Soft Limits and the Double-Copy

Based on
Arxiv:2106.12600 with Chi, A. Herderschee, C. Jones, S. Paranjape
and Arxiv:1611.07534 w/ C. Jones and S. Naculich

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Celestial Amplitudes from Soft Limits

When $\quad p^\mu \to \epsilon p^\mu \quad$ with $\quad \epsilon \to 0$

Photons $$A_{n+1}^{\mathrm{ph}} = \left(\frac{S^{(0)}}{\epsilon^2} + \frac{S^{(1)}}{\epsilon}\right) A_n$$

Gravity $$A_{n+1}^{\mathrm{grav}} = \left(\frac{\mathcal{S}^{(0)}}{\epsilon^3} + \frac{\mathcal{S}^{(1)}}{\epsilon^2} + \frac{\mathcal{S}^{(2)}}{\epsilon}\right) A_n$$

Weinberg (1965)

Cachazo & Strominger (2014)

The soft factors are UNIVERSAL.

Celestial Amplitudes from Soft Limits

When $p^\mu \to \epsilon p^\mu$ with $\epsilon \to 0$

Photons
$$A_{n+1}^{\mathrm{ph}} = \left(\frac{S^{(0)}}{\epsilon^2} + \frac{S^{(1)}}{\epsilon}\right) A_n + \frac{\tilde{S}^{(1)}}{\epsilon} \tilde{A}_n + O(\epsilon^0)$$

Gravity
$$A_{n+1}^{\mathrm{grav}} = \left(\frac{\mathcal{S}^{(0)}}{\epsilon^3} + \frac{\mathcal{S}^{(1)}}{\epsilon^2} + \frac{\mathcal{S}^{(2)}}{\epsilon}\right) A_n + \frac{\tilde{\mathcal{S}}^{(2)}}{\epsilon} \tilde{A}_n + O(\epsilon^0)$$

Weinberg (1965)

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New terms from certain-cubic field EFT operators

Jones, Naculich, HE (2016)

Motivated by the question of loop-corrections:
The effect of massive particles are encoded in EFT operators.

The soft factors are UNIVERSAL.

Divergent Soft Limit Master Formula

In 4d, the universal soft limits can be derived from a Master Formula Jones, Naculich, HE (2016)

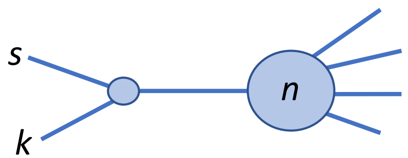
$$\hat{A}_{n+1}(z,\epsilon) = \sum_{k,h_P,c} \frac{g_{H_k}[sk]^{2h_s-\mathsf{a}}\langle X s\rangle^{1-\mathsf{a}}\,\hat{A}_n^{(k)}(z)}{\epsilon\, z^{\mathsf{a}-1}\langle sk\rangle^{2-\mathsf{a}}\left(1-\frac{z}{\epsilon}\frac{\langle Xk\rangle}{\langle sk\rangle}\right)} + O(\epsilon^0) \qquad \mathsf{a} \equiv h_s - h_k - h_P + 1$$

where we use 4d spinor helicity formalism $p = -|p\rangle[p|$ and a positive-helicity particle *s* is taken soft holomorphically:

$p_s = -|s\rangle[s| \to -\epsilon|s\rangle[s|$ with $|s\rangle \to \epsilon|s\rangle$ and $|s] \to |s]$

The formula is derived from very basic principles (locality & unitarity)

- Only cubic interactions can give rise to divergent soft limits
- Tree amplitudes factorize on their simple poles
- 3-pt amplitudes of massless particles are uniquely fixed by the particle helicities *h*



Above X is a reference spinor and z is an auxiliary variable.

Divergent Soft Limit Master Formula

There can be no poles at z=0

=>

- charge conservation
- equivalence principle
- Massless spin > 2 cannot couple consistently gravitons
- Massless spin 3/2 must couple supersymmetrically to gravitons

Jones, Naculich, HE (2016)

Well-known statements, but derived from same compact formula.

