On nonsupersymmetric Pati-Salam string models

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work in colaboration with I. R, I. Florakis and K. Violaris–Gkountonis, arXiv:1608.04582 [hep-th], arXiv:1703.09272 [hep-th], work in progress

Outline

- Introduction
- Non-supersymmetric string models - Coordinate dependent compactifications
- Non-supersymmetric Pati–Salam string models
- One loop potential for the moduli fields - Cosmological constant
- Conclusions

Introduction

The Standard Model of particle physics has been proved remarkably successful in interpreting experimental results. However, it is considered as an effective theory as it leaves a number of unanswered questions including: charge quantization, neutrino masses, dark matter, hierarchy problem, gravity.

Supersymmetry is a well studied, compelling Standard Model extension that could help to resolve some of these issues. The introduction of SUSY at a few TeV leads also to coupling unification (with minimal content). However, as of today, experiments have not provided any evidence in favour of supersymmetry.

Non-supersymmetric strings

Space-time supersymmetry is not required for consistency in string theory.

From the early days of the first string revolution it was known that heterotic strings in 10D comprise both the supersymmetric $E_8 \times E_8$ and $SO(32)$ models and the non-supersymmetric tachyon free $SO(16) \times SO(16)$ theory.

However, non-supersymmetric string phenomenology has not received much attention until recently*.

* * see e.g. S. Abel, K. R. Dienes and E. Mavroudi (2015,2017) , J. R. and I. Florakis (2016,2017) , Y. Sugawara, T. Wada (2016) , A. Lukas, Z. Lalak and E. E. Svanes (2015) , S.G. Nibbelink, O. Loukas, A. Mütter, E. Parr, P. K. S. Vaudrevange (2017), Faraggi et all (2020) , T. Coudarchet, E. Dudas, H. Partouche (2021) , R. Perez-Martinez, S. Ramos-Sanchez and P. K. S. Vaudrevange (2021)

Non-supersymmetric strings

Any scenario of supersymmetry breaking in the context of string theory has to address some important issues, as

- Resolve M_W/M_P hierarchy
- Compatibility with gauge coupling evolution (“unification”)
- Account for the smallness of the cosmological constant
- Resolve possible instabilities (tachyons)
- Moduli field stabilisation

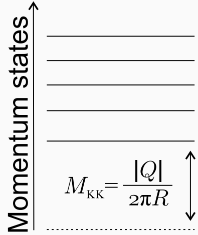
Coordinate dependent compactifications

The Scherk–Schwartz compactification provides an elegant mechanism to break SUSY in the context of String Theory. A (minimal) implementation of a stringy Scherk–Schwartz mechanism requires an extra dimension X^5 and a conserved charge Q. Upon compactification

$$\Phi\left(X^5 + 2\pi R\right) = e^{iQ}\Phi\left(X^5\right)$$

we obtain a shifted tower of Kaluza–Klein states for charged fields, starting at $M_{KK} = \frac{|Q|}{2\pi R}$

$$\Phi(X^5) = e^{\frac{iQX^5}{2\pi R}} \sum_{n\in Z} \Phi_n \, e^{i\,nX^5/R}$$



Coordinate dependent compactifications

$Q =$ Fermion number $\Rightarrow$ leads to different masses for fermions-bosons (lying in the same supermultiplet) and thus to spontaneous breaking of supersymmetry.

SUSY breaking related to the compactification radius $M \sim \frac{1}{R}$

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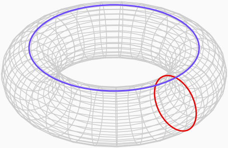
see e.g.
J. Scherk and J. H. Schwarz (1978,1979) , R. Rohm (1984) , C. Kounnas and M. Porrati (1988) , S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner (1989) , C. Kounnas and B. Rostand (1990) , C. Kounnas, H. Partouche (2017)

Gravitino mass

We consider compactifications of the six internal dimensions in three separate two-tori parametrised by the $T^{(i)}, U^{(i)}, i = 1, 2, 3$ moduli. For simplicity, we will assume that the Scherk–Schwartz mechanism is realised utilising the $T^{(1)}, U^{(1)}$ torus.

