Asymptotic Freedom
&
High Derivative Gauge Theories

Manuel Asorey

Centro de Astropartículas y Física de Altas Energías
Universidad Zaragoza

Departamento de Física Teórica
Universidad Zaragoza

WORKSHOP ON SM AND BEYOND

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High Derivative Theories

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- UV completion of Standard Model may involve higher derivatives ($s = 0, s = 1/2, s = 1$)
- HD give rise to renormalizable quantum gravity theories
- Inflationary models involving high derivative theories provide the best fits of the scalar/tensor relations
- Problems with ghosts, causality and unitarity

High Derivative Gauge Theories

$$S = \frac{1}{4g^2}\int d^4x\, F^a_{\mu\nu}F^{\mu\nu a} + \frac{1}{4g^2\Lambda^{2n}}\int d^4x\, F^a_{\mu\nu}\,\Delta^n\, F^{\mu\nu a}\,,$$

where

$$\Delta = d_A^* d_A + d_A d_A^*$$

is Hodge-covariant Laplacian operator

$$\Delta^{\nu\,b}_{\mu\,a} = -D^2\,\delta^\nu_\mu\delta^b_a + 2f^b{}_{ca}\,F_\mu{}^{\nu c}$$

Instantons are minima in each topological sector

Perturbation Theory

Gauge fixing [α-gauge]

$$S_\alpha = \frac{\alpha}{2g^2\Lambda^{2n}} \int d^4x\, \partial^\mu A^a_\mu (-\partial^\sigma \partial_\sigma)^n \partial^\nu A^a_\nu$$

One-loop divergences

$$\Gamma^{ab}_{\mu\nu}(p) = -c_n \frac{C_2(G)}{16\pi^2 \varepsilon} i\, \delta^{ab} \left(p^2 \eta_{\mu\nu} - p_\mu p_\nu \right)$$

with

$$c_n = \frac{29}{3} - 23n + 5n^2 \qquad n \geqslant 2,$$

$$c_1 = -\frac{43}{3}, \qquad c_0 = \alpha - \frac{13}{3}$$

Perturbation Theory

One-loop renormalization

$$S_{\text{count}} = c_n \frac{C_2(G)}{128\pi^2} \left(\frac{2}{\varepsilon} + \log \frac{\Lambda^2_{\text{QCD}}}{\Lambda^2} \right) F^a_{\mu\nu} F^{\mu\nu a},$$

β-function of the coupling constant

$$\beta_n = c_n \frac{g^3 C_2(G)}{32\pi^2} \quad n \geq 2$$

Asymptotic freedom only for $n = 0, 1, 2, 3, 4$

$$\widetilde{\mathbf{c}}_0 = -\frac{\mathbf{22}}{\mathbf{3}}, \mathbf{c}_1 = -\frac{\mathbf{43}}{\mathbf{3}}, \mathbf{c}_2 = -\frac{\mathbf{49}}{\mathbf{3}}, \mathbf{c}_3 = -\frac{\mathbf{43}}{\mathbf{3}}, \mathbf{c}_4 = -\frac{\mathbf{7}}{\mathbf{3}}, c_5 = \frac{59}{3}$$

Perturbation Theory

One-loop form factor

$$\Gamma^{ab}_{\mu\nu}(p) = -\frac{C_2(G)}{32\pi^2} i\delta^{ab} \left(p^2 \eta_{\mu\nu} - p_\mu p_\nu\right) \Pi(p^2)$$

with

$$\Pi(p^2) = \left(b_n \log \frac{p^2 + \Lambda^2}{\Lambda^2} + c_0 \log \frac{p^2}{\Lambda^2_{\mathrm{QCD}}} \right),$$

$$b_n = 14 - \alpha - 23n + 5n^2 \quad \text{for } n \geqslant 2$$

$$b_0 = 0, \quad b_1 = -10 - \alpha$$

Scaling Regimes

There are two different asymptotic regimes with two different beta functions:

