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Random Fuzzy Spaces in the Spectral Triple formalism

20/09/2019

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Corfu Summer Institute 2019, Corfu, Greece
Workshop on Quantum Geometry, Field Theory and Gravity

Random Fuzzy Spaces

Geometry is encoded in spectral data

$$(A, H, D; J, \gamma)$$

Random Fuzzy Spaces

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The particle content of the Standard Model is described by the following data

$$(A_F, H_F, D_F; J_F, \gamma_F)$$

$$A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$$
$$H_F = \mathbb{C}^{96}$$

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To get Einstein-Hilbert plus the Standard Model one considers the spectral triple obtained by tensoring the commutative manifold with the internal non-commutative finite space

$$(C^\infty(M) \otimes A_F,\ \ L^2(S) \otimes H_F,\ \ D_M \otimes 1 + \gamma_5 \otimes D_F;\ \ J_M \otimes J_F,\ \ \gamma_M \otimes \gamma_F)$$

bosonic action $\mathrm{Tr}\left(f\left(\frac{D}{\Lambda}\right)\right) \sim \int \mathcal{L}_M + \mathcal{L}_{g.f.} + \mathcal{L}_h$

fermionic action $\langle J\psi, D\psi\rangle$

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The idea: replace the commutative manifold with a non-commutative analogue

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Fuzzy spaces as spectral triples:

$$\mathcal{A} = M_n(\mathbb{C})$$

(arXiv:1502.05383)
Barrett

$$\mathcal{H} = \mathbb{C}^c \otimes M_n(\mathbb{C})$$

$$D = \sum_i \alpha_i \otimes \{H_i, \cdot\} + \sum_j \tau_j \otimes [L_i, \cdot]$$

Outline

- The random matrix model
- Preliminary studies in a symmetry-breaking potential
- The (1,3) type
- Why should geometry emerge
- Conclusions and outlook

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Path integration over geometries is then implemented by integration over the space of Dirac operators

$$\langle f(D) \rangle = \int f(D) e^{-S[D]} dD$$

The simplest non-trivial choice for an action

$$S = g_2 \mathrm{Tr} D^2 + \mathrm{Tr} D^4$$

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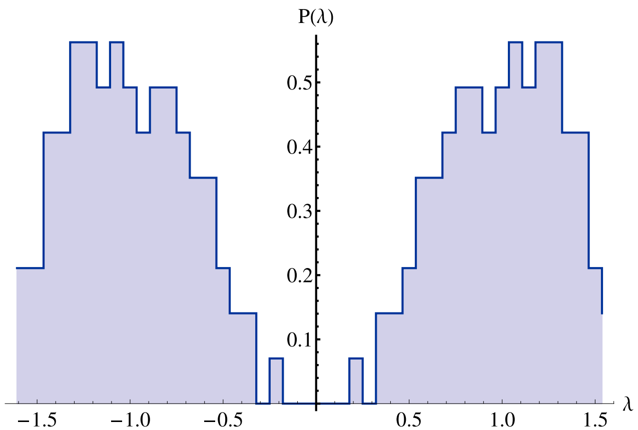
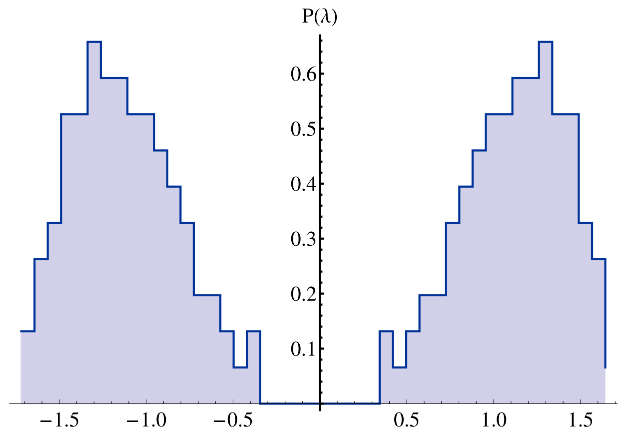
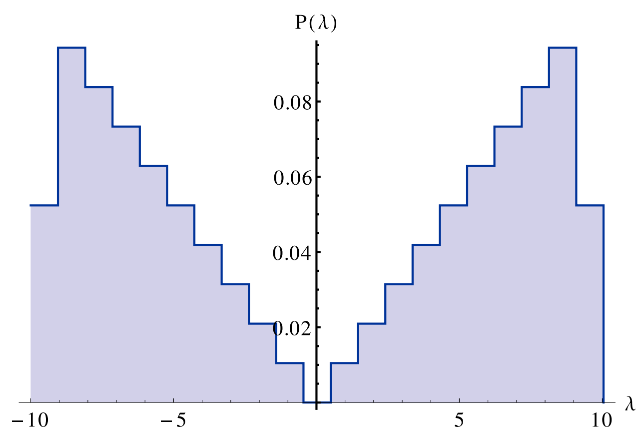
First indication of emergent geometry



