Status of the Quantum Statistical Parton Distributions.

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Summary

1) THE PROPOSAL BY NIEGAWA AND SISIKI AND BY FEYNMAN AND FIELD : IN THE PROTON $\bar{d}$ LARGER THAN $\bar{u}$ AS A CONSEQUENCE OF PAULI PRINCIPLE .

2) THE DEFECT IN THE GOTTFRIED SUM RULE AND DRELL-YAN PAIRS PRODUCTION IN PP AND PD SCATTERING CONFIRM THE PROPOSAL .

3) FERMI-DIRAC FUNCTIONS FOR FERMION PARTONS AND PLANCK FOR GLUONS IN THE VARIABLE x .

4) CONSEQUENCES OF THE EQUILIBRE CONDITIONS WITH RESPECT TO THE ELEMENTAR QCD PROCESSES : THE ISOSPIN AND SPIN ASYMMETRIES IN THE PROTON SEA ARE DERIVED BY THE VALENCE PARTON DISTRIBUTIONS .

3) EXTENSION TO THE TRANSVERSE DEGREES OF FREEDHOM OF THE PARTON DISTRIBUTIONS PROPOSED IN 2002 BY CLAUDE BOURRELY, F. B. AND JACQUES SOFFER : THE CONSEQUENCE OF THE ABSENCE AT SMALL x OF THE STRANGE PARTONS AS A CONSEQUENCE OF THEIR MASS

4) COMPARISON WITH THE HERA AND NNPDF FITS FOR THE DISTRIBUTIONS OF THE LIGHT FERMIONS AND OF THE GLUONS

5) PION PARTON DISTRIBUTIONS FROM DRELL-YAN PAIRS PRODUCTION IN PION NUCLEON SCATTERING

6) CONCLUSIONS

GENERAL QCD PROPERTIES

The sign of the β function for the renormalization group imples that QCD accounts for the CONFINEMENT of the quarks (''INFRARED SLAVERY'') and for the scale invariance for the structure functions, which describe DEEP INELASTIC SCATTERING (''ASYMPTOTIC FREEDHOM''). The protone and the other baryons, which at small Q^2 behave as states with three quarks combined into a color singlet, at high Q^2 appear as an incoherent set of quarks, gluons and antiquarks with distributions, which obey the sum rules of the parton model as the condition that at high p_z :

$$\int_0^1 \Sigma_i x p_i(x) dx = 1$$

whree x is the fraction of the proton momentum carried by the parton i .

ALTARELLI AND PARISI EQUAtIONS

QCD implies logarithmic violations of scale invariance described by Altarelli and Parisi equations, which allow to deduce the parton distributions at a Q^2 larger than a sufficiently high Q_0^2 from the ones at Q_0^2, for which one assumes a standard form:

$Ax^B(1-x)^C P(x)$

with the parameter A, B e C and the polynome P(x)depending on the parton and such a form holds for the non polarized and for the polarized distributions .

FORM OF THE DISTRIBUTIONS AT Q_0^2

Parton model and the consequent scale invariance hold for large values of Q^2 and $(p+q)^2 = M^2 + Q^2(\frac{1}{x} - 1)$ larger than M^2 and therefore the values $x = 0$ e $x = 1$ are exscluded as well as their neighboroods with amplitudes decreasing with Q^2 .

Therefore to fix the power behaviour around these points has not a strong motivation .

To fix the distributions at Q_0^2 one may be ispired by experiment, which suggests a role of quantum statistical mechanics .

FORM OF THE DISTRIBUTIONS AT Q_0^2

The role of Pauli principle leads to the proposal of quanrum statistical parton distributions for the partons as boundary condition for the DGLAP equation at a Q_0^2, which separates the non perturbative and the perturbative regimes of the evolution .

