai, Kitazawa, Tsuchiya, 1996

a conjectured nonperturbative formulation of superstring theory

$$\begin{aligned} S_{\mathrm{b}} &= -\frac{1}{4g^2}\mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu]) \\ S_{\mathrm{f}} &= -\frac{1}{2g^2}\mathrm{tr}(\Psi_\alpha(\mathcal{C}\,\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]) \end{aligned}$$

SO(9,1) symmetry

$N \times N$ Hermitian matrices

$A_\mu \quad (\mu = 0, \cdots, 9)$ Lorentz vector

$\Psi_\alpha \quad (\alpha = 1, \cdots, 16)$ Majorana-Weyl spinor

Lorentzian metric $\eta = \mathrm{diag}(-1, 1, \cdots, 1)$
is used to raise and lower indices.

Wick rotation $(A_0 = -iA_{10}\ , \quad \Gamma^0 = i\Gamma_{10})$

➡ Euclidean matrix model SO(10) symmetry

Anagnostopoulos, et al. JHEP 06 (2020) 069 , arXiv: 2002.07410 [hep-th]

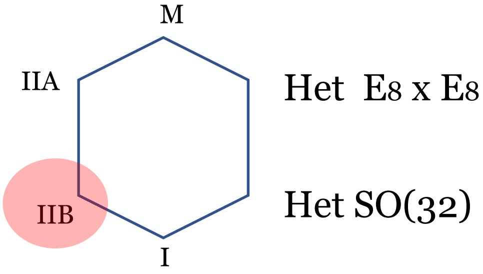
Crucial properties of the IKKT matrix model

as a nonperturbative formulation of superstring theory

- The connection to perturbative formulations can be seen manifestly by considering type IIB superstring theory in 10d.

worldsheet action, light-cone string field Hamiltonian, etc.

- It is expected to be a nonperturbative formulation of the unique theory underlying the web of string dualities.



- The model has $10D \mathcal{N}=2$ SUSY , which cannot be realized in quantum field theories without gravity. The low energy effective theory should inevitably include quantum gravity !

In the SUSY algebra, translation is realized as $A_\mu \mapsto A_\mu + \alpha_\mu \mathbf{1}$,

which suggests that the space-time is represented as the eigenvalue distribution of A_μ .

Geometry emerges from matrix degrees of freedom dynamically in this approach .

Plan of the talk

0. Introduction
1. Brief review of the Euclidean IKKT model
2. How to define the Lorentzian IKKT model
3. How to investigate the model
4. Results of the CL simulations
5. Summary and discussions

1. Brief review of the Euclidean IKKT model

the Euclidean IKKT model

"Wick rotation" : $A_0 = -iA_{10}$

$S_b \propto \mathrm{tr}\,(F_{\mu\nu})^2$ positive semi-definite!

$F_{\mu\nu} = -i[A_\mu, A_\nu]$

The flat direction : $[A_\mu, A_\nu] \sim 0$

Lifted in the bosonic case due to quantum effects.

Bhanot-Heller-Neuberger '82

It survives in the SUSY case if one neglects the fermionic zero modes.

In the original IKKT paper : $|\text{eigenvalues of } A_\mu| < \Lambda$

In fact, fermionic zero modes lift the flat directions.

Aoki-Iso-Kawai-Kitazawa-Tada '99

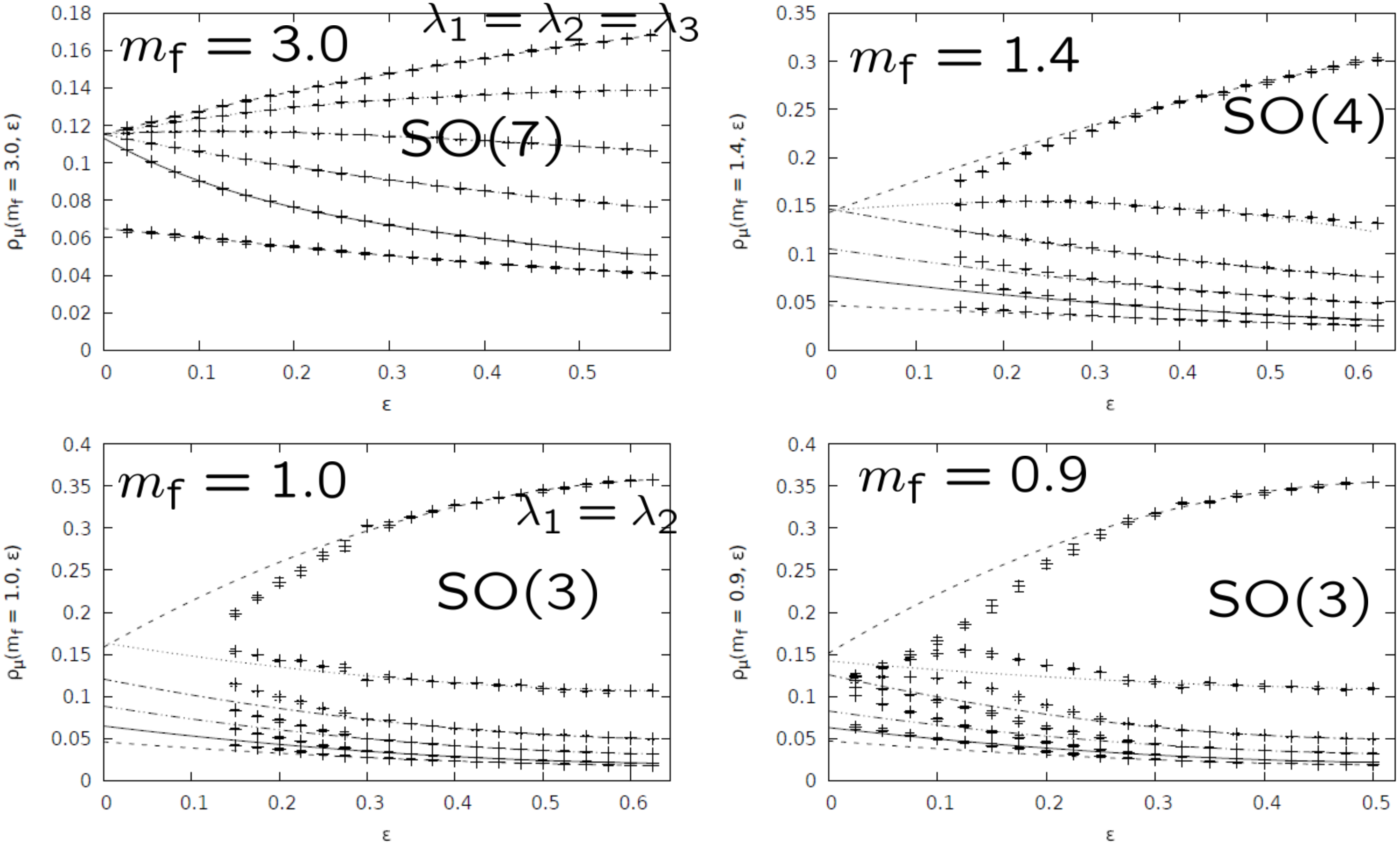
Euclidean model is well defined without any cutoff.

Krauth-Nicolai-Staudacher ('98),
Austing-Wheater ('01)

Results for the Euclidean IKKT model $\mathrm{SO}(10) \overset{\text{SSB}}{\to} \mathrm{SO}(3)$

SSB of SO(10) observed by decreasing the deformation parameter m_{f}.

ten eigenvalues of $T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu)$



$\lambda_1 = \lambda_2$

sym breaking parameter

$\lambda_1 = \lambda_2$

Anagnostopoulos-Azuma-Ito-JN-Okubo-Papadoudis, JHEP 06 (2020) 069 , arXiv: 2002.07410 [hep-th]

2. How to define the Lorentzian IKKT model

Partition function of the Lorentzian IKKT model

partition function

$$Z = \int dA\, d\Psi\, e^{i(S_{\rm b}+S_{\rm f})} = \int dA\, e^{iS_{\rm b}} \mathrm{Pf}\mathcal{M}(A)$$

This seems to be natural from the connection to the worldsheet theory.

c.f.) $$S = \int d^2\xi \sqrt{g}\left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\bar{\Psi}\gamma^\mu\{X^\mu, \Psi\}\right)$$

$$\xi_0 \equiv -i\xi_2$$

(The worldsheet coordinates should also be Wick-rotated.)

