Heterotic Unification and the GUT scale

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Outline

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Introduction

String Theory: UV complete framework for addressing questions pertinent to quantum gravity $\rightarrow$ many formal developments.

A traditional goal: Unification of all interactions, including gravity.

(String pheno) String vacua as phenomenological extensions of SM, e.g. $\mathcal{N} = 1$, SUSY breaking, ...

+ Necessary to incorporate quantum corrections

Introduction

Best studied: F^2 in heterotic effective action at 1-loop (in g_s)

- running of gauge couplings
- String Unification: $M_U =?$, $g_U =?$ (compare M_{GUT}, g_{GUT})

Compute 2-point function of gauge bosons on Σ_2 and split into

- massless contributions $\rightarrow$ logarithmic (field theory)
- heavy string states $\rightarrow$ threshold correction Δ_a

Introduction

Running coupling $g_a(\mu)$ for gauge group factor G_a in $\overline{DR}$

$$\frac{16\pi^2}{g_a^2(\mu)} = k_a \frac{16\pi^2}{g_s^2} + b_a \log\left(\frac{\xi}{4\pi^2}\frac{M_s^2}{\mu^2}\right) + \Delta_a$$

and $\xi \equiv 8\pi e^{1-\gamma}/3\sqrt{3}$

String scale data: M_s, g_s not independent

M_P does not renormalise at any loop!

$$M_s = g_s \frac{M_P}{\sqrt{32\pi}}$$

Moduli dependence in Δ_a via KK/winding masses

Gauge thresholds and Universality in $\mathcal{N} = 2$

Calculating Δ_a even at one loop is non-trivial.

Properties best visible in $\mathcal{N} = 2$ vacua: e.g. K3$\times T^2$

- One-loop exact in g_s
- Realised as $T^4/\mathbb{Z}_N \times T^2$ orbifold, $N = 2, 3, 4, 6$
- For simplicity $W = 0$: factorised T^2 and Kac-Moody lattices
- Only T^2 moduli appear: T, U

With these assumptions, $\mathcal{N} = 2$ universality

Gauge thresholds and Universality in $\mathcal{N} = 2$

Δ_a decomposes into

$$\Delta_a^{\mathcal{N}=2} = -k_a \hat{Y} + b_a \hat{\Delta}$$

$\hat{Y}$ known as the "Universal part"

- due to presence of gravitational sector
- independent of charges under G_a

$\hat{\Delta}$ known as the "Running part"

- multiplied by $\mathcal{N} = 2$ beta function
- charged heavy states running in the loop

Gauge thresholds and Universality in $\mathcal{N}=2$

Modularity, holomorphy and 6d gravitational anomalies uniquely fix

$$\hat{Y} = \frac{1}{12}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\,\Gamma_{2,2}(T,U)\left(\frac{\hat{\bar{E}}_2\bar{E}_4\bar{E}_6-\bar{E}_4^3}{\bar{\eta}^{24}}+1008\right)$$

$$\hat{\Delta} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\left(\Gamma_{2,2}(T,U)-\tau_2\right)$$

With some work, these modular integrals can be computed

$$\hat{Y} = \frac{1}{2}\log|j(T)-j(U)|^4+\frac{4\pi}{3T_2}E(2;U)+O(e^{-2\pi T_2})$$

$$\hat{\Delta} = -\log\left[\xi\, T_2 U_2 |\eta(T)\eta(U)|^4\right]$$

Gauge thresholds and Universality in $\mathcal{N} = 2$

Decomposition $\Delta_a^{\mathcal{N}=2} = -k_a \hat{Y} + b_a \hat{\Delta}$ has physical consequences

Natural unification of all gauge couplings

$$M_U = \frac{\xi M_P}{2\pi} g_s \exp(\hat{\Delta}/2) \quad , \quad g_s = g_U \left(1 + \frac{g_U^2}{16\pi^2} \hat{Y}\right)^{-1/2}$$

- All couplings automatically unify at $\mu = M_U$
- Common coupling $g_a(M_U) = g_U/\sqrt{k_a}$
- Moduli dependent values for M_U and g_U (via $\hat{Y}$, $\hat{\Delta}$)

Gauge thresholds and Universality in $\mathcal{N} = 2$

Question
Assuming Desert, how do we choose T, U such that String Unification M_U, g_U match corresponding GUT values?

$$M_U = M_{GUT} \sim 2 \times 10^{16}\,\mathrm{GeV} \quad , \quad g_U^2 = g_{GUT}^2 = 4\pi/25$$

Explicit expressions for $\hat{Y}, \hat{\Delta}$ reveals no value in (T, U) compatible with this requirement

- What is the origin of this discrepancy?

