The Gravity Dual of the Berkooz-Douglas matrix model

V. Filev

IMI, Bulgarian Academy of Sciences

related to work with Y. Asano, D. O'Connor and S. Kovacik

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Outline

1. AdS/CFT correspondence
   - Pure Super Yang-Mills
   - Adding flavours
2. D0/D4 system
   - Lower dimensional correspondence
   - Berkooz-Douglas matrix model
3. Back-reacted D0/D4 background
   - Uplift to 11D supergravity
   - BPS equations
   - Solution

AdS/CFT correspondence

Type IIB String Theory on

$AdS_5 \times S^5$

$\mathcal{N} = 4 \; SU(N_c)$ SUSY YM

- Gubser-Klebanov-Polyakov-Witten formula:

$$\langle e^{\int d^d x \phi_0(x) \langle \mathcal{O}(x) \rangle} \rangle_{\rm CFT} = \mathcal{Z}_{\rm string}[\phi_0(x)]$$

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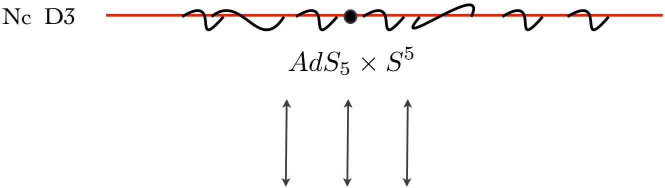

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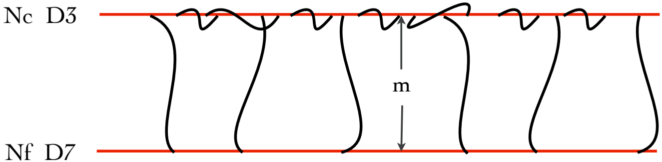
Generalizing the correspondence



	0	1	2	3	4	5	6	7	8	9
D3	-	-	-	-	·	·	·	·	·	·
D7	-	-	-	-	-	-	-	-	·	·

- Adding N_f massive $\mathcal{N} = 2$ Hypermultiplets:

$$m_q \int d^2\theta\, \tilde{Q}\, Q \to \mathrm{SYM} \quad \text{with} \quad m_q = m/2\pi\alpha'$$

Adding flavours D3/D7 Karch & Katz

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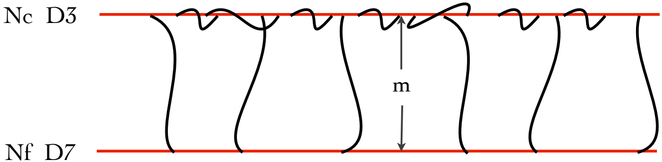

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String spectrum

3-3 strings	pure $\mathcal{N}$=4 SYM adjoint of $SU(N_c)$
3-7 strings	Q_i fundamental chiral field
7-3 strings	$\tilde{Q}^i$ anti-fundamental chiral field
7-7 strings	gauge field on the D7 brane frozen by infinite volume

Probe approximation $N_f \ll N_c$

- The probe is described by a Dirac-Born-Infeld action $S \propto \int d^7\xi \, e^{-\Phi}\sqrt{||G_{ab} - 2\pi\alpha'\mathcal{F}_{ab}||}$
- The profile of the D-brane encodes the fundamental condensate of theory. The semi-classical fluctuations correspond to meson-like excitations.
- The D-brane gauge field can describe: external electromagnetic field, chemical potential, electric current etc.
- Numerous applications: thermal and quantum phase transitions, chiral symmetry breaking, magnetic catalysis etc.

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Lower dimensional correspondence

- The D3/D7 system is T-dual to the D0/D4 system, share many common properties (meson melting transition, meson spectra)
- The dual theory of the D0/D4 set-up is a flavoured version of the BFSS matrix model - the Berkooz-Douglas (BD) matrix model.
- The BD matrix model is 1D quantum mechanics and is super renormalisable, avoiding the fine tuning problem.
- Aspects of the BD model in the probe approximation are studied in [arXiv:1512.02536, 1605.05597, 1612.09281] by Y. Asano, V. Filev, S. Kovacik and D. O'Connor.

