Black Holes and Wormholes in Einstein-scalar-Gauss-Bonnet Theories

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Outline

- Black Holes and Wormholes in General Relativity
- Alternative Theories of Gravity
  - The Einstein-Scalar-Gauss-Bonnet Theories
- Black Holes in the EsGB Theories
- Wormholes in the EsGB Theories
- The synergy between the Ricci and GB terms
- Conclusions

Based on 1711.03390 [PRL 2018], 1711.07431 [PRD 2018], 1812.06941 [PRD 2019], and 1904.13091 [hep-th], in collaboration with G. Antoniou (Minnesota U), A. Bakopoulos (Ioannina U.), B. Kleihaus & J. Kunz (Oldenburg U.) and N. Pappas (Athens U.)

Black Holes and Wormholes in General Relativity

Einstein's General Theory of Relativity based on his field equations

$$\boxed{R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}}$$

may determine, upon choosing the form of the energy-momentum tensor $T_{\mu\nu}$, the form of spacetime around:

- an ordinary massive body such as the Sun
- a black hole
- a wormhole

GR was experimentally verified at Solar System scales but black holes and wormholes were considered as merely mathematical curiosities

Black Holes and Wormholes in General Relativity

General Relativity admits three families of BH solutions:

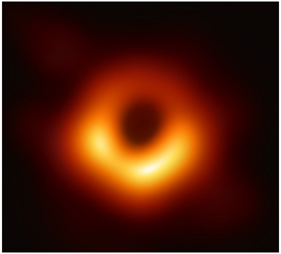
- Schwarzschild (1916): static, spherically-symmetric, neutral BH
- Reissner-Nordstrom (1921): static, spherically-symmetric, charged BH
- Kerr(-Newman) (1963): rotating, axially-symmetric (charged) BH

According to the "no-hair" theorems of GR (Birkhoff; Israel; Carter; Price; Hartle; Teitelboim; Bekenstein), a BH may be characterized only by its mass M, electromagnetic charge Q and angular-momentum J. A BH has no colour, baryon or lepton number....

... or scalar charges, according to the scalar "no-hair" theorems applying in minimally and non-minimally-coupled scalar fields (Bekenstein, 1995)

Black Holes and Wormholes in General Relativity

Today, we believe that black holes are legitimate astrophysical objects (of the Kerr family) that populate our universe and may give valuable information about the strong(er)-gravity regime



BH in M87

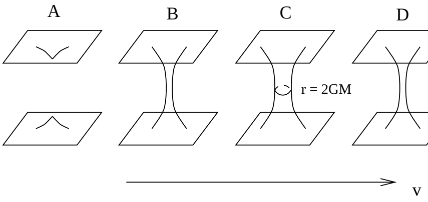
(Event Horizon Telescope Coll.)

BH merger (artist's view)

(www.universetoday.com)

Black Holes and Wormholes in General Relativity

What about wormholes? They are well motivated since, in GR, they hide inside the horizon of a black hole:



The region inside the horizon of a Schwarzschild BH, $r < r_h = 2M$, is dynamical and a throat appears in place of the singularity as time goes by

But the throat closes so quickly that not even a light signal can pass through (Einstein-Rosen, 1935; Wheeler, 1955)

The Reissner-Nordstrom and Kerr geometries have generically internal tunnels - but the internal Cauchy horizons are unstable

Black Holes and Wormholes in General Relativity

Looking for a traversable wormhole, Morris & Thorne (1988) disconnected the wormhole from the black hole. Using an ansatz of the form

$$ds^2 = -e^{2\Phi(r)}\,dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2),$$

they demanded

- an asymptotically-flat regime: $\Phi \to 0$ and $b/r \to 0$, for $r \to \infty$
- the absence of a horizon or singularity: $\Phi(r)$ everywhere regular
- the presence of a throat at $r_{min} = b_0$ where $b(r_{min}) = b_0$