The order $O(z^0)$ terms in the Master Formula

=>

the universal soft theorems

and

specify precisely which 3-field EFT operators contribute at subleading order to the soft photon theorem and at subsubleading order to the soft graviton theorem

Soft behavior

Under the holomorphic shift of a massless soft particle,

$p_s = -|s\rangle[s|$ with $|s\rangle \to \epsilon|s\rangle$ and $|s] \to |s]$

a (tree) amplitude generally behaves as

$$A_n \to O(\epsilon^\sigma)$$

where σ is the ***soft weight***

We saw divergent soft limits

Photons: $\sigma = -2$

Gravitons: $\sigma = -3$

Also

Gluons: $\sigma = -2$

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When $\sigma > 0$ the amplitude has vanishing soft limits.
Often (but not always) the case for Goldstone bosons (Adler zeros)

Examples: NLSM (such as chiral perturbation theory) $\sigma = 1$

DBI (Dirac-Born-Infeld) $\sigma = 2$

Special Galileon $\sigma = 3$

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Also models with $\sigma = 0$ e.g.

Born-Infeld (BI)
N=2 SUSY CP^1 NLSM
Conformal-DBI

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Double-Copy

The double-copy maps gluon amplitudes to gravity amplitudes

gravity^{+} = (Yang-Mills) x (Yang Mills)

So…. How can it be that

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When naively one might have expected

$$\sigma_{\text{grav}} \ ? =? \ \sigma_{\text{YM}} + \sigma_{\text{YM}}$$

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Answer: DOUBLE-COPY **KERNEL**

$$M_4 = -s A_4[1234] A_4[1243]$$

$$\sigma_{\rm grav} = 1 + \sigma_{\rm YM} + \sigma_{\rm YM}$$

Mandelstam variables

$$s = (p_1 + p_2)^2$$
$$t = (p_1 + p_3)^2$$
$$u = (p_1 + p_4)^2$$

The double-copy in field theory: KLT form

Color-ordered tree amplitudes

$$\boxed{A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]}$$

[Kawai-Lewellen-Tye 1985]

a and b are choices of (n-3)! color orderings

KLT kernel a function of Mandelstams

Ex 4pt: $A_4^{\text{gravity}} = A_4^{\text{YM}}[1234]\, S_4[1234|1243]\, A_4^{\text{YM}}[1243]$

with $S_4[1234|1243] = -s$

It remarkable that this works!!

Color-ordered YM gluon amplitudes: $A_4[1234]$ has simple poles in s and u, but not t.
$A_4[1243]$ has simple poles in s and t, but not u.

$$s = (p_1 + p_2)^2$$
$$t = (p_1 + p_3)^2$$
$$u = (p_1 + p_4)^2$$

Graviton amplitudes have no color-structure, so $M_4(1234)$ has simple poles in the s, t and u channels.

How can a product of A_4's possibly get even the pole structure of M_4 right??? *And* avoid double-poles?

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$$M_4 = -s A_4[1234] A_4[1243]$$

$$M_4 = -\frac{su}{t} A_4[1234] A_4[1234]$$

The double-copy kernel:
1) Eliminates double-poles from $A_4 * A_4$
2) Provides "missing" poles

Another important aspect of field theory KLT: KKBCJ relations

$$M_4 = -s A_4[1234] A_4[1243] \qquad \textit{and} \qquad M_4 = -\frac{su}{t} A_4[1234] A_4[1234]$$

then their difference must be zero, i.e.

$$0 = A_4[1243] - \frac{u}{t} A_4[1234]$$

And this **is** true for YM amplitudes.

This is an example of a **BCJ** (Bern-Carrasco-Johansson) **relation** at 4-point.

$$\left.\begin{array}{l} \text{Kleiss-Kuijf} \left\{ \begin{array}{ll} \text{Trace-reversal:} & \mathcal{A}_4[1432] = \mathcal{A}_4[1234]\,, \quad \textit{etc} \\ U(1)\text{-decoupling:} & \mathcal{A}_4[1234] + \mathcal{A}_4[1243] + \mathcal{A}_4[1423] = 0\,, \end{array} \right. \\ \qquad\qquad\qquad \text{BCJ:} \quad \mathcal{A}_4[1234] - \dfrac{t}{u}\mathcal{A}_4[1243] = 0\,. \end{array} \right\} \text{“KKBCJ relations”}$$

A ***n*-point**

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

choices of (n-3)! color orderings

KLT kernel

and associated **KKBCJ relations** that ensure that the result of the double-copy is *independent* of the choice of (n-3)! color-orders out of the (n-1)! possible in the KLT sum.