S. Sohaily, F. Tramontano and F. B.

EXPERIMENTAL FACTS

There are other phenomenological facts beyond the role of Pauli principle to explain the isospin asymmetry in the proton sea : $\bar{d}(x)$ larger $\bar{u}(x)$), which suggest that the functions, which give the probability that a parton, defined by its "flavor" and helicity, carries the percentage x of the hadron momentum in deep inelastic scattering, are fixed by quantum statistical mechanics .

EXPERIMENTAL FACTS

There is a correlation between the first moments of the valence partons and the shape of their distributions, which are broader in x for the partons with higher first moment (as for the Fermi sphere, which implies an increasing mean energy with the number of the fermions)

3) The common Boltzmann behaviour $\exp\frac{-x}{\bar{x}}$ for x larger of the highest "potential" , $Xu^{\uparrow} = 0.46$

PARTON DISTRIBUTIONS FIXED BY QUANTUM STATISTICAL MECHANICAS

Quantum statisticaL mechanics implies that the parton fermion distributions are the products of Fermi-Dirac functions of the variables, which appear in the sum rules for the longitudinal component of the momentum and for the transverse energy, while the gluon parton distributions are Planck functions (Bose-Einstein functions with vanishing potential) . The equilibre with respect to the elementary QCD processes, which give rise to the DGLAP evolution equations, relate the "potentials" of the valence partons to the ones of their antiparticles with opposite helicity and fix the "potential" of the gluons .

CHOICE OF x AS THE VARIABLE

Let us remind that the choice of the energy as the variable , which appears in statistical mechanics, follows from its presence in the constraint on the total energy :

$\Sigma n_i \epsilon_i = E$

For the partons the constraint is the sum rule, which implies that they carry the hadrone momentum :

$\int_0^1 \Sigma_i x p_i(x) dx = 1$

The role of Pauli principle suggests to write the proper functions of quantum statistical mechanics (Fermi-Dirac for the quarks and Planck for the gluons) in terms of the x variable , which is the one, which appears in the sum rules of the parton model .

SUM RULE FOR THE TRANSVERSE ENERGY

For the transverse distributions a sum rule has been proposed for the transverse energy , defined as the difference between the energy and the longitudinal component of the momentum . For the hadron of the target it is given by $P_0 - P_z$, approximately equal at largeP_z to $\frac{M^2}{2P_z}$. For a massless parton with the longitudinal component of the momentum, which is xP_z , and the transverse p_T, the transverse energy is given by :

$$\frac{p_T^2}{p_z + \sqrt{p_z^2 + p_T^2}} = \frac{p_T^2}{P_z(x + \sqrt{x^2 + \frac{p_T^2}{P_z^2}})}$$

Multiplying $\times 2P_z$ we obtain a sum rule with M^2 in the right hand side .

If we define P_z the momentum of the initial hadron in the refery system of the final hadrons, one has, neglecting terms in $(xM)^2$:

$$P_z^2 = \frac{Q^2}{4x(1-x)}$$

PREDICTIONS OF THE STATISTICAL APPROACH

The statistical approach predicts the isospin and spin asymmetries of the proton sea :

$\bar{d}(x)$ larger than $\bar{u}(x)$

$\bar{u}(x)$ positive

$\bar{d}(x)$ negative

confirmed by the defect in the Gottfried sum rule and by the asymmetries in Drell-Yan pairs produced in pp and pd scattering and in the production of W's in the polarized scattering at RHIC .

It allows to distinguish the contributions of the valence partons and their antiparticles and describes the x dependence of the ratios :

$\frac{F_2^n(x)}{F_2^p(x)}$, $\frac{\Delta u(x)}{u(x)}$ and $\frac{\Delta d(x)}{d(x)}$,

Predicts the Boltzmann behaviour exp $\frac{-x}{\bar{x}}$ for x larger than the highest potential , $X(\tilde{u}^{\uparrow})$, in good agreement with experiment.

