No-Go Theorems for Compactifications to De Sitter Space

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Based on
J.R. and P. Townsend, arxiv:1811.03660 (Class.Quant.Grav. 36 (2019) 095008)
arxiv:1904.11967 (JHEP 1906 (2019) 097)

Observations show that the expansion of the Universe is accelerating (SNe, PLANCK).

Is this compatible with theory?

$$ds^2_{FLRW} = -dt^2 + S^2(t)\left((1-kr^2)^{-1}dr^2 + r^2 d\Omega_2^2\right) \quad , \qquad k = -1,0,1$$

Computing the Ricci tensor, one obtains

$$R_{00} \equiv -\frac{3\partial_t^2 S}{S}$$

Therefore an accelerating universe requires $R_{00} < 0$

However, from the Einstein equations

$$R_{00} \equiv \tfrac{1}{2}(T_{00} + g^{ij}T_{ij})$$

The Strong Energy Condition (SEC) on the matter stress tensor requires that RHS be non-negative.

(Physically, it means that gravity is attractive; the source of the Newtonian potential is always of the same sign. SEC rules out antigravity).

SEC is satisfied by the stress tensor of D=10 and D=11 supergravities.

The acceleration of the Universe thus presents a challenge to String/M-theory.

But a no-go theorem establishes that there are no compactifications to de Sitter space in 10/11 supergravities

Gibbons (1985) & hep-th/0301117; Maldacena-Nuñez hep-th/0007018

The basic idea is as follows. Consider a compactification

$$ds_D^2 = \Omega^2(y)\, ds_{FLRW}^2 + ds_n^2(y) \quad , \qquad D = 4 + n$$

$$\rightarrow \qquad R_{00}^{(D)} = R_{00}(x) - \tfrac{1}{4}\Omega^{-2}\nabla_y^2\Omega^4$$

$$\rightarrow \qquad \left(\int_B \Omega^2\right) R_{00}(x) = \int_B \Omega^2 R_{00}^{(D)}$$

Therefore $R_{00} > 0$ if the Strong Energy Condition is satisfied in D-dimensions

This, however, does not rule out acceleration: Here we were looking at **time-independent** compactifications.

Since we are interested in cosmological solutions, it is natural to assume that the warp factor and the internal metric are also time dependent.

From the four-dimensional viewpoint, this implies that there are time-dependent scalar fields.

The GMN no-go theorem rules out de Sitter solutions arising from *static* compactifications.

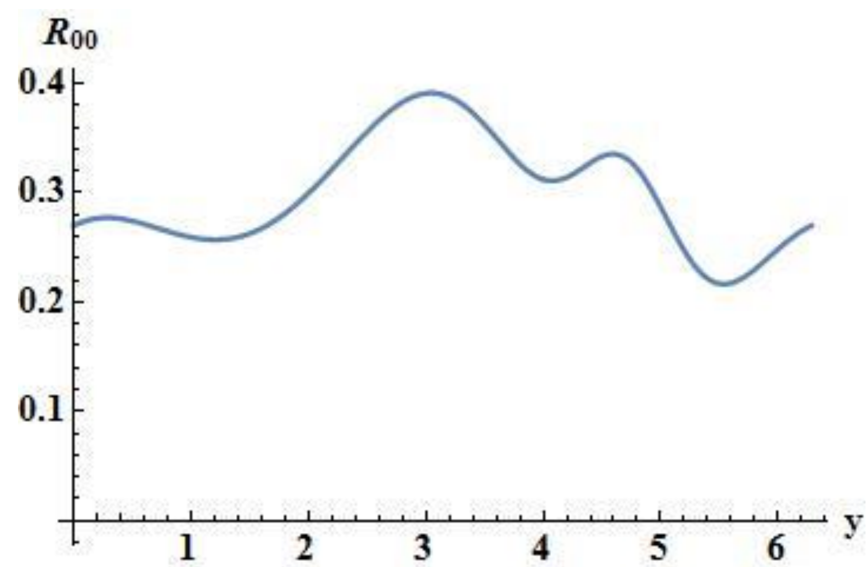
Are there non-singular time-dependent compactifications to de Sitter space obeying SEC?