Field theory double-copy `selection criterium'

In order to be "double-copyable", a theory's tree amplitudes must obey the KK and BCJ relations.

reduces the number of color-orderings from (n-1)! to (n-2)!

reduces the number of color-orderings from (n-2)! to (n-3)!

A new way to explore the landscape of field theories: *which theories can be input/output of the double-copy?*

Which theories obey the field theory KK&BCJ relations?

YM theory ✓

Chiral perturbation theory ✓

Super YM theory ✓

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Double-copy multiplication table

FT ⊗ FT	YM	$\mathcal{N}=4$ SYM	χPT
YM	gravity+	$\mathcal{N}=4$ SG	BI
$\mathcal{N}=4$ SYM	$\mathcal{N}=4$ SG	$\mathcal{N}=8$ SG	$\mathcal{N}=4$ sDBI
χPT	BI	$\mathcal{N}=4$ sDBI	sGalileon

$$n_{\mathrm{L}\otimes\mathrm{R}} = n_{\mathrm{L}} n_{\mathrm{R}}$$

Helicity maps as $h_{\mathrm{L}\otimes\mathrm{R}} = h_{\mathrm{L}} + h_{\mathrm{R}}$

Softness maps as $\sigma_{\mathrm{L}\otimes\mathrm{R}} = 1 + \sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}$

Example: all 70 scalars of N=8 supergravity have vanishing soft limits with $\sigma = 1$

Does FT x FT -> FT have an identity element?

$$\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \quad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,, \quad \mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.$$

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Color structure

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

no color structure

single color

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$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b] \longrightarrow m_n[\gamma|\delta] = \sum_{\alpha,\beta} m_n[\gamma|\alpha]\, S_n[\alpha|\beta]\, m_n[\beta|\delta]$$

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double color-structure

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Helicity maps as $h_{\mathrm{L}\otimes\mathrm{R}} = h_{\mathrm{L}} + h_{\mathrm{R}}$

Softness maps as $\sigma_{\mathrm{L}\otimes\mathrm{R}} = 1 + \sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}$

Spectrum has to map to self
=> only allow a single scalar in double-adjoint rep $\phi^{aa'}$

$$\sigma = 1 + 2\sigma \implies \sigma = -1$$

ϕ^3 interaction

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Which model is it?

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ϕ^3 interaction

$$f^{abc}\tilde{f}^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'} \quad ?$$

$$d^{abc}\tilde{d}^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'} \quad ?$$

$$i\,f^{abc} = \mathrm{Tr}\left[T^a[T^b, T^c]\right]$$

$$d^{abc} = \mathrm{Tr}\left[T^a\{T^b, T^c\}\right]$$

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$$f^{abc}\tilde{f}^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'} \quad ?$$

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$$\cancel{d^{abc}\tilde{d}^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'}} \quad ?$$

$$\cancel{d^{abc} = \mathrm{Tr}\left[T^a\{T^b, T^c\}\right]}$$

Cubic Bi-Adjoint Scalar model (BAS)

$$\mathcal{L}_{\mathrm{BAS}} = -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - g\, f^{abc}\tilde{f}^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'}$$

structure constants

Scalar in adjoint of (say) U(N) and U(N')

So that takes care of $\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$

What about the rest of the algebra?

The double-copy is a **map** FT x FT -> FT

The kernel defines the **multiplication rule** of this map

FT $\otimes$ FT	YM	$\mathcal{N}=4$ SYM	χPT	BAS
YM	gravity+	$\mathcal{N}=4$ SG	BI	YM
$\mathcal{N}=4$ SYM	$\mathcal{N}=4$ SG	$\mathcal{N}=8$ SG	$\mathcal{N}=4$ sDBI	$\mathcal{N}=4$ SYM
χPT	BI	$\mathcal{N}=4$ sDBI	sGalileon	χPT
BAS	YM	$\mathcal{N}=4$ SYM	χPT	BAS

String KLT *also* has an identity element:
BAS + very specific higher-derivative operators.

KLT algebra

$$\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \qquad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,, \qquad \mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.$$

Consider now the double-copy in EFT contexts

Which theories obey the field theory KKBCJ relations?

YM theory ✓ Chiral perturbation theory ✓

Super YM theory ✓ Bi-adjoint scalar model ✓

What about higher-derivative operators in EFTs?