Regularizing the Lorentzian model

- Unlike the Euclidean model,
  the Lorentzian model is NOT well defined as it is.

$$Z = \int dA\, d\Psi\, e^{i(S_{\mathrm{b}}+S_{\mathrm{f}})} = \int dA\, e^{iS_{\mathrm{b}}} \underbrace{\mathrm{Pf}\,\mathcal{M}(A)}_{\text{polynomial in } A}$$

pure phase factor

- Wick rotation

(Yuhma Asano '19, private communication)

$$S_{\mathrm{b}} \mapsto \tilde{S}_{\mathrm{b}} = N e^{i\frac{\pi}{2}s} \left\{ \frac{1}{2} e^{-i\pi k} \mathrm{tr}\,[\tilde{A}_0, \tilde{A}_i]^2 - \frac{1}{4} \mathrm{tr}\,[\tilde{A}_i, \tilde{A}_j]^2 \right\}$$

on the worldsheet

in the target space

This corresponds to deforming the integration contour in the Lorentzian model.

$$\left\{ \begin{array}{lclcl} A_0 & = & \mathrm{e}^{i\frac{\pi}{8}s - i\frac{\pi}{2}k} \tilde{A}_0 & = & \mathrm{e}^{-i\frac{3\pi}{8}u} \tilde{A}_0 \\ A_i & = & \mathrm{e}^{i\frac{\pi}{8}s} \tilde{A}_i & = & \mathrm{e}^{i\frac{\pi}{8}u} \tilde{A}_i \end{array} \right.$$

$$s = k (= u)$$

$u = 0$: Lorentzian

$u = 1$: Euclidean

Path deformed theory is well-defined for $0 < u \leq 1$

(Yuhma Asano ’19, private communication)

$$e^{iS_{\mathrm{b}}(A)} = e^{-S(\tilde{A})} \qquad \begin{cases} A_0 = \mathrm{e}^{-i\frac{3}{8}\pi u}\tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{1}{8}\pi u}\tilde{A}_i \end{cases} \qquad \tilde{F}_{\mu\nu} = -i[\tilde{A}_\mu, \tilde{A}_\nu]$$

$$S(\tilde{A}) \sim 2e^{i\frac{\pi}{2}(1-u)}\mathrm{tr}\,(\tilde{F}_{0i})^2 + e^{-i\frac{\pi}{2}(1-u)}\mathrm{tr}\,(\tilde{F}_{ij})^2$$

positive real part for $0 < u \leq 1$

$$\boxed{\mathrm{Re}\, S(\tilde{A}) \geq 0}$$

$S(\tilde{A})$: real positive at $u = 1$ (Euclidean).

According to Cauchy’s theorem,
$\langle \mathcal{O}(\mathrm{e}^{-i\frac{3}{8}\pi u}\tilde{A}_0, \mathrm{e}^{i\frac{1}{8}\pi u}\tilde{A}_i)\rangle_u$ is independent of u.

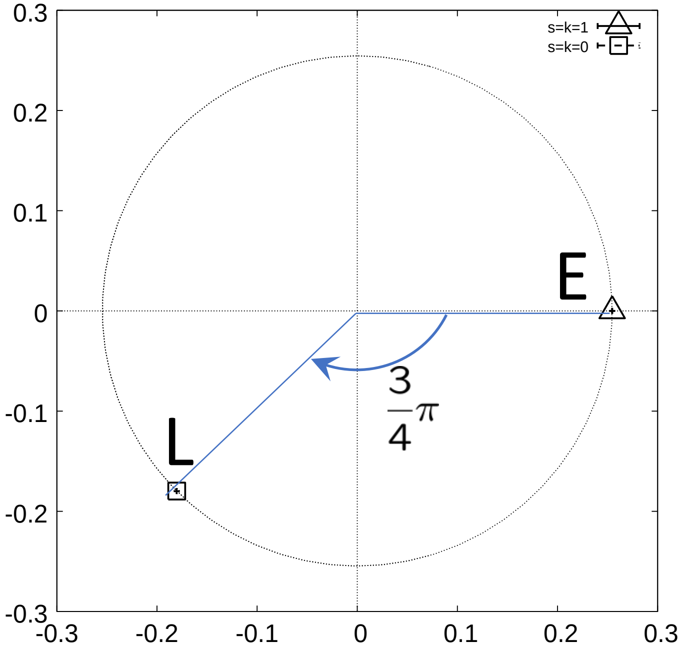
➡ If we define the Lorentzian model by taking the $u \to +0$ limit,

$$\langle \mathcal{O}(A_0, A_i)\rangle_{\mathrm{L}} = \langle \mathcal{O}(\mathrm{e}^{-i\frac{3\pi}{8}}\tilde{A}_0, \mathrm{e}^{i\frac{\pi}{8}}\tilde{A}_i)\rangle_{\mathrm{E}}$$

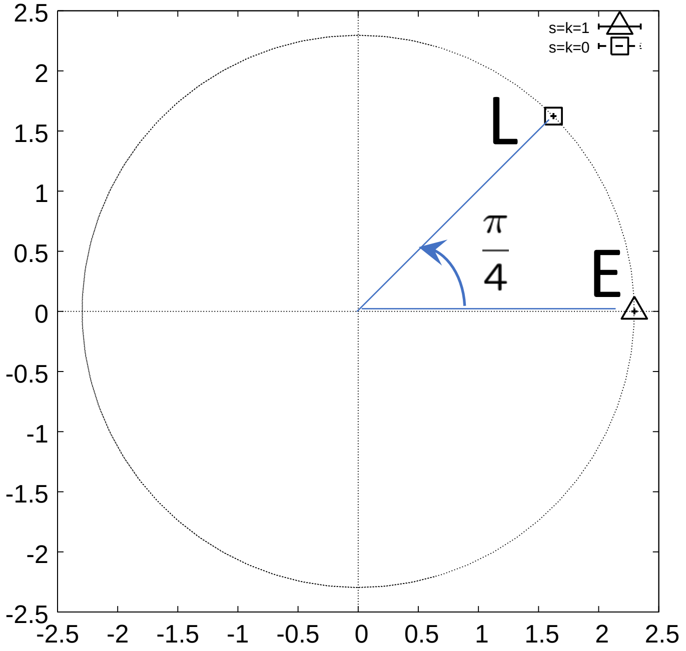
Confirmation of the equivalence by CL simulation

10D bosonic model

$$\left\langle \frac{1}{N}\mathrm{tr}(A_0)^2 \right\rangle_{\mathrm{L}} = e^{-\frac{3\pi}{4}i} \left\langle \frac{1}{N}\mathrm{tr}(\tilde{A}_0)^2 \right\rangle_{\mathrm{E}}$$



$$\left\langle \frac{1}{N}\mathrm{tr}(A_i)^2 \right\rangle_{\mathrm{L}} = e^{\frac{\pi}{4}i} \left\langle \frac{1}{N}\mathrm{tr}(\tilde{A}_i)^2 \right\rangle_{\mathrm{E}}$$



$$\begin{cases} A_0 = \mathrm{e}^{-i\frac{3}{8}\pi u}\tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{1}{8}\pi u}\tilde{A}_i \end{cases}$$

$$u = 0 \mapsto u = 1$$

Lorentzian Euclidean

The emergent space-time should be interpreted as being Euclidean !

Can we introduce some “boundary condition” ?