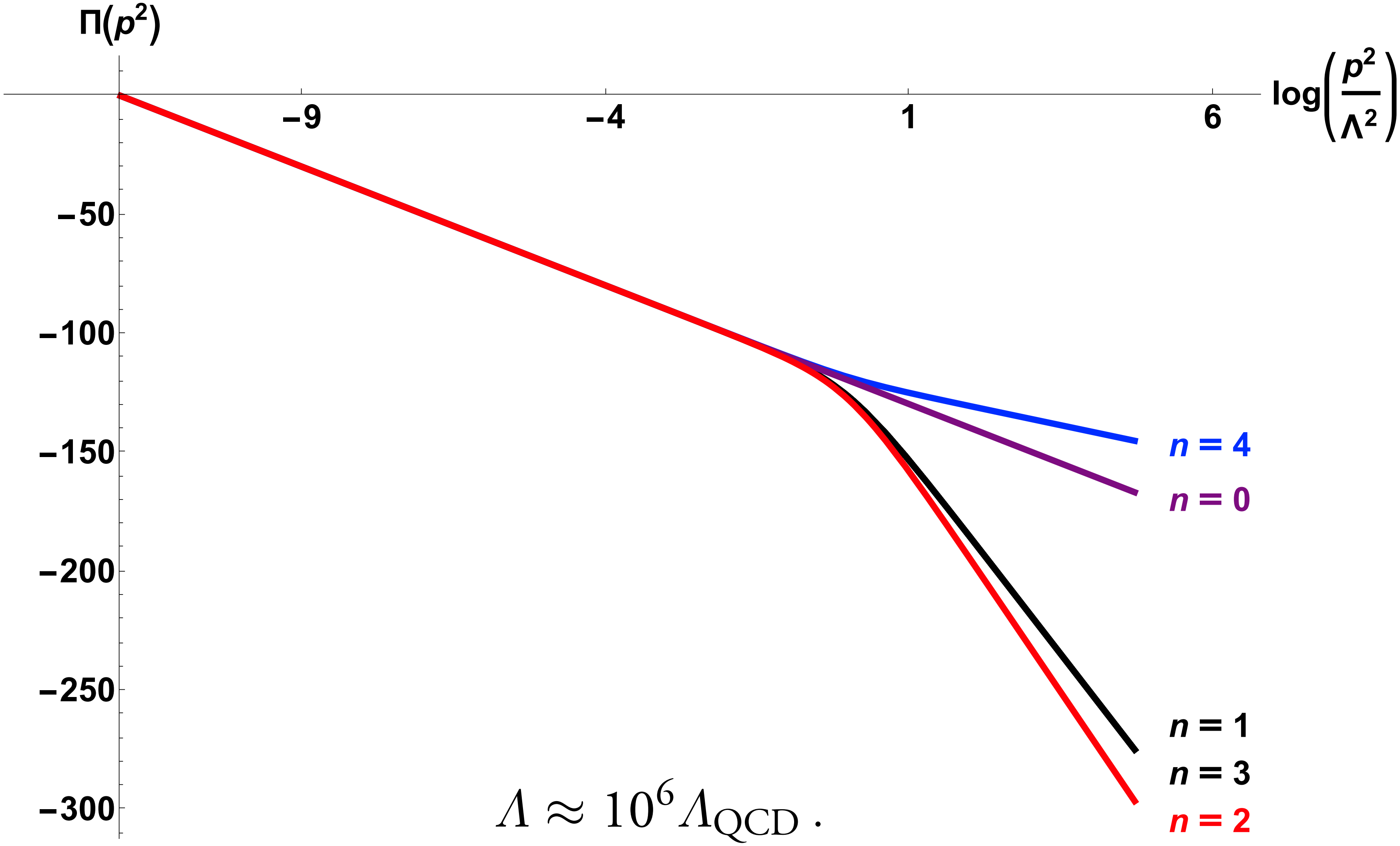
- UV regime $p \gg \Lambda$

$$\beta_{\mathrm{UV}} = c_n \frac{g^3 C_2(G)}{32\pi^2}$$

- IR regime $\Lambda_{\mathrm{QCD}} < p \ll \Lambda$

$$\beta_{\mathrm{IR}} = -\frac{22}{3}\,\frac{g^3 C_2(G)}{32\pi^2}$$

Two-point form factor



$\Lambda \approx 10^6 \Lambda_{\rm QCD}$.

Generalization

Replace the Hodge-covariant Laplacian operator by a generalized Laplacian

$$\Delta \quad \Rightarrow \quad {}^{\lambda}\Delta$$

$${}^{\lambda}\Delta_{\mu\,a}^{\nu\,b} = -\delta_a^b\,\delta_\mu^\nu\,D^2 + 2\,\lambda f^b{}_{ca}\,F_\mu{}^{\nu c}$$

