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#### **Emergent Axions**

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with P. Betzios, M. Bianchi, D. Consoli, E. Kiritsis

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## Plan of the talk

- Motivation
- Framework
- Emergent gravity
- Emergent axions
- Phenomenological considerations
- Conclusions

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- Until now, the only theory that gives a consistent description of quantum gravity is string theory, which provides a natural cutoff, the string scale, protecting the theory from divergences.
- \* On the other hand, new holographic ideas relate large-N gauge theories to string theories (which contain gravity) providing a different approach to this problem.

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- They are protected by their topological invariance and therefore they do not acquire heavy masses in contrary to other scalar operators.





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- \* The two separate sectors are connected via messenger fields.



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- At weak coupling (IR) the hidden theory contains: vectors  $\hat{A}^{\mu}$ , scalars  $\hat{\phi}$  and spin-1/2 particles  $\hat{\psi}$  (the simplest QFTs).



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- In contrary to other scalar operators, they do not acquire masses of the messenger scale M and therefore they can be light and visible at low energies.
- \* Our goal is to study their properties in various different cases and compare with data.

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- \* Therefore we certainly have an ALP (axion-like-particle).
- \* Whether it is a QCD axion (potential has a min at 0) has to be checked.



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The effective action finally is

$$S_{eff} = -\frac{g_{SM}^2 g_{QFT}^2}{90(4\pi)^2 M^4} \int d^4x \Big[ (F \cdot F)(\hat{F} \cdot \hat{F}) + 2(F \cdot \hat{F})^2 + \frac{7}{4} (F \wedge F)(\hat{F} \wedge \hat{F}) + \frac{7}{2} (F \wedge \hat{F})^2 \Big]$$

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• Our goal is to focus in this coupling.



# Effective theory

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- \* Consider the interaction between two theories  $T_1$  (= Hidden) and  $T_2$  (= SM)

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 Following the standard procedure (Schwinger functional, Legendre transformations) we get

 $i \langle O_1(p) O_1(-p) \rangle \Big|_{total} = \frac{G_{11}(p)}{1 - \lambda^2 G_{11}(p) G_{22}(p)} = G_{11}(p) + \lambda^2 \frac{G_{22}(p) G_{11}^2(p)}{1 - \lambda^2 G_{11}(p) G_{22}(p)}$ where  $G_{ii}(p) = \langle O_i(p) O_i(-p) \rangle_i$ .

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\* This is the propagator of the  $O_1$ , in the presence of the interaction term  $\lambda O_1 O_2$ .

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- \* In order to "read" the mass  $m_a$  and decay constant  $f_a$  we need the to expand the G's.
- We have several different options / regimes.

# Fixing $m_a \& f_a$

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- \* We have the following options to explore:
  - $p \ll m_{SM}, m_h \ll M$
  - $m_h \ll p \ll m_{SM}$
  - $m_{SM} \ll p \ll m_h$
  - $m_{SM}, m_h \ll p \ll M$

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\* For generic scalar operator, with a scale m (mass gap) and an UV scale M, the UV scale is dominant and we have ( $m \ll M$ )

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$$a_n \sim M^{2(\Delta-2)-n} \left[ 1 + \mathcal{O}\left(\frac{m}{M}\right) \right] \longrightarrow m_a^2 \sim M^2 , \quad f_a^2 \sim \frac{M^2}{\lambda_0^2}$$

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$$a_n \sim M^{2(\Delta-2)-n} \left[ 1 + \mathcal{O}\left(\frac{m}{M}\right) \right] \longrightarrow m_a^2 \sim M^2 , \quad f_a^2 \sim \frac{M^2}{\lambda_0^2}$$

\* However, instanton densities  $O \sim Tr[F \land F]$  are protected by symmetries and they are UV insensitive Vicari Panagopoulos, Gursoy Kitirsis Nitti

$$a_n \sim m^{2(\Delta-2)-n}$$

For 
$$p \ll m_{SM}, m_h \ll M$$

$$\frac{1 - \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$$

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\* Therefore, their masses are given by the mass gap of the hidden theory and not by the large messenger masses *M*.