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Berkooz-Douglas matrix model

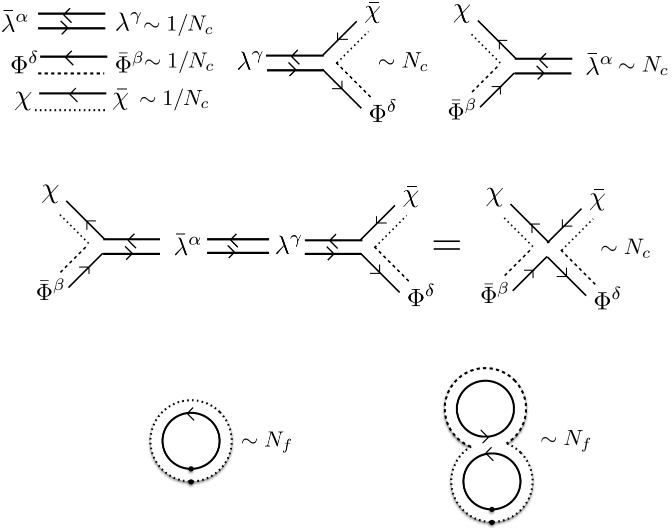
- Original motivation -M_5 brane density hep-th/9610236 (Berkooz & Douglas).
- Reducing the D5/D9 system (Van Raamsdonk, hep-th/0112081):

$$\begin{aligned}\mathcal{L} &= \frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{2}D_0X^aD_0X^a + \frac{i}{2}\lambda^{\dagger\,\rho}D_0\lambda_\rho + \frac{1}{2}D_0\bar{X}^{\rho\dot\rho}D_0X_{\rho\dot\rho} + \frac{i}{2}\theta^{\dagger\dot\rho}D_0\theta_{\dot\rho}\right)\\ &+\frac{1}{g^2}\mathrm{tr}\left(D_0\bar\Phi^\rho D_0\Phi_\rho + i\chi^\dagger D_0\chi\right) + \mathcal{L}_{\rm int}\end{aligned}$$

where:

$$\begin{aligned}\mathcal{L}_{\rm int} &= \frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[X^a,X^b][X^a,X^b] + \frac{1}{2}[X^a,\bar X^{\rho\dot\rho}][X^a,X_{\rho\dot\rho}] - \frac{1}{4}[\bar X^{\alpha\dot\alpha},X_{\beta\dot\alpha}][\bar X^{\beta\dot\beta},X_{\alpha\dot\beta}]\right)\\ &-\frac{1}{g^2}\mathrm{tr}\left(\bar\Phi^\rho(X^a-m^a)(X^a-m^a)\Phi_\rho\right)\\ &+\frac{1}{g^2}\mathrm{tr}\left(\bar\Phi^\alpha[\bar X^{\beta\dot\alpha},X_{\alpha\dot\alpha}]\Phi_\beta + \frac{1}{2}\bar\Phi^\alpha\Phi_\beta\bar\Phi^\beta\Phi_\alpha - \bar\Phi^\alpha\Phi_\alpha\bar\Phi^\beta\Phi_\beta\right)\\ &+\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{2}\bar\lambda^\rho\gamma^a[X^a,\lambda_\rho] + \frac{1}{2}\bar\theta^{\dot\alpha}\gamma^a[X^a,\theta_{\dot\alpha}] - \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar\theta^{\dot\alpha}[X_{\beta\dot\alpha},\lambda_\alpha]\right)\\ &+\frac{1}{g^2}\mathrm{tr}\left(\bar\chi\gamma^a(X^a-m^a)\chi + \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar\chi\lambda_\alpha\Phi_\beta - \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar\Phi^\alpha\bar\lambda_\beta\chi\right)\end{aligned}$$

Quenched versus dynamical



Uplift to 11D supergravity

- Both the D0 and D4–branes can be uplifted to 11D.
- The D0–brane is a KK - brane (uplifts to KK momentum).
- The D4–brane uplifts to a $\mathcal{M}5$-brane.
- Therefore the D0/D4 system uplifts to a $\mathcal{M}5$-brane background with quantised momentum along the $\mathcal{M}$-theory circle.
- A magnetic analogue of this system, the D2/D6 system has been already studied in the literature by Cherkiz and Hashimoto (hep-th/0210105). The uplifted background us a $\mathcal{M}2$-membrane background with a Taub-NUT geometry in the transverse space.
- We will consider the most general ansatz consistent with the symmetries and use SUSY to restrict it.

