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Multiple modular symmetries as the origin of flavour

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See e.g. https://arxiv.org/pdf/1706.08749 Feruglio

Consider a complex field τ, $(\mathrm{Im}\,\tau > 0)$
a symmetry Γ with elements γ acts

$$\gamma : \tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d},$$

It is convenient to represent as 2×2

$$\overline{\Gamma} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} /(\pm\mathbf{1}),\ a, b, c, d \in \mathbb{Z},\quad ad - bc = 1 \right\}.$$

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The group has 2 Gens

$$S_\tau^2 = (S_\tau T_\tau)^3 = \mathbf{1}.$$

$$S_\tau : \tau \to -\frac{1}{\tau}, \qquad T_\tau : \tau \to \tau + 1,$$

$$S_\tau = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad T_\tau = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$ ③

A subgroup $\overline{\Gamma}(N)$

$$a = k_a N + 1, \; d = k_d N + 1, \; b = k_b N, \qquad c = k_c N,$$

$$\overline{\Gamma}(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in PSL(2, \mathbb{Z}), \; \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod N \right\}.$$

Make factor group

$$\overline{\Gamma}/\overline{\Gamma}(N) \sim \Gamma_N$$

$\Gamma_2 \sim S_3$, $\Gamma_3 \sim A_4$, $\Gamma_4 \sim S_4$, $\Gamma_5 \sim A_5$ ④

Reminder : Factor Group

G with Inv. subgroup H ; Make Cosets

H, g_iH, . . . These form a group

where multiplication is

$g_i H \cdot g_j H = (g_i \cdot g_j) H$

$S_3 : \{ e, (12), (23), (31) ; (123), (321) \}$

$Z_3 : \{ e, (123), (321) \} \equiv H$ $\quad S_3/Z_3 \sim Z_2$

$\{ H, (12)H \} \sim Z_2$

(5)

In a Γ_N invariant theory,
chiral superfield ϕ

$$\phi_i(\tau) \to \phi_i(\gamma\tau) = (c\tau + d)^{-2k_i} \rho_{I_i}(\gamma)\phi_i(\tau) \, ,$$

SUSY action

$$\mathcal{S} = \int d^4x d^2\theta d^2\overline{\theta} K(\phi_i, \overline{\phi}_i; \tau, \overline{\tau}) + \left[\int d^4x d^2\theta W(\phi_i; \tau) + \text{h.c.} \right] ,$$

$$\begin{aligned} K(\phi_i, \overline{\phi}_i; \tau, \overline{\tau}) &\to K(\phi_i, \overline{\phi}_i; \tau, \overline{\tau}) + f(\phi_i, \tau) + \overline{f}(\overline{\phi}_i, \overline{\tau}) \, , \\ W(\phi_i; \tau) &\to W(\phi_i; \tau) \, . \end{aligned}$$

invariant W

(6)

$$W(\phi_i;\tau) = \sum_n \sum_{\{i_1,\cdots,i_n\}} \sum_{I_Y} (Y_{I_Y}\phi_{i_1}\cdots\phi_{i_n})_{\mathbf{1}}\,.$$

T

"coefficients"

Complicated way of writing a general form

"coefficients" transform

$$Y_{I_Y}(\tau) \to Y_{I_Y}(\gamma\tau) = (c\tau + d)^{2k_Y}\rho_{I_Y}(\gamma)Y_{I_Y}(\tau)$$

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Stabilisers

$$\rho_I(\gamma)Y_I(\tau_\gamma) = (c\tau_\gamma + d)^{-2k}Y_I(\tau_\gamma)\,. \tag{31}$$

This equation lead us to the following important properties for the stabiliser and the modular form:

- A modular form at a stabiliser $Y_I(\tau_\gamma)$ is an eigenvector of the representation matrix $\rho_I(\gamma)$ with respective eigenvalue $(c\tau_\gamma + d)^{-2k}$.
- The stabiliser τ_γ satisfies $|c\tau_\gamma + d| = 1$ since $(c\tau_\gamma + d)^{-2k}$ is an eigenvalue of a unitary matrix.

A special case is that when $(c\tau_\gamma + d)^{-2k} = 1$ is satisfied, $\boxed{\rho_I(\gamma)Y_I(\tau_\gamma) = Y_I(\tau_\gamma)}$ and we recover the residual flavour symmetry generated by γ. In general, the eigenvalue does not need to be fixed at 1 in the framework of modular symmetry.