YM: $\mathrm{tr}\,F^2$ ✓ $\mathrm{tr}\,F^3$ ✓ $\mathrm{tr}\,F^4$ 1✗ $\mathrm{tr}\,D^2F^4$ 1✓1✗ $\mathrm{tr}\,D^4F^4$ 1✓2✗ . . .

Here just MHV counting

Dixon & Broedel

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Here only MHV counting

χPT: $\mathrm{tr}\,\partial^2\phi^n$ ✓ $\mathrm{tr}\,\partial^4\phi^4$ 2✗ $\mathrm{tr}\,\partial^6\phi^4$ 1✓1✗ $\mathrm{tr}\,\partial^8\phi^4$ 1✓2✗ $\mathrm{tr}\,\partial^{10}\phi^4$ 1✓2✗ ...

Why are some operators allowed and not others? ***Is this the most general story?***

YM + h.d.

YM + h.d.

Gravity$^+$ + h.d.

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

YM + h.d.

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Include **higher-derivative corrections in the double-copy kernel**

String theory does that!

String theory KLT

KLT originally came from closed string = (open string)^2 at tree-level

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

string KLT kernel

The KLT kernel is deeply linked with the open string amplitudes to ensure correct pole structure in the closed string amps.

Upon expansion in alpha’, this translates to very particular higher-derivative corrections of the kernel:
not the most general options and tuned exactly to the alpha’ corrections in the open string.

Example: $$S_4[1234|1243] = -\sin(\pi\alpha' s) = -\pi\alpha' s + \frac{1}{6}(\pi\alpha' s)^3 + \ldots$$

Only *s*-dependence, no *t* or *u*; why?

Only odd powers in *s*; why?

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

What are the rules for generalizing the KLT kernel?

The generalized double-copy kernel should

1) eliminate double-poles
2) provide “missing” poles
3) not introduce spurious poles

We propose a new framework for systematically analyzing generalizations of the double-copy kernel: the KLT bootstrap

2106.12600 with Chi, A. Herderschee, C. Jones, S. Paranjape

The proposal is based on the **KLT algebra**

KLT algebra

$$\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \quad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,, \quad \underbrace{\mathbf{1} = \mathbf{1} \otimes \mathbf{1}}_{\text{KLT Bootstrap Equation}}\,.$$

When the multiplication rule is changed,
the identity element is changed, and vice versa:
The kernel and the identity model are uniquely linked!

KLT algebra

$$\underbrace{\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \quad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,,}_{\text{Generalize the KKBCJ / monodromy relations}} \quad \underbrace{\mathbf{1} = \mathbf{1} \otimes \mathbf{1}}_{\text{KLT Bootstrap Equation}}\,.$$

Generalize the **KKBCJ / monodromy** relations

(will not derive here; see 2106.12600)

KLT Bootstrap Equation

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the identity element is changed, and vice versa:
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Bi-Adjoint Scalar model (BAS)

$$\mathcal{L}_{\mathrm{BAS}} = -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - g f^{abc} \tilde{f}^{a'b'c'} \phi^{aa'} \phi^{bb'} \phi^{cc'}$$

Statement BAS = BAS x BAS --- or $\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ can be written as

$$m_n[\gamma|\delta] = \sum_{\alpha,\beta} m_n[\gamma|\alpha]\, S_n[\alpha|\beta]\, m_n[\beta|\delta]$$

or in **matrix form** $m_n = m_n.S_n.m_n$

(n-3)! x *(n-3)! submatrices*

Double-sum over *(n-3)!* color orderings

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or in **matrix form** $m_n = m_n . S_n . m_n$

So multiplying from both the left and right with inverses of matrices of BAS amplitudes gives

$$S_n = \left(m_n\right)^{-1}$$

[Cachazo et al]

The field theory KLT kernel is the inverse of an *(n-3)!* x *(n-3)!* submatrix of BAS amplitudes!

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4-point case

Tree amplitudes color-ordered wrt both color-factors, e.g.

$$m_4[1234|1234] = \frac{g^2}{s} + \frac{g^2}{u}\,, \qquad m_4[1234|1243] = -\frac{g^2}{s}$$

=>

$$S_4[1234|1234] = \left(m_4[1234|1234]\right)^{-1} = -\frac{su}{tg^2}\,,$$

$$S_4[1234|1243] = \left(m_4[1243|1234]\right)^{-1} = -\frac{s}{g^2}\,.$$

Strings KLT kernel

The string theory KLT kernel is the inverse of an *(n-3)!* x *(n-3)!* submatrix of BAS+ (*specific* h.d.) amplitudes!