The above demand in turn an energy-momentum tensor

$$T_{tt} = \rho\,, \qquad T_{rr} = -\tau\,, \qquad T_{\theta\theta} = T_{\varphi\varphi} = p$$

satisfying $\tau \geq \rho \;\Rightarrow$ violation of energy conditions $\Rightarrow$ Exotic Matter

Alternative Theories of Gravity

Today, the formulation of an alternative theory of gravity, beyond GR, is in high demand! There is a plethora of such theories in the literature:

Scalar-tensor theories, $f(R)$ theories, Higher-derivative theories, Chern-Simons gravity, Einstein-aether theory, Massive gravity, Gravitational aether, $f(T)$ theories, TeVes, ...

In particular, the first four types can be described by the action

$$S = \int d^4x \sqrt{-g} \left[f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}, \phi) + \mathcal{L}(\phi) \right]$$

which arises

- as part of the string effective action at low energies
- as part of a Lovelock effective theory in four dimensions
- as part of a modified scalar-tensor (Horndeski or DHOST) theory

Alternative Theories of Gravity

We focus on the following Einstein-scalar-Gauss-Bonnet theory of gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + f(\phi) \, R^2_{GB} \right],$$

with $f(\phi)$ a coupling function between a scalar field ϕ and the GB term

$$R^2_{GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

This theory has a number of attractive points:

- It contains a quadratic gravitational term, the next important term in strong-curvature regimes in the effective approach
- it leads to field equations with up to 2nd-order derivatives, and with no Ostrogradski instabilities or ghosts
- it leads to interesting cosmological, singularity-free solutions for $f(\phi) \sim \ln\left[2e^\phi \eta^4(ie^\phi)\right]$ (Antoniadis, Rizos & Tamvakis, 1994) and $f(\phi) = \phi^{2n}$ (P.K., Rizos & Tamvakis, 1998) - a constructive synergy of linear and quadratic terms at regimes of strong curvature

Black Holes in EsGB Theory

In the EsGB theory, the no-hair theorem was evaded for $f(\phi) \sim e^{\phi}$ (P.K, Mavromatos et al, 1996; 1998) and $f(\phi) \sim \phi$ (Sotiriou & Zhou, 2014)

For what other forms of the coupling function $f(\phi)$ can one get a static, spherically-symmetric black-hole solution with a non-trivial scalar field?

$$ds^2 = -e^{A(r)}dt^2 + e^{B(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

The equations of motion read

$$\nabla^2\phi + \dot{f}(\phi)R^2_{GB} = 0\,, \qquad R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}\,R = T_{\mu\nu}$$

where

$$T_{\mu\nu} = -\frac{1}{4}g_{\mu\nu}(\partial\phi)^2 + \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(g_{\rho\mu}g_{\lambda\nu} + g_{\lambda\mu}g_{\rho\nu})\eta^{\kappa\lambda\alpha\beta}\tilde{R}^{\rho\gamma}{}_{\alpha\beta}\nabla_\gamma\partial_\kappa f\,,$$

$$\Downarrow$$

$$\boxed{A'' = \frac{P}{S}\,, \qquad \phi'' = \frac{Q}{S}\,, \qquad P, Q, S = g(r, \phi, \phi', A')}$$

Black Holes in EsGB Theory

For the existence of a regular black-hole horizon we demand that

$$e^{A(r)} \to 0, \quad e^{-B(r)} \to 0, \quad \phi(r) \to \phi_h$$

Demanding that ϕ'' is also finite at the horizon r_h, we find the constraint

$$\phi'_h = \frac{r_h}{4\dot{f}_h}\left(-1 \pm \sqrt{1 - \frac{96\dot{f}_h^2}{r_h^4}}\right), \qquad \dot{f}_h^2 < \frac{r_h^4}{96}$$