3. How to investigate the model

Complex Langevin equation

J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077
[arXiv:1904.05919 [hep-th]]

The effective action

$$\begin{aligned} S_{\text{eff}} &= -i\,N\left\{\frac{1}{2}\text{tr}\,[A_0, A_i]^2 - \frac{1}{4}\text{tr}\,[A_i, A_j]^2\right\} \\ &\quad - \log\Delta(\alpha) - \sum_{a=1}^{N-1}\tau_a \end{aligned}$$

Complex Langevin equation

$$\left\{ \begin{aligned} \frac{d\tau_a}{dt} &= -\frac{\partial S_{\text{eff}}}{\partial \tau_a} + \eta_a \\ \frac{d(\mathcal{A}_i)_{ab}}{dt} &= -\frac{\partial S_{\text{eff}}}{\partial(\mathcal{A}_i)_{ba}} + (\eta_i)_{ab} \end{aligned} \right.$$

τ_a : complex variables, $\mathcal{A}_i$: general complex matrices.

In this work, we omit the fermionic matrices
to reduce computation time

➡ bosonic model

Defining “time” in the IKKT model in complex Langevin simulation

J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077
[arXiv:1904.05919 [hep-th]]

Fixing the U(N) symmetry: $A_\mu \mapsto U A_\mu U^\dagger$

$$Z = \int dA_0 \, dA_i \, e^{-S} = \int d\alpha \, dA_i \, \Delta(\alpha) \, e^{-S}$$

$A_0 = \mathrm{diag}(\alpha_1, \cdots, \alpha_N)$

$\alpha_1 < \alpha_2 < \cdots < \alpha_N$

$\Delta(\alpha) = \prod_{a>b} (\alpha_a - \alpha_b)^2$: van der Monde determinant

We make the change of variables

$$\alpha_1 = 0 \ , \quad \alpha_2 = e^{\tau_1} \ , \quad \alpha_3 = e^{\tau_1} + e^{\tau_2} \ , \quad \cdots \ , \quad \alpha_N = \sum_{a=1}^{N-1} e^{\tau_a} \ ,$$

to introduce the “time ordering” respecting holomorphicity.

The expectation value of the time coordinates

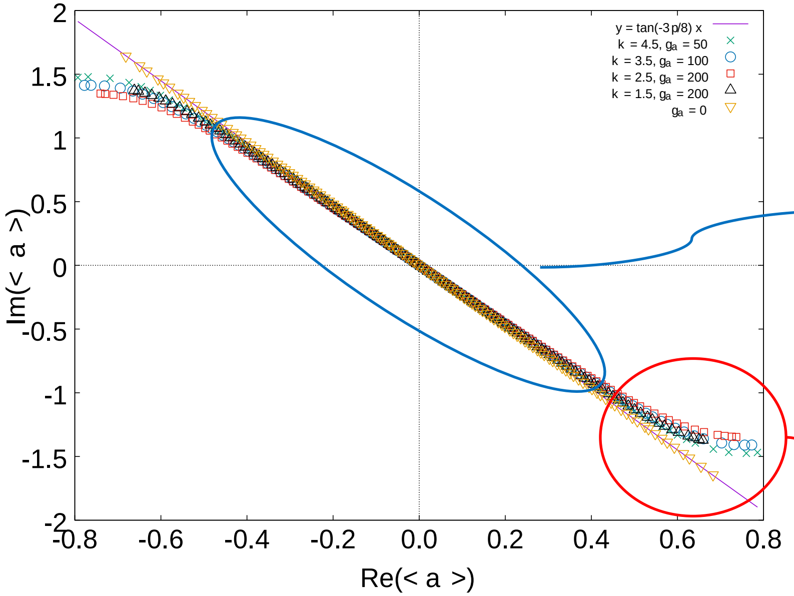
$$\langle \mathcal{O}(A_0, A_i)\rangle_{\mathrm{L}} = \langle \mathcal{O}(\mathrm{e}^{-i\frac{3\pi}{8}}\tilde{A}_0, \mathrm{e}^{i\frac{\pi}{8}}\tilde{A}_i)\rangle_{\mathrm{E}}$$

$$\Downarrow$$

$$\langle \alpha_i\rangle_{\mathrm{L}} = e^{-i\frac{3\pi}{8}} \boxed{\langle \tilde{\alpha}_i\rangle_{\mathrm{E}}} \in \mathbb{R}$$

We introduce a constraint : $\alpha_N - \alpha_1 = \sqrt{\kappa} \in \mathbb{C}$

One cannot deform the contour as we did above.
The model is not equivalent to the Euclidean model any more.



$$\Delta\alpha_i \equiv \alpha_{i+1} - \alpha_i$$

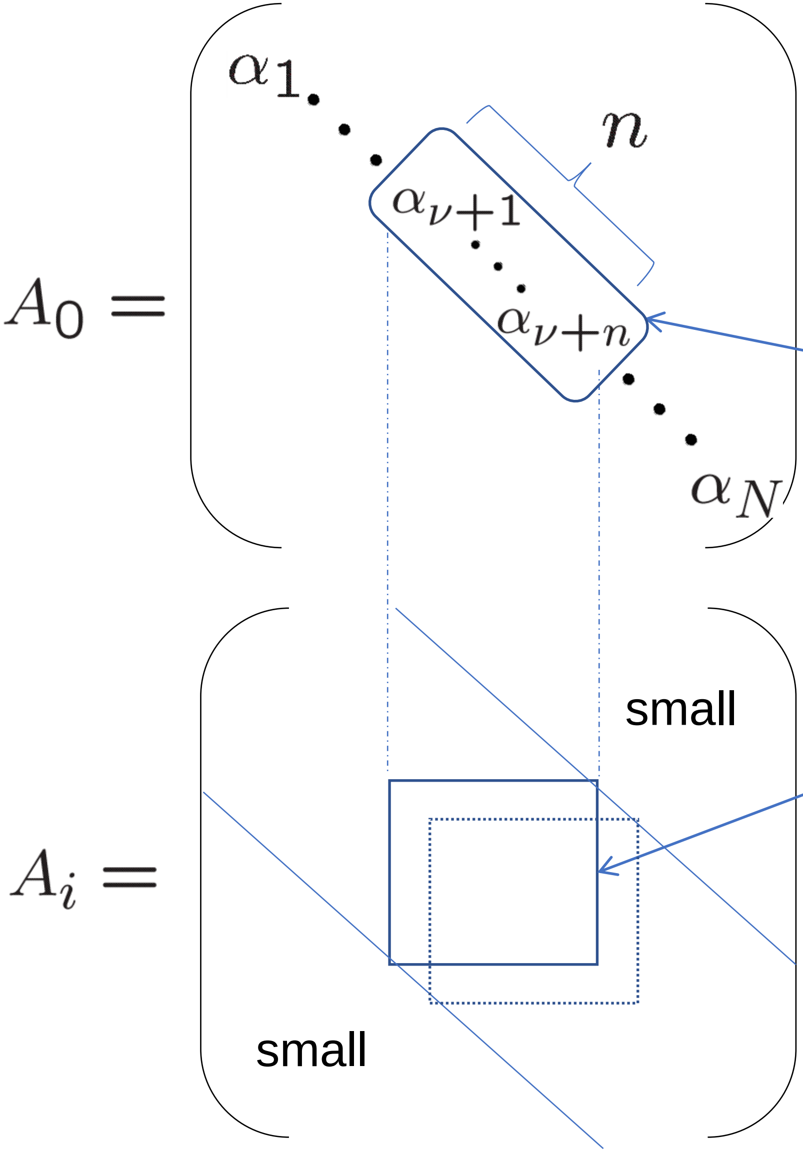
$\Delta\alpha_i \propto e^{-i\frac{3\pi}{8}}$ (Euclidean regime)

$\Delta\alpha_i > 0$ (Lorentzian regime)

How about space ?