SHORT ANSWER: YES

$$ds_D^2 = \Omega^2(y+t)\,ds_{\mathrm{dS}}^2 + \varphi^2(y+t)dy^2 \quad , \qquad y \approx y + 2\pi L$$

$$\Omega = 2A\left(1 + \tfrac{1}{5}\sin[(t+y)/L]\right)$$
$$\varphi = A\left(1 + \tfrac{2}{5}\sin[(t+y)/L]\right)$$

$$\boxed{R_{00} \geq 0}$$



The Strong Energy Condition is satisfied

However, as we will discuss, the 5D stress tensor that supports this solution of the 5D Einstein equations does not satisfy the Dominant Energy Condition (DEC)

No-Go theorems for general (time-dependent) warped compactifications

Consider a time-dependent compactification

$$ds_D^2 = \Omega^2(y,t)\, ds_{FLRW}^2 + h_{\alpha\beta}(y,t) dy^\alpha dy^\beta \quad , \qquad \alpha, \beta = 1, \cdots, n \quad , \quad D = 4 + n$$

$$ds_{FLRW}^2 = -dt^2 + S^2(t) \bar{g}_{ij} dx^i dx^j \quad , \qquad i, j = 1, \cdots, 3$$

The condition for the 4d metric to be in the Einstein-frame is

$$\int_B d^n y \sqrt{\det h}\ \ \Omega^2 = \frac{G_D}{G_4}$$

The Einstein-frame condition is required since otherwise the Newton constant would be time-dependent, and this would be in conflict with observations.

It is convenient to introduce the notation

$$\langle \Phi \rangle = \frac{G_4}{G_D} \int_B d^n y \sqrt{\det h}\ \ \Omega^2\ \Phi$$

$$\langle 1 \rangle = 1$$

$$\langle \Phi \rangle = \Phi \quad if \qquad \partial_y \Phi = 0$$

For simplicity, let us first consider the case when

$$h_{\alpha\beta} = \varphi^2(t, y)\,\delta_{\alpha\beta}$$

Then

$$R_{00} = -n\left(\frac{\ddot{\varphi}}{\varphi} - \frac{\dot{\varphi}\,\dot{\Omega}}{\varphi\,\Omega}\right) - 3\left(\frac{\ddot{S}}{S} + \frac{\ddot{\Omega}}{\Omega} - \frac{\dot{\Omega}^2}{\Omega^2} + \frac{\dot{\Omega}}{\Omega}\frac{\dot{S}}{S}\right) + \frac{1}{4}\Omega^{-2}\nabla^2\Omega^4$$

The STRONG ENERGY CONDITION (SEC) is

$$\boxed{R_{00} \geq 0}$$

We can get rid of the last term by integrating over *y*-coordinates. This gives

$$\langle R_{00}\rangle = -n\left\langle\frac{\ddot{\varphi}}{\varphi} - \frac{\dot{\varphi}\,\dot{\Omega}}{\varphi\,\Omega}\right\rangle - 3\left(\frac{\ddot{S}}{S} + \left\langle\frac{\ddot{\Omega}}{\Omega}\right\rangle - \left\langle\frac{\dot{\Omega}^2}{\Omega^2}\right\rangle + \left\langle\frac{\dot{\Omega}}{\Omega}\right\rangle\frac{\dot{S}}{S}\right) \quad \geq \quad 0$$

The inequality can be further simplified by using the Einstein-frame condition.
We first assume that time-independence is imposed *before* integrating:

$$\sqrt{\det h}\;\Omega^2 = \varphi^n \Omega^2 = \omega(y) \quad , \qquad \int_B d^n y \;\; \omega(y) \; = \frac{G_D}{G_4}$$

$$\rightarrow \qquad \frac{n\dot{\varphi}}{\varphi} = -\frac{2\dot{\Omega}}{\Omega}$$

Then

$$\langle R_{00} \rangle = -\frac{3\ddot{S}}{S} - \frac{3\dot{S}}{S}\left\langle \frac{\dot{\Omega}}{\Omega} \right\rangle - \left\langle \frac{\ddot{\Omega}}{\Omega} \right\rangle - \left(1 + \frac{4}{n}\right)\left\langle \frac{\dot{\Omega}^2}{\Omega^2} \right\rangle \quad \geq \quad 0$$

Suppose that there is a time t_0 for which Omega' = 0 and S' = 0. Then the above SEC inequality implies that, at this time,

$$-\frac{3\ddot{S}}{S} - \left\langle \frac{\ddot{\Omega}}{\Omega} \right\rangle \geq 0 \qquad\qquad (t = t_0) \qquad\qquad (1)$$

which is the inequality found by [Vafa et al, arXiv:1806.08362].

1) It does not exclude acceleration. Indeed S'' can be positive if Omega'' is negative. This is precisely the mechanism for cosmologies exhibiting *transient* acceleration: the acceleration occurs as the volume of the compact space passes through a minimum value, corresponding to a maximum of Omega(t).