For 
$$p \ll m_{SM}, m_h \ll M$$

$$\frac{1 - \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$$

$$iG_{h,h}(p) = m_h^{2\Delta_h - 4} \left[ \bar{a}_0 - \bar{a}_2 \frac{p^2}{m_h^2} + \mathcal{O}\left(\frac{p^4}{m_h^4}\right) \right]$$
$$iG_{SM,SM}(p) = m_{SM}^{2\Delta_{SM} - 4} \left[ \bar{b}_0 - \bar{b}_2 \frac{p^2}{m_{SM}^2} + \mathcal{O}\left(\frac{p^4}{m_{SM}^4}\right) \right]$$

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$$iG_{_{SM},SM}(p) = m_{_{SM}}^{2\Delta_{_{SM}} - 4} \left[ \bar{b}_0 - \bar{b}_2 \frac{p^2}{m_{_{SM}}^2} + \mathcal{O}\left(\frac{p^4}{m_{_{SM}}^4}\right) \right]$$

\* We finally get

$$\begin{split} m_a^2 &= \bar{a}_0 m_h^2 \left( 1 + \frac{\bar{b}_0 \bar{a}_0 \lambda_0^2}{\bar{a}_2} \frac{m_h^4 m_{_{SM}}^4}{M^8} - \frac{\bar{b}_2 \bar{a}_0^2 \lambda_0^2}{\bar{a}_2} \frac{m_h^6 m_{_{SM}}^2}{M^8} + \cdots \right) \\ f_a^2 &= \frac{\bar{a}_0^2 \bar{a}_2}{\lambda_0^2} \frac{M^8}{m_h^6} - \bar{b}_2 \ m_{_{SM}}^2 \end{split}$$

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\* Assume that  $\bar{a}_i, \bar{b}_i \sim 1$  we have  $m_a \& f_a$  as functions of our parameters  $m_{SM}, m_h, M$ .

For 
$$p \ll m_{SM}, m_h \ll M$$

$$\frac{1 - \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$$

$$iG_{h,h}(p) = m_h^{2\Delta_h - 4} \left[ \bar{a}_0 - \bar{a}_2 \frac{p^2}{m_h^2} + \mathcal{O}\left(\frac{p^4}{m_h^4}\right) \right]$$
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We finally get

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- \* Assume that  $\bar{a}_i, \bar{b}_i \sim 1$  we have  $m_a \& f_a$  as functions of our parameters  $m_{SM}, m_h, M$ .
- \* Therefore, the mass  $m_a$  has two contributions:

For 
$$p \ll m_{SM}, m_h \ll M$$

$$\frac{1 - \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$$

$$iG_{h,h}(p) = m_h^{2\Delta_h - 4} \left[ \bar{a}_0 - \bar{a}_2 \frac{p^2}{m_h^2} + \mathcal{O}\left(\frac{p^4}{m_h^4}\right) \right]$$
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We finally get

$$\begin{split} m_a^2 &= \bar{a}_0 m_h^2 \left( 1 + \frac{\bar{b}_0 \bar{a}_0 \lambda_0^2}{\bar{a}_2} \frac{m_h^4 m_{_{SM}}^4}{M^8} - \frac{\bar{b}_2 \bar{a}_0^2 \lambda_0^2}{\bar{a}_2} \frac{m_h^6 m_{_{SM}}^2}{M^8} + \cdots \right) \\ f_a^2 &= \frac{\bar{a}_0^2 \bar{a}_2}{\lambda_0^2} \frac{M^8}{m_h^6} - \bar{b}_2 \ m_{_{SM}}^2 \end{split}$$

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- \* Therefore, the mass  $m_a$  has two contributions:
  - the SM quantum effects ~  $\Lambda_{QCD}^2/f_a$  as with standard axions and