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Symmetries and stabilisers in modular invariant flavour models

Ivo de Medeiros Varzielas, Miguel Levy, Ye-Ling Zhou

The idea of modular invariance provides a novel explanation of flavour mixing. Within the context of finite modular symmetries Γ_N and for a given element $\gamma \in \Gamma_N$, we present an algorithm for finding stabilisers (specific values for moduli fields τ_γ which remain unchanged under the action associated to γ). We then employ this algorithm to find all stabilisers for each element of finite modular groups for $N = 2$ to 5, namely, $\Gamma_2 \simeq S_3$, $\Gamma_3 \simeq A_4$, $\Gamma_4 \simeq S_4$ and $\Gamma_5 \simeq A_5$. These stabilisers then leave preserved a specific cyclic subgroup of Γ_N. This is of interest to build models of fermionic mixing where each fermionic sector preserves a separate residual symmetry.

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Subjects: **High Energy Physics - Phenomenology (hep-ph)**; High Energy Physics - Theory (hep-th)
DOI: 10.1007/JHEP11(2020)085
Cite as: arXiv:2008.05329 **[hep-ph]**
(or arXiv:2008.05329v2 **[hep-ph]** for this version)

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E.g. $\tau = i$ is a stabiliser for S_2

$$S_2(i) = -\frac{1}{i} = i$$

$\tau = i\infty$ is a stabiliser for T_2

$$T_2[i\infty] : 1 + i\infty = i\infty$$

Stabilisers usually depend on N

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Method

We know what τ stabilises

1. Take $\tau = \tau_i$, where $\tau_i = \gamma_i \tau_i, i = 1, ..., 4$ is a stabiliser of $\mathcal{D}$;
2. Act γ on τ: $\tau' = \gamma\tau$. Compute γ^{-1}; — Go from τ to τ' with γ
3. The element that stabilises $\underline{\tau'}$ is given by $\underline{\gamma^{-1}\gamma_i}\gamma$.

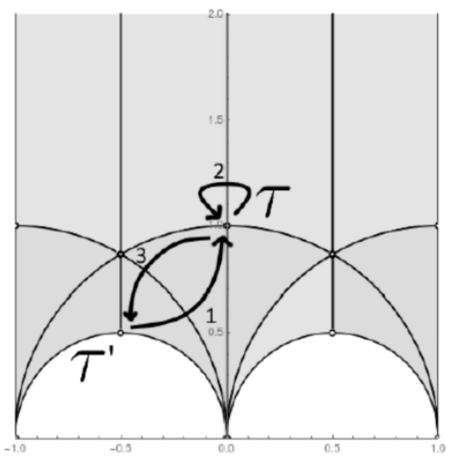

Figure 1: An example of the applied methodology to find the stabilisers of Γ_N. The example shown is for Γ_2, where the arrows denote the actions of different elements, γ^{-1}, γ_i, γ, for 1,2,3 respectively, following the convention of the text.

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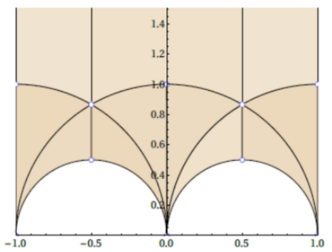
$\Gamma_2 \sim S_3$



Domain

Figure 2: The fundamental domain $\mathcal{D}(2)$ of $\bar{\Gamma}(2)$ (i.e., the full target space of $\Gamma_2 \simeq S_3$) with the stabilisers of modular transformations of Γ_2 denoted as dots.

	γ	τ_γ
$\mathcal{C}_2$	$T_\tau C_\tau$	$0, 1+i$
	T_τ	$i\infty, \frac{1}{2}+\frac{i}{2}$
	S_τ	$i, 1$
$\mathcal{C}_3$	$T_\tau S_\tau$	$-\frac{1}{2}+\frac{i\sqrt{3}}{2}, \frac{1}{2}+\frac{i\sqrt{3}}{2}$
	C_τ	$-\frac{1}{2}+\frac{i\sqrt{3}}{2}, \frac{1}{2}+\frac{i\sqrt{3}}{2}$

Stabilisers

Table 1: The non-identity elements of Γ_2 and respective stabilisers.