2) It does not even exclude late-time *eternal* acceleration. The assumption that there is a specific time t_0 where the first time derivatives vanish does not apply to the late-time power law attractor.
Here we disagree with the conclusion in Vafa et al that SEC rules out accelerating cosmologies.
In that paper use is made of the bound (1) for the late-time attractor of an exponential potential. But this has non zero first derivatives Omega' and S' .
Indeed, we will see late time acceleration is compatible with SEC

No-go theorem for dS

Search now for de Sitter solutions

In the case of *time-independent* compactifications, one would just get

$$\langle R_{00} \rangle \geq 0 \quad \Rightarrow \quad -\frac{\ddot{S}}{S} \geq 0$$

In particular

$$S = \exp(Ht) \quad \Rightarrow \quad -\frac{3\ddot{S}}{S} = -3H^2 \geq 0$$

Therefore, $H = 0$, as known.

Consider now a time-dependent compactification with $S = \exp(Ht)$ at late times.
the S'' term will dominate unless Omega also increases exponentially.
So we consider the ansatz:

$$S = \exp(Ht)\,, \quad \Omega = \exp(Jt) \quad \Rightarrow \quad 3H^2 + 3HJ + 2\left(1 + \frac{2}{n}\right)J^2 \leq 0$$

But this equation has no real solutions.
This completely rules out de Sitter compactifications of the *assumed* form.
What about our example?
The Einstein-frame condition is implemented in *integrated* form. In this case, there are compactifications to de Sitter space consistent with SEC.

Einstein frame condition in the integrated form

The inequality that we used to rule out de Sitter was obtained considering compactifications implementing the Einstein-frame condition *prior* to integration over *y:*

In general, the integrand may depend on time: $\sqrt{\det h}\,\Omega^2 = \varphi^n \Omega^2 = \omega(y,t)$

The time dependence may disappear only upon integration.
The only general requirement is the Einstein-frame condition in integrated form:

$$\int_B d^n y \sqrt{\det h}\;\; \Omega^2 = \frac{G_D}{G_4}$$

The resulting inequality

$$R_{00} = -n\left(\frac{\ddot{\varphi}}{\varphi} - \frac{\dot{\varphi}\,\dot{\Omega}}{\varphi\,\Omega}\right) - 3\left(\frac{\ddot{S}}{S} + \frac{\ddot{\Omega}}{\Omega} - \frac{\dot{\Omega}^2}{\Omega^2} + \frac{\dot{\Omega}}{\Omega}\frac{\dot{S}}{S}\right) + \frac{1}{4}\Omega^{-2}\nabla^2\Omega^4 \quad \geq \quad 0$$

involves terms of different signs and does not lead to any definite conclusions.

Einstein frame condition in our de Sitter example:

$$\int_0^{2\pi L} dy\, \varphi(t+y)\Omega^2(t+y) = \frac{G_5}{G_4}$$

It is time-independent as a consequence of periodicity.
And $R_{00} > 0$ at all times!!

GENERAL NO GO THEOREM – USING DOMINANT ENERGY CONDITION

$$T_{00} = -\rho g_{00} \quad , \qquad T_{ij} = P g_{ij} \ , \qquad i, j = 1,2,3$$

$$\rho + P \geq 0 \quad \Rightarrow \quad -2\dot{X} + 2HX + X^2 \geq \tfrac{1}{4}\left(\mathrm{tr}\, h^{-1}\dot{h}\right)^2 + \tfrac{1}{2}\,\mathrm{tr}\left(h^{-1}\dot{h}\right)^2$$

$$X \equiv \tfrac{1}{2}\,\mathrm{tr}\left(h^{-1}\dot{h}\right) + 2\dot{\Omega}/\Omega$$

Note that *X* := 0 if the Einstein-frame condition is implemented prior to integration.
In this case, we see that the above inequality is satisfied if the compact space metric *h* is time independent.
But we have already established that such compactifications violate SEC.