For 
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$$iG_{_{SM},SM}(p) = m_{_{SM}}^{2\Delta_{_{SM}} - 4} \left[ \bar{b}_0 - \bar{b}_2 \frac{p^2}{m_{_{SM}}^2} + \mathcal{O}\left(\frac{p^4}{m_{_{SM}}^4}\right) \right]$$

We finally get

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- \* Assume that  $\bar{a}_i, \bar{b}_i \sim 1$  we have  $m_a \& f_a$  as functions of our parameters  $m_{SM}, m_h, M$ .
- \* Therefore, the mass  $m_a$  has two contributions:
  - the SM quantum effects ~  $\Lambda_{QCD}^2/f_a$  as with standard axions and
  - a contribution from the hidden theory order  $m_{hidden}$  (unlike fundamental axions).

For  $m_{SM} \ll p \ll m_h$ 

$$\frac{1 - \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$$

\* In that case, SM glueballs are fat and unstable and hidden ones are pointlike with expansions

$$\begin{split} & iG_{h,h}(p) = m_h^{2\Delta_h - 4} \left[ \bar{a}_0 - \bar{a}_2 \frac{p^2}{m_h^2} + \mathcal{O}\left(\frac{p^4}{m_h^4}\right) \right] \\ & iG_{_{SM},SM} = p^4 \log \frac{p^2}{m_{_{SM}}^2} \left[ \hat{b}_0 + \hat{b}_2 \frac{m_{_{SM}}^2}{p^2} + \mathcal{O}\left(\frac{m_{_{SM}}^4}{p^4}\right) \right] \quad , \quad p \gg m_{_{SM}} \end{split}$$

For  $m_{SM} \ll p \ll m_h$ 

$$\frac{1 - \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$$

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\* In that regime

 $\frac{1}{G_{hh}(p)} \gg \lambda^2 G_{SMSM}(p)$ 

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In that regime

$$\frac{1}{G_{hh}(p)} \gg \lambda^2 G_{SMSM}(p)$$

\* And we get (the SM contribution is tiny)

$$m_a^2 = \frac{\bar{a}_0}{\bar{a}_2} m_h^2 \quad , \quad f_a^2 = \frac{\bar{a}_2}{\bar{a}_0^2} \frac{m_h^2}{\lambda_0^2} \left(\frac{M}{m_h}\right)^{2\Delta_h}$$

For  $m_{SM} \ll p \ll m_h$ 

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$$m_a^2 = \frac{\bar{a}_0}{\bar{a}_2} m_h^2 \quad , \quad f_a^2 = \frac{\bar{a}_2}{\bar{a}_0^2} \frac{m_h^2}{\lambda_0^2} \left(\frac{M}{m_h}\right)^{2\Delta_h}$$

 The result is similar to the previous case. The leading contribution is coming from the hidden theory.

For 
$$m_h \ll p \ll m_{SM}$$

$$\frac{1 - \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$$

\* In that case, hidden glueballs are fat and unstable but SM ones are pointlike with

$$iG_{h,h}(p) = p^4 \log \frac{p^2}{m_h^2} \left[ -\hat{a}_0 + \hat{a}_2 \frac{m_h^2}{p^2} + \mathcal{O}\left(\frac{m_h^4}{p^4}\right) \right] \quad , \quad p \gg m_h$$
$$iG_{_{SM},_{SM}}(p) = m_{_{SM}}^{2\Delta_{_{SM}}-4} \left[ \bar{b}_0 - \bar{b}_2 \frac{p^2}{m_{_{SM}}^2} + \mathcal{O}\left(\frac{p^4}{m_{_{SM}}^4}\right) \right]$$