We may then construct a regular, near-horizon solution of the form

$$e^A = a_1(r - r_h) + \dots, \qquad e^{-B} = b_1(r - r_h) + \dots,$$

$$\phi = \phi_h + \phi'_h(r - r_h) + \phi''_h(r - r_h)^2 + \dots$$

Black Holes in EsGB Theory

At large distances from the horizon, we assume a power series expansion in $1/r$, and by substituting in the equations of motion, we find

$$\phi = \phi_\infty + \frac{D}{r} + \frac{MD}{r^2} + \frac{32M^2D - D^3}{24r^3} + \frac{12M^3D - 24M^2\dot{f} - MD^3}{6r^4} + \dots$$

$$e^A = 1 - \frac{2M}{r} + \frac{MD^2}{12r^3} + \frac{24MD\dot{f} + M^2D^2}{6r^4} + \dots$$

$$e^B = 1 + \frac{2M}{r} + \frac{16M^2 - D^2}{4r^2} + \frac{32M^3 - 5MD^2}{4r^3} + \mathcal{O}\left(\frac{1}{r^4}\right) + \dots$$

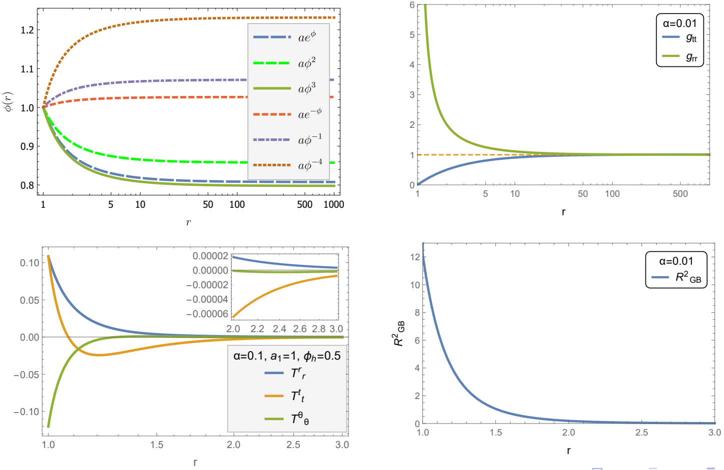
It is in order $\mathcal{O}(1/r^4)$ that the explicit form of the coupling function $f(\phi)$ first makes its appearance

Thus, a general coupling function f does not interfere with the existence of an asymptotically-flat limit for the spacetime

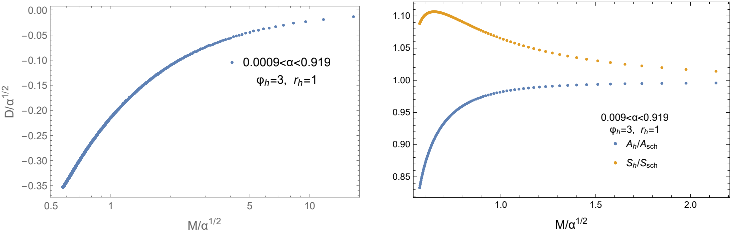
Black Holes in EsGB Theory

Choosing $f(\phi)$ and then (ϕ_h, ϕ'_h), we found numerous BH solutions:

(Antoniou, Bakopoulos & P.K., PRL 2018, PRD 2018)



Black Holes in EsGB Theory



- The scalar charge is a "secondary" conserved quantity
- In the limit of large mass, all GB black holes reduce to the Schwarzschild solution
- The entropy of the GB black holes may at times exceed that of the Schwarzschild solution (shown that of $f(\phi) \sim 1/\phi$)
- All GB black holes are smaller than the corresponding Schwarzschild solution and have a minimum mass

Doneva & Yazadjiev, 1711.01187 ($f \sim 1 - e^{-\phi^2}$), Silva et al, 1711.02080 ($f \sim \phi^2$)