DISADVANTAGES OF THE STANDARD PARAMETRIZATION

The forma $Ax^B(1-x)^C P(x)$ for the different parton distributions has the disavantage that the high x behaviour for each distribution is fixed by the exponent C, which comes out different for the different valence quarks with the conseguence that the limit $\frac{d(x)}{u(x)}$ for $x \to 1$ is 0 or infinity. In the fit by Hera the parameter C is larger for u than for d, while for the sea is still smaller with the consequence to be dominant in that limit. To agree with the experimental behaviour of the ratio $\frac{d(x)}{u(x)}$ the factor ad-hoc $(1+9.7x^2)$ is introduced for the parton u .

PARTON DISTRIBUTIONS INSPIRED BY QUANTUM STATISTICAL MECHANICS

The massless quark parton distributions in the variables x and p_T^2 depend on the longitudinal and transverse potentials, X_q and Y_q, where q denotes both "flavor" and helicity :

$$xq(x) = \frac{A'}{\mu^2} x^{b-1} \frac{1}{(\exp \frac{x - X_q}{\bar{x}} + 1)} \frac{1}{[\exp (\frac{2P_z(p_0 - p_z)}{\mu^2} - Y_q) + 1]}$$

which with the transformation : $P_T^2 = \frac{\mu^2 \eta (x + \sqrt{x^2 + \frac{p_T^2}{P_z^2}})}{2}$ gives rise to the integral in the variable η, which has the value :

$$\ln (1 + \exp Y_q) + \frac{(1-x) 2\mu^2}{Q^2} Poly(-2, -\exp Y_q)$$

PARTON DISTRIBUTIONS INSPIRED BY QUANTUM STATISTICAL MECHANICS

The parameter (μ^2) is fixed by the sum rule for the transverse energy to be 0.200 (GeV^2) and is proportional the denominator of the gaussian for the transverse distribution approaches $(\mu)^2 x$ for $p_T^2 x$ larger than $\mu^2 x Y_q$.

The second term contribues to the higher twist .

THE DIFFRACTIVE CONTRIBUTION

At small x the main contribution to the parton distributions is diffractive, probably related to the presence of the gluons, which implies an infinite number of partons :

$q_D(x)$ proportional at $x^{-1.25}$

The consistency with the sum rules :

$u - \bar{u} = 2$

$d - \bar{d} = 1$

$\Delta u + \Delta \bar{u} - \Delta d + \Delta \bar{d} = \frac{G_A}{G_V} = 1.26$

implies that the diffractive contribution is the same for particles and antiparticles and does not contribute to the Bjorken sum rule.

THE EXTENSION TO THE TRANSVERSE DEGREES OF FREEDHOM MAY REPRODUCE THE "AD HOC" FACTORS INTRODUCED IN THE 2002 WORK BY BBS

To describe the distributions

$$xq(x)$$

one had to modify the Fermi-Dirac functions

$$\frac{1}{(\exp \frac{x-\tilde{X}_q}{\bar{x}}+1)}$$

where $\bar{x}$ is the"temperature" and $\tilde{X}_q$ the "potential of the parton, which depends on its "flavor" and its helicity, with the factor :

$$A\tilde{X}_q x^b$$

and add the diffractive contribution :

$$\frac{\tilde{A}x^{\tilde{b}}}{(\exp \frac{x}{\bar{x}}+1)}$$

isoscalar and unpolarized to avoid infinite contributions to the sum rules of the parton model, if $\tilde{b}$ is less than 0 .

PARTON DISTRIBUTIONS INSPIRED BY QUANTUM STATISTICAL MECHANICS

For the antiparticles of the valence partons we have the same diffractive contribution :

$$\frac{\tilde{A}x^{\tilde{b}}}{(\exp\frac{x}{\bar{x}}+1)}$$

to be added to :

$$\frac{\bar{A}x^{\bar{b}}}{\tilde{X}_q}\frac{1}{(\exp\frac{x+\tilde{X}_q}{\bar{x}}+1)}$$

with opposite helicity for q e $\bar{q}$:

