# To positivity and beond where Higgs-dilaton has never gone before

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#### The talk is based on the paper

M. Herrero-Valea, I.Timiryasov and A. Tokareva, "To Positivity and Beyond, where Higgs-Dilaton Inflation has never gone before," arXiv:1905.08816 [hep-ph]. Motivation: why the scale invariance is so interesting?

- At high energies the Standard Model without gravity is scale invariant - insight in UV physics which might correspond to a scale invariant theory
- Problem of quadratic divergencies can be solved: no scale no quadratic divergences (dimentional regularization is preferred since it doesn't provide with additional scale.)
- Drawback: scale symmetry is anomalous:  $T^{\mu}_{\mu} \propto eta(g) F^2_{\mu
  u}$
- Go out of that: There is a way to define a quantum theory in such a way that exact scale symmetry is preserved at quantum level.

# Spontaneousely broken scale invariance: Higgs-dilaton model

Classical action providing with spontaneous breaking of the scale invariance

$$egin{aligned} S &= \int d^4 x \sqrt{|g|} \left( -rac{1}{2} (2\xi_h arphi^\dagger arphi + \xi_\chi \chi^2) R + rac{1}{2} \partial_\mu \chi \partial^\mu \chi - V(arphi, \chi) 
ight) + S_{ ext{SM}} \ V(arphi, \chi) &= \lambda \left( arphi^\dagger arphi - rac{lpha}{2\lambda} \chi^2 
ight)^2 \end{aligned}$$

Definition of quantum theory which preserves quantum scale invariance: dimensional regularization  $d = 4 - \epsilon$ ,

$$V(\varphi,\chi) = \lambda \left(\varphi^{\dagger}\varphi - \frac{\alpha}{2\lambda}\chi^{2}\right)^{2}\chi^{\epsilon}$$

Defined in this way, the model is manifestly scale invariant in d dimensions.

Drawback: theory becomes non-renormalizable even without gravity.

J. Garcia-Bellido, J. Rubio, M. Shaposhnikov and D. Zenhausern, Phys. Rev. D 84, 123504 (2011) [arXiv:1107.2163 [hep-ph]]. M. E. Shaposhnikov and F. V. Tkachov, arXiv:0905.4857 [hep-th]

Scale symmetry corresponds to the shift symmetry of the dilaton field. The kinetic term for Higgs and dilaton fields is

$$\begin{split} \mathcal{K} &= \left(\frac{1+6\xi_h}{\xi_h}\right) \frac{\partial_\mu \rho \partial^\mu \rho}{\sin^2 \theta + \kappa \cos^2 \theta} + \frac{(1+6\xi_h)}{(1+6\xi_\chi)} \frac{M_P^2}{\xi_h} \frac{\left(\tan^2 \theta + \xi_\chi/\xi_h\right) \partial_\mu \theta \partial^\mu \theta}{\cos^2 \theta \left(\tan^2 \theta + \kappa\right)^2} \\ \kappa &= \frac{(1+6\xi_h)\xi_\chi}{(1+6\xi_\chi)\xi_h} \end{split}$$

The potential

$$U(\theta) = \frac{\lambda M_{P}^{4}}{4\xi_{h}^{2}} \left(\frac{\sin^{2}\theta}{\sin^{2}\theta + \kappa \cos^{2}\theta}\right)^{2}$$

J. Garcia-Bellido, J. Rubio, M. Shaposhnikov and D. Zenhausern, Phys. Rev. D 84, 123504 (2011) [arXiv:1107.2163 [hep-ph]].

# Higgs-dilaton inflation

Potential



 $\xi \lesssim 0.001$  - in order to satisfy Planck constraints

For canonically normalized inflaton





# Higgs-dilaton model as an EFT

#### Field-dependent cutoff scale



F. Bezrukov, G. Karananas, et. al, arXiv:1212.4148

# Positivity constraints for EFT or IR obstruction to UV completion

Is it always possible to have a UV completion for a particular EFT? Desired properties of the full theory:

- Unitarity
- Causality
- Crossing symmetry
- Lorentz invariance
- Martins-Froissart bound:  $\mathcal{A}(s) < s \log^2 s, \ s o \infty$
- Locality