Wormholes in EsGB Theory

An additional family of solutions was found in the EdGB theory, i.e the EsGB theory with $f(\phi) = \alpha e^{\phi}$ (P.K., Mavromatos et al, 1996) where

$$e^A \simeq a_0 + ..., \quad e^{-B} = b_1(r - r_0) + \dots , \quad \phi \simeq \phi_0 + \phi_1\sqrt{r - r_0} + \dots$$

All components of $T_{\mu\nu}$ and scalar invariants were finite

Under a redefinition $l^2 = r^2 - r_0^2$, the line-element becomes:

(P.K., Kleihaus & Kunz, PRL 2011, PRD 2012)

$$ds^2 = -e^{A(l)}dt^2 + e^{B(l)}dl^2 + (l^2 + r_0^2)\,(d\theta^2 + \sin^2\theta d\varphi^2)$$

where now

$$e^A = a_0 + a_1 l + \dots , \quad e^{B(r)} = b_0 + b_1 l + \dots$$

$$\phi(l) = \phi_0 + \phi_1 l + \dots$$

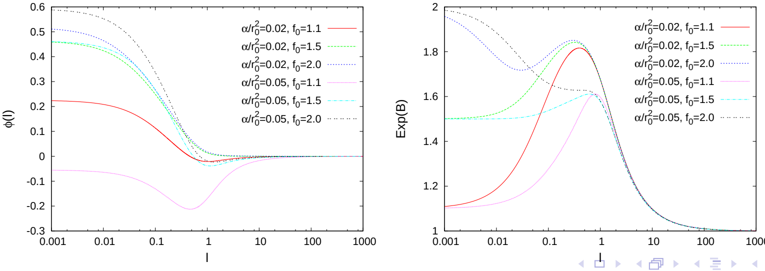
The above describes a wormhole with the throat at $r = r_0$ or $l = 0$

Wormholes in EsGB Theory

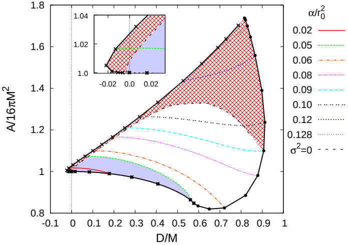
At large distances, we obtain

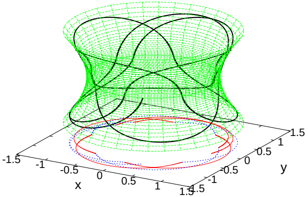
$$e^A \simeq 1 - \frac{2M}{l} + ..., \qquad e^B = 1 + \frac{2M}{l} + \dots,$$
$$\phi \simeq \phi_\infty + \frac{D}{l} + \dots,$$

where M and D are the mass and scalar charge of the wormhole. The complete numerical solutions have the form:



Wormholes in EsGB Theory





- The dilatonic WHs are bounded by the dilatonic BHs (for $b_0 \to 1$); they even share the same linear stability under radial perturbations
- Regular symmetric solutions arise if a perfect fluid (non-exotic!) and a gravitational source term are introduced at the throat

$$S = \int d^3x\sqrt{h}\,(\lambda_1 + \lambda_0 e^{\phi}\tilde{R})$$

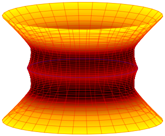
- The WHs are traversable both by massless and massive particles

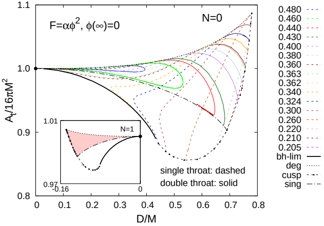
Wormholes in EsGB Theory

In the context of the EsGB theory with an arbitrary $f(\phi)$, we searched for wormholes by employing a new set of coordinates:

$$ds^2 = -e^{A(\eta)}dt^2 + e^{\Gamma(\eta)}\left[d\eta^2 + (\eta^2+\eta_0^2)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$$