Finally for the gluon we have the Planck formula , a Bose-Einstein formula with vanishing potential :

$$xG(x)=\frac{A_G x^{b_G}}{(\exp\frac{x}{\bar{x}}-1)}$$

THE EQUILIBRIUM CONDITIONS FOR THE QCD PROCESSES

The equilibrium with respect to the two processes elementary QCD, the emission of a gluon by a fermion and the conversion of a gluon into a $q\bar{q}$ pair with opposite helicities, has the important conseguences to predict a vanishing potential for the gluons and opposite values for the potentials of the quarks and of their antiparticles with opposite helicity . The Bose-Einstein formula for the gluons $xG(x)$ becomes a Planck formula :

$$\frac{1}{(\exp \frac{x}{\bar{x}} - 1)}$$

and $\Delta G(x) = 0$

THE EQUILIBRIUM CONDITIONS FOR THE QCD PROCESSES

The constraint :

$X_q^h + X_{\bar{q}}^{-h} = 0$

allows to disentangle the quark and antiquark contribution to the electromagnetic deep inelastic scattering. While for the non polarized distributions the separation is obtained by the quark number sum sules :

$$u - \bar{u} = 2$$

$$d - \bar{d} = 1$$

for the polarized distributions the equilibrium conditions allow to determine the polarization of the light antiquarks, $\bar{u}$ and $\bar{d}$, from the knowledge of the shapes of the distributions of the valence partons.

COMPARISON OF THE VALUES OF THE PARAMETERS FIXED IN 2002 WITH THE HERA FIT

In the first column of the following table we report the values of some parameters obtained by the comparison with a set of precise data on Deep Inelastic Scattering, while in the second one the ones obtained by the comparison with the HERA fit for the non polarized and by requiring the best agreement with the 2002 polarized distributions in very good agreement with the experiments performed after . Finally in the third one the coefficients depending on the transverse potentials , Y_q, in very good agreement with the"ad hoc" factors, X_q, introduced in 2002.

COMPARISON OF THE VALUES OF THE PARAMETERS FIXED IN 2002 WITH THE HERA FIT

2002 2014

$\bar{x}$ = .099 ; .102
$X_u^{\uparrow}$ = .461 .446 .465
$X_d^{\downarrow}$ = .301 .320 .3115
$X_u^{\downarrow}$ = .298 .297 .2975
$X_d^{\uparrow}$ = .228 .222 .235
b = .41 .43
$\tilde{b}$ = -.25 -.25

This comparison has been inspired by Jacques Soffer, who immediately realized the similarity of the distributions found in 2002 with the result of the HERA fit .

THE SEA POLARIZATION

As expected, the largest potential is $\tilde{X}_u^{\uparrow}$ and the snallest$\tilde{X}^{\uparrow}{}_d$.

The equilibrium conditions imply :

$\Delta\bar{u}(x)$ positive and $\Delta\bar{d}(x)$ negative

in agreement with the asymmetries in the production of $W^{\pm}$ in the polarized experiments at RHIC and implyng a positive "sea" contribution to the Bjorken sum rule .

COMPARISON WITH THE STANDARD FORM FOR THE PARTON DISTRIBUTIONS

Despite the fact that $x = 0$ ($Q^2 = 0$) and the neighborood of $x = 1$ (elastic scattering and production of resonances) do not belong to the domain of the Deep Inelastic Scattering, the standard parametrization for the parton distributions is ;

$Ax^B(1-x)^C P(x)$

with A, B ,C and P(x) fixed by the comparison with experiment for each parton distribution and a separate study of the non polarized and polarized distrinbutions. While the diffractive part, which is the dominant one at small x, has a singular behaviour, a negative power for $x \to 0$, the valence partons, the dominannt ones at intermediate and high x, have a different power behaviour at small x . The positive values of C give rise to a different decrease at high x for the valence partons, 2 (u or d) for the non polarized distribution or 4, if one considers also the polarized .