Extracting time-evolution from the Lorentzian model

Kim-J.N.-Tsuchiya PRL 108 (2012) 011601 [arXiv:1108.1540]



$$\alpha_1 < \cdots < \alpha_N$$

definition of "time"

$$\bar{\alpha}_\nu = \frac{1}{n}\sum_{i=1}^{n} \alpha_{\nu+i} \in \mathbf{C}$$

$$t_\rho = \sum_{\nu=1}^{\rho} |\bar{\alpha}_{\nu+1} - \bar{\alpha}_\nu|$$

The state of the universe $\bar{A}_i(t)$ at time t

A_i has a band diagonal structure

non-trivial dynamical property

cf.) Klinkhamer, arXiv: 2105.05831 [hep-th]

The extent of space

$$R^2(t) = \left\langle \frac{1}{n} \mathrm{tr} \left(\bar{A}_i(t) \right)^2 \right\rangle$$

$$\langle \mathcal{O}(A_0, A_i) \rangle_{\mathrm{L}} = \langle \mathcal{O}(\mathrm{e}^{-i\frac{3\pi}{8}} \tilde{A}_0, \mathrm{e}^{i\frac{\pi}{8}} \tilde{A}_i) \rangle_{\mathrm{E}}$$

$$\Downarrow$$

$$\left\langle \frac{1}{N} \mathrm{tr}(A_i)^2 \right\rangle_{\mathrm{L}} = e^{i\frac{\pi}{4}} \left\langle \frac{1}{N} \mathrm{tr}(\tilde{A}_i)^2 \right\rangle_{\mathrm{E}}$$

This is not true any more
once we introduce the constraint $\alpha_N - \alpha_1 = \sqrt{\kappa} \in \mathbb{C}$

$$R^2(t) = \left\langle \frac{1}{n} \mathrm{tr} \left(\bar{A}_i(t) \right)^2 \right\rangle$$
$$\propto e^{i\frac{\pi}{4}} \ \text{(Euclidean regime)}$$
$$> 0 \ \text{(Lorentzian regime)}$$

Thus, the signature of the space time can change dynamically in the IKKT model.

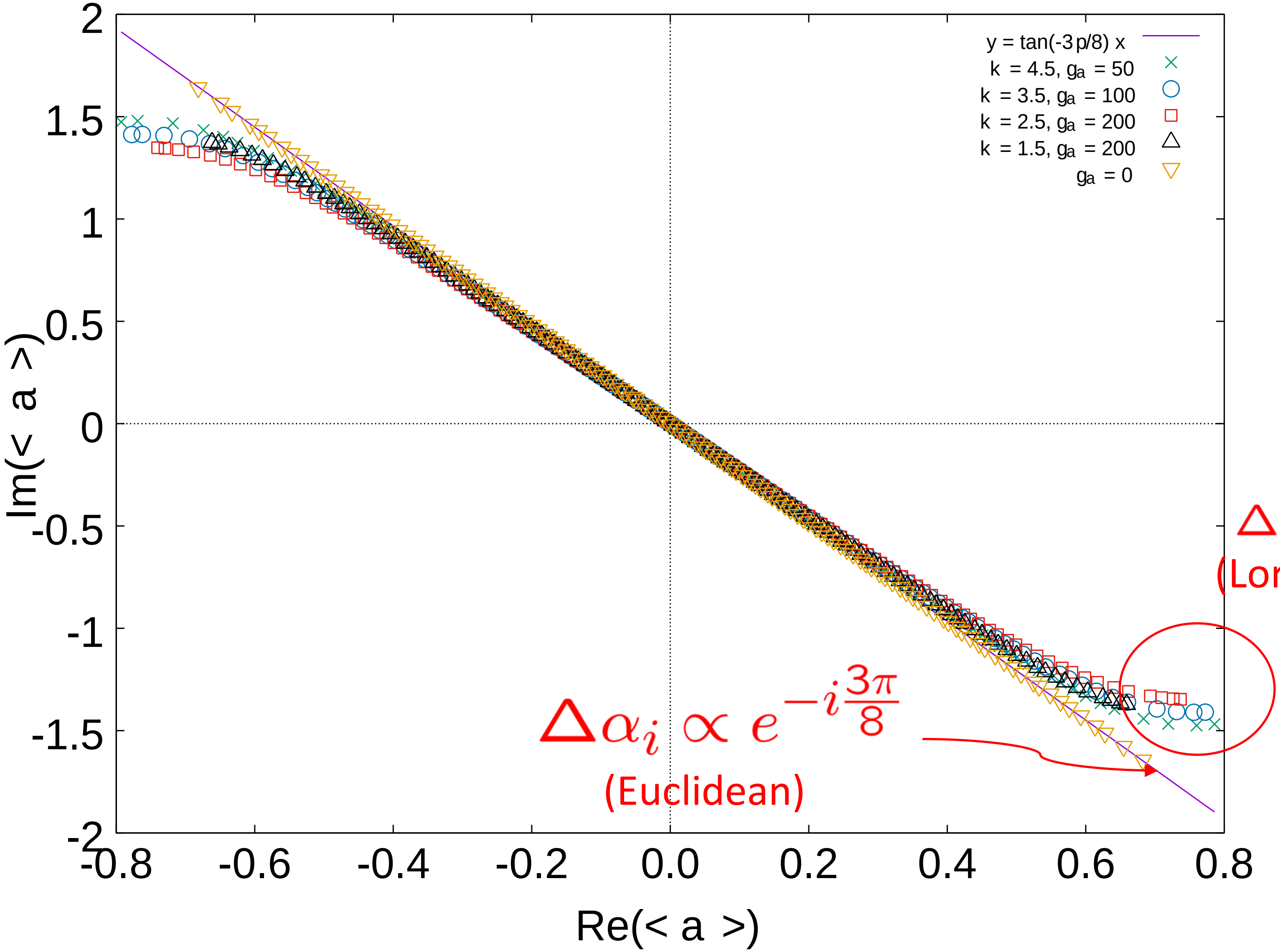
previous work based on classical solutions: Cheney-Lu-Stern ('17)
Sperling-Steinacker ('18)

4. Results of the CL simulations

Eigenvalues of A_0

$N = 128$

(preliminary results)



$\Delta\alpha_i > 0$

(Lorentzian)

$\Delta\alpha_i \propto e^{-i\frac{3\pi}{8}}$

(Euclidean)

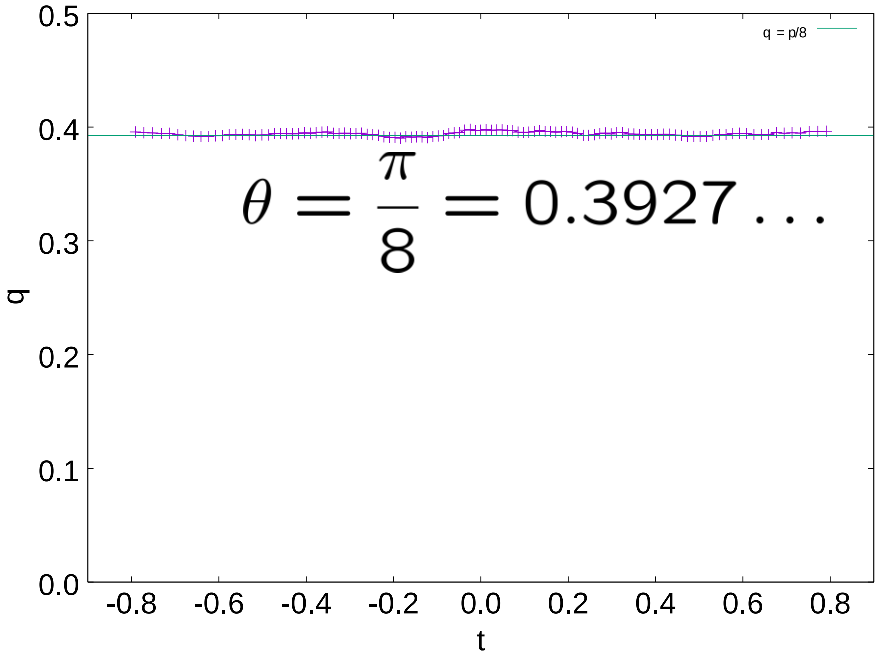
the time evolution of space

$N = 128$ (preliminary results)

