Quantum spacetime, Schwarzschild solution and gravity in the IKKT matrix model

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Humboldt Kolleg Frontiers in Physics, september 2019 constructive formulation of Planck scale physics?

guidelines:

- finite dof per volume $\rightarrow$ non-geometric model
- gauge theory (Minkowski signature)
- space-time & gravity should emerge from fundamental d.o.f.
- simple

Matrix Models (Yang-Mills)

$S = Tr([X^\mu, X^\nu][X_\mu, X_\nu] \; + ...)$ provide suitable framework !

- simple
- describe dynamical (NC) spaces, gauge theory $X^\mu \to U^{-1} X^\mu U$
- quantization: $\int dX \, e^{-S[X]}$

  IKKT model: protected from UV/IR mixing (maximal SUSY)

  cf. critical string
- requires new mechanism for 3+1D gravity

summary: (significant progress!)

- (3+1)-dim. covariant quantum space-time solution
  (FRW cosmology, Big Bounce)
- tower of higher-spin modes, truncated at n
  Yang-Mills-type action, no ghosts
- spin 2 $\rightarrow$ dynamical metric, graviton
- linearized Schwarzschild solution & scalar mode

HS, arXiv:1606.00769
HS, arXiv:1710.11495
M. Sperling, HS arXiv:1806.05907
M. Sperling, HS arXiv:1901.03522
HS arXiv:1905.07255, arXiv:1909.xxxxx

The IKKT model

IKKT or IIB model — Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S[Y,\Psi] = -Tr\Big([Y^a,Y^b][Y^{a'},Y^{b'}]\eta_{aa'}\eta_{bb'} + m^2 Y^a Y_a + \bar\Psi\gamma_a[Y^a,\Psi]\Big)$$

$$Y^a = Y^{a\dagger} \in Mat(N,\mathbb{C}), \qquad a = 0,...,9, \qquad N \to \infty$$

gauge invariance $Y^a \to UY^aU^{-1}$, $SO(9,1)$, ~~SUSY~~

- quantized Schild action for IIB superstring
- reduction of $10D$ SYM to point
- equations of motion:

  $$\Box Y^a + m^2 Y^a = 0, \qquad \Box \equiv \eta_{ab}[Y^a,[Y^b,.]]$$

- quantization: $Z = \int dYd\Psi\, e^{iS[Y]}$

strategy:

- solutions X^μ $\to$ space(time)
  cf. branes, generically non-commutative
- fluctuations $X^\mu + \mathcal{A}^\mu$ $\to$ gauge theory
  dynamical geometry $\to$ gravity ?! (not holographic)
- $\int dX$ = path integral, including geometry

(brane interactions consistent with IIB, cf. IKKT, BFSS)

numerical studies possible & underway

evidence for emergent 3+1D expanding space-time

Nishimura, Tsuchiya 1904.05919, Kim, Nishimura, Tsuchiya arXiv:1108.1540 ff

cf. talks by J. Nishimura, S. Papadoudis in Workshop on Quantum Geometry, Field Theory and Gravity examples:

1) Moyal-Weyl quantum plane $\mathbb{R}^{3,1}_\theta$:

$$[X^\mu, X^\nu] = i\theta^{\mu\nu}\,\mathbf{1}$$

admits translations $X^\mu \to X^\mu + c^\mu \mathbf{1}$, rotation invariance broken

fluctuations $X^\mu + \mathcal{A}^\mu$ in IKKT $\to$ NC $\mathcal{N} = 4$ SYM

2) fuzzy 2-sphere S^2_N

$$X_1^2 + X_2^2 + X_3^2 = R_N^2, \qquad [X_i, X_j] = i\epsilon_{ijk} X_k$$

fully covariant under $SO(3)$ (Hoppe, Madore) more generally: fuzzy spaces = quantized symplectic manifolds

$$X^a \sim x^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^{9,1}$$

$[X^a, X^b] \sim \{x^a, x^b\} = i\theta^{ab}(x)$ ... (quantized) Poisson bracket

algebra of functions on NC (=fuzzy) space:

$$\phi(X) \in \mathrm{End}(\mathcal{H}) \cong \mathcal{C}^\infty(\mathcal{M}), \qquad \dim \mathcal{H} \sim \mathrm{Vol}(\mathcal{M})$$