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&\quad K_2(u,v)(du^2 + u^2 d\Omega_3^2) + K_4(u,v)(dv^2 + v^2 d\Omega_4^2)\ , \\
\mathcal{F}_{(4)} &= F'(v)\,v^4\ \sin^3\psi\ \sin\tilde\alpha\ \cos\tilde\alpha\ d\psi\wedge d\tilde\alpha\wedge d\tilde\beta\wedge d\tilde\gamma\ , \\
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- Charge conservation fixes $F'(v)$:

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Gravitino variation

- We demand that the gravitino variation vanish:

$$\delta\psi_\mu = \nabla_\mu \varepsilon - \frac{1}{6}\frac{F'(v)}{K_4(u,v)^2}\Gamma_\mu \bar{\Gamma}^{7\,8\,9\,10}\varepsilon \quad \text{for} \quad \mu \in S^4 \,,$$

$$\delta\psi_\mu = \nabla_\mu \varepsilon + \frac{1}{12}\frac{F'(v)}{K_4(u,v)^2}\Gamma_\mu \bar{\Gamma}^{7\,8\,9\,10}\varepsilon \quad \text{for} \quad \mu \notin S^4 \,.$$

- We use the conventions / projections:

$$\bar{\Gamma}^{0\,1\,2\,3\,4\,5\,6\,7\,8\,9\,10} = \delta_1$$

$$\bar{\Gamma}^{0\,1\,2\,3\,4\,5}\varepsilon = \delta_2\,\varepsilon$$

$$\bar{\Gamma}_{0\,1}\varepsilon = \delta_3\,\varepsilon$$

- where $\bar{\Gamma}^i$ are flat and we have:

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- Clearly the background preserves 1/4 of the original SUSY consistent with the description of SUSY brane intersection.

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BPS equations

- This boils down to solving:

$$
\begin{aligned}
0 &= \frac{\partial_u K_1}{K_1} - \delta_3 \left(\frac{K_3}{K_1}\right)^{1/2} \partial_u A_0 \\
0 &= \frac{\partial_v K_1}{K_1} - \delta_3 \left(\frac{K_3}{K_1}\right)^{1/2} \partial_v A_0 + \delta_1\, \delta_2\, \frac{1}{3}\, \frac{F'(v)}{K_4^{3/2}} \\
0 &= \frac{\partial_u K_1}{K_1} + \frac{\partial_u K_3}{K_3} \\
F'(v) &= -\frac{3}{2}\delta_1\, \delta_2\, K_4^{3/2} \left(\frac{\partial_v K_1}{K_1} + \frac{\partial_v K_3}{K_3}\right) \\
F'(v) &= -3\, \delta_1\, \delta_2\, K_4^{3/2}\, \frac{\partial_v K_2}{K_2} \\
F'(v) &= \delta_1\, \delta_2\, \frac{3}{2}\, K_4^{1/2}\, \partial_v K_4 \\
\partial_u K_2 &= 0 \\
\partial_u K_4 &= 0
\end{aligned}
$$

Solution

- With the choice $\delta_1 = \delta_2$ and $\delta_3 = 1$ we arrive at:

$$
\begin{aligned}
K_1 &= \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} H(u,v)^{-1} \\
K_2 &= \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} \\
K_3 &= \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} H(u,v) \\
K_4 &= \left(1 + \frac{v_5^3}{v^3}\right)^{2/3} \\
A_0(u,v) &= H(u,v)^{-1} - 1 \\
F(v) &= 1 + \frac{v_5^3}{v^3}
\end{aligned}
$$

The metric

- The resulting metric is given by:

$$ds_{11}^2 = \left(1+\frac{v_5^3}{v^3}\right)^{-1/3}\left(-\frac{dt^2}{H(u,v)} + H(u,v)\left(dx_{11} + \left(H(u,v)^{-1}-1\right)dt\right)^2 + du^2 + u^2\,d\Omega_3^2\right) + \left(1+\frac{v_5^3}{v^3}\right)^{2/3}\left(dv^2 + v^2\,d\Omega_4^2\right) .$$