At tree level the gravitino receives a mass

$$m_{3/2} = \frac{|U^{(1)}|}{\sqrt{T_2^{(1)} U_2^{(1)}}} = \frac{1}{R_1}$$



for a square torus: $T = \imath R_1 R_2, U = \imath R_2/R_1$
All $T^{(i)}, U^{(i)}$ moduli remain massless.

At $R_1 \to \infty$ we have $m_{3/2} = 0$ and the supersymmetry is restored.

One loop partition function

$$
\begin{aligned}
Z = \frac{1}{\eta^2\bar\eta^2}\,\frac{1}{2^4} \sum_{\substack{h_1,h_2,H,H'\\ g_1,g_2,G,G'}} \frac{1}{2^3}\sum_{\substack{a,k,\rho\\ b,\ell,\sigma}} \frac{1}{2^3}\sum_{\substack{H_1,H_2,H_3\\ G_1,G_2,G_3}} (-1)^{a+b+HG+H'G'+\Phi} \\
\times \frac{\vartheta[^a_b]}{\eta}\,\frac{\vartheta[^{a+h_1}_{b+g_1}]}{\eta}\,\frac{\vartheta[^{a+h_2}_{b+g_2}]}{\eta}\,\frac{\vartheta[^{a-h_1-h_2}_{b-g_1-g_2}]}{\eta} \\
\times \frac{\bar\vartheta[^k_\ell]^3}{\bar\eta^3}\,\frac{\bar\vartheta[^{k+H'}_{\ell+G'}]}{\bar\eta}\,\frac{\bar\vartheta[^{k-H'}_{\ell-G'}]}{\bar\eta}\,\frac{\bar\vartheta[^{k+h_1}_{\ell+g_1}]}{\bar\eta}\,\frac{\bar\vartheta[^{k+h_2}_{\ell+g_2}]}{\bar\eta}\,\frac{\bar\vartheta[^{k-h_1-h_2}_{\ell-g_1-g_2}]}{\bar\eta} \\
\times \frac{\bar\vartheta[^{\rho+H'}_{\sigma+G'}]}{\bar\eta}\,\frac{\bar\vartheta[^{\rho-H'}_{\sigma-G'}]}{\bar\eta}\,\frac{\bar\vartheta[^\rho_\sigma]^2}{\bar\eta^2}\,\frac{\bar\vartheta[^{\rho+H}_{\sigma+G}]^4}{\bar\eta^4} \\
\times \frac{\Gamma^{(1)}_{2,2}[^{H_1}_{G_1}|^{h_1}_{g_1}](T^{(1)},U^{(1)})}{\eta^2\bar\eta^2}\,\frac{\Gamma^{(2)}_{2,2}[^{H_2}_{G_2}|^{h_2}_{g_2}](T^{(2)},U^{(2)})}{\eta^2\bar\eta^2}\,\frac{\Gamma^{(3)}_{2,2}[^{H_3}_{G_3}|^{h_1+h_2}_{g_1+g_2}](T^{(3)},U^{(3)})}{\eta^2\bar\eta^2}\,,
\end{aligned}
$$

where $T^{(i)} = T_1^{(i)} + iT_2^{(i)}$, $U^{(i)} = U_1^{(i)} + iU_2^{(i)}$ are the moduli of the three two tori, $\eta(\tau)$ is the Dedekind eta function and $\vartheta[^\alpha_\beta](\tau)$ stand for the Jacobi theta functions.