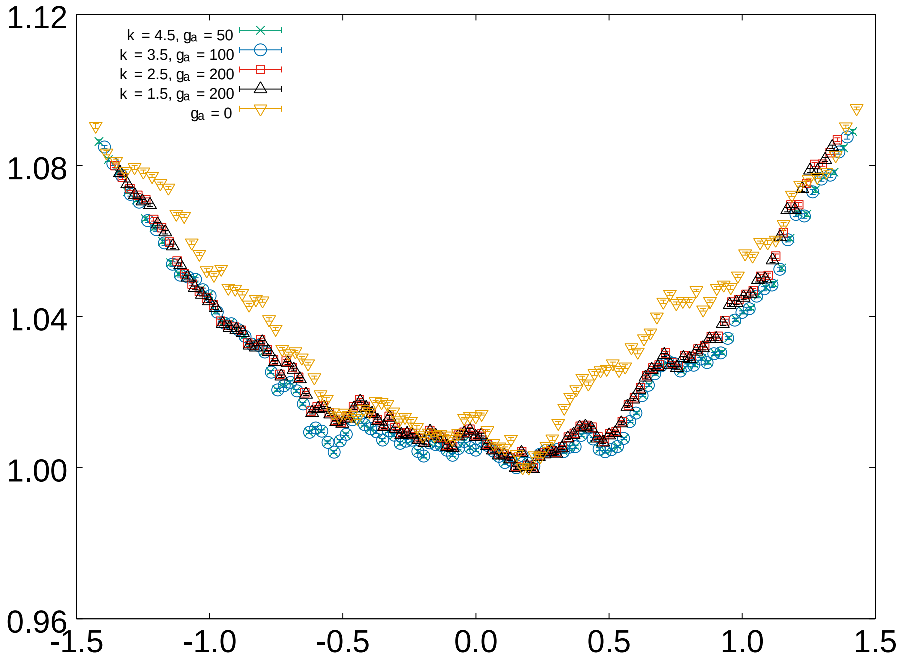
block size $n = 16$

$$t_\rho = \sum_{\nu=1}^{\rho} |\bar{\alpha}_{\nu+1} - \bar{\alpha}_\nu|$$

$|\langle R^2(t)\rangle|/\Lambda^2$ $\quad$ $\Lambda^2 = |\langle R^2(0)\rangle|$



$$\theta = \frac{\pi}{8} = 0.3927\ldots$$



t/Λ

$$R^2(t) = \left\langle \frac{1}{n}\mathrm{tr}\left(\bar{A}_i(t)\right)^2 \right\rangle = e^{2i\theta}|R^2(t)|$$

$\propto e^{i\frac{\pi}{4}}$ (Euclidean regime)

Scaling behavior observed !

5. Summary and Discussions

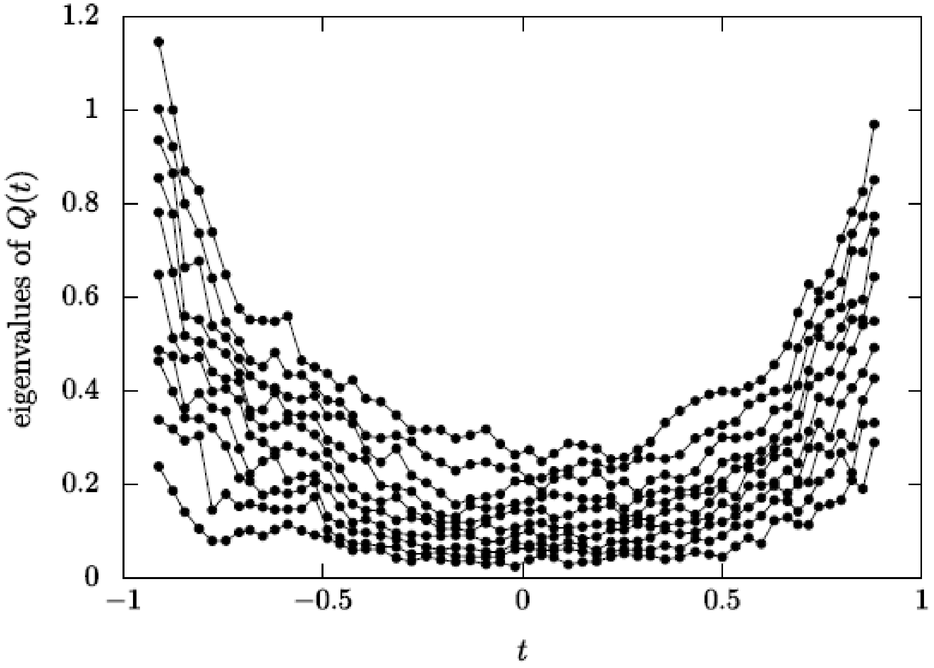
Summary

- IKKT matrix model = a nonperturbative formulation of superstring theory
- The Euclidean model exhibits SSB of SO(10) to SO(3)
  due to the phase of the fermion determinant (or Pfaffian).
- In fact, the Lorentzian model becomes equivalent to Euclidean model
  if we define it by deformation of the integration contour.
- We introduce a "boundary condition"
  on both ends of the eigenvalue spectrum of A_0 .
- Signature change from Euclidean to Lorentzian may naturally occur.
  (a bouncing universe scenario unlike Hartle-Hawking's no boundary)
- Can we make the duration of the real time regime longer
  by the boundary condition at larger N ?

  Does the space become "real" at late times ?
  Does the expanding behavior (like inflation) show up ?

Discussions

- The expanding behavior observed in the solution to the EOM of the Lorentzian model with the IR cutoffs.



We expect similar expanding behavior to show up at late times.

Hatakeyama-Matsumoto-J.N.-Tsuchiya-Yosprakob, *PTEP* 2020 (2020) 4, 043B10

- Including fermionic matrices (SUSY model).
  Does the SSB in the Euclidean model imply that SO(3) is realized in the present model as well ?

6. Backup slides

3. Complex Langevin method

The complex Langevin method

Parisi ('83), Klauder ('83)

$$Z = \int dx\, w(x) \qquad\qquad x \in \mathbb{R}$$

complex

MC methods inapplicable
due to sign problem !

Complexify the dynamical variables, and consider their (fictitious) time evolution :

$$z^{(\eta)}(t) = x^{(\eta)}(t) + i\, y^{(\eta)}(t)$$

defined by the complex Langevin equation

Gaussian noise (real)

$$\frac{d}{dt} z^{(\eta)}(t) = \boxed{v(z^{(\eta)}(t))} + \eta(t)$$

probability $\propto \mathrm{e}^{-\frac{1}{4}\int dt\, \eta(t)^2}$

$$\langle \mathcal{O} \rangle \overset{?}{=} \lim_{t\to\infty} \langle \boxed{\mathcal{O}(z^{(\eta)}(t))} \rangle_\eta$$

$$v(x) \equiv \frac{1}{w(x)} \frac{\partial w(x)}{\partial x}$$

Rem 1 : When w(x) is real positive, it reduces to one of the usual MC methods.

Rem 2 : The drift term $v(x) \equiv \frac{1}{w(x)} \frac{\partial w(x)}{\partial x}$ and the observables $\mathcal{O}(x)$ should be evaluated for complexified variables by analytic continuation.