[Mizera]

$$m_4^{(\alpha')}[1234|1243] = -\frac{1}{\sin(\pi\alpha' s)} = -\underbrace{\frac{1}{\pi\alpha' s}}_{\text{BAS}} - \frac{1}{6}\pi\alpha' s - \frac{7}{360}(\alpha'\pi s)^3 + \dots .$$

How to generalize the double-copy kernel?

Which terms are allowed in BAS+h.d.?

KLT bootstrap

n=4 => (n-1)! = 6 single-trace color-orderings: 1234, 1243, 1324, 1342, 1423, 1432

Recall that $\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ means $m_n = m_n . S_n . m_n$ and this implies $S_n = \left(m_n\right)^{-1}$

Written out for rank (4-3)!=1 at 4-point means, *for example*:

KLT bootstrap equation

$$m_n[1234|1234] = m_n[1234|1243] \frac{1}{m_4[1243|1243]} m_n[1243|1234]$$

$$\Longrightarrow \quad m_n[1234|1234] m_4[1243|1243] - m_n[1234|1243] m_n[1243|1234] = 0$$

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So this condition is that a 2x2 minor of the 6x6 **matrix of m_4[a|b] amplitudes** have to vanish:

$$\begin{pmatrix} \boxed{\begin{matrix} m_4[1234|1234] & m_4[1234|1243] \\ m_4[1243|1234] & m_4[1243|1243] \end{matrix}} & \cdots \\ \cdots & \end{pmatrix}$$

Similarly, all 2x2 minors must vanish! But that's just saying that we must have a rank 1 system. **Aha!**

KLT bootstrap at 4-pt

$$m_4[1234|1234] = f_1(s,t) \quad \text{with} \quad f_1(s,t) = f_1(-s-t,t)\,,$$
$$m_4[1234|1243] = f_2(s,t)\,,$$
$$m_4[1234|1324] = f_3(s,t) = f_2(-s-t,t)\,,$$
$$m_4[1234|1342] = f_4(s,t) = f_2(s,t)\,,$$
$$m_4[1234|1423] = f_5(s,t) = f_2(-s-t,t)\,,$$
$$m_4[1234|1432] = f_6(s,t) \quad \text{with} \quad f_6(s,t) = f_6(-s-t,t)\,,$$

Cyclic symmetry & momentum relabeling

6 x 6 matrix for these amplitudes has rank 6.

4-point KLT bootstrap equations

Imposing the vanishing of all 2x2 minors =>

$$\boxed{f_1(s,t) = \frac{f_2(s,t)f_2(-s-t,s)}{f_2(t,s)}\,,} \quad \boxed{f_6(s,t) = f_1(s,t)\,.}$$
$$\boxed{f_2(s,t)f_2(-s-t,s)f_2(t,-s-t) = f_2(t,s)f_2(-s-t,t)f_2(s,-s-t)\,.}$$

Solved by BAS and the strings BAS+h.d. amplitudes.

What else solves it?

Most general rank (n-3)! kernel at 4-point

Write the most general ansatz for f_2: $f_2(s,t) = -\frac{g^2\Lambda^2}{s} + \sum_{k=0}^{N} \sum_{r=0,k} \frac{a_{k,r}}{\Lambda^{2k}} s^r\, t^{k-r}$

Solve the **KLT bootstrap equations** order by order. Impose **locality**. Result:

$$f_2(s,t) = -\frac{g^2\Lambda^2}{s} + \frac{1}{\Lambda^2}(a_{1,0}t + a_{1,1}s) + \frac{a_{2,0}}{\Lambda^4}t(s+t) + \frac{1}{\Lambda^6}\left[a_{3,0}t^3 + a_{3,1}st^2 + a_{3,2}s^2t + a_{3,3}s^3\right] + \mathcal{O}\left(\frac{1}{\Lambda^8}\right)$$

Strings result recovered for

$$a_{1,1} = -\frac{1}{6}, \qquad a_{3,3} = -\frac{7}{360}, \ldots$$

and all other $a_{i,j} = 0$

New double-copy kernel much more general.