Twisted/shifted lattices

The Scherk–Schwarz breaking is implemented utilising orbifold shifts parametrised by $G_i, H_i, i = 1, 2, 3$

$$\Gamma_{2,2}\left[{}^{H_i}_{G_i}\middle|{}^{h}_{g}\right](T,U) = \begin{cases} \left|\frac{2\eta^3}{\vartheta\left[{}^{1-h}_{1-g}\right]}\right|^2 & ,\ (H_i, G_i) = (0,0) \text{ or } (H_i, G_i) = (h,g) \\ \Gamma^{\text{shift}}_{2,2}\left[{}^{H_i}_{G_i}\right](T,U) & ,\ h = g = 0 \\ 0 & ,\ \text{otherwise} \end{cases}$$

$$\Gamma^{\text{shift}}_{2,2}\left[{}^{H_i}_{G_i}\right](T,U) = \sum_{\substack{m_1,m_2\\ n_1,n_2}} (-1)^{G_i(m_1+n_2)}\, q^{\frac{1}{4}|P_L|^2}\, \bar{q}^{\frac{1}{4}|P_R|^2}\,,$$

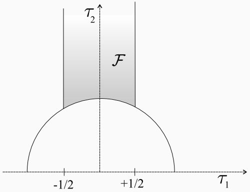
with

$$P_L = \frac{m_2 + \frac{H_i}{2} - Um_1 + T(n_1 + \frac{H_i}{2} + Un_2)}{\sqrt{T_2U_2}}\,,$$

$$P_R = \frac{m_2 + \frac{H_i}{2} - Um_1 + \bar{T}(n_1 + \frac{H_i}{2} + Un_2)}{\sqrt{T_2U_2}}\,.$$

One loop potential

The effective potential at one loop, as a function moduli $t_I = T^{(i)}, U^{(i)}$, is obtained by integrating the string partition function $Z(\tau_1, \tau_2; t_I)$ over the worldsheet torus Σ_1



$$V_{\rm one-loop}(t_I) = -\frac{1}{2(2\pi)^4}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3}\, Z(\tau, \bar{\tau}; t_I)\,,$$

where $\mathcal{F}$ is the fundamental domain .

For given values of the moduli

$$Z = \sum_{\substack{n\in\mathbb{Z}/2 \\ n\geq -1/2}} \sum_{m\in\mathbb{Z}} Z_{n,m}\, q_{\rm r}^n\, q_{\rm i}^m = \sum_{\substack{n\in\mathbb{Z}/2 \\ n\geq -1/2}} \left[\sum_{m=-[n]-1}^{[n]+2} Z_{n,m}\, q_{\rm i}^m\right] q_{\rm r}^n\,.$$

where $q_{\rm r} = e^{-2\pi\tau_2}$ and $q_{\rm i} = e^{2\pi i\tau_1}$

One loop potential: Large volume limit

The asymptotic behaviour of the one loop potential is

$$\lim_{T_2 \gg 1} V_{\text{one-loop}}(T, U) = -\frac{(n_B - n_F)}{2^4 \pi^7 T_2^2} \sum_{m_1, m_2 \in \mathbb{Z}} \frac{U_2^3}{\left|m_1 + \frac{1}{2} + U\, m_2\right|^6} + \mathcal{O}\left(e^{-\sqrt{2\pi T_2}}\right)$$

$$\lim_{T_2 \gg 1} V_{\text{one-loop}}(T, U) = \xi\, \frac{(n_B - n_F)}{T_2^2} + \text{exponentially supressed}$$

where ξ is a constant and n_B, n_F stand for the number of massless bosonic and fermionic degrees of freedom respectively, and $T_2 = R^2$ for a square torus.

Cosmological constant is exponentially small for large R for models with fermion-boson degeneracy $n_B = n_F$ (super-no-scale models).