- And the reduced 10D metric is:

$$ds_{10}^2 = -H(u,v)^{-1/2}\left(1+\frac{v_4^3}{v^3}\right)^{-1/2} dt^2 + H(u,v)^{1/2}\left[\frac{\left(du^2+u^2\,d\Omega_3^2\right)}{\left(1+\frac{v_4^3}{v^3}\right)^{1/2}} + \left(1+\frac{v_4^3}{v^3}\right)^{1/2}\left(dv^2+v^2\,d\Omega_4^2\right)\right]$$

$$e^{\Phi} = \left(1+\frac{v_4^3}{v^3}\right)^{-1/4} H(u,v)^{3/4},$$

$$C_0 = \left(H(u,v)^{-1}-1\right)dt\,,\quad F_4 = -3\,v_4^3\,\omega_{S^4}$$

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Harmonic equation

- $H(u, v)$ is not fixed by SUSY. We define the angular momentum along $\xi = \partial/\partial x_{11}$:

$$J_{x_{11}} \propto \int_{\partial\Sigma} \star(\nabla_\mu \xi_\nu \, dx^\mu \wedge dx^\nu) = \int_{\Sigma} d\star(\nabla_\mu \xi_\nu \, dx^\mu \wedge dx^\nu) \ .$$

- Independence on Σ leads to:

$$\delta J_{x_{11}} = \int_{\delta\Sigma} d\star(\nabla_\mu \xi_\nu \, dx^\mu \wedge dx^\nu) = 0 \ .$$

- The resulting harmonic equation can written as:

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Perturbative solution at infinity

- It is natural to consider the expansion:

$$H(u,v) = \sum_{n=0}^{\infty}\left(\frac{v_5^3}{v^3}\right)^n H_n(u,v)$$

- and Fourier transform each component:

$$H(u,v) = 1 + \frac{r_0^7}{(2\pi)^4}\int d^4p\, e^{i\,\vec{p}.\vec{u}}\, h(p,v) = 1 + \frac{r_0^7}{4\,\pi^2}\int\limits_0^{\infty} dp\, p^2\, \frac{J_1(p\,u)}{u}\, h(p,v)$$

- $h_n(p,v)$ satisfy the recursive relation:

$$\partial_v^2 h_n(p,v) + \frac{4-6\,n}{v}\partial_v h_n(p,v) + \left(\frac{9\,n\,(n-1)}{v^2} - p^2\right) h_n(p,v) = p^2\, h_{n-1}(p,v)$$

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$$h_n(p,v) = A_n\, e^{-p\,v}\,(p\,v)^{3\,n}\,\frac{1+p\,v}{v^3} + B_n\, e^{p\,v}\,(p\,v)^{3\,n}\,\frac{1-p\,v}{v^3}\ ,$$

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First order solution

- To first order we obtain:

$$H(u,v) = 1 + \frac{r_0^7}{(u^2+v^2)^{7/2}}\left\{1 + \frac{v_5^3}{v^3}\left(\frac{-3u^2+4v^2}{4(u^2+v^2)} + \right.\right.$$
$$\left.\left. +3\,A_1\,\frac{u^6+24u^4v^2-144u^2v^4+64v^6}{(u^2+v^2)^3}\right)\right\}$$

- How to fix the constant A_1?
- We could compare with a result from the probe brane calculation. To leading order in N_f they should agree (we have $v_5^3 \sim N_f/Nc\,\lambda$).
- We fix A_1 by demanding that the ADM mass of the D4-branes remains unchanged in the decoupling limit. This leads to $A_1 = -1/192$.

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General solution

- To obtain the general solution on needs to use information in the IR. In the 11D uplift for small v the metric is:

$$ds_{11}^2 = \frac{z^2}{L^2}\left(\frac{dt^2}{H} + H(dx_{11} + (H^{-1} - 1)dt)^2 + du^2 + u^2 d\Omega_3^2\right) + \frac{L^2\, dz^2}{z^2} + \frac{L^2}{4} d\Omega_4^2$$

- Where $z = 2\sqrt{v_5\, v}$ and $L = 2v_5$.
- This is an $AdS_7 \times S^4$ space-time with a spherical wave.
- The independent solutions are given in terms of Bessel K_3 and I_3 functions. Only the K_3 function is regular at the origin, however it is the I_3 function that corresponds to the perturbative solution at infinity.
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Summary

- We reviewed some checks of the AdS/CFT duality with flavour.
- We solved the BPS equations for the uplifted backreacted D0/D4 geometry.
- We obtained perturbative solution at infinity.
- We are working on a full (numerical) solution
- Obvious generalisations are separating the D0 and D4–branes and introducing temperature.
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