Complex Langevin equation

J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077
[arXiv:1904.05919 [hep-th]]

The effective action

$$
\begin{aligned}
S_{\text{eff}} &= -i\,N\left\{\frac{1}{2}\text{tr}\,[A_0, A_i]^2 - \frac{1}{4}\text{tr}\,[A_i, A_j]^2\right\} \\
&\quad - \log\Delta(\alpha) - \sum_{a=1}^{N-1}\tau_a + \boxed{\frac{1}{4}\gamma_\alpha\,(\alpha_N - \sqrt{\kappa})^4}
\end{aligned}
$$

constraint for our “boundary condition”

Complex Langevin equation

$$
\left\{
\begin{aligned}
\frac{d\tau_a}{dt} &= -\frac{\partial S_{\text{eff}}}{\partial \tau_a} + \eta_a \\
\frac{d(A_i)_{ab}}{dt} &= -\frac{\partial S_{\text{eff}}}{\partial (A_i)_{ba}} + (\eta_i)_{ab}
\end{aligned}
\right.
$$

τ_a : complex variables, A_i : general complex matrices.

In this work, we omit the fermionic matrices
to reduce computation time

➡ bosonic model

5. Comments on our previous work

A phase diagram of the IKKT model

$$e^{iS_{\mathrm{b}}(A)} = e^{-S(\tilde{A})}$$

$$\begin{aligned} S(\tilde{A}) &= N\beta\, e^{-i\frac{\pi}{2}(1-s)} \left\{ \frac{1}{2} e^{-ik\pi} \mathrm{tr}\,[\tilde{A}_0, \tilde{A}_i]^2 - \frac{1}{4} \mathrm{tr}\,[\tilde{A}_i, \tilde{A}_j]^2 \right\} \\ &= 2\boxed{e^{i\frac{\pi}{2}(1+s-2k)}}\mathrm{tr}\,(\tilde{F}_{0i})^2 + \boxed{e^{-i\frac{\pi}{2}(s-1)}}\mathrm{tr}\,(\tilde{F}_{ij})^2 \end{aligned}$$

	Real part	Real part
I	(+)	(+)
II	(+)	(-)
III	(-)	(+)
IV	(-)	(-)

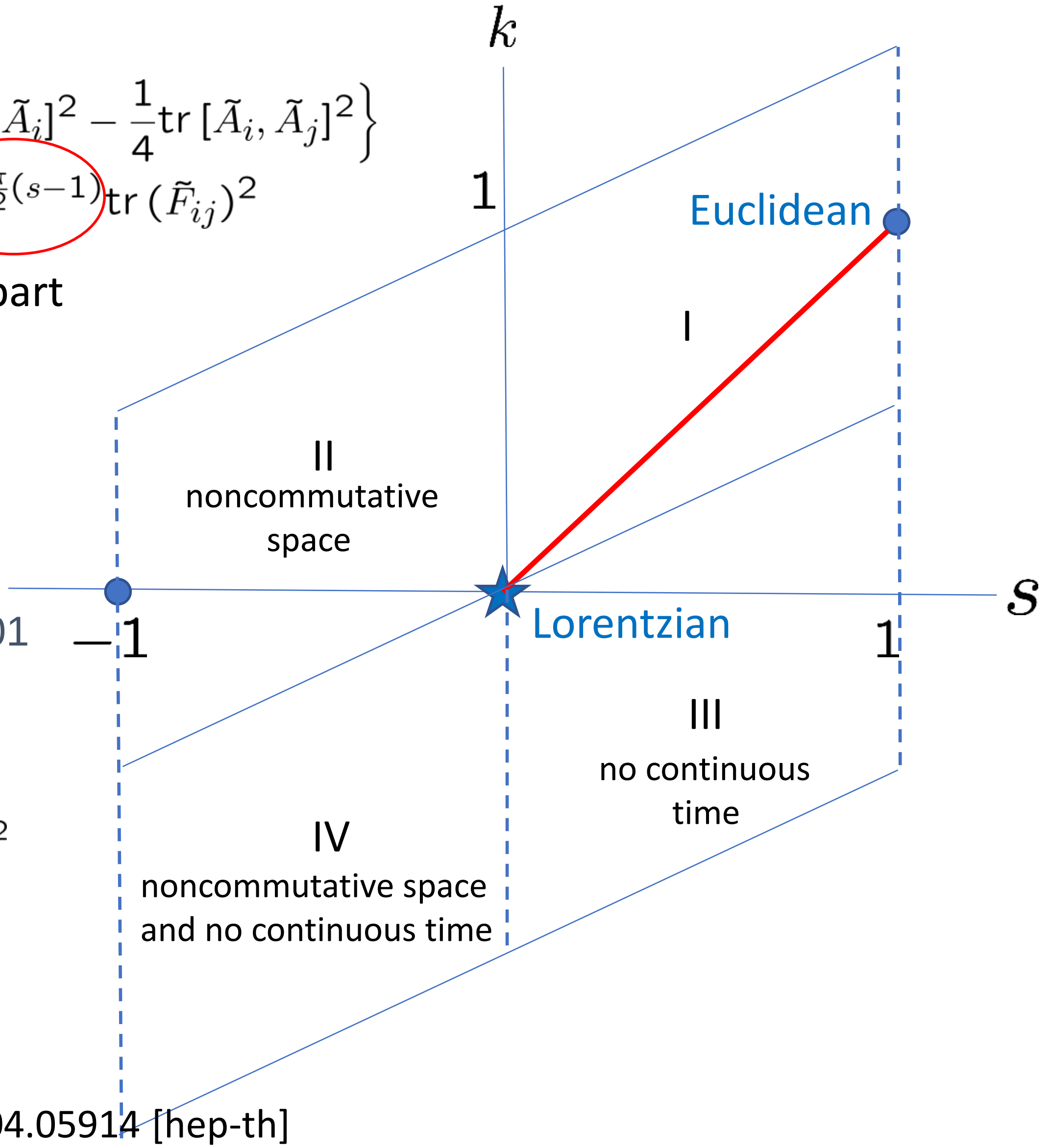
Kim-J.N.-Tsuchiya PRL 108 (2012) 011601
[arXiv:1108.1540]

MC simulations at (s,k)=(-1,0)

with IR cutoffs on $\mathrm{tr}\,(A_0)^2$ and $\mathrm{tr}\,(A_i)^2$

Expanding 3D space emerges,
but the space is not continuous…

Aoki-Hirasawa-J.N.-Ito-Tsuchiya,
PTEP 2019 (2019) 9, 093B03, arXiv:1904.05914 [hep-th]