The non-supersymmetric Pati-Salam model

Based on "Lepton Number as the Fourth Color", J. C. Pati and A. Salam (1974)

$$\text{Gauge symmetry} : SU(4)\times SU(2)_L\times SU(2)_R$$

SM Fermions:

$$F_L(\mathbf{4},\mathbf{2},\mathbf{1}) = Q(\mathbf{3},\mathbf{2},-1/6) + L(\mathbf{1},\mathbf{2},1/2)\,,$$

$$\overline{F}_R(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) = u^c(\overline{\mathbf{3}},\mathbf{1},2/3) + d^c(\overline{\mathbf{3}},\mathbf{1},-1/3) + e^c(\mathbf{1},\mathbf{1},-1) + \nu^c(\mathbf{1},\mathbf{1},0)$$

Extra triplets: $(\mathbf{6},\mathbf{1},\mathbf{1})$

Pati-Salam Higgs scalars: $H\,(\mathbf{4},\mathbf{1},\mathbf{2})$

SM Higgs scalars:

$$h\,(\mathbf{1},\mathbf{2},\mathbf{2}) = H_u\left(\mathbf{1},\mathbf{2},+\frac{1}{2}\right) + H_d\left(\mathbf{1},\mathbf{2},-\frac{1}{2}\right)$$

Pati–Salam string models

Our starting point is the free fermionic formulation of the heterotic string. In this context all world-sheet bosonic coordinates are fermionised (except the ones associated with 4D space-time).
In the standard notation the fermionic coordinates in the light-cone gauge are:

left:	$\psi^\mu, \chi^{1,\dots,6},$	$y^{1,\dots,6}, \omega^{1,\dots,6}$	
right:		$\bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6},$	$\bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \quad \bar{\phi}^{1,\dots,8}$

In this framework a model is defined by a set of basis vectors which encode the parallel transport properties of the fermionic fields along the non-contractible loops of the world-sheet torus, and a set of phases associated with generalised GSO projections (GGSO).

Pati–Salam string models

A class of Pati-Salam models can be generated by the basis

$$\beta_1 = \mathbf{1} = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\},$$
$$\beta_2 = S = \{\psi^\mu, \chi^{1,\dots,6}\},$$
$$\beta_3 = T_1 = \{y^{12}, \omega^{12} | \bar{y}^{12}, \bar{\omega}^{12}\},$$
$$\beta_4 = T_2 = \{y^{34}, \omega^{34} | \bar{y}^{34}, \bar{\omega}^{34}\}$$
$$\beta_5 = T_3 = \{y^{56}, \omega^{56} | \bar{y}^{56}, \bar{\omega}^{56}\},$$
$$\beta_6 = b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1}\},$$
$$\beta_7 = b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{2}\},$$
$$\beta_8 = z_1 = \{\bar{\phi}^{1,\dots,4}\},\ \beta_9 = z_2 = \{\bar{\phi}^{5,\dots,8}\},\ \beta_{10} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\},$$

and a set of $10(10-1)/2 + 1 = 46$ GGSO phases $c\begin{bmatrix}\beta_i\\ \beta_j\end{bmatrix} = \pm 1$.

This class compises $2^{46} \approx 7 \times 10^{13}$ models.

Gauge group:

$$G = \{SU(4) \times SU(2)_L \times SU(2)_R\}_{\text{observable}} \times U(1)^3 \times SU(2)^4 \times SO(8)$$

Phenomenological criteria

(a) Absence of physical tachyons in the string spectrum

(b) Existence of complete chiral fermion generations

(c) Existence of Pati–Salam and SM symmetry breaking scalar Higgs fields

(d) Absence of observable gauge group enhancements

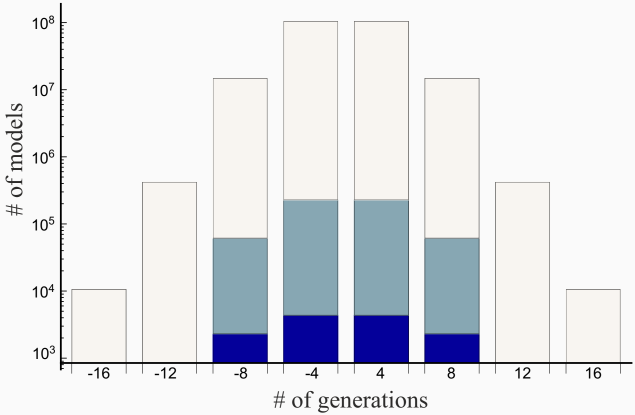
(e) Vector-like fractionally charged exotic states