(a) Type $(1,1)$, $g_2 = -2.5$



(b) Type $(2,0)$, $g_2 = -3$



(c) Type (1,3) Fuzzy S^2

(arXiv:1510.01377)
Barrett, Glaser

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Order parameter for the phase transition

$$F = \frac{1}{\mathcal{N}} \sum_i (\mathrm{Tr}\, H_i)^2$$

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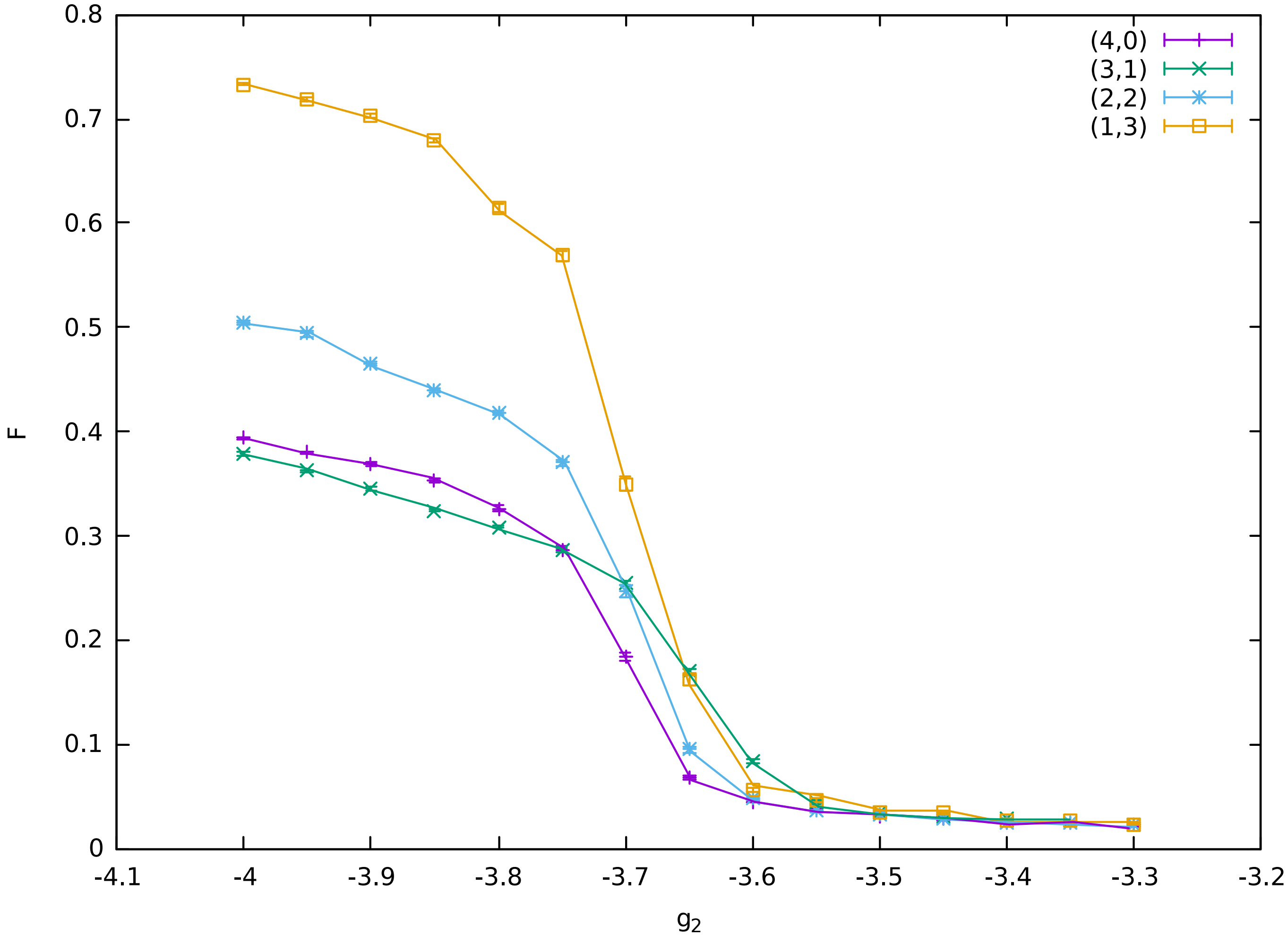