Gauge thresholds and Universality in $\mathcal{N} = 2$

Inspect ratio of String Unification to GUT scale

$$\frac{M_U}{M_{GUT}} = \frac{\xi}{4(2\pi)^{3/2}} \frac{M_P}{M_{GUT}} \frac{g_{GUT}}{\sqrt{1 + \frac{g_{GUT}^2}{16\pi^2}\,\hat{Y}}} \exp(\hat{\Delta}/2)$$

$M_P/M_{GUT} \sim 6.1 \times 10^2$, so we need suitable values for $\hat{Y}, \hat{\Delta}$ to lower string unification scale down to GUT scale

This turns out to be impossible due to unbroken $O(2,2)$

$$O(2,2;\mathbb{Z}) = SL(2;\mathbb{Z})_T \times SL(2;\mathbb{Z})_U \ltimes \mathbb{Z}_2$$

- T-duality symmetry in both $\hat{Y}$ and $\hat{\Delta}$
- Thresholds have extrema at fixed points
- Minimum at $T = U = e^{2\pi i/3}$ gives $\hat{Y} \sim 27.6$, $\hat{\Delta} \sim 0.068$

Gauge thresholds and Universality in $\mathcal{N} = 2$

In $\mathcal{N} = 2$ universality with $O(2, 2; \mathbb{Z})$

String Unification overshoots GUT scale by factor ~ 20

This is a well known story but the role of unbroken $O(2, 2; \mathbb{Z})$ was not fully appreciated in the past

GUT scale Mismatch and the Decompactification problem

Let's forget SU-GUT scale mismatch for a moment

A related problem arises at large volume

$$T_2 = \mathrm{Im}\, T = \mathrm{vol}(T^2) \gg M_s^{-2}$$

KK scale $M_{KK} \sim 1/\sqrt{T_2}$: much lower than M_s or even M_{GUT}

M_U is pushed above M_P exponentially fast

Effectively 6d physics: gauge coupling has dimensions of length

$$\hat{\Delta} \sim \frac{\pi}{3} T_2 \quad , \quad \hat{Y} \sim 4\pi T_2$$

Thresholds grow linearly with T^2 volume

Depending on $\mathrm{sgn}(b_a)$, either decoupling or non-perturbative

GUT scale Mismatch and the Decompactification problem

Non-perturbative regime: theory loses predictability

“Decompactification problem”

Technically, linear growth arises from Dedekind and Klein functions

$$\eta(T) = q^{1/24} \prod_{n>0} (1 - q^n) \quad , \quad j(T) = \frac{1}{q} + 196884q + \dots$$

where $q = \exp(2\pi i T)$

- $T_2 |\eta(T)|^4$ and $j(T)$ are automorphic functions of $SL(2;\mathbb{Z})_T$
- They enter $\hat{Y}$ and $\hat{\Delta}$ and reflect T-duality symmetry

GUT scale Mismatch and the Decompactification problem

One (obvious) solution:
Keep moduli close to string scale: $M_s^2 T_2 \sim 1$

- SU-GUT scale mismatch persists
- In $\mathcal{N} = 1$, large volume is necessary
  (cf. Ibanez-Luest, Nilles-Stieberger,. . .)
- SUSY breaking: potential may lead to large volume

So this won't do. . .