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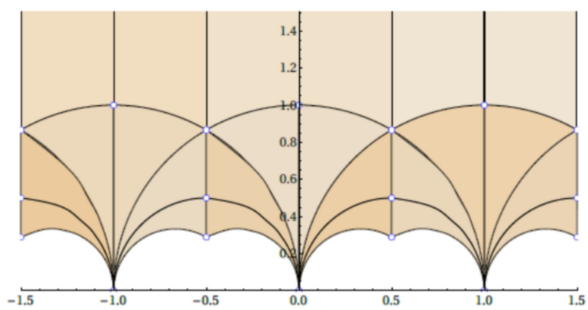
$\Gamma_3 \sim A_4$

Stabilisers

Domain



	γ	τ_γ
$\mathcal{C}_2$	C_τ^2	$-\frac{1}{2}+\frac{i\sqrt{3}}{2}, 1$
	T_τ^2	$i\infty, \frac{3}{2}+\frac{i}{2\sqrt{3}}$
	$T_\tau C_\tau$	$0, \frac{3}{2}+\frac{i\sqrt{3}}{2}$
	$C_\tau T_\tau$	$-1, \frac{1}{2}+\frac{i\sqrt{3}}{2}$
$\mathcal{C}_3$	C_τ	$-\frac{1}{2}+\frac{i\sqrt{3}}{2}, 1$
	T_τ	$i\infty, \frac{3}{2}+\frac{i}{2\sqrt{3}}$
	$C_\tau S_\tau$	$0, \frac{3}{2}+\frac{i\sqrt{3}}{2}$
	$T_\tau S_\tau$	$-1, \frac{1}{2}+\frac{i\sqrt{3}}{2}$
$\mathcal{C}_4$	$T_\tau^2 C_\tau$	$-1+i, \frac{1}{2}+\frac{i}{2}$
	S_τ	$i, \frac{3}{2}+\frac{i}{2}$
	$T_\tau C_\tau T_\tau$	$-\frac{1}{2}+\frac{i}{2}, 1+i$

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Multiples

Multiple modular symmetries as the origin of flavour

Ivo de Medeiros Varzielas, Stephen F. King, Ye-Ling Zhou

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Two A4 modular symmetries for Tri-Maximal 2 mixing

Ivo de Medeiros Varzielas, João Lourenço

We construct lepton flavour models based on two A_4 modular symmetries. The two A_4 are broken by a bi-triplet field to the diagonal A_4 subgroup, resulting in an effective modular A_4 flavour symmetry with two moduli. We employ these moduli as stabilisers, that preserve distinct residual symmetries, enabling us to obtain Tri-Maximal 2 (TM2) mixing with a minimal field content (without flavons).

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Subjects:	**High Energy Physics - Phenomenology (hep-ph)**
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Multiple Moduli

$$\gamma_J : \tau_J \to \gamma_J \tau_J = \frac{a_J \tau_J + b_J}{c_J \tau_J + d_J} .$$

$$\Gamma^1_{N_1} \times \Gamma^2_{N_2} \times \cdots \times \Gamma^M_{N_M} ,$$

Chiral Superfield

$$\begin{aligned}\phi_i(\tau_1, ..., \tau_M) \quad \to \quad & \phi_i(\gamma_1\tau_1, ..., \gamma_M\tau_M) \\ & = \prod_{J=1,...,M} (c_J\tau_J + d_J)^{-2k_{i,J}} \bigotimes_{J=1,...,M} \rho_{I_{i,J}}(\gamma_J)\phi_i(\tau_1, \tau_2, ..., \tau_M) ,\end{aligned}$$

SUSY Action :

$$\mathcal{S} = \int d^4x d^2\theta d^2\overline{\theta} K(\phi_i, \overline{\phi}_i; \tau_1, ..., \tau_M, \overline{\tau}_1, ..., \overline{\tau}_M) + \int d^4x d^2\theta W(\phi_i; \tau_1, ..., \tau_M) + \text{h.c.} ,$$

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$$W(\phi_i;\tau_1,...,\tau_M)=\sum_n \sum_{\{i_I,\cdots,i_n\}} \left(Y_{(I_{Y,1},...,I_{Y,M})}\phi_{i_1}\cdots\phi_{i_n}\right)_{\mathbf{1}},$$

where the "coefficients" are modular forms ...

$$\begin{aligned}Y_{(I_{Y,1},...,I_{Y,M})}(\tau_1,...,\tau_M) &\to Y_{(I_{Y,1},...,I_{Y,M})}(\gamma_1\tau_1,...,\gamma_M\tau_M)\\ &= \prod_{J=1,...,M}(c_J\tau_J+d_J)^{2k_{Y,J}} \bigotimes_{J=1,...,M}\rho_{I_{Y,J}}(\gamma_J)Y_{(I_{Y,1},...,I_{Y,M})}(\tau_1,...,\tau_M)\,.\end{aligned}$$

Complicated way of writing a general form!

