RELATING THE PHASES of CKM and PMNS MATRICES in 2HDM

Francisco J. Botella
(with G.C. Branco and M. Nebot, arXiv:1808.00493 and with J.M. Alves, G.C. Branco, F. Cornet-Gómez and M. Nebot, arXiv:2105.14054)

IFIC-U.Valencia & CSIC-

September 2021

Introduction I

- A relation between CP violation in the quark and lepton sectors, (between δ_{CKM} and δ_{PMNS}).
- Experimentally CKM complex, even if New Physics.
- Complex CKM $\not\Rightarrow$ complex Yukawa couplings (vacuum induced CP violation).
- To relate CP violation in CKM and PMNS, we **assume that CP is spontaneously broken** (the vacuum phases generate δ_{CKM} and δ_{PMNS}).

2HDM and SCPV I

- The first model of spontaneous CP violation (**SCPV**): **T.D. Lee in 1973**, a two Higgs doublets model (**2HDM**), with vacuum expectation values with a relative phase which violates T and CP.
- 2HDM in general has Scalar Flavour Changing Neutral Couplings (**SFCNC**) at tree level.
- SFCNC **eliminated by** Natural Flavour Conservation (NFC) -Glashow and Weinberg-. (Z_2)
- SFCNC **are controlled** with Minimal Flavour Violation **(MFV)** in Branco Grimus Lavoura (**BGL** models) by Z_4-$\left(m_\beta/v\right)\, V_{t\alpha} V^*_{t\beta}$.
- **But** SCPV and NFC generate **real CKM** (Branco)
- **BGL models cannot have SCPV** (Z_4 symmetry too strong constraint).

2HDM and SCPV II

- Keeping Z_2 softly broken allows for SCPV and the particular realization of the Z_2 -that defines **gBGL** models-

$$\begin{array}{rcl} \Phi_1 \to \Phi_1 & ; & \Phi_2 \to -\Phi_2 \\ Q_{L_{1,2}} \to Q_{L_{1,2}} & ; & Q_{L_3} \to -Q_{L_3} \\ u_R \to u_R & ; & d_R \to d_R \end{array}$$

does not meet NFC criteria. A complex CKM is generated.

The gBGL model with SCPV I

- Introducing neutrino right-handed (Dirac for simplicity) with real $\Gamma_i^{(f)}$

$$\begin{aligned} L_Y &= -\overline{Q}_L\left(\Gamma_1^{(d)}\Phi_1+\Gamma_2^{(d)}\Phi_2\right)d_R-\overline{Q}_L\left(\Gamma_1^{(u)}\widetilde{\Phi}_1+\Gamma_2^{(u)}\widetilde{\Phi}_2\right)u_R \\ &\quad -\overline{L}_L\left(\Gamma_1^{(e)}\Phi_1+\Gamma_2^{(e)}\Phi_2\right)e_R-\overline{L}_L\left(\Gamma_1^{(\nu)}\widetilde{\Phi}_1+\Gamma_2^{(\nu)}\widetilde{\Phi}_2\right)\nu_R+.h.c. \end{aligned}$$

- The **gBGL** model is defined by the previous Z_2 symmetry giving the flavour structure

$$\Gamma_1^{(d)}\sim\Gamma_1^{(u)}\sim\Gamma_1^{(e)}\sim\Gamma_1^{(\nu)}\sim\begin{pmatrix}\times&\times&\times\\ \times&\times&\times\\ 0&0&0\end{pmatrix}$$

$$\Gamma_2^{(d)}\sim\Gamma_2^{(u)}\sim\Gamma_2^{(e)}\sim\Gamma_2^{(\nu)}\sim\begin{pmatrix}0&0&0\\ 0&0&0\\ \times&\times&\times\end{pmatrix}$$

The gBGL model with SCPV II

$$\Gamma_2^{(f)} = P_3 \Gamma_2^{(f)}; \quad \Gamma_1^{(f)} = (I - P_3)\, \Gamma_1^{(f)}; \quad P_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- SSB and SCPV is trigger by $\langle \Phi_j \rangle^T = \begin{pmatrix} 0 & e^{i\theta_j} v_j / \sqrt{2} \end{pmatrix}$ with $\theta = \theta_2 - \theta_1$. With $v^2 = v_1^2 + v_2^2$, $c_\beta = v_1/v$, $s_\beta = v_2/v$, $t_\beta = v_2/v_1$, $\langle H_1 \rangle^T = \begin{pmatrix} 0 & v/\sqrt{2} \end{pmatrix}$, $\langle H_2 \rangle^T = \begin{pmatrix} 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} e^{-i\theta_1} \Phi_1 \\ e^{-i\theta_2} \Phi_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ s_\beta & -c_\beta \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$$