We found regular wormhole solutions for every form of $f(\phi)$ with an asymptotically-flat behaviour and with a single or double throat





All EsGB WHs are bounded by the corresponding BHs and are free of any exotic matter

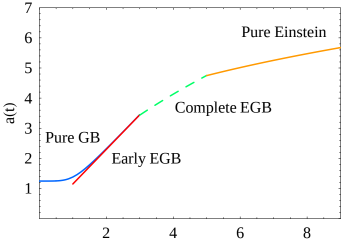
(Antoniou, Bakopoulos, P.K., Kleihaus, Kunz, 1904.13091 [hep-th])

Synergy between Ricci and GB terms

At large distances, the quadratic GB term is negligible – however, at small distances it plays a very important role: without the GB term, there is no scalarised BH or WH solutions

Can we claim that there is a class of BH or WH solutions that may be attributed almost solely to the GB term?

In a cosmological set-up, we have shown that, in the early universe, as $t \to 0$, R becomes negligible, R^2_{GB} dominates, and the field eqns may then be analytically solved yielding a class of singularity-free (or, inflationary) solutions



(P.K. Gannouji & Dadhich, 2015; Antoniadis, Rizos & Tamvakis, 1994; P.K., Rizos and Tamvakis, 1998)

Synergy between Ricci and GB terms

We have tried and failed to find "pure" GB black-hole solutions. If we ignore the Ricci term in the field eqns and assume that $e^B \to \infty$, as $r \to r_h$, we get

$$e^{-B} = 2\ln(r/r_h)\,, \qquad \phi'' \to \infty\,, \qquad\qquad (\times)$$

Therefore, the emergence of regular BH solutions with scalar hair demands the synergy between the Ricci and GB terms

If the GB exceeds a certain value expressed via the minimum-mass limit

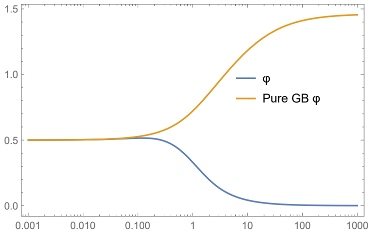
$$\phi'_h = \frac{r_h}{4\dot{f}_h}\left(-1 \pm \sqrt{1 - \frac{96\dot{f}_h^2}{r_h^4}}\right), \qquad |\dot{f}_h| \leq r_h^2/4\sqrt{6}$$

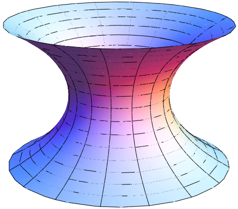
its repulsive effect is too strong and destroys the horizon – no such "barrier" exists in the cosmological set-up where the repulsive force of the GB term eliminates even the BB singularity

Synergy between Ricci and GB terms

A repulsive force is also necessary for the throat of a wormhole to remain open. In addition, no limit exists that restricts the magnitude of the GB term

Then, as in the cosmological case, regular pure GB wormhole solutions do emerge that in fact do not violate the energy conditions!





(A. Bakopoulos, P.K. and N. Pappas, in preparation)

Conclusions

• The Effective Alternative Theories of Gravity may be the way forward in gravitational physics

• The Einstein-scalar-Gauss-Bonnet theory is a particular type of a quadratic theory which so far has survived the observational constraints

• The presence of the GB term – independently of the exact form of its coupling function – always leads to novel scalarised BHs as well as to double/single-throat wormholes

• The emergence of the scalarised BHs demands the synergy between the Ricci and the GB terms whereas scalarised wormholes arise also in the regime of dominance of the GB term (as in cosmology)

Black Holes in EsGB Theory

- **<u>Old No-Hair Theorem:</u>** it uses the scalar equation

$$\int d^4x\,\sqrt{-g}\,f(\phi)\left[\nabla^2\phi+\dot{f}(\phi)\,R^2_{GB}\right]=0$$