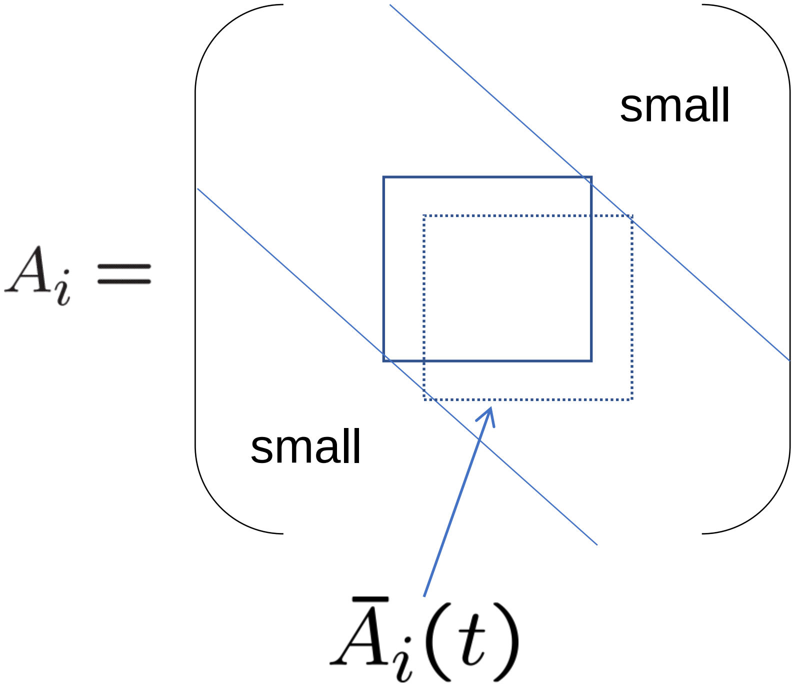


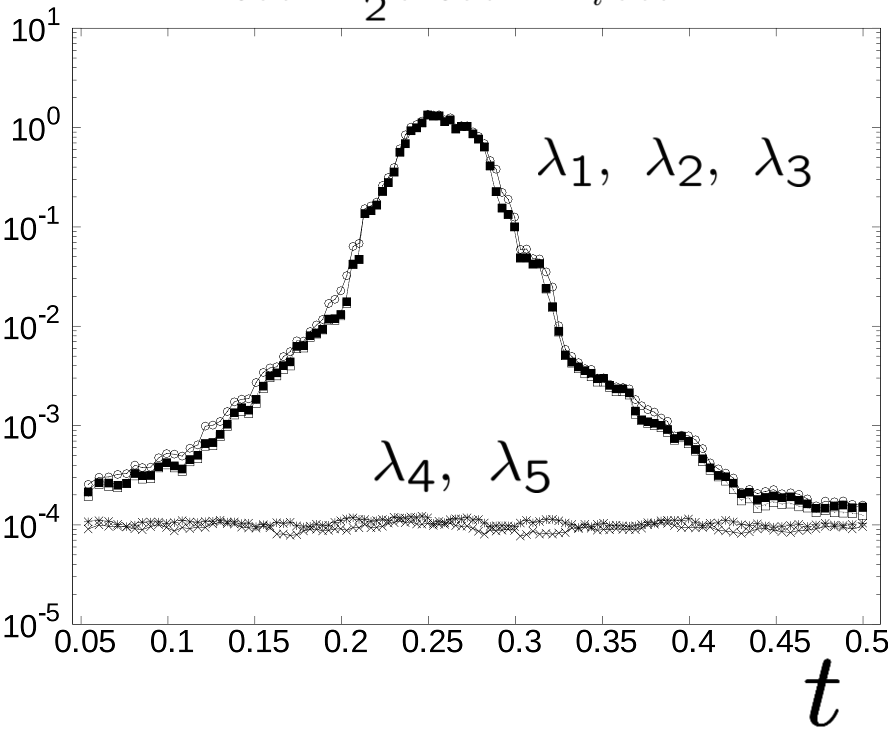
Emergence of (3+1)-dim. expanding behavior

6D bosonic model $N = 128\ ,\quad \kappa = 0.02\ ,\quad \beta = 8\ ,\quad (s,k) = (-1,0)\ ,\quad n = 16$

eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left\{X_i(t)X_j(t)\right\}$

$$X_i(t) = \frac{1}{2}(\bar{A}_i(t) + \bar{A}_i^{\dagger}(t))$$





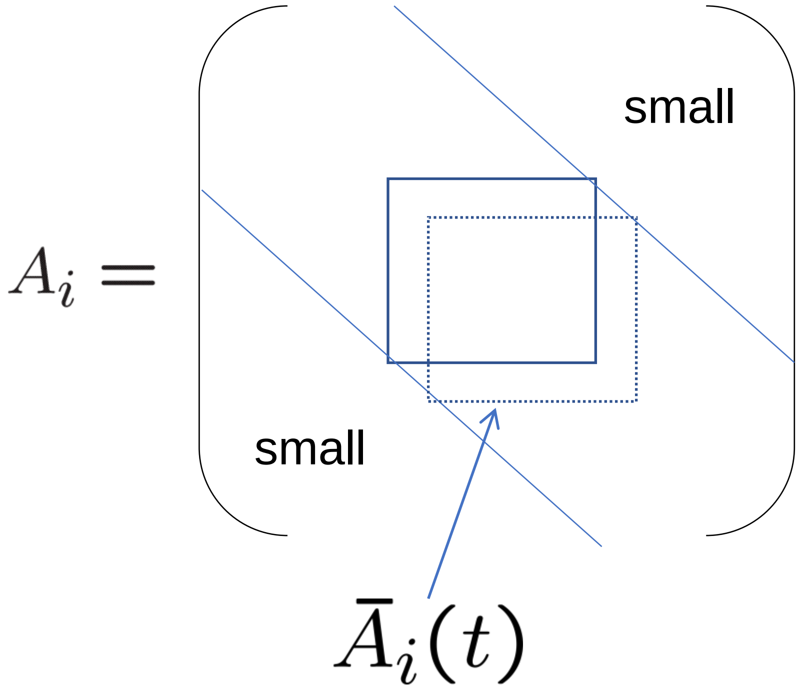
SSB : SO(5) → SO(3) occurs at some point in time.

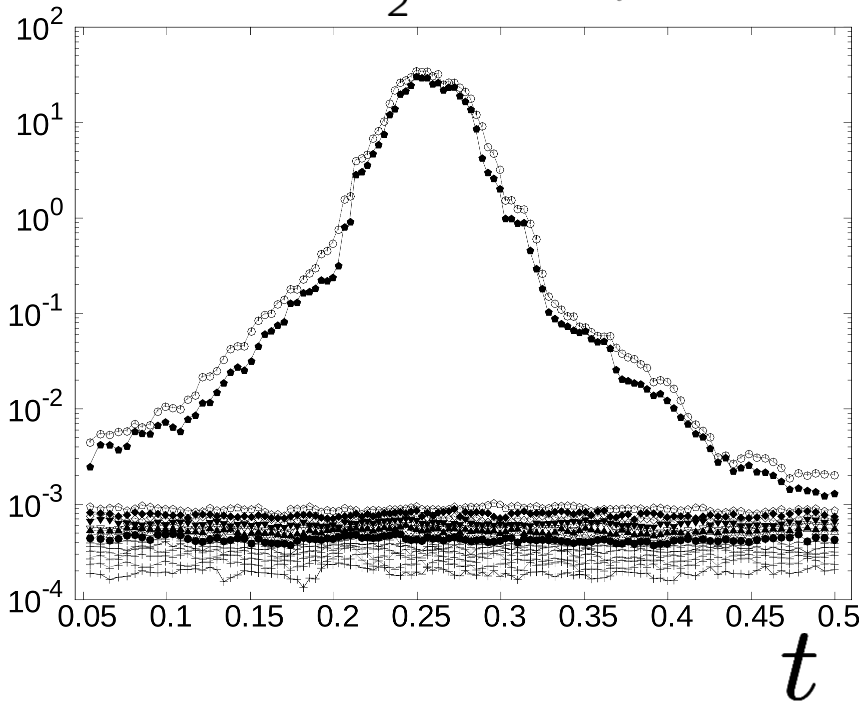
But the space is not continuous

6D bosonic model $N = 128\ , \quad \kappa = 0.02\ , \quad \beta = 8\ , \quad (s,k) = (-1,0)\ , \quad n = 16$

eigenvalues of $Q = \sum_{i=1}^{5} \left\{ X_i(t) \right\}^2$

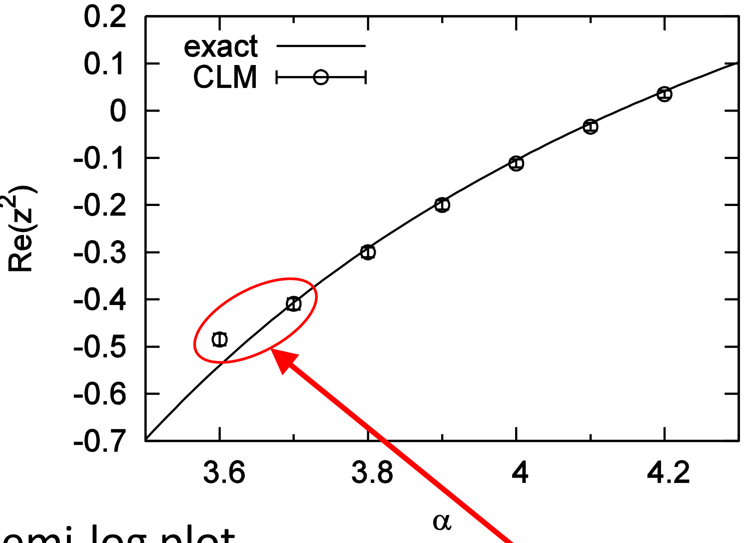
$$X_i(t) = \frac{1}{2}(\bar{A}_i(t) + \bar{A}_i^\dagger(t))$$





Only 2 Evs of Q become large suggesting the Pauli-matrix structure.