The Yukawa sector in the Higgs basis will be

$$\begin{aligned} L_Y \quad = \quad & -\overline{Q}_L \frac{\sqrt{2}}{v} \left(M_d^0 H_1 + N_d^0 H_2 \right) d_R - \overline{Q}_L \frac{\sqrt{2}}{v} \left(M_u^0 \widetilde{H}_1 + N_u^0 \widetilde{H}_2 \right) u_R \\ & - \overline{L}_L \left(M_l^0 H_1 + N_l^0 H_2 \right) e_R + \overline{L}_L \frac{\sqrt{2}}{v} \left(M_\nu^0 \widetilde{H}_1 + N_\nu^0 \widetilde{H}_2 \right) \nu_R + h.c \end{aligned}$$

The gBGL model with SCPV III

where

$$M_d^0 = \frac{v}{\sqrt{2}}\left(\Gamma_1^{(d)} c_\beta + \Gamma_2^{(d)} s_\beta e^{i\theta}\right); \quad M_u^0 = \frac{v}{\sqrt{2}}\left(\Gamma_1^{(u)} c_\beta + \Gamma_2^{(u)} s_\beta e^{-i\theta}\right)$$
$$M_l^0 = \frac{v}{\sqrt{2}}\left(\Gamma_1^{(l)} c_\beta + \Gamma_2^{(l)} s_\beta e^{i\theta}\right); \quad M_\nu^0 = \frac{v}{\sqrt{2}}\left(\Gamma_1^{(\nu)} c_\beta + \Gamma_2^{(\nu)} s_\beta e^{-i\theta}\right)$$

- Together with the important result

$$N_f^0 = \left[t_\beta I - \left(t_\beta + t_\beta^{-1}\right) P_3\right] M_f^0$$

Note that it will not be possible to bi-diagonalize both matrices N_f^0 and M_f^0 simultaneously.

- The matrices N_f^0 **control SFCNC**, in general present in all sectors.

The gBGL model with SCPV IV

- The Higgs potential is the standard for 2HDM with a Z_2 symmetry, with a soft breaking term, the possibility to have CP violation from the vacuum is open

$$\begin{aligned} V &= \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \mu_{22}^2 \Phi_2^\dagger \Phi_2 - \mu_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \\ &+ \left[\lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + h.c. \right] + 2\lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) \\ &+ 2\lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 \end{aligned}$$

$$\cos\theta = \frac{\mu_{12}^2}{2\lambda_5 v_1 v_2}, \;\; \theta \neq 0, \pm\frac{\pi}{2}, \pm\pi$$

Generation of CP violating CKM and PMNS matrices I

- From the structure of the mass matrices it is evident that

$$M_f^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\sigma_f} \end{pmatrix} \widehat{M}_f^0 \equiv \Phi_3\left(\sigma_f\right) \widehat{M}_f^0$$

with $\widehat{M}_f^0$ real mass matrices and $\sigma_f = \pm\theta \begin{smallmatrix} f=d,e \\ f=u,\nu \end{smallmatrix}$. Therefore

$$M_f = \mathcal{U}_{f_L}^{\dagger} M_f^0 \mathcal{O}_{f_R} = \mathcal{O}_{f_L}^{T} \widehat{M}_f^0 \mathcal{O}_{f_R} = diag\left(m_{f_1}, m_{f_2}, m_{f_3}\right)$$

with $\mathcal{O}_{f_R}, \mathcal{O}_{f_L}$ real orthogonal matrices, and

$$\mathcal{U}_{f_L} = \Phi_3\left(\sigma_f\right) \mathcal{O}_{f_L}$$

Generation of CP violating CKM and PMNS matrices II

- But $V_{CKM} = \mathcal{U}_{u_L}^{\dagger}\mathcal{U}_{d_L}$ and $U_{PMNS} = \mathcal{U}_{e_L}^{\dagger}\mathcal{U}_{\nu_L}$ and we have

$$\boxed{V \equiv V_{CKM} = \mathcal{O}_{u_L}^{T}\Phi_3\left(2\theta\right)\mathcal{O}_{d_L}}\ ;\ \boxed{U \equiv U_{PMNS} = \mathcal{O}_{e_L}^{T}\Phi_3\left(-2\theta\right)\mathcal{O}_{\nu_L}}$$