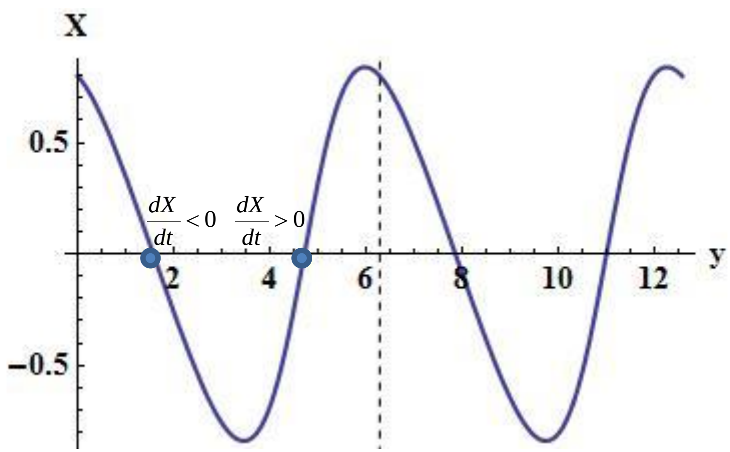
More generally, one has <*X*> = 0, as a consequence of the integrated Einstein frame condition.

The above inequality is violated by our de Sitter example, because any periodic function X whose integral is zero on a period, will have at least two zeros and in one of them dX/dt > 0.

$$ds_D^2 = \Omega^2(y+t)\, ds_{\mathrm{dS}}^2 + \varphi^2(y+t) dy^2 \quad , \qquad y \approx y + 2\pi L$$

$$\Omega = 2A\left(1 + \tfrac{1}{5}\sin[(t+y)/L]\right)$$
$$\varphi = A\left(1 + \tfrac{2}{5}\sin[(t+y)/L]\right)$$

$$X = \frac{d}{dt}\log\left(\Omega^2\varphi\right), \qquad \langle X \rangle = \int_0^{2\pi} dy\ X = 0$$

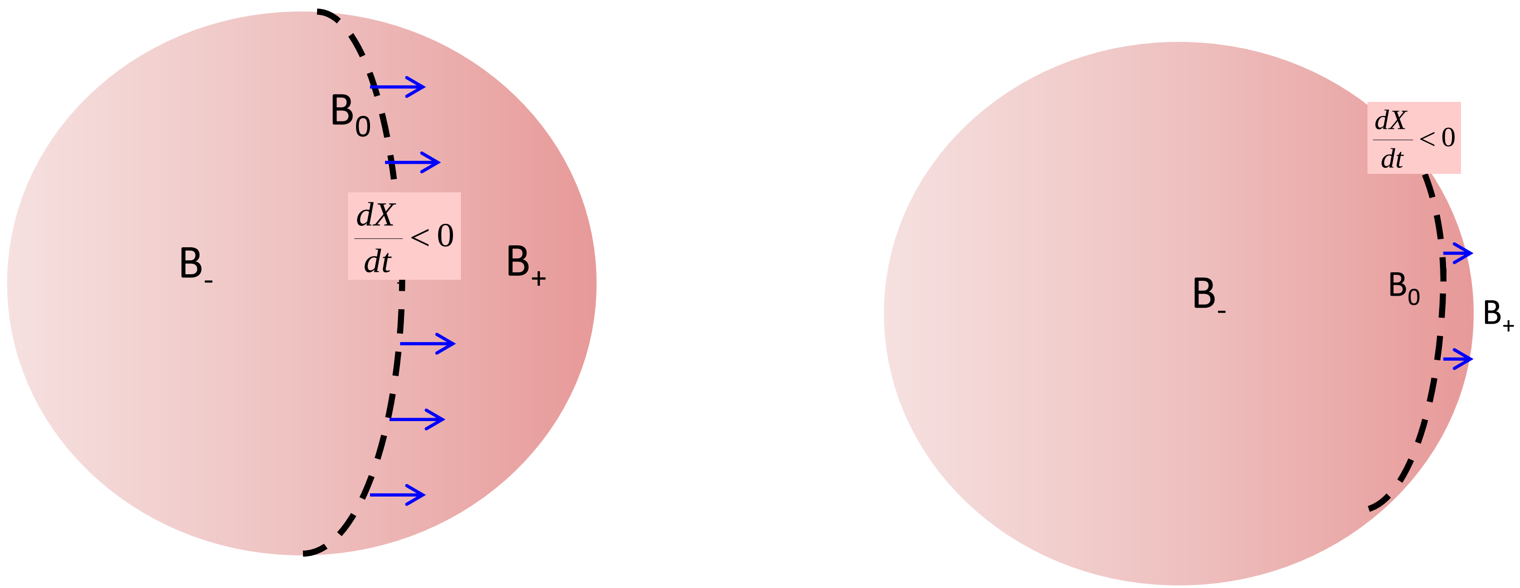


$$-2\dot{X} + 2HX + X^2 \geq 0$$ is violated in the second point

$\langle X \rangle = 0$ implies that there are regions in the compact space *B* where *X* is positive and regions where *X* is negative.