4-point result as BAS + h.d. Lagrangian

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - \frac{g}{6} f^{abc} \tilde{f}^{a'b'c'} \phi^{aa'} \phi^{bb'} \phi^{cc'} \\
&\quad - \frac{a_\mathrm{L} + a_\mathrm{R}}{2\Lambda^4} f^{abx} f^{cdx} f^{a'b'x'} f^{c'd'x'} (\partial_\mu \phi^{aa'})(\partial^\mu \phi^{bb'}) \phi^{cc'} \phi^{dd'} \\
&\quad + \frac{a_\mathrm{L}}{\Lambda^4} f^{abx} f^{cdx} d^{a'b'x'} d^{c'd'x'} (\partial_\mu \phi^{aa'}) \phi^{bb'} (\partial^\mu \phi^{cc'}) \phi^{dd'} \\
&\quad + \frac{a_\mathrm{R}}{\Lambda^4} d^{abx} d^{cdx} f^{a'b'x'} f^{c'd'x'} (\partial_\mu \phi^{aa'}) \phi^{bb'} (\partial^\mu \phi^{cc'}) \phi^{dd'} + \dots
\end{aligned}
$$

$$d^{abc} = \mathrm{Tr}\left[T^a \{T^b, T^c\}\right]$$

Observations

- There is no $d^{abc} d^{a'b'c'} \phi_{aa'} \phi_{bb'} \phi_{cc'}$ term: it does not solve the rank 1 bootstrap equations.
- There is no ϕ^4 term; does not solve the rank 1 bootstrap equations
- The d^{abc} terms modify the U(1) decoupling identities that are part of the field theory KK relations and generalize the strings monodromy relations.
- Known strings kernel has $a_L = a_R$. The generalization allows "heterotic"-type double-copy.

Double-copy of YM + h.d.

Impose generalized KKBCJ relations <=> $\mathbf{1} \otimes \mathbf{R} = \mathbf{R} \quad \mathbf{L} \otimes \mathbf{1} = \mathbf{L}$

on a general ansatz for MHV 4-pt YM + h.d. to find

$$\mathcal{A}_4^L[1^+2^+3^-4^-] = [12]^2\langle 34\rangle^2 \left[\frac{(g_{\text{YM}}^L)^2}{su} - \frac{1}{\Lambda^4}\left(\frac{(g_{\text{YM}}^L)^2}{g^2} a_L + (g_{F^3}^L)^2 \frac{t}{s} \right) - \frac{e_{3,1}^L}{\Lambda^6} t + \mathcal{O}\left(\frac{1}{\Lambda^8}\right) \right]$$

Usual YM

tr F^4

Pole term w/ two tr F^3 vertices

tr D^2F^4

Its coefficient is controlled by the generalized KLT kernel

And similarly for the R sector.

Summarizing the difference between admissible operators in ordinary field theory KLT vs. the new generalized KLT:

For YM + higher-derivatives

FT KLT YM: $\mathrm{tr}\,F^2$ ✓ $\mathrm{tr}\,F^3$ ✓ $\mathrm{tr}\,F^4$ 1✗ $\mathrm{tr}\,D^2F^4$ 1✓1✗ $\mathrm{tr}\,D^4F^4$ 1✓2✗ ...

Gen. KLT YM: $\mathrm{tr}\,F^2$ ✓ $\mathrm{tr}\,F^3$ ✓ $\mathrm{tr}\,F^4$ 1✓ $\mathrm{tr}\,D^2F^4$ 1✓1✗ $\mathrm{tr}\,D^4F^4$ 1✓2✓ ...

Green checkmark: operator allowed with arbitrary coefficient.
Blue checkmark: operator allowed with coefficient fixed by the parameters in the KLT kernel.

Summarizing the difference between admissible operators in ordinary field theory KLT vs. the new generalized KLT:

For YM + higher-derivatives

FT KLT	YM:	$\mathrm{tr}\,F^2$ ✓	$\mathrm{tr}\,F^3$ ✓	$\mathrm{tr}\,F^4$ 1✗	$\mathrm{tr}\,D^2F^4$ 1✓1✗	$\mathrm{tr}\,D^4F^4$ 1✓2✗ ...
Gen. KLT	YM:	$\mathrm{tr}\,F^2$ ✓	$\mathrm{tr}\,F^3$ ✓	$\mathrm{tr}\,F^4$ 1✓	$\mathrm{tr}\,D^2F^4$ 1✓1✗	$\mathrm{tr}\,D^4F^4$ 1✓2✓ ...