THE DIFFERENCES BETWEEN THE STATISTICAL AND THE STANDARD DISTRIBUTIONS

For the statistical distributions the decrease at high x is naturally explained with the Boltzmann behaviour of the parton distributions for x larger than the"potential" of each parton :

exp $\frac{-x}{\bar{x}}$

The variation of the ratios between the distributions of the different valence partons :

$\frac{d(x)}{u(x)}$, $\frac{\Delta u(x)}{u(x)}$ e $\frac{\Delta d(x)}{d(x)}$

is concentrated in the range between the lowest and the highest "potential" :

$(X_{d^{\uparrow}}, X_{u^{\uparrow}}) = (0.22, 0.46)$

while at higher x they approach the same Boltzmann behaviour and their ratios vary more slowly .

The parametrization standard has the opposite behaviour, since the effect of the different values of the C parameter for the power $(1-x)^C$ gets more important when $x \to 1$.

THE LIMIT OF $\frac{d(x)}{u(x)}$ WHEN $x \to 1$

At high x the ratio ;

$\frac{F_2^n(x)}{F_2^p(x)}$

depends on $\frac{d(x)}{u(x)}$

The difficulty in measuring the unpolarized neutron structure function at high x is due to the Fermi motion of the two nucleons in the deuteron, which has the consequence that to get the neutron parton distributios from the ones of the proton and of the deuteron is not easy . Therefore the small statistics and the standard parametrization imply a large uncertainty for the ratio $\frac{d(x)}{u(x)}$ at high x .

THE LIMIT OF $\frac{d(x)}{u(x)}$ WHEN $x \to 1$

Instead for the quantum statistical distributions the values of the parameters, which fix that ratio , the "temperature" and the "longitudinal and transverse potentials" :

$\bar{x}$, X_q e Y_q

are fixed by the measurements in the region with x in the range $(0.22, 0.46)$, where the statistics is large and the systematic error is small .
The perfect agreement of the prediction for :

$\frac{d(1)}{u(1)} = 0.22$

with the result of the accurate analysis by Orwell, Accardi and Melnitchouk is a good confirm for the parton statistical distributions.

THE GLUON DISTRIBUTION

THE PLANCK FORMULA

The equilibrium conditions imply a vanishing value for both the helicities of the gluon ''potentials'' with the conseguences :

$\Delta G(x) = 0$

and the Planck formula :

$xG(x) = \frac{A_G x^{b_G}}{[\exp \frac{x}{\bar{x}} - 1]}$

THE COMPARISON FOR THE GLUON DISTRIBUTION

The standard formula :

$Ax^B(1-x)^C P(x)$

implies that the decrease at x depends on the exponente C and becomes faster as x increases, while the Planck formula has a more regular behaviour, which approaches the proportionality to the exponential form :

$\exp \frac{-x}{\bar{x}}$

Since the gluon distribution in Deep Inelastic Scattering plays an important role for the logarithmic scale violations, a method to establish the agreement of the Planck distribution with the experimental information from HERA is to compare the HERA result at $Q^2 = 4$:

$\int_0^{0.2} xG(x)dx = 0.36$

$\int_{0.2}^{1} xG(x)dx = 0.05$

with the Planck formula proposed by the quantum statistical approach :

$\int_0^{0.2} \frac{A_g x}{[\exp x\bar{x} - 1]} = 0.34$

$\int_{0.2}^{1} \frac{A_g x}{[\exp x\bar{x} - 1]} = 0.125$

STANDARD OR PLANCK ?

The agreement is good for :

$\int_0^{0.2} xG(x)dx$

where most gluons are concentrated, while for x larger than .2 HERA gives a faster decrease .

Since for the valence fermions the high x decrease is described better from the statistical distributions, probably the fast decrease at high x proposed by HERA is the conseguence of their parametrization as appears from the comparison with NNPDF .