In general we have a partition $$B = B_- \cup B_0 \cup B_+$$

In neighborhoods of B_0, where $X = 0$, we must have $dX/dt < 0$ for all times to satisfy DEC. Then the space B_+ shrinks, because points in B_+ near B_0, where X is near zero, will move to B_-. As a result, the volume of B_+ shrinks to zero size leading to a delta function singularity.



Therefore it is not possible to have non-singular compactifications to dS satisfying DEC

Accelerating universes in String theory

DO THE STRONG AND DOMINANT ENERGY CONDITIONS RULE OUT ACCELERATION?

Consider first an exponential potential

$$S = \int d^4x \sqrt{-g}\left(\frac{R}{2\kappa^2} - \tfrac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_0\, e^{-2a\phi}\right) \quad , \qquad V_0 > 0$$

Many compactifications of M-theory lead to exponential potentials.

***a* < 1 gives rise to an eternally accelerating cosmology with a future event horizon.**
But this potential does not arise from string/M-theory compactifications.
Only *a* > 1 arises in practice.

Compactifications giving rise to a positive potential include:

- Flux compactifications: *a* > sqrt[3]

- Hyperbolic compactifications. They lead to 1 < *a* < sqrt[3]

While none of the examples can lead to late time (eternal) acceleration, they can lead to *transient* acceleration [Townsend, Wohlfarth, hep-th/03003097; Townsend hep-th/0308149]

The Einstein equations give

$$R_{00} \equiv 2\dot{\phi}^2 - 2V_0 e^{-2a\phi}$$

Therefore $R_{00} < 0$ in the time interval when $\dot{\phi} \approx 0$, implying an accelerating cosmology during this time interval.

For example, consider a compactification on a maximally symmetric space X_n *with flux:*

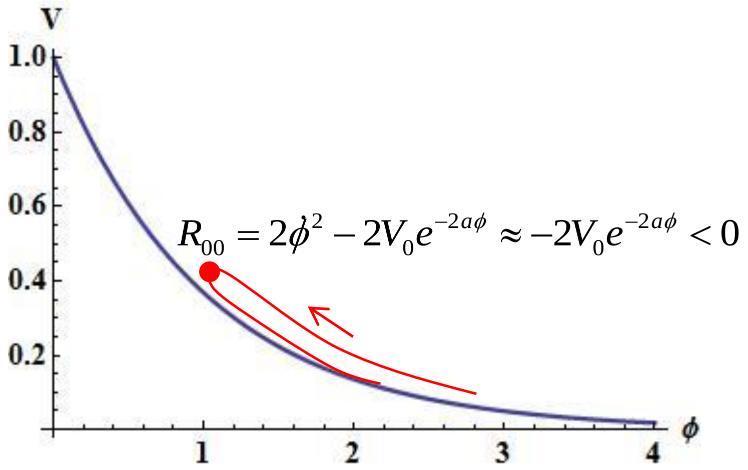
$$S = \frac{1}{16\pi G_D}\int d^D x\sqrt{-g^{(D)}}\left(R - \tfrac{1}{48}F_4^2\right) \quad , \qquad D = 4+n$$

$$ds_D^2 = e^{-n\phi(x)}\, g_{\mu\nu}(x)dx^\mu dx^\nu + e^{2\phi(x)}dX_n^2 \quad , \qquad *F_4 = b\,\mathrm{vol}(X_n)$$

$$\rightarrow \quad S = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\left(R(g) - \tfrac{n(n+2)}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{b^2}{2}e^{-3n\phi} + \varepsilon\, n(n-1)e^{-(n+2)\phi}\right)$$

Here epsilon = +/- 1 corresponds to sphere or hyperbolic space.