GUT scale Mismatch and the Decompactification problem

Look at these two different problems:

SU/GUT mismatch vs. Decompactification

At first sight, they look uncorrelated

- one is related to extrema of $\hat{Y}, \hat{\Delta}$, i.e. small volume
- the other arises at large volume

GUT scale Mismatch and the Decompactification problem

Closer look: both problems share a **common** origin

It all goes back to unbroken $SL(2;\mathbb{Z})_T \subset O(2,2;\mathbb{Z})$

Technically, symmetry implies $\hat{\Delta}, \hat{Y} \sim \int_{\mathcal{F}} \Gamma_{2,2}(T,U) \times$ stuff

The Narain lattice reflects O(2,2) and asymptotically

$$\Gamma_{2,2}(T,U) = \sum_{m,n\in\mathbb{Z}^2} q^{P_L^2/4}\bar{q}^{P_R^2/4} \to T_2 + \dots$$

GUT scale Mismatch and the Decompactification problem

Both problems can be solved simultaneously
(Angelantonj, I.F., 2019)

provided T-duality group is broken such that

$$SL(2;\mathbb{Z})_T \to \Gamma^1(N)_T$$

via the congruence subgroup

$$\Gamma^1(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2;\mathbb{Z}) \;\middle|\; a, d = 1 \,(\mathrm{mod}\, N),\ b = 0 \,(\mathrm{mod}\, N) \right\}$$

K3 and T^2 no longer factorise, rather elliptic fibration

Exactly solvable CFT realisation: freely acting $\mathbb{Z}_N$ orbifolds

Twists in K3 and shifts along non-trivial cycles of T^2

GUT scale Mismatch and the Decompactification problem

How does it look like?

Morally:

$$\int_{\mathcal{F}} \Gamma_{2,2} \times \left(\frac{1}{N} \sum_{h,g \in \mathbb{Z}_N} \mathcal{A}[^h_g] \right) \to \int_{\mathcal{F}} \left(\frac{1}{N} \sum_{h,g \in \mathbb{Z}_N} \Gamma_{2,2}[^h_g] \, \mathcal{A}[^h_g] \right)$$

- h : orbifold sectors
- g : projection
- momentum shift $\Gamma_{2,2}[^h_g] \leftrightarrow$ geometric X (not $\tilde{X}$)
- T-duality $SL(2;\mathbb{Z})_T \to \Gamma^1(N)_T$

GUT scale Mismatch and the Decompactification problem

Partial unfolding (cf. Angelantonj, I.F., Pioline)

$$\Delta_a = \int_{\mathcal{F}} \frac{1}{N} \Gamma_{2,2} \times \mathcal{A}[^0_0] + \int_{\mathcal{F}_N} \frac{1}{N} \Gamma_{2,2}[^0_1] \times \mathcal{A}[^0_1]$$

here $\mathcal{F}_N = \mathbb{H}^+/\Gamma_0(N)$ fundamental domain of Hecke congruence subgroup $\Gamma_0(N)_\tau \subset SL(2;\mathbb{Z})_\tau$

$$\Gamma_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2;\mathbb{Z}) \;\middle|\; c = 0 \, (\mathrm{mod}\, N) \right\}$$

Also: helicity supertrace in $\mathcal{A}[^0_0]$ vanishes $(\mathcal{N} = 4)$

$$\hat{\Delta} = \int_{\mathcal{F}_N} \frac{d^2\tau}{\tau_2^2} \, \Gamma_{2,2}[^0_1] \quad , \quad \hat{Y} = \int_{\mathcal{F}_N} \frac{d^2\tau}{\tau_2^2} \, \Gamma_{2,2}[^0_1](T,U) \, \Phi_N(\tau)$$

Momentum shift $X \to X + (\lambda_1 + \lambda_2 U)/N$ with $\lambda_i \in \mathbb{Z}_N$ selects residual $\Gamma^1(N)_T$ factor

GUT scale Mismatch and the Decompactification problem

Large volume behavior at most logarithmic

$$\hat{\Delta} \sim -\log\left(\xi f_N(U) T_2\right) + O(e^{-2\pi T_2}) \quad , \quad \hat{Y} \sim O(T_2^{-1})$$

f_N: automorphic function of U w.r.t. residual T-duality group $O(2,2;\mathbb{Z}) \to \Gamma^1(N)_T \times G(N)_U$

$M_{KK} \sim M_{SUSY} \sim 1/\sqrt{T}$ **: effectively** $\mathcal{N} = 4$ **above KK scale and eliminates linear growth in gauge thresholds**

This solves the Decompactification problem

(Kiritsis, Kounnas, Petropoulos, Rizos 1996)

GUT scale Mismatch and the Decompactification problem

However, the breaking to $\Gamma^1(N)_T$ also makes $\hat{\Delta}$ unbounded from below.