- danger: $\theta^{\mu\nu}$ in 3+1D breaks Lorentz invariance
- avoided on covariant quantum spaces

4D covariant quantum spaces

- prototype: fuzzy four-sphere S_N^4

  Grosse-Klimcik-Presnajder; Castelino-Lee-Taylor; Medina-o'Connor; Ramgoolam; Kimura; Abe; Karabail-Nair; Zhang-Hu 2001 (QHE); HS

- hyperboloid H_n^4 Hasebe 1207.1968 , M. Sperling, HS 1806.05907

- projection $H_n^4 \rightarrow \mathcal{M}_n^{3,1}$ cosmological space-time

  HS, 1710.11495, 1709.10480

  M. Sperling, HS 1901.03522, HS 1905.07255

... lead to higher-spin gauge theory in IKKT model !

cf. Hanada, Kawai, Kimura hep-th/0508211, ff

Euclidean fuzzy hyperboloid H_n^4 (=$EAdS_n^4$)

Hasebe 1207.1968; M. Sperling, HS 1806.05907

$\mathcal{M}^{ab}$ ... hermitian generators of $\mathfrak{so}(4,2)$,

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = i(\eta_{ac}\mathcal{M}_{bd} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad} + \eta_{bd}\mathcal{M}_{ac}) \ .$$

$$\eta^{ab} = \mathrm{diag}(-1,1,1,1,1,-1)$$

choose “short” discrete unitary irreps $\mathcal{H}_n$ (“minireps”, doubletons)

special properties:

- irreps under $\mathfrak{so}(4,1)$, multiplicities one, minimal oscillator rep.
- positive discrete spectrum

$$\mathrm{spec}(\mathcal{M}^{05}) = \{E_0, E_0+1, ...\}, \qquad E_0 = 1 + \frac{n}{2}$$

lowest eigenspace is $n+1$-dim.

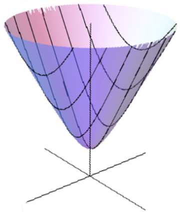
fuzzy hyperboloid H_n^4

5 hermitian generators

$$X^a := r\mathcal{M}^{a5}, \qquad a = 0, ..., 4$$

satisfy

$$\eta_{ab}X^aX^b = X^iX^i - X^0X^0 = -R^2\mathbf{1}, \qquad R^2 = r^2(n^2 - 4)$$

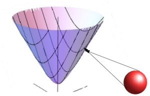


hyperboloid in $\mathbb{R}^{1,4}$, manifest $SO(4,1)$

noncommutative $[X^a, X^b] = ir^2\mathcal{M}^{ab} =: i\Theta^{ab}$

claim:

$H_n^4 \;=\;$ quantized $\mathbb{C}P^{1,2} = S^2$ -bundle $\{\theta^{\mu\nu}$ selfdual$\}$ over H^4



can be seen from oscillator construction:
4 bosonic oscillators $[\psi_\alpha, \bar\psi^\beta] = \delta_\alpha^\beta$

$\mathcal{H}_n$ = suitable irrep in Fock space

$$\mathcal{M}^{ab} = \bar\psi\Sigma^{ab}\psi, \qquad \gamma_0 = diag(1,1,-1,-1)$$
$$X^a = r\bar\psi\gamma^a\psi$$

$End(\mathcal{H}_n) \cong$ functions on $H_n^4 \;\cong\;$ harmonics on $S^2 \times$ functions on H^4

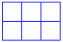
local stabilizer acts on $S^2 \;\Rightarrow\;$ harmonics = higher spin modes

fuzzy "functions" on H_n^4:

$\mathcal{C}^0$ = scalar functions on H^4: $\phi(X)$

$\mathcal{C}^1$ = selfdual 2-forms on H^4: $\phi_{ab}(X)\theta^{ab} =$ 

$\vdots$

$End(\mathcal{H}_n) \cong$ fields on H^4 taking values in $\mathfrak{hs} = \oplus$  $\ni \theta^{a_1 b_1} \ldots \theta^{a_s b_s}$

higher spin modes = would-be KK modes on S^2

can show:

$$\mathrm{End}(\mathcal{H}_n) = \mathcal{C}^0 \oplus \mathcal{C}^1 \oplus \ldots \oplus \mathcal{C}^n, \qquad \phi^{(s)} \in \mathcal{C}^s$$