  With seven parameters we can have arbitrary V and U, **except that CP violation in the quark sector and CP violation in the lepton sector, must vanish with $\theta \to 0$.**

- It is thus interesting to scrutinize the relation among the CP violating phases in V and U, δ_{CKM} and δ_{PMNS} respectively. δ_{CKM} and δ_{PMNS} will simply correspond to the CP phases in a standard parametrization.

CP violation in CKM and PMNS and SFCNC I

- To present the relation among δ_{CKM} and δ_{PMNS} the simplest approach would be to impose that SFCNC are absent, since there is no evidence yet of SFCNC beyond the SM. But this leads, as we will see, to a real CKM and thus SFCNC are necessary.
- The SFCNC are encoded in the N_f^0 matrices which control the Yukawa couplings of H_2. In the fermion mass bases, $N_f^0 \longrightarrow N_f$:

$$
\begin{aligned}
N_f &= \mathcal{U}_{f_L}^\dagger N_f^0 \mathcal{O}_{f_R} = \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) P_3^f \right] M_f \\
&= \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) P_3^f \right] diag\left(m_{f_1}, m_{f_2}, m_{f_3} \right)
\end{aligned}
$$

where we have introduced the projectors

$$
P_3^f = \mathcal{U}_{f_L}^\dagger P_3 \mathcal{U}_{f_L} = \mathcal{O}_{f_L}^T P_3 \mathcal{O}_{f_L}
$$

CP violation in CKM and PMNS and SFCNC II

the real projectors P_3^f generates the SFCNC and are

$$\left(P_3^f\right)_{ij} = \left(\mathcal{O}_{f_L}^T P_3 \mathcal{O}_{f_L}\right)_{ij} = \left(\mathcal{O}_{f_L}\right)_{3i}\left(\mathcal{O}_{f_L}\right)_{3j} \equiv \widehat{r}_{[f]i}\widehat{r}_{[f]j}$$

where $\widehat{r}_{[f]i}$ are the third rows of $\mathcal{O}_{f_L}$.

$$\mathcal{O}_{f_L} = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \widehat{r}_{[f]1} & \widehat{r}_{[f]2} & \widehat{r}_{[f]3} \end{pmatrix}$$

- Each $\widehat{r}_{[f]i}$ requires two independent parameters but because

$$P_3^u = V P_3^d V^\dagger \; ; \; P_3^e = U P_3^\nu U^\dagger$$

$$\left[\mathcal{O}_{u_L}^T P_3 \mathcal{O}_{u_L}\right] = \left[\mathcal{O}_{u_L}^T \Phi_3\left(2\theta\right)\mathcal{O}_{d_L}\right]\left[\mathcal{O}_{d_L}^T P_3 \mathcal{O}_{d_L}\right]\left[\mathcal{O}_{d_L}^T \Phi_3\left(-2\theta\right)\mathcal{O}_{u_L}\right]$$

we only need two parameters to control SFCNC in the quark sector in addition to V_{CKM}. (Similar in the lepton sector).

CP violation in CKM and PMNS and SFCNC III

- The only way to avoid SFCNC in P_3^f is to set one component $\widehat{r}_{[f]k} = 1$ and the others $\widehat{r}_{[f]j} = 0$ $j \neq k$, but then

$$\left(P_3^f\right)_{ij} = \delta_{ik}\delta_{jk} \equiv (P_k)_{ij}$$

and V_{CKM} is real if P_3^u or P_3^d has this form as we show bellow (The same in the leptonic sector). As an example in the lepton sector

$$\begin{aligned} U &= \mathcal{O}_{e_L}^T \Phi_3(-2\theta)\mathcal{O}_{\nu_L} = \mathcal{O}_{e_L}^T\left[I + \left(e^{-i2\theta} - 1\right)P_3\right]\mathcal{O}_{\nu_L} \\ &= \mathcal{O}_{e_L}^T\mathcal{O}_{\nu_L}\mathcal{O}_{\nu_L}^T\left[I + \left(e^{-i2\theta} - 1\right)P_3\right]\mathcal{O}_{\nu_L} \\ &= \mathcal{O}_{e_L}^T\mathcal{O}_{\nu_L}\left[I + \left(e^{-i2\theta} - 1\right)P_3^\nu\right] \end{aligned}$$

Then the PMNS matrix U is written as a real rotation times a diagonal matrix of phases, and thus there is no CP violation in the leptonic sector. Similar in the other cases.