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Example with S_4

$$S=T_\tau^2\,,\ T=S_\tau T_\tau\,,\ U=T_\tau S_\tau T_\tau^2 S_\tau\,.$$

$$\rho_I(S)=\begin{pmatrix}-1 & 2 & 2\\ 2 & -1 & 2\\ 2 & 2 & -1\end{pmatrix}/3\ ;$$

$$\boxed{\rho_I(\gamma)Y_I(\tau_\gamma)=Y_I(\tau_\gamma)}$$

$$Y_A(\langle \tau_A\rangle)=\begin{pmatrix}-1\\ 2\omega\\ 2\omega^2\end{pmatrix}$$

τ_A $\qquad\qquad\qquad$ τ_B

$$S=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix},\ T=\begin{pmatrix}0 & 1\\ -1 & -1\end{pmatrix},\ U=\begin{pmatrix}1 & -1\\ 2 & -1\end{pmatrix}.$$

τ_C

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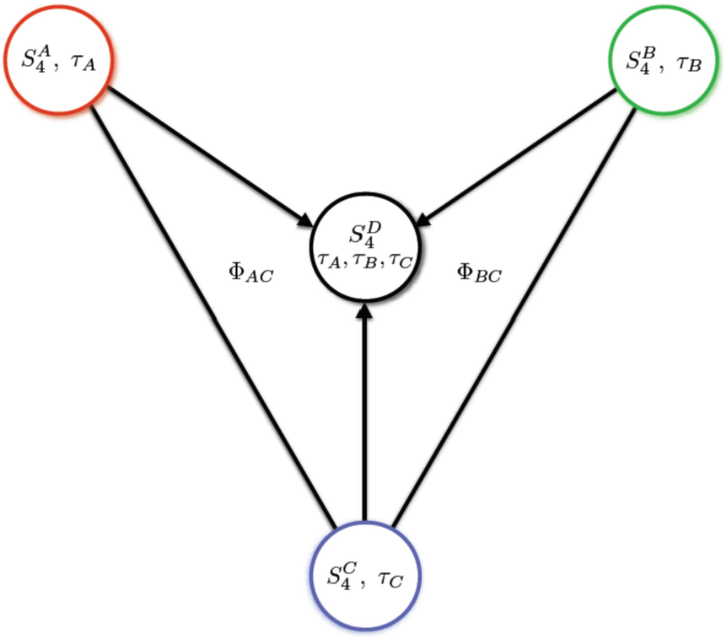

(18)

Field	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
L	**1**	**1**	**3**	0	0	0
e^c	**1**	**1**	**1**	0	0	-6
μ^c	**1**	**1**	**1**	0	0	-4
τ^c	**1**	**1**	**1**	0	0	-2
N_A^c	**1**	**1**	**1**	-6	0	0
N_B^c	**1**	**1**	**1**	0	-4	0
Φ_{AC}	**3**	**1**	**3**	0	0	0
Φ_{BC}	**1**	**3**	**3**	0	0	0

Yuk/Mass	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
$Y_e(\tau_C)$	**1**	**1**	**3**	0	0	6
$Y_\mu(\tau_C)$	**1**	**1**	**3**	0	0	4
$Y_\tau(\tau_C)$	**1**	**1**	**3**	0	0	2
$Y_A(\tau_A)$	**3**	**1**	**1**	6	0	0
$Y_B(\tau_B)$	**1**	**3**	**1**	0	4	0
$M_A(\tau_A)$	**1**	**1**	**1**	12	0	0
$M_B(\tau_B)$	**1**	**1**	**1**	0	8	0
$M_{AB}(\tau_A, \tau_B)$	**1**	**1**	**1**	6	4	0

Φ are bi-Triplets

$$
\begin{aligned}
w_\ell \;=\; & \frac{1}{\Lambda}\left[L\Phi_{AC}Y_A(\tau_A)N_A^c + L\Phi_{BC}Y_B(\tau_B)N_B^c\right]H_u \\
& + \left[LY_e(\tau_C)e^c + LY_\mu(\tau_C)\mu^c + LY_\tau(\tau_C)\tau^c\right]H_d \\
& + \frac{1}{2}M_A(\tau_A)N_A^cN_A^c + \frac{1}{2}M_B(\tau_B)N_B^cN_B^c + \underbrace{M_{AB}(\tau_A,\tau_B)N_A^cN_B^c}_{\text{cross-term allowed}}\,,
\end{aligned}
$$