For 
$$m_h \ll p \ll m_{SM}$$

$$\frac{1 - \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$$

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$$iG_{h,h}(p) = p^{4}\log\frac{p^{2}}{m_{h}^{2}}\left[-\hat{a}_{0} + \hat{a}_{2}\frac{m_{h}^{2}}{p^{2}} + \mathcal{O}\left(\frac{m_{h}^{4}}{p^{4}}\right)\right] , \quad p \gg m_{h}$$
$$iG_{_{SM},SM}(p) = m_{_{SM}}^{2\Delta_{_{SM}}-4}\left[\bar{b}_{0} - \bar{b}_{2}\frac{p^{2}}{m_{_{SM}}^{2}} + \mathcal{O}\left(\frac{p^{4}}{m_{_{SM}}^{4}}\right)\right]$$

\* In that regime,  $\langle aa \rangle^{-1} \sim M^8 \log |x|$  is a non-standard non-local axion kinetic term and

$$S_{eff} \simeq \frac{M^8}{2} \int d^4 x_1 d^4 x_2 \ a(x_1) \log \frac{|x_1 - x_2|}{m_h} a(x_2) + \int d^4 x \ \chi(x) O_{SM}(x)$$

which is valid only for  $m_{_{SM}}^{-1} < \ell < m_h^{-1}$ .

For 
$$m_h \ll p \ll m_{SM}$$

which i

$$\frac{1 - \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$$

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$$iG_{h,h}(p) = p^{4}\log\frac{p^{2}}{m_{h}^{2}}\left[-\hat{a}_{0} + \hat{a}_{2}\frac{m_{h}^{2}}{p^{2}} + \mathcal{O}\left(\frac{m_{h}^{4}}{p^{4}}\right)\right] , \quad p \gg m_{h}$$
$$iG_{SM,SM}(p) = m_{SM}^{2\Delta_{SM}-4}\left[\bar{b}_{0} - \bar{b}_{2}\frac{p^{2}}{m_{SM}^{2}} + \mathcal{O}\left(\frac{p^{4}}{m_{SM}^{4}}\right)\right]$$

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\* Such axions are still interesting and well-defined, however the experimental viability is different and standard experimental constrains do not directly apply.

For 
$$m_h \ll p \ll m_{SM}$$

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$$\frac{1 - \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$$

\* In that case, hidden glueballs are fat and unstable but SM ones are pointlike with

$$iG_{h,h}(p) = p^{4}\log\frac{p^{2}}{m_{h}^{2}}\left[-\hat{a}_{0} + \hat{a}_{2}\frac{m_{h}^{2}}{p^{2}} + \mathcal{O}\left(\frac{m_{h}^{4}}{p^{4}}\right)\right] , \quad p \gg m_{h}$$
$$iG_{SM,SM}(p) = m_{SM}^{2\Delta_{SM}-4}\left[\bar{b}_{0} - \bar{b}_{2}\frac{p^{2}}{m_{SM}^{2}} + \mathcal{O}\left(\frac{p^{4}}{m_{SM}^{4}}\right)\right]$$

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$$\begin{split} S_{eff} &\simeq \frac{M^8}{2} \int d^4 x_1 d^4 x_2 \,\, a(x_1) \log \frac{|x_1 - x_2|}{m_h} a(x_2) + \int d^4 x \,\, \chi(x) O_{_{SM}}(x) \\ \text{s valid only for } m_{_{SM}}^{-1} &< \ell < m_h^{-1}. \end{split}$$

- \* Such axions are still interesting and well-defined, however the experimental viability is different and standard experimental constrains do not directly apply.
- \* In this category we also have the case where the hidden theory is conformal  $(m_h \rightarrow 0)$ .