Recent development : the condition for correct convergence

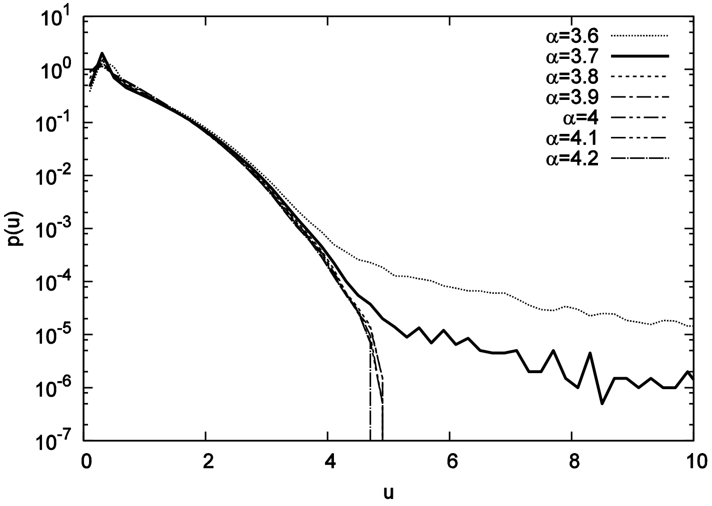


$$Z = \int dx\, w(x)$$
$$w(x) = (x+i\alpha)^p \, e^{-x^2/2}$$

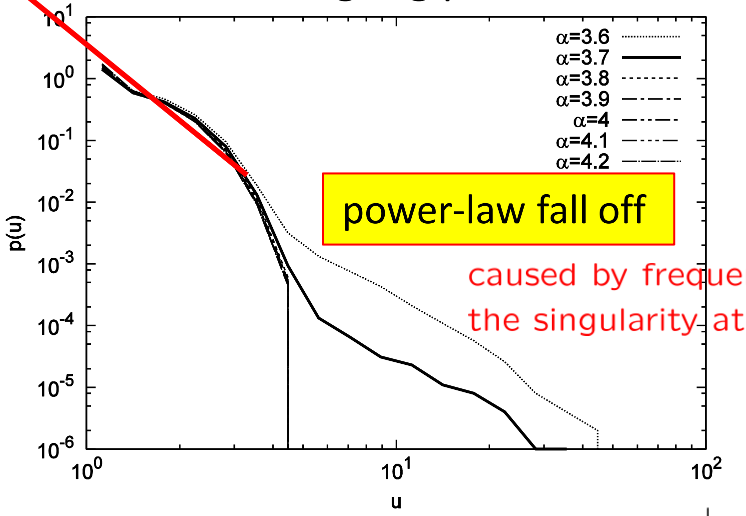
$$p = 4$$

In this model, CLM fails at $\alpha \lesssim 3.7$.

semi-log plot



log-log plot



power-law fall off

caused by frequent visits to the singularity at $z = -i\alpha$

The probability distribution of the magnitude of the drift term $u \equiv |v(z)| = \left| \frac{p}{z+i\alpha} - z \right|$ should be suppressed exponentially in order for the method to be justified.

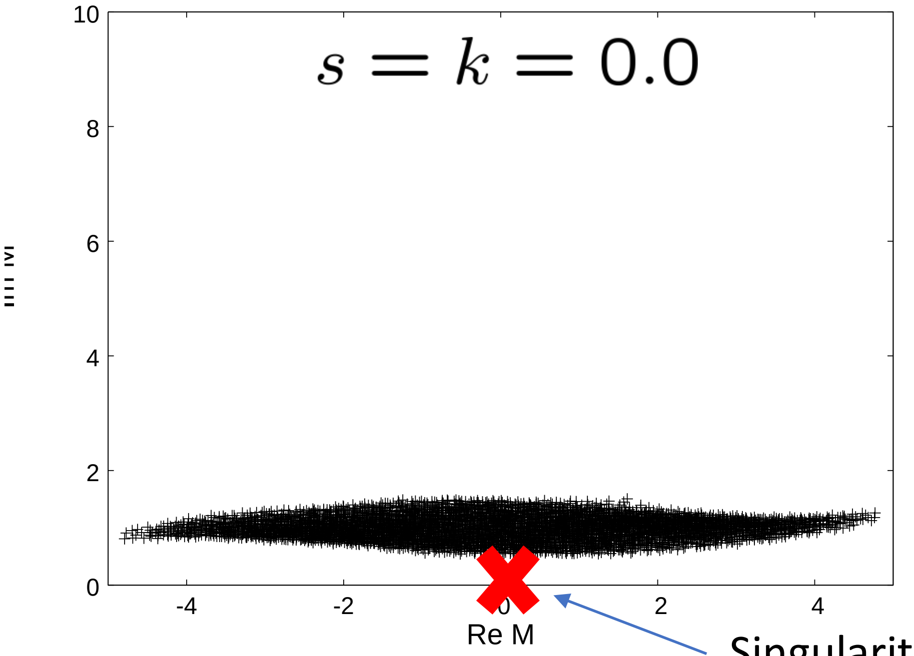
Nagata-J.N.-Shimasaki, Phys.Rev. D94 (2016) no.11, 114515.

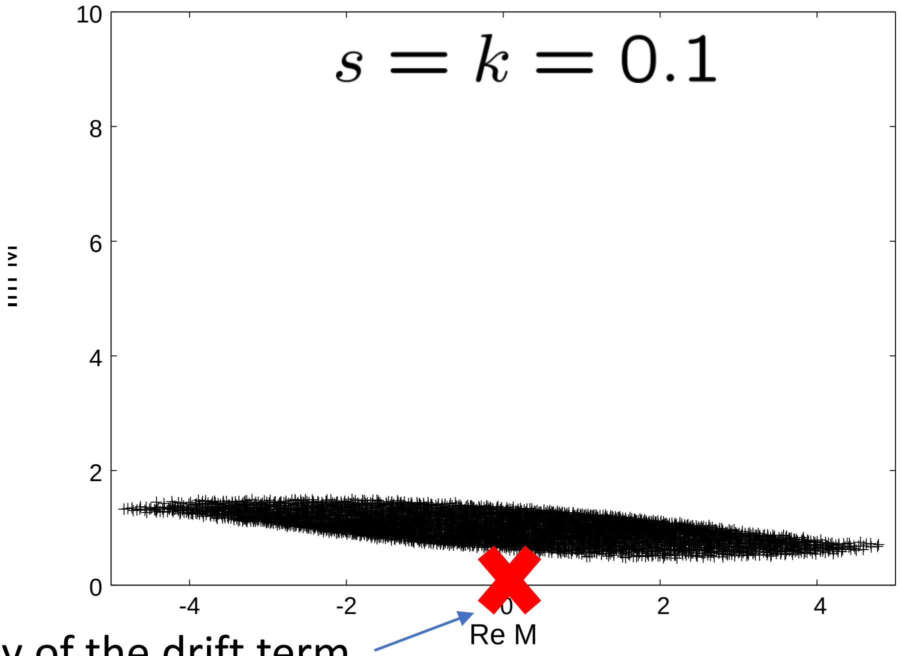
In the Lorenzian case, adding a small "mass" term in the Dirac operator can cure the problem !

The Dirac operator in the Lorentzian model has real eigenvalues for Hermitian configurations !

6D SUSY model with deformation parameter $m_{\mathrm{f}} = 1.0$

$N = 32 \ , \beta = 1.4 \ , \kappa = 1.0$





Singularity of the drift term