(19)

$$\langle \Phi_{AC} \rangle_{i\alpha} = v_{AC}(P_{23})_{i\alpha}\,, \quad \langle \Phi_{BC} \rangle_{m\alpha} = v_{BC}(P_{23})_{m\alpha}\,.$$

$$P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(these come from the Potential)

$$\begin{aligned}
L\Phi_{AC}Y_A(\tau_A)N_A^c &= L_1\left[(\Phi_{AC})_{11}(Y_A)_1 + (\Phi_{AC})_{21}(Y_A)_3 + (\Phi_{AC})_{31}(Y_A)_2\right]N_A^c \\
&+ L_2\left[(\Phi_{AC})_{13}(Y_A)_1 + (\Phi_{AC})_{23}(Y_A)_3 + (\Phi_{AC})_{33}(Y_A)_2\right]N_A^c \\
&+ L_3\left[(\Phi_{AC})_{12}(Y_A)_1 + (\Phi_{AC})_{22}(Y_A)_3 + (\Phi_{AC})_{32}(Y_A)_2\right]N_A^c\,, \\
&= (L_1, L_2, L_3)P_{23}\begin{pmatrix} (\Phi_{AC})_{11} & (\Phi_{AC})_{12} & (\Phi_{AC})_{13} \\ (\Phi_{AC})_{21} & (\Phi_{AC})_{22} & (\Phi_{AC})_{23} \\ (\Phi_{AC})_{31} & (\Phi_{AC})_{32} & (\Phi_{AC})_{33} \end{pmatrix}^T P_{23}\begin{pmatrix} (Y_A)_1 \\ (Y_A)_2 \\ (Y_A)_3 \end{pmatrix} N_A^c\,,
\end{aligned} \quad (20)$$

$$S_4^A \times S_4^B \times S_4^C \to S_4^D\,,$$

$$w_\ell^{\text{eff}} = \left[\frac{v_{AC}}{\Lambda}LY_A(\tau_A)N_A^c + \frac{v_{BC}}{\Lambda}LY_B(\tau_B)N_B^c\right]H_u$$

$$+\left[LY_e(\tau_C)e^c + LY_\mu(\tau_C)\mu^c + LY_\tau(\tau_C)\tau^c\right]H_d$$

$$+\frac{1}{2}M_A(\tau_A)N_A^cN_A^c + \frac{1}{2}M_B(\tau_B)N_B^cN_B^c + M_{AB}(\tau_A,\tau_B)N_A^cN_B^c\,,$$

where e.g. $LY_A(\tau_A)N_A^c = \left[L_1(Y_A)_1 + L_2(Y_A)_3 + L_3(Y_A)_2\right]N_A^c\,,$

$$Y_e(\langle\tau_C\rangle) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad Y_\mu(\langle\tau_C\rangle) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad Y_\tau(\langle\tau_C\rangle) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},$$

$$Y_A(\langle\tau_A\rangle) = \begin{pmatrix} -1 \\ 2\omega \\ 2\omega^2 \end{pmatrix}, \quad Y_B(\langle\tau_B\rangle) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad (21)$$

$$M_N = \begin{pmatrix} M_A & M_{AB} \\ M_{AB} & M_B \end{pmatrix} . \qquad V = e^{i\alpha_3} \begin{pmatrix} \hat{c}_R & \hat{s}_R^* \\ -\hat{s}_R & \hat{c}_R^* \end{pmatrix} ,$$

cross-term ?

$$\begin{aligned} M_\nu \;&=\; (\mu_1 \hat{c}_R^2 + \mu_2 \hat{s}_R^{*2}) \begin{pmatrix} 1 & -2\omega^2 & -2\omega \\ -2\omega^2 & 4\omega & 4 \\ -2\omega & 4 & 4\omega^2 \end{pmatrix} + (\mu_1 \hat{s}_R^2 + \mu_2 \hat{c}_R^{*2}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \\ &+ (\mu_1 \hat{c}_R \hat{s}_R - \mu_2 \hat{c}_R^* \hat{s}_R^*) \begin{pmatrix} 0 & -1 & 1 \\ -1 & 4\omega^2 & 2i\sqrt{3} \\ 1 & 2i\sqrt{3} & -4\omega \end{pmatrix} , \end{aligned}$$

→ PMNS is TM_1

$$U_{\mathrm{TM}_1} = \begin{pmatrix} \frac{2}{\sqrt{6}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \end{pmatrix} .$$

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Good Fit

BF	Para.		χ^2	α_1	α_2	θ_R	μ_1	μ_2
			0.74	64.53°	20.38°	43.01°	0.00633 eV	0.0114 eV
	Obs.	θ_{12}	θ_{13}	θ_{23}	δ	m_2	m_3	m_{ee}
		34.33°	8.61°	49.6°	290°	0.00860 eV	0.0502 eV	0.00206 eV

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Conclusions

- Modular Symmetries are promising as the origin of Flavour
- Stabilisers play a key role
- Multiple Modular Symmetries have specific advantages

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T

Thanks

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