For 
$$m_{SM}, m_h \ll p \ll M$$

$$\frac{1 - \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$$

\* In that case, *both* glueballs are fat and unstable with expansions

$$\begin{split} &iG_{h,h}(p) = p^4 \log \frac{p^2}{m_h^2} \left[ -\hat{a}_0 + \hat{a}_2 \frac{m_h^2}{p^2} + \mathcal{O}\left(\frac{m_h^4}{p^4}\right) \right] \quad , \quad p \gg m_h \\ &iG_{_{SM},SM} = p^4 \log \frac{p^2}{m_{_{SM}}^2} \left[ \hat{b}_0 + \hat{b}_2 \frac{m_{_{SM}}^2}{p^2} + \mathcal{O}\left(\frac{m_{_{SM}}^4}{p^4}\right) \right] \quad , \quad p \gg m_{_{SM}} \end{split}$$

and the couplings are expected to be non-local.

For 
$$m_{SM}, m_h \ll p \ll M$$

- $\frac{1 \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$
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and the couplings are expected to be non-local.

\* In that case

$$\langle \chi \chi \rangle^{-1} = \frac{M^8}{\lambda_0^2 G_{h,h}} - G_{_{SM},_{SM}} = -\frac{M^8}{\lambda_0^2 p^4 \log \frac{p^2}{m_h^2} \left[\hat{a}_0 + \cdots\right]} + \cdots$$

we get again the non-local non-standard

$$S_{eff} \simeq \frac{M^8}{2} \int d^4 x_1 d^4 x_2 \ a(x_1) \log \frac{|x_1 - x_2|}{m_h} a(x_2) + \int d^4 x \ \chi(x) O_{SM}(x)$$

For 
$$m_{SM}, m_h \ll p \ll M$$

- $\frac{1 \lambda^2 G_{hh}(p) G_{SMSM}(p)}{G_{hh}(p)} = f_a^2 (p^2 + m_a^2) + \mathcal{O}[p^4]$
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and the couplings are expected to be non-local.

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\* Again, deeper analysis is needed for these models.

## Phenomenological Windows

Dark Matter axions

$$10^{-25} \text{ eV} < m_a^{DM} < 10^{-18} \text{ eV}$$

Dark Energy axions

 $10^{-33} \text{ eV} < m_a^{DE} < 10^{-30} \text{ eV}$ 

- Axions as Inflatons: (very much model dependent)
- Heavy Axions  $(m_a > 1 \text{eV})$

 $m_a > 10 \text{ MeV}$  and  $\tau_{a\gamma} < 10^{-2} \text{ s}$  or  $m_a < 10 \text{ eV}$  and  $\tau_{a\gamma} > 10^{24} \text{ s}$ 

QCD axions

 $10^{-12} \text{ eV} < m_a^{QCD} < 10^{-3} \text{ eV}$  $10^9 \text{ GeV} < f_a^{QCD} < 10^{15} \text{ GeV}$ 



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- \* If in addition we consider the relation between  $m_{SM}$ ,  $m_h$  we have
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- \* For QCD axions we have:  $(m_a f_a)^{1/2} \sim (m_u \Lambda_{QCD}^3)^{1/4} \sim 10^{-1} GeV \sim m_{SM}$

In our case  $M^2 \sim m_a (m_a f_a)^{1/2} \sim m_h m_{SM}$  violating our initial assumption.

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- \* In this frame work gravity is emerging via the stress tensor of the hidden sector.
- As a byproduct we have axions (instanton densities) which, in contrary to other scalar operators, are not suppressed by the large messenger masses.
- \* The hidden instanton density generates an emergent axion coupled to the SM. The characteristic decay constant of the emergent axion is

$$f \sim m_{\rm hidden} \left(\frac{M}{m_{\rm hidden}}\right)^4 \gg m_{\rm hidden}$$

where  $m_{hidden}$  the characteristic scale of the hidden theory.

- \* The mass of the emergent axion has two contributions.
  - One due to SM quantum effects ~  $\Lambda_{QCD}^2/f_a$  as with standard axions.
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  - One due to SM quantum effects ~  $\Lambda_{QCD}^2/f_a$  as with standard axions.
  - In addition (unlike fundamental axions), has also a contribution from the hidden theory order m<sub>hidden</sub>.
- Some regions of the parameter space of our models provide non-standard non-local kinetic terms and deeper study is required.