(selected by spin Casimir S^2)

i.e. higher spin theory, truncated at n

M. Sperling, HS 1806.05907

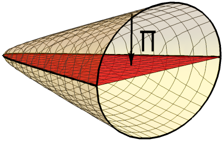
FRW space-time $\mathcal{M}^{3,1}$ from H_n^4

HS 1710.11495

$\mathcal{M}_n^{3,1}$ = H_n^4 projected to $\mathbb{R}^{1,3}$ via

$$Y^\mu \sim y^\mu : \ \mathbb{C}P^{1,2} \to H^4 \subset \mathbb{R}^{1,4} \xrightarrow{\Pi} \mathbb{R}^{1,3} \ .$$

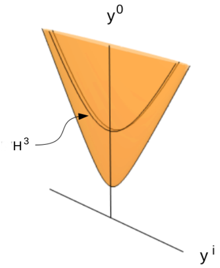
metric has Minkowski signature !



algebraically: $\mathcal{M}_n^{3,1}$ generated by

$$\boxed{Y^\mu := X^\mu, \quad \text{for} \ \ \mu = 0,1,2,3 \qquad \text{(drop } X^4 \text{)}}$$

geometric properties:



- manifest $SO(3,1)$ $\Rightarrow$ foliation into space-like 3-hyperboloids H^3_τ
- double-covered FRW space-time ($k=-1$)

$$ds^2 = d\tau^2 - a(\tau)^2 d\Sigma^2$$

covariant quantum space-time $\mathcal{M}^{3,1}$:

generators: $Y^\mu = r\mathcal{M}^{\mu 5} \sim y^\mu, \quad T^\mu = \frac{1}{R}\mathcal{M}^{\mu 4} \sim t^\mu$

commutation relations:

$$\begin{aligned}
[T^\mu, Y^\nu] &= i\frac{X_4}{R}\eta^{\mu\nu} \quad \sim i\{t^\mu, y^\nu\} \\
[Y^\mu, Y^\nu] &= i\Theta^{\mu\nu} \quad \sim i\{t^\mu, t^\nu\} \\
[T^\mu, T^\nu] &= -\frac{i}{r^2R^2}\Theta^{\mu\nu} \sim i\{t^\mu, t^\nu\}
\end{aligned}$$

semi-class. constraints

$$\begin{aligned}
y_\mu y^\mu &= -R^2\cosh^2(\eta), \qquad x^4 = R\sinh(\eta) \\
t_\mu t^\mu &= r^{-2}\cosh^2(\eta)\,, \\
t_\mu y^\mu &= 0, \\
\theta^{\mu\nu} &= c(y^\mu t^\nu - y^\nu t^\mu) + b\epsilon^{\mu\nu\alpha\beta}y_\alpha t_\beta
\end{aligned}$$

t^μ ... space-like S^2 fiber

covariant under $SO(3,1)$
finite density of microstates (NC version)

spin s "functions" on $\mathcal{M}_n^{3,1}$:

$$\phi^{(s)} = \phi_{\mu_1\ldots\mu_s}(y) t^{\mu_1} \ldots t^{\mu_s}$$

$t_\mu y^\mu = 0 \;\Rightarrow$ "space-like gauge" $\boxed{y^{\mu_i}\phi_{\mu_1\ldots\mu_s} = 0}$

($\rightarrow$ no ghosts!)

space-like $SO(3,1)$ manifest

$SO(4,2)$ - invariant integral = trace

$$\langle\phi,\phi'\rangle := \int\limits_{\mathbb{C}P^{1,2}} \omega^{\wedge 3}\, \phi\phi' = \int\limits_{H^4} dV[\phi\phi']_0$$

$SO(3,1)$-invariant substructure: derivation

$$
\begin{aligned}
D: \quad \mathcal{C}^s &\to \mathcal{C}^{s+1} \oplus \mathcal{C}^{s-1} \\
\phi &\mapsto D^+\phi + D^-\phi = \{\theta^{45}, \phi\}
\end{aligned}
$$

explicitly

$$
D(\phi) = \nabla^{(3)}_\mu \phi_{\mu_1 \ldots \mu_s}(y)\, t^{\mu_1} \ldots t^{\mu_s} t^{\mu}
$$

def.

$$
\begin{aligned}
\mathcal{C}^{(s,0)} &= \{\phi \in \mathcal{C}^s;\ D^-\phi = 0\} && \text{... primal fields} \\
\mathcal{C}^{(s+k,k)} &= (D^+)^k \mathcal{C}^{(s,0)} && \text{... descendants}
\end{aligned}
$$

(cf. CFT, no highest weight modules!)