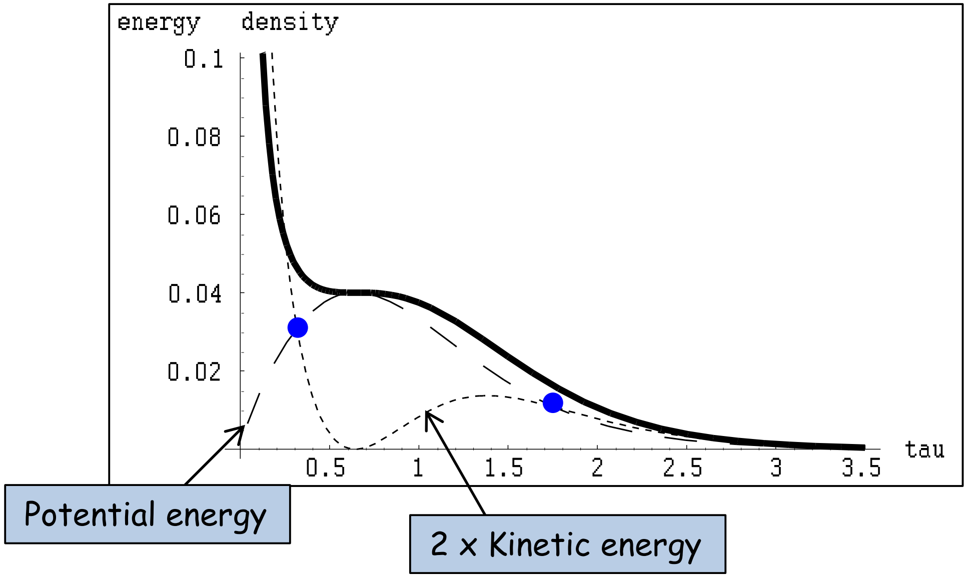
Even when b = 0, i.e. no flux, compactification on the hyperbolic space leads to transient periods of acceleration (the potential is monotonic so necessarily there is a turning point).



The equations of motion are

$$\ddot{\phi} + \frac{3\dot{S}\dot{\phi}}{S} - 2aV_0 e^{-2a\phi} = 0, \qquad \frac{\dot{S}^2}{S^2} = \frac{1}{6}\left(\frac{1}{2}\dot{\phi}^2 + V_0 e^{-2a\phi}\right)$$

They can be solved exactly [J.R., hep-th/0403010]



Recalling that $\frac{1}{2}R_{00} \equiv 2\frac{1}{2}\dot{\phi}^2 - V_0 e^{-2a\phi}$

the period of accelerated expansion corresponds to the time interval between the intersection points of 2 x kinetic energy and the potential energy

For $a > 1$, at late times the kinetic energy dominates and the expansion is decelerating.

Is there any way to have an accelerating universe **at late times** in String theory?

Power-law acceleration

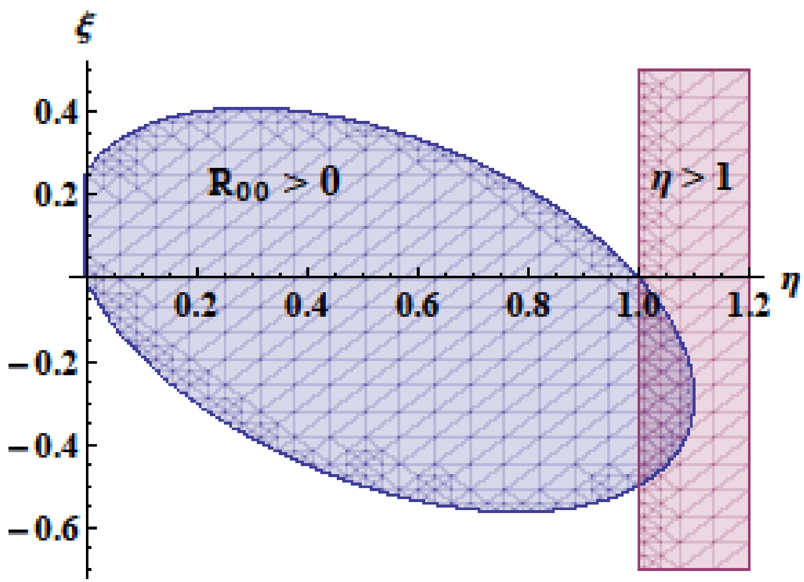
Let us return to our compactification and now assume a late-time behavior

$$S = t^{\eta} \quad , \qquad \Omega = \Omega_0(y)\, t^{\xi}$$

The expansion is accelerating provided eta > 1.

The Strong Energy Condition now becomes $\quad \langle R_{00} \rangle = -\frac{3\ddot{S}}{S} - \frac{3\dot{S}}{S}\left\langle \frac{\dot{\Omega}}{\Omega} \right\rangle - \left\langle \frac{\ddot{\Omega}}{\Omega} \right\rangle - \left(1+\frac{4}{n}\right)\left\langle \frac{\dot{\Omega}^2}{\Omega^2} \right\rangle \quad \geq \quad 0$

$$-\eta^2 - \eta\xi - c\xi^2 + \eta + \tfrac{1}{3}\xi \geq 0 \quad , \qquad c \equiv \frac{2}{3}\left(1+\frac{2}{n}\right)$$



$$1 < \eta_{\max} < \frac{4}{3} \qquad\qquad -\frac{n}{n+2} < \xi < 0$$

Thus there are accelerating cosmologies satisfying the Strong Energy Condition.