CP violation in CKM and PMNS and SFCNC IV

- **Therefore, in this model, to have CP violation in the CKM matrix, there must be tree level SFCNC both in the up and in the down quark sectors.**
- **In order to have a non-vanishing CP violating phase in the PMNS matrix, there must be tree level SFCNC both in the neutrino and in the charged lepton sectors.**

The relation between the CKM and PMNS phases I

- Our CKM and PMNS matrices are

$$V \equiv V_{CKM} = \mathcal{O}_{u_L}^T \Phi_3 \left(2\theta\right) \mathcal{O}_{d_L} \quad ; \quad U \equiv U_{PMNS} = \mathcal{O}_{e_L}^T \Phi_3 \left(-2\theta\right) \mathcal{O}_{\nu_L}$$

with $c_x = \cos x$, $s_x = \sin x$ and

$$R_{12}\left(x\right) = \begin{pmatrix} c_x & s_x & 0 \\ -s_x & c_x & 0 \\ 0 & 0 & 1 \end{pmatrix} ; R_{13}\left(x\right) = \begin{pmatrix} c_x & 0 & s_x \\ 0 & 1 & 0 \\ -s_x & 0 & c_x \end{pmatrix}$$

$$R_{23}\left(x\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_x & s_x \\ 0 & -s_x & c_x \end{pmatrix}$$

The relation between the CKM and PMNS phases II

we can chose

$$
\begin{aligned}
V &= \mathcal{O}_{u_L}^{T}\Phi_3\left(2\theta\right)\mathcal{O}_{d_L} \\
\mathcal{O}_{u_L} &= R_{12}\left(p_1^u\right)R_{23}\left(p_2^u\right)R_{13}\left(p_3^u\right) \\
\mathcal{O}_{d_L} &= R_{12}\left(p_1^d=0\right)R_{23}\left(p_2^d\right)R_{13}\left(p_3^d\right)
\end{aligned}
$$

V will depend on $R_{12}^{T}\left(p_1^u\right)R_{12}\left(p_1^d\right)=R_{12}^{T}\left(p_1^u-p_1^d\right)$. Without lose of generality, $p_1^d=0$ in such a way that **the number of independent parameters in the quark sector are six** $\left\{p_1^u,p_2^u,p_3^u,p_2^d,p_3^d,\theta\right\}$ matching the four standard CKM $\left\{\theta_{12}^q,\theta_{13}^q,\theta_{23}^q,\delta_q\right\}$ and two from SFCNC $\left\{\widehat{r}_{[u]1},\widehat{r}_{[u]2}\right\}$; (or, equivalently, of 2 independent $\widehat{r}_{[u]j}$; $\widehat{r}_{[d]k}$)

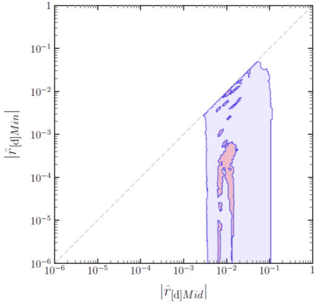
- The same happens in the lepton sector, six independent parameters in our PMNS U, $\left\{p_1^e,p_2^e,p_3^e,p_2^\nu,p_3^\nu,\theta\right\}$ should match the four standard PMNS $\left\{\theta_{12}^l,\theta_{13}^l,\theta_{23}^l,\delta_l\right\}$ and two from SFCNC $\left\{\widehat{r}_{[e]1},\widehat{r}_{[e]2}\right\}$.