#### The answer is: NO

Well-known example:

$$L = \frac{1}{2} (\partial_{\mu} \phi)^{2} + \frac{c}{\Lambda^{4}} (\partial_{\mu} \phi \partial_{\mu} \phi)^{2} - \frac{1}{2} m^{2} \phi^{2}$$

No 'good' UV completion for c < 0 (superluminality). A. Adams,

N. Arkani-Hamed, S. Dubovsky, A. Nicolis and R. Rattazzi, JHEP 0610, 014 (2006) [hep\_th/0602178], = >

# Analytic properties of forward scattering amplitudes

 $2 \rightarrow 2$  amplitude for the scalar with mass  $m^2$  in the limit t = 0.



The amplitude is analytical everywhere except

- poles at the real axis
- two branch cuts corresponding to the production of particles

### Positivity constraints

Let's define

$$\Sigma_{\rm IR} = \frac{1}{2\pi i} \oint_{\Gamma_{\rm IR}} ds \frac{\mathcal{A}(s)}{(s - \mu^2)^3},$$

$$\prod_{\rm m^2} \mu^2 s_{\rm th}} \prod_{\rm m^2} \mu^2 \prod_{\mu^2} \sigma_{\rm s}^{\rm ab \to X}(s) = \sigma_{\rm s}^{\rm ab \to X}(s)$$

$$\Sigma_{\rm IR} = \sum_{X} \int_{s_{\rm th}^2}^{\infty} \frac{ds}{\pi} \sqrt{u(s)s} \left( \frac{\sigma^{ab \to X}(s)}{(s-\mu^2)^3} + \frac{\sigma_{X}^{ab \to X}(s)}{(\mu^2 - u(s))^3} \right) > 0$$

### Beyond positivity bounds

Idea: the low energy part of the right hand side is calculable within the EFT. For example, if

$$\mathcal{A} = c_1 \frac{s^2 + t^2 + u^2}{\Lambda^4} + c_2 \frac{stu}{\Lambda^6}$$

we can perform the integral up to the energy  $E < \Lambda$ 

$$\Sigma_{\rm IR} > \sum_X \int_{s_{\rm th}^2}^{E^2} \frac{ds}{\pi} \sqrt{u(s)s} \left( \frac{\sigma^{ab \to X}(s)}{(s-\mu^2)^3} + \frac{\sigma_{\times}^{ab \to X}(s)}{(\mu^2 - u(s))^3} \right)$$

and obtain a non-trivial constraint

$$c_2 > rac{c_1^2}{128\pi^2} \left(rac{E}{\Lambda}
ight)^8$$

B. Bellazzini, F. Riva, J. Serra and F. Sgarlata, Phys. Rev. Lett. **120**, no. 16, 161101 (2018) [arXiv:1710.02539 [hep-th]].

We can rewrite the lagrangian in the form, in order to avoid loop contributions to  $s^2/\Lambda^4$ ,... from the potential terms ( $\phi^6/\Lambda^2$ )

$$U(\phi) = \frac{\lambda}{4} \phi'$$

$$\mathcal{K} = \frac{1}{2} \left( 1 + \frac{\xi_{\chi} - \xi_h}{M_p^2 (1 + 6\xi_{\chi})} \, \phi^2 \right) \partial_\mu \varrho \partial^\mu \varrho + \frac{1}{2} \left( 1 + \frac{2\xi_h + 6\xi_h^2 - \xi_{\chi}}{M_p^2 (1 + 6\xi_{\chi})} \, \phi^2 + \mathcal{O}(\phi^4) \right) \partial_\mu \phi \partial^\mu \phi.$$

In this way, the loop contribitions from the potential are subleading for the terms growing with momenta

At the leading order, the most general form of the terms with four derivatives and four fields is

$$\frac{A}{\Lambda^{4}}\partial_{\mu}\varrho\partial^{\mu}\varrho\partial_{\nu}\phi\partial^{\nu}\phi + \frac{B}{\Lambda^{4}}\partial_{\mu}\varrho\partial_{\nu}\varrho\partial^{\mu}\phi\partial^{\nu}\phi + \frac{C}{\Lambda^{4}}\partial_{\mu}\varrho\partial^{\mu}\varrho\partial_{\nu}\varrho\partial^{\mu}\phi\partial^{\nu}\phi + \frac{D}{\Lambda^{4}}\partial_{\mu}\phi\partial^{\mu}\phi\partial_{\nu}\phi\partial^{\nu}\phi$$

We account for

- $\bullet\,$  symmetries  $\phi \to -\phi, \; \rho \to -\rho$
- dilaton shift symmetry  $\rho \to \rho + {\rm const}$

What can we really constrain?