(f) Consistency with the Scherk–Schwarz SUSY breaking

(g) Compliance with the super-no-scale condition, that is translated to equality of the fermionic and bosonic degrees of freedom

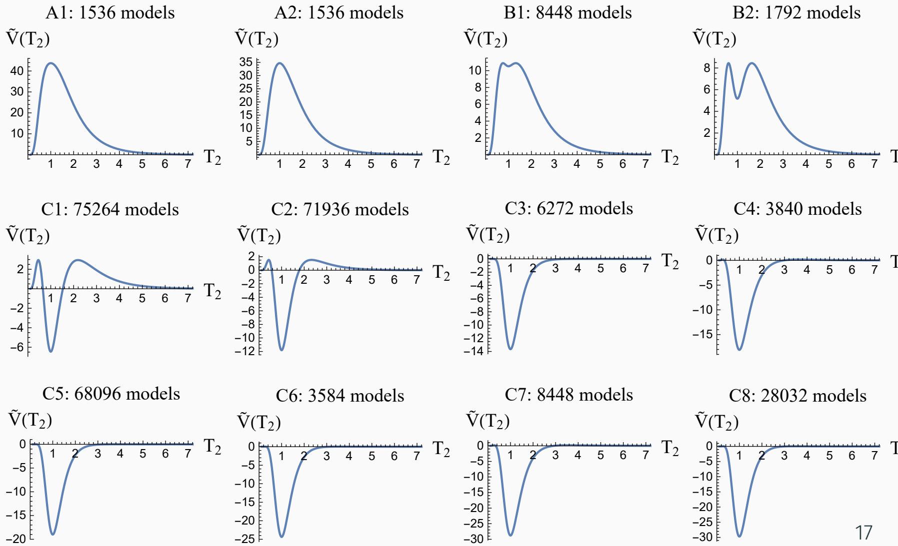
Phenomenologically promising Pati–Salam string models

A comprehensive computer scan over the full parameter space (1.7×10^{10} models) yields

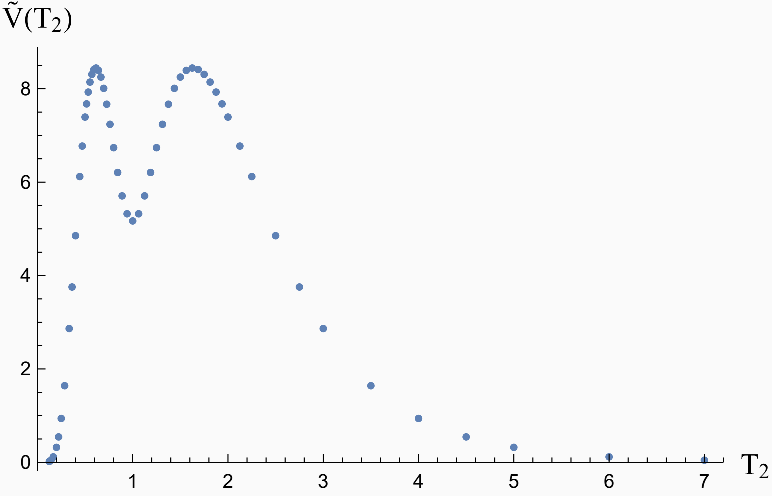


Light shaded bars: (a)-(c) 2.4×10^8 *models, Medium shaded bars (a)-(g)* 5.6×10^5 *models, Dark shading bars:* 1.4×10^4 *models*

One-loop potentials



Typical class B model potential



Conclusions

We have shown the existence of non-supersymmetric Pati-Salam string models with the following interesting properties

- Spectra with realistic characteristics (Fermion chirality, PS and SM Higgs scalars)
- SUSY breaking via the Scherk–Schwarz mechanism at scales $M_{susy} \sim \frac{1}{R} \ll M_{Planck}$
- Fermion-boson degeneracy (super-no-scale condition) that leads to exponentially small cosmological constant at the large volume limit
- Examine more realistic configurations employing real fermions … etc