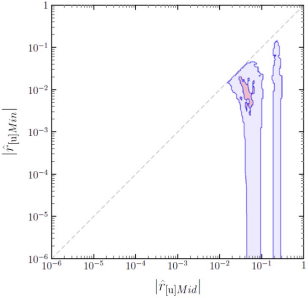
The relation between the CKM and PMNS phases III

- In summary, the experimental information constrains $\left\{\theta_{12}^{q}, \theta_{13}^{q}, \theta_{23}^{q}, \delta_{q}, \widehat{r}_{[u]1}, \widehat{r}_{[u]2}\right\}$ and could fix the model parameters $\left\{p_1^u, p_2^u, p_3^u, p_2^d, p_3^d, \theta\right\}$ (a full analysis along these lines was presented in Eur. Phys. J. C (2019) 79:711).
- The most important aspect here is that, ideally, one can fix θ with this procedure, since CP violation is well established in the quark sector.
- And finally with the well-known PMNS mixing angles $\left\{\theta_{12}^{l}, \theta_{13}^{l}, \theta_{23}^{l}\right\}$ together with the knowledge of $\left\{\widehat{r}_{[e]1}, \widehat{r}_{[e]2}\right\}$, from $h \longrightarrow l_i \bar{l}_j$, and incorporating θ **we should be able to predict** δ_l.

Quark sector analysis results I

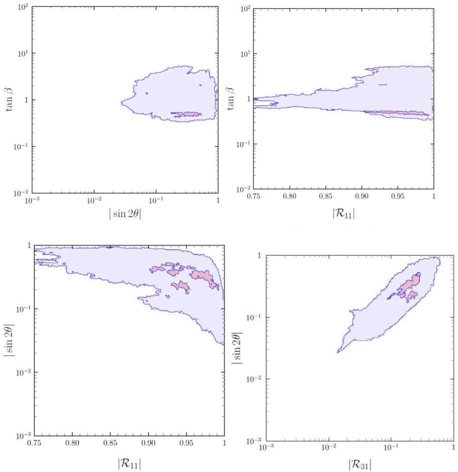
- In the quark sector the model is viable after surmounting flavour constraints, Higgs constraints, electroweak constraints and overall that, as we have shown, SFCNC cannot be eliminated to produce a correct δ_{CKM}
- $\widehat{r}_{[d]}$ and $\widehat{r}_{[u]}$





Quark sector analysis results II

- $\tan\beta$ vs $|\sin 2\theta|$ and $|\mathcal{R}_{11}|$ and $|\sin 2\theta|$ vs $|\mathcal{R}_{11}|$ and $|\mathcal{R}_{31}|$



Quark sector analysis results III

- Surprisingly, we have still **a lot of room** in the SFCNC and consequently **in the value of** θ. Therefore, generalizing the full analysis to include the leptonic sector does not look the more promising way to begin with, specially if we are trying to show how it works the connection among δ_{CKM} and δ_{PMNS} in this kind of models.

Simplified models (Quark sector) I

- The idea is to restrict the model by making simplifying assumptions about the SFCNC sector, guided by experimental data.
- We cannot assume the absence of SFCNC. The way of eliminating as much as possible SFCNC is to impose a zero in the vector $\widehat{r}_{[u]}$ and a zero in the vector $\widehat{r}_{[d]}$:

$\widehat{r}_{[u]}$	$(0, \times, \times)$	$(\times, 0, \times)$	$(\times, \times, 0)$
$\widehat{r}_{[d]}$	$(0, \times, \times)$	$(\times, 0, \times)$	$(\times, \times, 0)$

  only one type of SFCNC in each sector $(d_i \leftrightarrow d_j$ and $u_l \leftrightarrow u_m)$.
- **These models incorporate** the **MFV** ansatz, only four parameters as in CKM.
- In fact **the still allowed SFCNC**, in each sector, will be **fixed by one of the 3 mixing angles** of the V_{CKM} matrix.

Simplified models (Quark sector) II

- We have 9 models in the quark sector and 9 models in the lepton sector. **81 models has been analyzed. Only one survives the experimental data**, so we present this model in the quark sector.
- The surviving model has only $t \rightleftarrows c$ and $d \rightleftarrows b$ SFCNC

$$\widehat{r}_{[u]} = \left(0, -\sin p_2^u, \cos p_2^u\right) \; , \widehat{r}_{[d]} = \left(-\sin p_2^d, 0, \cos p_2^d\right)$$

$$\begin{aligned} O_{uL} &= R_{12}\left(p_1^u\right) R_{23}\left(p_2^u\right) = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix} \\ O_{dL} &= R_{13}\left(p_2^d\right) = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & 0 & \times \end{pmatrix} \end{aligned}$$

$$V = R_{23}^T\left(p_2^u\right) \Phi_3\left(2\theta\right) R_{12}^T\left(p_1^u\right) R_{13}\left(p_2^d\right)$$