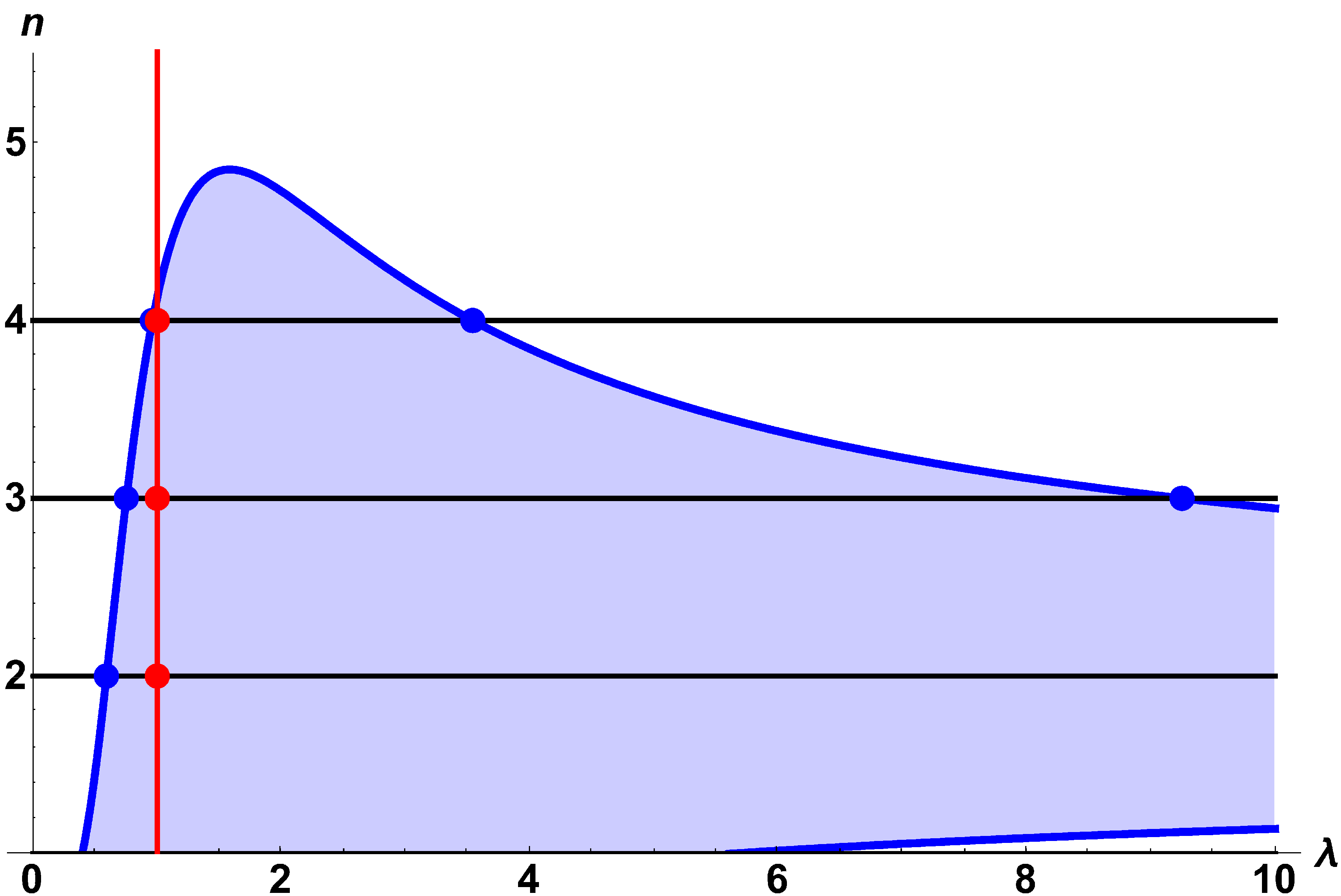
One-loop β-function coefficients

$$c_n = -\frac{7}{3}+5n+4n^2-\left(4+10n+4n^2\right)\lambda+\left(16-18n+5n^2\right)\lambda^2$$

$$n \geq 2$$

$$c_1 = \frac{38}{3} - 18\lambda - 9\lambda^2$$

Two-point form factor



UV Finite theories

The range of asymptotically free theories is more restrictive for $\lambda \neq 1$

For some values of $\lambda \neq 1$ the theory is finite

$$\begin{array}{llll} n=1 & \lambda_1 = -2.55 & \lambda_2 = 0.55 \\ n=2 & \lambda \;\, = \;\;\, 0.59 & \\ n=3 & \lambda_1 = \;\;\, 0.75 & \lambda_2 = 9.25 \\ n=4 & \lambda_1 = \;\;\, 0.96 & \lambda_2 = 3.54 \end{array}$$

The theory is free of UV divergences

No instanton solutions $\Rightarrow$ new QCD potentials for axions

Higher Derivative Ghosts

- BRST symmetry is preserved
- High derivative terms are not renormalized
- $\beta_\Lambda = -\frac{1}{n}\beta_g$
- Effective ghost masses run to infinite if $\beta_g \neq 0$
- Unitarity and Causality might be recovered in UV

M.A., F. Falceto and L. Rachwal
JHEP 05(2021)075

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- Unitarity and Causality might be recovered
- Implications for high derivative theories of quantum gravity
- Are there similar phenomena in gravity?