Integrating by parts, we obtain

$$\int d^4x\,\sqrt{-g}\,\dot{f}(\phi)\left[\partial_\mu\phi\,\partial^\mu\phi-f(\phi)\,R^2_{GB}\right]=0$$

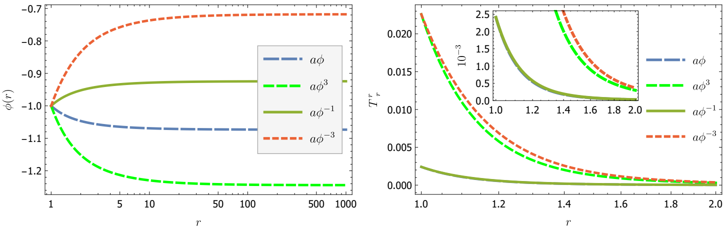
Since, $\partial_\mu\phi\,\partial^\mu\phi>0$, the above holds only for $f(\phi)\,R^2_{GB}>0$. Silva et al, with a slightly different manipulation, found instead that $\ddot{f}\,R^2_{GB}>0$.

Should we combine both constraints? How *many constraints* are there? How can we impose this *integral* constraint before we determine the solutions (the novel no-hair theorem is a *local* one)?

If the old no-hair theorem is trustable, BH's with $f(\phi)<0$ do not exist...

Black Holes in EsGB Theory

Surprise! They do....!



Solutions that *violate* the old no-hair theorem but *respect* the novel no-hair theorem do arise for several $f(\phi)$'s.

But the sign of $f(\phi)$ affects directly the entropy of the BH solution

$$S = \frac{A_h}{4} + 4\pi f(\phi_h) > 0$$

If, for the same M, BHs arise with both positive and negative $f(\phi)$, these will have different entropies with $S_+ > S_-$

Black Holes in EsGB Theory

Can we smoothly connect the two asymptotic solutions? Bekenstein's *Novel No-Hair* theorem (1995) said no, because:

- "at radial infinity: T_r^r is positive and decreasing"

Indeed, even in the presence of the GB term: $T_r^r \simeq \phi'^2/4 \simeq D^2/4r^4 + \ldots$

- "near the BH horizon: T_r^r is negative and increasing"

If true, the smooth connection of the two demands an extremum - this is *excluded* by the positivity of energy in ordinary scalar-tensor theories

However, in the Einstein-scalar-Gauss-Bonnet theories with general f, the second clause is not true. Instead, we find that

$$sign(T_r^r)_h = -sign(\dot{f}_h \phi'_h) = 1 \mp \sqrt{1 - 96\dot{f}^2/r_h^4}) > 0$$

The regularity of the horizon automatically guarantees the positivity of T_r^r

Bounds on the EsGB Theory

Up to now, GR has proven to be compatible with every observable and every process in our universe

But, as we explore objects and processes that involve a higher curvature, the small deformations of GR should start being important in agreement with the effective approach

If deformations do not appear, then bounds may be set on the parameters of the alternative theory - here, we focus on the GB coupling α defined through the relation $f(\phi) = \alpha \tilde{f}(\phi)$ with $[\alpha] = L^2$

• **Bounds from Solar System:** Using the Shapiro time delay and the uncertainty in the Mercury's orbit, the GB coupling α must obey the constraints (Amendola, Charmousis & Davis, 2007)

$$\sqrt{\alpha} \leq 1.3 \times 10^{12}\,\mathrm{cm} \qquad \text{and} \qquad \sqrt{\alpha} \leq 1.9 \times 10^{13}\,\mathrm{cm}$$

Bounds on the EsGB Theory

- **Bounds from Observed Black Holes:** The EsGB theory predicts a minimum mass for the BHs through the constraint (P.K., Mavromatos et al, 1996)

$$\dot{f}^2 \leq \frac{r_h^4}{96} \Rightarrow \sqrt{\alpha} \leq 1.2 \left(\frac{M}{M_\odot}\right) \times 10^5\,\mathrm{cm}$$

for $f(\phi) = e^{\phi}$. For $M \simeq 5 M_\odot$, then $\sqrt{\alpha} \leq 6.2 \times 10^5\,\mathrm{cm}$