Simplified models (Quark sector) III

- The result of the fit of V to the experimental data (to the known V_{CKM}) gives

2θ	p_1^u	p_2^u	p_2^d
$1.077\,\left(^{+0.039}_{-0.031}\right)$	$0.22694\,(52)$	$4.235\,(59)\cdot 10^{-2}$	$3.774\,(98)\cdot 10^{-3}$

  In order to relate δ_{CKM} and δ_{PMNS} **it is specially relevant that the quark sector fixes θ**

- In addition our fit fixes SFCNC with

$$\widehat{r}_{[u]} = (0, -0.0423, 0.9991)$$
$$\widehat{r}_{[d]} = (-0.0038, 0, 0.9999)$$

  A non-trivial result is that these values are within the allowed regions of the previous figures. The precise effects in specific processes depend on other parameters like t_β and $\mathcal{R}_{11}$ that is the corresponding element of the Higgs mixing matrix, in particular it is the mixing

Simplified models (Quark sector) IV

among the 125GeV Higgs and the Higgs with SM Higgs couplings. From the previous figures and taking from the θ value $|\sin(2\theta)| = 0.88$ we get

$$\mathcal{R}_{11} \in (0.82, 0.90)\,, \quad t_\beta \in (0.5, 1.8)$$

- The most relevant prediction of this model in the SFCNC concerns the transition $t \to ch$

$$Br\,(t \to ch) = 0.1306\,(1 - \mathcal{R}_{11}^2)\left(t_\beta + t_\beta^{-1}\right)^2 r_{[u]2}^2\, r_{[u]3}^2$$

$$\boxed{2.7 \times 10^{-4} \leq Br\,(t \to ch) \leq 4.3 \times 10^{-4}}$$

In the $d \rightleftarrows b$ SFCNC we get $B_r\left(h \to b\overline{d} + d\overline{b}\right) \sim 10^{-6}$.

Simplified models (Lepton sector) I

- $Br(\mu \to e + \gamma) < 4.2 \times 10^{-13}$ is controlled by $\left|U_{ei} U_{\mu i}\right|^2$. Requiring a fine tuning at the level of $10^{-4} - 10^{-5}$ among the neutral scalar and pseudoscalar contributions in the 2 loop Barr-Zee graphs. It is mandatory to put a zero in $\widehat{r}_{[e]1}$ or in $\widehat{r}_{[e]2}$.
- Still in the neutrino sector we have three possibilities of putting the zero in each one of the components of $\widehat{r}_{[\nu]}$. Therefore we are left with 6 different models in the leptonic sector.
- Out of these six cases the only one allowed experimentally is:

$$\widehat{r}_{[e]} = \left(-\sin p_2^e, 0, \cos p_2^e\right) \ , \ \widehat{r}_{[\nu]} = \left(-\sin p_2^\nu, \cos p_2^\nu, 0\right)$$

$$\begin{aligned} O_{eL} &= R_{12}\left(p_1^e\right) R_{13}\left(p_2^e\right) \\ O_{\nu L} &= P_{23} R_{12}\left(p_2^\nu\right) \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

Simplified models (Lepton sector) II

$$U = R_{13}^T\left(p_2^l\right)\Phi_3\left(-2\theta\right)R_{12}^T\left(p_1^l\right)P_{23}R_{12}\left(p_2^\nu\right)$$

The PMNS matrix is fully fixed by three mixing angles and the CP violating phase θ already fixed by the quark sector.