So let us examine the implications of this cosmology.

Decompactification

Recall $$h_{\alpha\beta} = \varphi^2(t,y)\,\delta_{\alpha\beta} \quad , \qquad \varphi^n \Omega^2 = 1 \quad \rightarrow \quad \varphi = t^\sigma , \quad \sigma = -\frac{2\xi}{n}$$

$$-\frac{n}{n+2} < \xi < 0 \quad \rightarrow \quad 0 < \sigma < \frac{2}{n+2}$$

This implies that the compact space is expanding.

Therefore the accelerating expansion of the universe requires decompactification.

Additionally, sigma < 1 tells us that *this* expansion is decelerating.

NO GO-THEOREM FOR POWER-LAW ACCELERATION

What is the *four*-dimensional stress tensor that supports this FLRW metric?

Computing Einstein tensor for the FLRW metric one obtains

$$\boxed{T_{00} = t^{-2}\rho \quad , \qquad T_{ij} = t^{-2} p\, g_{ij}}$$

where

$$\rho = 3\eta^2 \quad , \qquad p = -\eta(3\eta - 2)$$

The four-dimensional stress tensor is that of a perfect fluid with mass density \rho and pressure *p* satisfying the continuity equation

$$\dot{\rho} = -3(\rho + p)\dot{S}/S$$

The equation of state is

$$p = w\rho \quad , \qquad w = -1 + \frac{2}{3\eta}$$

Using, the bound on \eta, we obtain

$$\eta < \eta_{\max} < \frac{4}{3} \qquad \rightarrow \qquad \boxed{w > -\frac{1}{2}}$$

The expansion is accelerated when w < -1/3. This shows, in particular, that the SEC can be violated in the lower dimension even though it is not violated in the higher dimension.

The standard no-go theorem forbids de Sitter cosmologies having *w* = -1.
We have now arrived at a much stronger restriction on *w* (for the late time attractor).

This may be compared with current experimental bounds

Considering the Dark Energy as a dynamical fluid with equation of state $p = w\rho$
PLANCK adopts the parametrization

$$w(a) = w_0 + (1 - S(t))w_1$$

$$w_0 = -0.96 \pm 0.08 \quad , \qquad w_1 = -0.28^{+0.31}_{-0.27} \qquad \text{Planck} + \text{BAO} + \text{SNe}$$

$$w_0 = -0.76 \pm 0.20 \quad , \qquad w_1 = -0.72^{+0.62}_{-0.54} \qquad \text{Planck} + \text{BAO} \,/\, \text{RSD, WL}$$

[Planck Collaboration, "Planck 2018 results. VI. Cosmological parameters," arXiv:1807.06209].

w = -1 is consistent with a cosmological constant.

Thus *w* is near *w* = -1, but, according to the best experimental fit,
***w* seems to be increasing with time**, as w_1 appears to be negative.

Late-time Stress Tensor

The required stress tensor supporting this cosmology is determined from the Einstein equations. Computing the Einstein tensor, one obtains

$$\boxed{T_{00} = t^{-2}\rho_0 \quad , \qquad T_{ij} = t^{2(\eta-1)} P_0 \, \bar{g}_{ij} \quad , \qquad T_{\alpha\beta} = t^{-2-2\frac{(n+2)}{n}} P_0^{(\text{int})} \, \delta_{\alpha\beta}}$$

where

$$\rho_0 = 3\eta^2 - \left(1+\frac{2}{n}\right)\xi^2 \quad , \qquad P_0 = -3\eta^2 - \left(1+\frac{2}{n}\right)\xi^2 + 2\eta$$

$$P_0^{(\text{int})} = -6\eta^2 - 3\left(1+\frac{2}{n}\right)\xi\,\eta - \left(1+\frac{2}{n}\right)\xi^2 + 3\eta + \left(1+\frac{2}{n}\right)\xi$$

The stress tensor is conserved by the Bianchi identities $\nabla^{\mu} G_{\mu\nu} = 0 \quad , \qquad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

Weak Energy Condition: Using the bounds on \eta and \xi, it follows that the energy density is positive definite.

$$-\frac{n}{n+2} < \xi < 0 \quad , \quad \eta > 1 \quad \Rightarrow \qquad \rho_0 > 0$$

Similarly, the bounds on \eta and \xi imply that both pressures are *negative definite*.