- **Bounds from Scalar Dipole Radiation:** From the phase evolution of the gravitational waveform, due to the emission of scalar dipole radiation, as found in GW151226 and GW170608, it was found (Nair et al, 2019)

$$\sqrt{\alpha} \leq 5.4 \times 10^5\,\mathrm{cm}$$

We may define a dimensionless coupling $\zeta \equiv \alpha/M^2$, which is independent of the mass scale of the observed system. Then, the bound by Nair et al translates to $\zeta \leq 0.54$ - this must be compared with the theoretical bound $\zeta \leq 0.69$ (P.K., Mavromatos et al, 1996; Pani & Cardoso, 2009)

Bounds on the EsGB Theory

• **Bounds from the Speed of GWs:** The constraint derived from the almost simultaneous detection of gravitational and EM radiation from the BNS GW170817 on the speed of GWs (Abbott et al, 2017)

$$-3 \times 10^{-15} \leq c_g/c - 1 \leq 7 \times 10^{-16}$$

imposed a very strong bound on cosmological models in alternative theories. In the case of localised gravitational solutions, though, we obtain the result (Kobayashi et al, 2012)

$$c_g/c - 1 = \frac{8D\dot{f}(\phi_\infty)}{r^3}$$

which, even if $\dot{f}(\phi_\infty) \neq 0$ and $D \neq 0$, decays very fast.

It should also be stressed that weak-gravitating bodies, including neutron stars, do not acquire a scalar charge - only BH do (Yagi, Stein & Yunes, 2016)

Bounds on the EsGB Theory

• **Bounds from Quasi-Normal Modes:** Differ by the GR prediction by a few %, thus a bound may be set (Blazquez-Salcedo et al, 2016)

$$\sqrt{\alpha} \leq 10 \left(\frac{50}{\rho}\right)^{1/4} \left(\frac{M}{M_\odot}\right) \times 10^5 \,\mathrm{cm}$$

For LVC detections, the signal-to-noise ratio is $\rho \simeq 10$, leading to $\sqrt{\alpha} \leq 7.5 \times 10^5 \,\mathrm{cm}$. For the Einstein Telescope, $\rho \simeq 100$, leading to $\sqrt{\alpha} \leq 4.2 \times 10^5 \,\mathrm{cm}$.

• **Bounds from Binary Orbital Decay:** From the observed period decay rate of the BH low-mass x-ray binary A0620-00, which should be increased by the emission of scalar dipole radiation, it is found (Yagi, 2012)

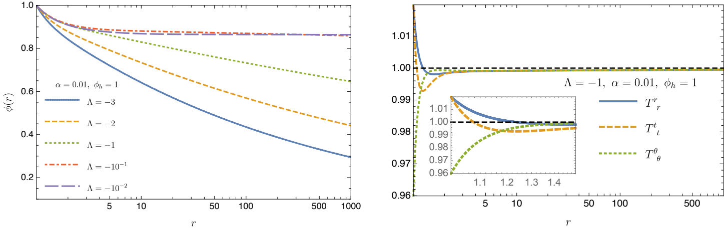
$$\sqrt{\alpha} \leq 1.9 \times 10^5 \,\mathrm{cm}$$

Black Holes in EsGB Theory with Λ

We may extend the previous theory by adding a cosmological constant

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + f(\phi)\,R^2_{GB} - \Lambda\right],$$

For $\Lambda < 0$, we find complete BH, asymptotic Anti-de Sitter solutions:



For $\Lambda > 0$, no complete BH, asymptotic de Sitter solution was found. The GB term does not work well with a constant positive distribution of energy or with inflation...