- Now we can fit U to the experimental information on PMNS encoded in $\left\{\theta_{12}^l, \theta_{13}^l, \theta_{23}^l\right\}$. We fix the quark fit result $2\theta = 1.077$. Different PMNS analyses show some sensitivity to the phase δ_l, we do not include that information. The fit gives two solutions:

Solution 1:	$p_1^e = 0.7496$,	$p_2^e = 1.3541$,	$p_2^\nu = 0.8974$
Solution 2:	$p_1^e = 2.3889$,	$p_2^e = 1.3541$,	$p_2^\nu = 1.0542$

SFCNC are controlled in both cases by $\hat{r}_{[e]} = (-0.9765, 0, 0.2156)$

Simplified models (Lepton sector) III

- Most important, the solutions differ in the values of the (unique) CP violating imaginary part of the Jarlskog invariant quartet

$$J_{PMNS} = \mathrm{Im}\left(U_{e1}U_{\mu 2}U^*_{e2}U^*_{\mu 1}\right)$$

and the phase $\delta_{PMNS} = \delta_I$,

Case	J_{PMNS}	$\delta_{PMNS} = \delta_I$	$\Delta\chi^2_{NO}(\delta_{PMNS})$	$\Delta\chi^2_{IO}(\delta_{PMNS})$
Solu 1	-0.0316	1.629π (**293º**)	5	0
Solu 2	0.0282	0.679π (**126º**)	13	> 20

$\Delta\chi^2_{NO}(\delta_{PMNS})$ and $\Delta\chi^2_{IO}(\delta_{PMNS})$ show the values that correspond to δ_{PMNS} attending to the $\Delta\chi^2$ profiles for δ_I obtained for normal and inverted neutrino mass ordering.

Simplified models (Lepton sector) IV

- **Using the information on CP violation in the quark sector, we have been able to predict the phase in PMNS using the connection that SCPV provides in this model; in particular, Solution 1 has** $\delta_{PMNS} = 1.629\pi$, **which is in good agreement with the most likely values in PMNS analyses**.
- We have also the parameters that control the SFCNC in the $\tau \leftrightarrow e$ sector, $r_e = -0.9765$ and $r_\tau = 0.2156$. These figures give rise again to a definite prediction for $h \to e\overline{\tau} + \tau\overline{e}$, through the equation

$$B_r\left(h \to e\tau\right) = \left(1 - \mathcal{R}_{11}^2\right)\left(t_\beta + t_\beta^{-1}\right)^2 r_e^2 r_\tau^2 B_r^{SM}\left(h \to \tau\overline{\tau}\right)\left(\frac{\Gamma_{SM}\left(h\right)}{\Gamma\left(h\right)}\right)$$

Taking the allowed regions of $\mathcal{R}_{11}^2$ and t_β, we have the sharp prediction

$$\boxed{3 \times 10^{-3} < \left(\frac{\Gamma(h)}{\Gamma_{SM}(h)}\right) B_r\left(h \to e\tau\right) < 5 \times 10^{-3}}$$

Simplified models (Lepton sector) V

Note that this result should be seen or disproved soon because the actual experimental bound is $B_r\left(h \to e\tau\right) < 4.7 \times 10^{-3}$ (CMS has announced $B_r\left(h \to e\tau\right) < 2.2 \times 10^{-3}$, see De Roeck talk).

Conclusions I

- We have discussed the possibility of having a framework where there is a connection between the CP violations in the quark and the lepton sectors.
- We have used gBGL 2HDM with SCPV.
- We have shown that in order to generate a complex CKM matrix, one has to have SFCNC both in the up and down quark sectors (similar in the lepton sector).
- We have shown that within those gBGL models, there is a connection between δ_{CKM} and δ_{PMNS}. The interplay among CPV and the existence of SFCNC makes these relations quite involved implying connections or predictions for processes mediated by SFCNC in all the sectors: up, down quarks and charged leptons.

Conclusions II

- To clarify all these relations we have worked with models that have the minimal amount of SFCNC needed to keep SCPV. These simplified models verify the MFV ansatz. Because they are controlled by the four unit vectors $\widehat{r}_{[u]}$, $\widehat{r}_{[d]}$, $\widehat{r}_{[e]}$, $\widehat{r}_{[\nu]}$ having a zero in some entry, there are $3^4 = 81$ possible models of this type.
- In the model that is in agreement with all constraints the connection between CKM and PMNS gives a prediction for PMNS in agreement with recent PMNS analyses, together with

$$
\begin{aligned}
2.7 \times 10^{-4} \quad &\leq \quad Br\left(t \to ch\right) \leq 4.3 \times 10^{-4} \\
3 \times 10^{-3} \quad &< \quad \left(\frac{\Gamma\left(h\right)}{\Gamma_{SM}\left(h\right)}\right) B_r\left(h \to e\tau\right) < 5 \times 10^{-3}
\end{aligned}
$$