- φφ → φφ does not provide with any constraints: the leading contribution (not suppressed by Λ) to the rhs will come from the imaginary part of the fish diagram (~ λ<sup>2</sup>).
- $\rho\phi \to \rho\phi$  and  $\rho\rho \to \rho\rho$  scatterings provide with bounds on coefficients near the higher derivative operators
- it is impossible to say something about the terms  $\phi^2 (\partial_\mu \rho)^2 / \Lambda^2$ because they do not contribute to  $\Sigma_{IR}$ . However, they give the main contribution to the rhs of the inequality

# $\rho\phi ightarrow \rho\phi$ scattering

The scattering amplitude

$$\mathcal{A}(s,t,u) = \frac{2A(t^2+u^2) + B(2s^2+t^2+u^2)}{2\Lambda^4} + \frac{(t+u)(\xi_h - \xi_{\chi})}{M_P^2(1+6\xi_{\chi})}$$
$$\Sigma_{IR} = \frac{1}{2}\frac{\partial^2 \mathcal{A}(s)}{\partial s^2} = \frac{2A+3B}{2\Lambda^4}$$

Right hand side:



$$\int_{E_{\rm IR}^2}^{E_{\rm UV}^2} \frac{ds}{\pi} \sqrt{u(s)s} \left( \frac{\sigma(s)}{(s-\mu^2)^3} + \frac{\sigma(s)}{(u(s)+\mu^2)^3} \right) = \frac{(\xi_h - \xi_\chi)^2 \log\left(\frac{E_{\rm UV}}{E_{\rm IR}}\right)}{4\pi^2 M_P^4 (1+6\xi_\chi)^2},$$

### The resulting bound

$$rac{2A+3B}{\Lambda^4}\gtrsim rac{(\xi_h-\xi_\chi)^2}{2\pi^2 M_P^4(1+6\xi_\chi)^2}\log\left(rac{E_{
m UV}}{E_{
m IR}}
ight)$$



In the limit of Higgs inflation and with  $\Lambda = M_P/\xi_h$ 

$$2A+3B\gtrsim rac{1}{2\pi^2\xi_h^2}\log\left(rac{E_{
m UV}}{E_{
m IR}}
ight)$$
 , where  $E$  is the equation of the second s

•  $\rho\rho\to\rho\rho$  scattering provides with the bound

$$rac{24C}{\Lambda^4} \gtrsim rac{(\xi_h-\xi_\chi)^2\log\left(rac{E_{
m UV}}{E_{
m IR}}
ight)}{4\pi^2 M_P^4 (1+6\xi_\chi)^2}$$

- For the coeffition D in front of the term  $(\partial \phi)^4$  we can obtain only the familiar positivity bound D > 0
- Summary of the bounds

$$rac{2A+3B}{\Lambda^4}\gtrsim rac{(\xi_h-\xi_\chi)^2}{2\pi^2 M_P^4(1+6\xi_\chi)^2}\log\left(rac{E_{
m UV}}{E_{
m IR}}
ight)$$
 $rac{48C}{\Lambda^4}\gtrsim rac{(\xi_h-\xi_\chi)^2}{2\pi^2 M_P^4(1+6\xi_\chi)^2}\log\left(rac{E_{
m UV}}{E_{
m IR}}
ight),\quad D>0$ 

# Conclusions

- If we require that quantum scale invariant models should have 'nice' UV completion than we arrive to a set of constraints on some operators which appear in the low energy theory: IR theory knows something about the very existance of UV completion!
- A novel approach to use positivity and beyond positivity constraints provides with non-trivial bounds on Wilson coefficients in front of the 4-derivative operators in Higgs-dilaton model
- These bounds do not require any operators which can sighnificantly affect Higgs-dilaton inflation - not obvious from the beginning

# Thanks for your attention!