Independently of new extrema of $\hat{\Delta}$, one can always choose T_2 such that $M_U = M_{GUT}$

$$T_2 \simeq \frac{g^2_{GUT}}{128\pi^3\, f_N(U)} \left(\frac{M_P}{M_{GUT}}\right)^2$$

Assuming $f_N(U) = O(1)$ as in typical orbifolds, we find $T_2 \sim 50$

This also resolves the SU/GUT scale mismatch!

$\mathcal{N} = 1$ and Chirality

So far, we assumed unbroken $\mathcal{N} = 2$ SUSY $\to$ universality

We now want to apply this to chiral $\mathcal{N} = 1$ vacua

$$\Delta_a = d_a + \sum_i \left(-k_a \hat{Y}^{(i)} + \beta_{a,i} \hat{\Delta}^{(i)} \right)$$

- d_a moduli independent $\mathcal{N} = 1$ constants
- i labels $\mathcal{N} = 2$ subsectors
- $\beta_{a,i}$ beta function coeffs for i subsector (relations to 6d anomaly) Derendinger, Ferrara, Kounnas, Zwirner 1992

Unification is no longer automatic

$\mathcal{N} = 1$ and Chirality

Additional constraints on charged spectrum required

Define

$$k_a \Phi_a \equiv b_a \log\left(\frac{\xi}{4\pi^2}\frac{M_s^2}{M_U^2}\right) + d_a + \sum_i \beta_{a,i}\hat{\Delta}^{(i)}$$

and impose

$$\Phi_a = \Phi_b = \dots$$

for all unifying gauge group factors $G_a, G_b, \dots$

- Case $d_a = 0$, $\Phi_a = 0$ reduces to Ibanez-Luest 1992
- General case applies to both 'mirage' and 'true' unification
- For 'true', conditions trivialise $\rightarrow$ choose T_i to match GUT
- For 'mirage' with 3 G_as, can always satisfy Φ-conditions and match GUT by tuning T_is

$\mathcal{N} = 1$ and Chirality

Now consider: heterotic $\mathcal{N} = 1$ as T^6/Γ limits of CY, with Γ preserving 4 Killing spinors

Thresholds are **moduli independent** unless Γ contains elements preserving 8 supercharges: "$\mathcal{N} = 2$ subsectors"

Again, they decompose

$$\Delta_a = d_a + \sum_i \left(-k_a \hat{Y}^{(i)} + \beta_{a,i} \hat{\Delta}^{(i)} \right)$$

In general, this runs into Decompactification problem

Need to break $SL(2;\mathbb{Z})_T \to \Gamma^1(N)_T$ for all $\mathcal{N} = 2$ subsectors

Challenge: do this without spoiling chirality (non-trivial)

$\mathcal{N} = 1$ and Chirality

This is impossible in $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds - or even $(\mathbb{Z}_2)^n$
Kiritsis, Kounnas, Petropoulos, Rizos 1996 and Faraggi, Kounnas, Partouche 2015

To get $\Gamma^1(N)_T$ in all $\mathcal{N} = 2$ subsectors, we need free action

- twisted sectors are massive
- untwisted sectors are non-chiral (real action of $\mathbb{Z}_2$)

so chirality is lost

Exception to this no-go
Balance $\hat{Y}$ against $\hat{\Delta}$ (I.F. and Rizos, 2017)
See talk by J. Rizos

An explicit example

Incompatibility between $\Gamma^1(N)_T$ and chirality

Can be lifted by choosing T^6/Γ with complex action Γ on untwisted fermions

An example $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3'$ at fixed $U_i = e^{2\pi i/6}$