p+q = 4

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Fuzzy sphere Dirac operator

$$D_{fs} = \gamma^0 \otimes I + \sum_{j<k=1}^{3} \gamma^0 \gamma^j \gamma^k \otimes [L_{jk}, \cdot]$$

Standard generators of su(2)

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(1, 3) Dirac operator

$$D_{13} = \gamma^0 \otimes \{H_0, \cdot\} + \gamma^1\gamma^2\gamma^3 \otimes \{H_{123}, \cdot\} +$$

$$\sum_{i=1}^{3} \gamma^i \otimes [L_i, \cdot] + \sum_{j<k=1}^{3} \gamma^0\gamma^j\gamma^k \otimes [L_{jk}, \cdot]$$

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Do they reduce to su(2) generators dynamically?

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Using the Frobenius inner product

$$\langle A, B\rangle = \mathrm{Tr} A^{\dagger} B = ||A|| \cdot ||B|| \cos\theta$$

we can test whether the algebra of the L matrices shrinks, for example:

$$[L_{jk}, L_{lm}] \propto L_{rs}$$

by measuring the angle between them

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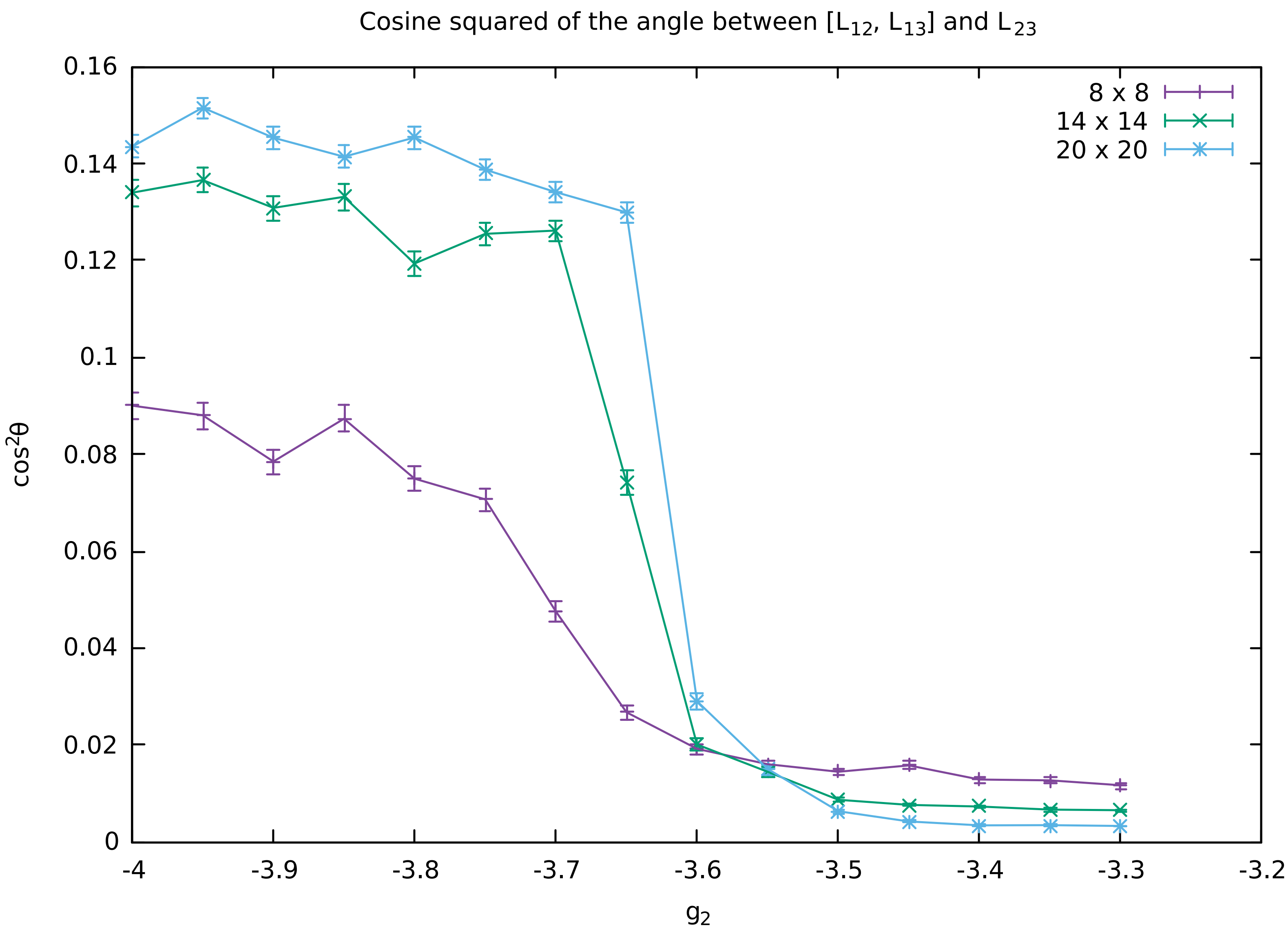

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Consider the (0,3) model

$$D = \sum_i \sigma_i \otimes [M_i, \cdot] + I \otimes \{H_0, \cdot\}$$

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Action