For chiPT + higher-derivatives

FT KLT	χPT:	$\mathrm{tr}\,\partial^2\phi^n$ ✓	$\mathrm{tr}\,\partial^4\phi^4$ 2✗	$\mathrm{tr}\,\partial^6\phi^4$ 1✓1✗	$\mathrm{tr}\,\partial^8\phi^4$ 1✓2✗	$\mathrm{tr}\,\partial^{10}\phi^4$ 1✓2✗
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Summarizing the difference between admissible operators in ordinary field theory KLT vs. the new generalized KLT:

For YM + higher-derivatives

FT KLT	YM: $\mathrm{tr}\,F^2$ ✓	$\mathrm{tr}\,F^3$ ✓	$\mathrm{tr}\,F^4$ 1✗	$\mathrm{tr}\,D^2F^4$ 1✓1✗	$\mathrm{tr}\,D^4F^4$ 1✓2✗	...
Gen. KLT	YM: $\mathrm{tr}\,F^2$ ✓	$\mathrm{tr}\,F^3$ ✓	$\mathrm{tr}\,F^4$ 1✓	$\mathrm{tr}\,D^2F^4$ 1✓1✗	$\mathrm{tr}\,D^4F^4$ 1✓2✓	...

For chiPT + higher-derivatives

FT KLT	χPT: $\mathrm{tr}\,\partial^2\phi^n$ ✓	$\mathrm{tr}\,\partial^4\phi^4$ 2✗	$\mathrm{tr}\,\partial^6\phi^4$ 1✓1✗	$\mathrm{tr}\,\partial^8\phi^4$ 1✓2✗	$\mathrm{tr}\,\partial^{10}\phi^4$ 1✓2✗
Gen. KLT	χPT: $\mathrm{tr}\,\partial^2\phi^n$ ✓	$\mathrm{tr}\,\partial^4\phi^4$ 2✗	$\mathrm{tr}\,\partial^6\phi^4$ 1✓1✓	$\mathrm{tr}\,\partial^8\phi^4$ 1✓2✗	$\mathrm{tr}\,\partial^{10}\phi^4$ 1✓2✓

Green checkmark: operator allowed with arbitrary coefficient.
Blue checkmark: operator allowed with coefficient fixed by the parameters in the KLT kernel.

For FIXED choice of kernel, this LINKS the coefficients of $\mathrm{tr}\,F^4$ with that of one of the $\mathrm{tr}\,\partial^6\phi^4$ operators.

Double-copy of YM + h.d. -> Gravity⁺ + h.d.

Usual Einstein gravity

Pole term from exchanges of dilaton *and* axion!

$$\mathcal{M}_4(1^+2^+3^-4^-) = [12]^4\langle 34\rangle^4 \times\Bigg[-\frac{(g^L_{\rm YM})^2(g^R_{\rm YM})^2}{g^2\Lambda^2}\frac{1}{stu} + \frac{((g^L_{\rm YM})^2(g^R_{F^3})^2+(g^R_{\rm YM})^2(g^L_{F^3})^2)}{g^2\Lambda^6}\frac{1}{s} + \frac{1}{\Lambda^8}\left(\frac{(g^L_{\rm YM})^2(g^R_{\rm YM})^2}{g^4}a_{2,0} + \frac{1}{g^2}((g^R_{\rm YM})^2 e^L_{3,1} + (g^L_{\rm YM})^2 e^R_{3,1})\right) + \mathcal{O}\left(\frac{1}{\Lambda^{10}}\right)\Bigg]$$

vanishes in string theory

local R^4 contribution

In the field theory or strings double copy, there is less freedom in the coefficient of R^4.

The result of the double-copy: in all cases checked, *same operators produced but with shifts of their coefficients.*

Higher-point

Necessary to test consistency by going to higher point:

What if the KLT bootstrap at 5-point further fixed some of the 4-point kernel coefficients $a_{i,j}$?
(Then we'd be in trouble!)

For n=5 => (n-1)! = 4! = 24 distinct orderings.

Cyclic symmetry + momentum relabelings => parameterized by 8 functions $g_i(s,t)$, i=1,2,…,8.

We impose the rank (n-3)! = 2 conditions equivalent to $\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ on this 24x24 system and solve.