PION PARTON DISTRIBUTIONS

Recently pion parton distributions have been determined by studying the production of Drell-Yan pairs in pion nucleon scattering (C. Bourrely, J. C. Peng and F. B.). The non diffractive part of the valence partons gives the most important contribution to the process and has been determined to be :

$$\frac{A_U X_U x^{b_U}}{\exp \frac{(x-X_U)}{\bar{x_\pi}} + 1)}$$

with $A_U = 0.776$, $X_U = 0.756$, $b_U = 0.5$ and $\bar{x_\pi} = 0.106$.

While b_U and $\bar{x_\pi}$ are near to the values found for the nucleon, the "potential" X_U is larger than the "potentials" found for the valence partons in the nucleon in agreement with the dominance at high x_F of the Drell-Yan pairs produced with incident negative pions with respect to antiprotons scattered with nucleons .

PION PARTON DISTRIBUTIONS

The high value X_U implies a negligible non diffractive part for the antiparticles of the valence partons and for the quark number sum rule a first moment for the non-diffractive part of the valence partons very near to 1 . As long as for the second moments the two valence partons carry about half the pion momentum, while the remaining half is carried by the gluons and by the diffractive contribution . To get a better knowledge of the gluon contribution the same authors are studying with the help of Wen-Chen Chang the production of J/ψ particles, to which contributes the gluon-gluon scattering .

PLANCK FORMULA FOR THE GLUON PARTON DISTRIBUTION IN THE NUCLEON

A glance to a non-official proposal for the gluon parton distributions obtained, keeping into account also data at LHC, leads to verify the perfect agreement with the Planck formula :

$$\frac{A_G b^G}{(\exp \frac{x}{\bar{x}} - 1}$$

with the same $\bar{x} = 0.099$ (!) and almost the same percentage of the moment of the nucleon found in 2002 . The gluon distribution turns from a power behaviour at small x into a Boltzmann exponential behaviour at high x :

"a smoking gun for the statistical approach" .

CONCLUSIONS

1) The agreement of the fermion distributions found in 2002 inspired by quantum statistical mechanics with the ones found by Hera is an important confirm of the validity of the statistical approach, which has been motivated by the idea of Niegawa, Sisiki, Feynman and Fields that the Pauli principle implies the isospin asymmetry in the proton sea .

2) The theory has been improved with the extension to the transverse components of the momentum and with the hypothesis that the statistical distributions are the boundary condition at low Q^2 of the Altarelli e Parisi equations (DGLAP).

3) The agreement for the values of the parameters with the ones found in 2002 by Claude Bourrely, Jacques Soffer and F. B. is another point in favor of the statistical approach .

4) As long as for the dependence on the transverse moment p_T at high values approaches the behaviour :

$$\sqrt{p_T}\,\mathrm{exp}\,\frac{-2p_T}{\mu\sqrt{\bar{x}}}$$

CONCLUSIONS

5) The decreasing at high x and the ratios between the distributions of the valence partons are better described by the statistical distribuzions than by the che standard ones:

$Ax^B(1-x)^C P(x)$.

In fact the ratios change faster in the range :

$X_{d^\uparrow}, X_{u^\uparrow}$ (0.22, 0.46)

than for values larger than $X_{u^\uparrow}$.

CONCLUSIONS

6) An interesting property of the statistical approach is the fact that the high x behaviour depends on free parameters, which are fixed by measurements in a region of x, where both the statistical and the systematic errors are small, the region (0.22, 0.46), where the valence partons dominate, in such a way to provide the factor for the Boltzmann behaviour exp $\frac{-x}{\bar{x}}$.

Also it predicts the isospin and spin asymmetries of the sea .

CONCLUSIONS

7) As long as for the gluons the difference at high x between the Planck form and the result of HERA depends on its standard parametrizzation $Ax^B(1-x)^C$, since the first differs from the one found by NNPDF less than the second for values of x larger than 0.2 and gives a perfect agreement with the unofficial result at LHC.

8) The statistical approach is sucessfully applied to the production of Drell-Yan pairs in pion nucleon scattering with a large ""potential" found for the valence partons, which explains the dominance at high x_F with incident negative pion with respect to incident antiproton .