$$S[D] = \frac{1}{4} \operatorname{Tr} D^4 + \frac{1}{3} g_3 \operatorname{Tr} D^3 + \frac{1}{4} g_2 \operatorname{Tr} D^2$$

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$$S[D(M_i)] = -\frac{1}{4}\operatorname{Tr}[M_i, M_j]^2 - \frac{1}{2}\operatorname{Tr} M_i^2 M_j^2 + \frac{2}{3} i g_3 \epsilon_{ijk} \operatorname{Tr} M_i M_j M_k + \frac{1}{2} g_2 \operatorname{Tr} M_i^2$$

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Consider the (0,3) model

$$D = \sum_i \sigma_i \otimes M_i$$

EOM:

$$[M_j, [M_j, M_i]] - \{M_j^2, M_i\} + i g_3 \epsilon_{ijk} [M_j, M_k] + g_2 M_i = 0$$

Conclusions

- If one looks at the Standard Model in the non-commutative framework, replacing the manifold with a fuzzy space is just about the most natural progression
- Random fuzzy spaces exhibit phase transitions in certain potentials
- Interesting behaviour potentially emerges at the critical point
- No geometric input

To do

- Analytical understanding

  Recent progress in the simplest model using topological recursion

  (arXiv:1906.09362)
  S. Azarfar, M. Khalkhali

- Matter fields

  Natural way to include fermions: $\langle J\psi, D\psi \rangle$

- Lorentzian version

  In the IKKT model CLM has given promising results

Thank you for listening