Found consistent solution for the bootstrapped 5pt (BAS+h.d.) amplitudes; **no** constraints placed on 4-pt coefficients; in fact, up to quadratic order in Mandelstams, the amplitudes are completely fixed by 4-pt input.

Tested for 5pt +++++ YM+h.d.

Summary

- We have investigated the algebraic structure of the KLT multiplication rule.
- The KLT algebra gives a systematic way to generalize the double-copy in the KLT form: the **double-copy bootstrap.**
- Solved as BAS + most general h.d. terms for minimal rank $(n-3)!$ at 4- and 5-point.
- Tested in examples with YM and chiPT.

$$\underbrace{\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \quad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,,}_{\text{Generalize the }\textbf{KKBCJ / monodromy}\text{ relations}} \quad \underbrace{\mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.}_{\textbf{KLT Bootstrap Equation}}$$

Outlook

1) To the orders checked, the generalized double-copy produces the **same** h.d. operators in the double-copy LxR amplitude, but with some shifted Wilson coefficients: why?

   small multiplicity / low-enough dim effect? or something more fundamental?

   => Currently studying similarity transformations from ``hybrid" double-copy kernels, finding interesting algebraic structures. [Alan Chen & H.E., work in progress].

2) **The method is more than BAS+hd**. It is a ***framework for exploring more general forms of the double-copy***:

   - Does there exist other form of the double-copy without the cubic BAS interaction?
   - Is minimal rank (n-3)! fundamental?
   - Initiated study of non-minimal rank examples in our paper, more to do.

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3) Also, recent work on higher-derivative terms in the color-factors in the BCJ formulation

    [Carrasco, Rodina, Zekioglu, Z.Yin (2019+2021)]

    => their BCJ-form => BAS + h.d. also with rank (n-3)! (in the examples we have checked)

    => have translated a few examples to their form to ours

    The relationship should be studied more.

Example of exact kernel solution

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - \frac{g}{6} f^{abc}\tilde{f}^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'} \\
&\quad - \frac{\cancel{a_{\mathrm{L}}} + a_{\mathrm{R}}}{2\Lambda^4} f^{abx} f^{cdx} f^{a'b'x'} f^{c'd'x'} (\partial_\mu \phi^{aa'})(\partial^\mu \phi^{bb'})\phi^{cc'}\phi^{dd'} \\
&\quad \cancel{+ \frac{a_{\mathrm{L}}}{\Lambda^4} f^{abx} f^{cdx} d^{a'b'x'} d^{c'd'x'} (\partial_\mu \phi^{aa'})\phi^{bb'}(\partial^\mu \phi^{cc'})\phi^{dd'}} \\
&\quad + \frac{a_{\mathrm{R}}}{\Lambda^4} d^{abx} d^{cdx} f^{a'b'x'} f^{c'd'x'} (\partial_\mu \phi^{aa'})\phi^{bb'}(\partial^\mu \phi^{cc'})\phi^{dd'} \cancel{+ \ldots}
\end{aligned}
$$

Exact minimal rank solution at 4pt and 5pt

- L sector unmodified BAS KKBCJ relations
- R sector has modified KKBCJ relations

Kernel

$$S_4[1234|1234] = (m_4[1234|1234])^{-1} = -\frac{g^2 t}{su} - 4\frac{a_{\mathrm{R}}}{\Lambda^4}\, t$$

YM + h.d.

$$\mathcal{A}_4^{\mathrm{L}}[1^+2^+3^-4^-] = [12]^2\langle 34\rangle^2 \frac{(g_{\mathrm{YM}}^{\mathrm{L}})^2}{su}$$

$$\mathcal{A}_4^{\mathrm{R}}[1^+2^+3^-4^-] = (g_{\mathrm{YM}}^{\mathrm{R}})^2[12]^2\langle 34\rangle^2\left[\frac{1}{su} + \frac{4a_{\mathrm{R}}}{g^2\Lambda^4}\right]$$

Double-copy:

$$\mathcal{M}_4(1^+2^+3^-4^-) = \kappa^2\frac{[12]^4\langle 34\rangle^4}{stu}$$

So: GR = YM x (YM + F^4)   !!!

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   => have translated a few examples to their form to ours

   The relationship should be studied more.

4) Positivity constraints? EFT-hedron? UV completability? What makes the strings kernel special?

Outlook

5) The double-copy also has a celestial version Casali + Puhm 2007

Is there a celestial formulation of the double-copy bootstrap?

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Thank you