- $\mathbb{Z}_3$: $v = (\frac{1}{3}, \frac{1}{3}, \frac{2}{3})$ - "Z-orbifold" Dixon, Harvey, Vafa, Witten 1985
- standard embedding, W=0
- $\mathbb{Z}_3'$: $w = (\frac{1}{3} + \delta, -\frac{1}{3} + \delta, \delta)$
- opposite rotations in first two T^2s
- order 3 shifts $z_i \to z_i + (1 + U_i)/3$ on all three 2-tori

Chirality is generated already by $T^6/\mathbb{Z}_3$, without $\mathcal{N} = 2$ sectors
When $\mathbb{Z}_3'$ acts, its untwisted sector remains chiral

An explicit example

In the full $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3'$ there are three $\mathcal{N} = 2$ subsectors

- residual T-duality $\prod_{i=1}^{3} \Gamma^1(3)_{T_i}$
- theory has unbroken $\mathcal{N} = 1$
- non-abelian $E_6 \times E_8$
- charged chiral matter

An explicit example

Gauge thresholds decompose via partial unfolding

$$\Delta_{E_8} = d_8 + \sum_{i=1,2,3} \left(\hat{Y}^{(i)} - 20\hat{\Delta}^{(i)} \right)$$

$$\Delta_{E_6} = d_6 + \sum_{i=1,2,3} \left(\hat{Y}^{(i)} - 8\hat{\Delta}^{(i)} \right)$$

d_8, d_6 constant contributions from Z-orbifold

$$Y^{(i)} = \frac{1}{144} \int_{\mathcal{F}_3} \frac{d^2\tau}{\tau_2^2} \Gamma_{2,2}[{}^0_1](T_i, U_i) \left[\frac{\hat{E}_2 E_4 (3E_4 X_3 - 2E_6)}{2\eta^{24}} \right. \\ \left. + \frac{E_4(2E_4^2 - 3X_3 E_6)}{2\eta^{24}} + 1152 \right]$$

$$\hat{\Delta}^{(i)} = \int_{\mathcal{F}_3} \frac{d^2\tau}{\tau_2^2} \Gamma_{2,2}[{}^0_1](T_i, U_i)$$

An explicit example

Can be evaluated with some work

$$\hat{\Delta}^{(i)} = -\log\left[\frac{\xi}{27}\, T_{i,2} U_{i,2} \left| \frac{\eta^3(T_i/3)}{\eta(T_i)} \frac{\eta^3(\frac{1+U_i}{3})}{\eta(U_i)} \right|^2 \right]$$
$$\sim -\log\left(\frac{\xi}{27} T_{i,2} f_3(U_i)\right) + O(e^{-2\pi T_{i,2}/3})$$

As expected, only logarithmic growth in in $\hat{\Delta}$ and

$$\hat{Y}^{(i)}_{singular} \sim \log\left[\frac{|j(T_i) - 744|^{1/3}}{|j_\infty(T_i/3) + 3|} \left| \frac{j_\infty(T_i/3) + 231}{j_\infty(T_i/3) - 12} \right|^9 \right]$$

linear growth cancels out non-trivially, and no logarithmic growth ($\hat{Y}$ is IR finite)

An explicit example

Behavior at large volume

$$\hat{\Delta}^{(i)} \sim -\log\left(\frac{\xi}{27} T_{2,i} f_3(U_i)\right) \quad , \quad \hat{Y}^{(i)} \sim \frac{c_3(U_i)}{T_{i,2}}$$

$f_3(U)$, $c_3(U)$ of order one

This large volume behavior is a generic property of the breaking to $\prod_i \Gamma^1(N)_{T_i}$

Again, appropriate choice of T_i can match GUT scale

Gravitational R^2 thresholds: similar analysis $\to$ logarithmic growth

Conclusions

Unification of gauge couplings at M_{GUT} is an appealing possibility and already much studied in string literature

- However, past treatments required either $W \neq 0$ or faced decompactification problem
- The latter drives theory non-perturbative very close to GUT scale

Conclusions

Key idea: break T-duality group to

$$\prod_i \Gamma^1(N)_{T_i}$$

It is possible to precisely match SU and GUT scales

- $\mathcal{N} = 1$ and $\mathcal{N} = 2$ vacua
- even with $W = 0$
- without too many restrictions on charged spectrum
- can preserve chirality
- Decompactification problem is solved simultaneously