DOMINANT ENERGY CONDITION

We must require that

$$\rho_0 \pm P_0 \geq 0 \quad , \qquad \rho_0 \pm P_0^{(\mathrm{int})} \geq 0$$

The Dominant Energy Condition (DEC) is needed for causality.
In particular, DEC ensures that there is no superluminal propagation, as v_s = Sqrt[*p*/rho] < 1.
In addition, DEC is needed for the proof of positive energy in GR, which is a necessary condition for stability of the Minkowski space [Christodoulou, Klainerman, 1993].

Note, in particular, that DEC implies the weak energy condition $\rho_0 \geq 0$

For the present accelerated cosmology, the stress tensor satisfies the Dominant energy condition automatically once SEC is satisfied. To prove this, one has to take into account the bound on xi.

Consider for example

$$\rho_0 + P_0 = 2\left(\eta - \frac{n+2}{n}\xi^2\right) \; > \; 2\left(1 - \frac{n}{n+2}\right) = \frac{4}{n+2} > 0$$

Similarly one can prove the other conditions.

Thus the accelerating cosmology is supported by *physical* matter, described by a conserved stress tensor satisfying both strong and dominant energy conditions.

Accelerating cosmologies with two scalar fields

Another way to circumvent the limitations to acceleration implied by the positivity bounds is to consider two interacting scalar fields. An example is provided by the following Lagrangian for a dilaton and an axion parametrizing the hyperbolic space H_2

$$L = R - \tfrac{1}{2}\left((\partial\sigma)^2 + e^{-\mu\sigma}(\partial\chi)^2\right) - \Lambda e^{-\lambda\sigma}$$

For constant \chi, there is no accelerating cosmologies if \lambda >1.

Recall that $$R_{00} = 2(\partial_t\sigma)^2 - 2\Lambda e^{-\lambda\sigma}$$

If the kinetic term dominates at late times, then $R_{00} > 0$.
As a result,

$$-R_{00} = \frac{3\partial_t^2 S}{S} < 0$$

and the expansion is decelerating.

To get late-time acceleration, we need that the interaction with the other scalar field slows down the \sigma field rolling down the potential, so that the kinetic term is always smaller than the potential.

So let us study solutions with non-constant \chi , to see if kinetic energy can be transferred through the interaction so that \sigma is slowed down and R_{00} becomes negative at late times

There is in fact a FLRW solution with non-constant \chi , found by
[J. Sonner and P. K. Townsend, hep-th/0608068]

$$\sigma = ct \quad , \qquad \chi = const.\exp(2c\mu t)\,, \qquad c = \frac{\sqrt{3}}{\sqrt{\mu(\mu+\lambda)}}$$

$$S = t^{\eta} \quad , \qquad \eta = \frac{\lambda+\mu}{3\lambda}$$

Accelerated expansion occurs when

$$\eta > 1 \qquad \rightarrow \qquad \lambda < \frac{\mu}{2}$$

Thus there are accelerating cosmologies for any \lambda, as long as \mu > 2\lambda.

The stronger the coupling \mu, the more damped is the rolling down the exponential potential due to transfer of potential energy to the axion, which raises the critical value of V'/V corresponding to a crossover from deceleration to acceleration.

A mechanic analog is the rolling of a disc of large moment of inertia down a hill under the influence of gravity; potential energy is transferred not only to kinetic energy of downward motion but also to rotational kinetic energy, which slows the downward motion.

CONCLUSIONS

De Sitter Space

• The Strong Energy Condition rules out compactification to de Sitter space if the compact space is time-independent.

• We have shown that allowing for time dependence, the resulting SEC inequality does *not* rule out compactifications to de Sitter.
We have presented a compactification to dS supported by a stress tensor satisfying SEC.

• Combining SEC and DEC, we have proven a general no-go theorem excluding any (non-singular) time-dependent compactification to dS space.

Accelerating cosmologies

• We have seen that the *SEC, by itself, does not prevent compactification to universes that undergo late-time (eternal) accelerated expansion*.

• Our compactifications lead to FLRW universes filled with a perfect fluid that has a constant $w = p/\text{rho}$, on which the SEC imposes $w > -1/2$.

• An interesting property of our examples is that accelerated expansion of the FLRW universe requires expansion of the compact space. The universe will eventually decompactify.