$SO(3,1)$ spin s on H^3 $\leftrightarrow$ $SO(4,1)$ spin $s+k$ on H^4

$\mathcal{M}^{3,1}$ solution in IKKT model:

background solution:

$$T^\mu := \frac{1}{R}\mathcal{M}^{\mu 4}$$

satisfies e.o.m.

$$\Box T^\mu = \frac{3}{R^2} T^\mu, \qquad \Box = [T^\mu, [T_\mu, .]]$$

- $[\Box, \mathcal{S}^2] = 0, \quad \mathcal{S}^2$ ... spin Casimir, selects spin sectors $\mathcal{C}^s$
- $\Box \sim \alpha^{-1}\Box_G$ encodes eff. FRW metric $ds_G^2 = -dt^2 + a(t)^2 d\Sigma^2$,

fluctuations & higher spin gauge theory

$$S[Y] = Tr([Y^\mu, Y^\nu][Y_\mu, Y_\nu] + \frac{6}{R^2} Y^\mu Y_\mu) = S[U^{-1} Y U]$$

background: $\overline{Y}^\mu = T^\mu$ ... space-time $\mathcal{M}_n^{3,1}$

add fluctuations $Y^\mu = \overline{Y}^\mu + \mathcal{A}^\mu,$

$\mathcal{A}_\mu$ ... $\mathfrak{hs}$-valued 1-form on $\mathcal{M}^{3,1}$, incl. spin 2

expand action to second oder in $\mathcal{A}_\mu$

$$S[Y] = S[\overline{Y}] + \frac{2}{g^2} \mathrm{Tr} \mathcal{A}_\mu \Big(\underbrace{(\Box - \frac{3}{R^2})\delta^\mu_\nu + 2[[\overline{Y}^\mu, \overline{Y}^\nu], .]}_{\mathcal{D}^2} - \underbrace{[\overline{Y}^\mu, [\overline{Y}^\nu, .]]}_{g.f.} \Big) \mathcal{A}_\nu$$

generic eigenmodes: M. Sperling, HS: 1901.03522, HS 1909.xxxxx

can show (using $\mathfrak{so}(4,2)$):

- 4 generic eigenmodes for each $\phi = \phi^{(s)}$ with $\Box\phi = \lambda\phi$:

$$\mathcal{D}^2 \mathcal{A}_\mu^{(i)}[\phi] = \lambda\, \mathcal{A}_\mu^{(i)}[\phi], \qquad i \in \{+,-,n,g\}$$

$$\mathcal{A}_\mu^{(i)}[\phi] = \begin{cases} \mathcal{A}_\mu^{(+)}[\phi] & := \{y_\mu, D^-\phi\}_+ \\ \mathcal{A}_\mu^{(-)}[\phi] & := \{y_\mu, D^+\phi\}_- \\ \mathcal{A}_\mu^{(n)}[\phi] & := D^+\{y_\mu, \phi\}_- \\ \mathcal{A}_\mu^{(g)}[\phi] & := \{t_\mu, \phi\} \qquad \text{(pure gauge)} \end{cases}$$

- 4 towers of regular on-shell modes ($s > 0$)

$$\mathcal{D}^2 \mathcal{A}^{(i)}[\phi] = 0 \qquad \text{for} \qquad \Box\phi = 0, \qquad i \in \{+,-,n,g\}$$

universal propagation $\Box \sim \Box_G$

- 2 spin 0 modes

inner product matrix

$$\mathcal{G}^{(i,j)} = \left\langle \mathcal{A}^{(i)}[\phi'], \mathcal{A}^{(j)}[\phi] \right\rangle, \qquad i,j \in \{+,-,n,g\}$$

signature $(+++-)$

gauge-fixing $\{t^\mu, \mathcal{A}_\mu\} = 0$

physical Hilbert space

$$\mathcal{H}_{\text{phys}} = \{\mathcal{D}^2\mathcal{A} = 0, \;\; \mathcal{A} \;\text{ gauge fixed}\}/_{\{\text{pure gauge}\}}$$

results:

- generically 2 physical modes for each $\Box\phi^{(s)} = 0, \;\; s \geq 1$
  would-be massive, $m^2 = 0$
- no ghosts, no tachyons
- same propagation for all modes

  (even though Lorentz invar only partially manifest)

vielbein, metric & dynamical geometry

effective metric $G_{\mu\nu}$ extracted from kinetic term

$$-Tr[T^\alpha,\phi][T_\alpha,\phi] \sim \int e^\alpha\phi e_\alpha\phi = \int \sqrt{|G|}G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

vielbein

$$e^\alpha \quad := \{T^\alpha,.\} = \sinh(\eta)\eta^{\alpha\mu}\partial_\mu$$

metric

$$G^{\mu\nu} \quad \sim \eta_{\alpha\beta}e^{\alpha\mu}e^{\beta\nu}$$

encoded in Laplacian $\Box_Y = [Y_\mu,[Y^\mu,.]] \sim -\frac{1}{\sqrt{|G|}}\partial_\mu(\sqrt{|G|}G^{\mu\nu}\partial_\nu.)$:

$\rightarrow$ FLRW metric: $\quad ds^2 = d\tau^2 - a(\tau)^2 d\Sigma^2$

- late times: $a(t) \approx \frac{3}{2}t, \qquad t \to \infty$ . ... coasting universe (not bad !)
- big bounce: $a(t) \propto (t-t_0)^{\frac{1}{5}}$

perturbed vielbein: $Y^\alpha = T^\alpha + \mathcal{A}^\alpha$

$$e^\alpha = \{T^\alpha + \mathcal{A}^\alpha, .\} = e^{\alpha\mu}[\mathcal{A}]\partial_\mu$$

metric fluctuation:

$$h_{\mu\nu}[\mathcal{A}] \sim \{\mathcal{A}_\mu, y_\nu\}_0 + (\mu \leftrightarrow \nu)$$

coupling to matter:

$$S[\text{matter}] \sim \int\limits_{\mathcal{M}} d^4y\, h^{\mu\nu} T_{\mu\nu}$$

gauge transformations

-functions: $\delta_\Lambda \phi = \{\Lambda, \phi\}, \qquad \Lambda \in End(\mathcal{H})$

spin 1 trafos: $\Lambda = v^\mu(y) t_\mu \ \rightarrow$ 3 (rather than 4) diffeos !

(invar. symplectic volume on $\mathbb{C}P^{1,2}$)

-fluctuations: $\delta_\Lambda \mathcal{A}^\mu = [\Lambda, \mathcal{A}^\mu] + [\Lambda, \overline{Y}^\mu]$

-metric:

$$\boxed{\delta_\Lambda G_{\mu\nu} = \nabla_\mu \mathcal{A}_\nu + \nabla_\mu \mathcal{A}_\nu,} \qquad \mathcal{A}_\mu = \{y_\mu, \Lambda\}_-$$

$$\nabla_\alpha \mathcal{A}^\alpha = \frac{1}{x_4^2}(y \cdot \mathcal{A}) \qquad \approx \text{ volume preserving}$$

linearized metric fluctuation modes

$$h_{\mu\nu}[\mathcal{A}] \propto \{\mathcal{A}_\mu, y_\nu\} + (\mu \leftrightarrow \nu)$$

off-shell: most general metric fluct.

- $\mathcal{A}^{(-)}[\phi^{(2)}]$ ... 5, $\approx$ massive spin 2
- $\mathcal{A}^{(+)}[\phi^{(0)}], \mathcal{A}^{(n)}[D\phi^{(0)}]$ ... 2 scalars
- $\nabla_\mu \mathcal{A}_\nu + \nabla_\mu \mathcal{A}_\nu$ ... 3 (!) pure gauge

physical (vacuum):

$\mathcal{A}^-[\phi^{(2,0)}]$ ...2 graviton modes (massless !)

$\mathcal{A}^-[\phi^{(2,1)}]$ ...2 ”vector“ modes, $\approx$ pure gauge

$\mathcal{A}^-[\phi^{(2,2)}]$ ...”scalar“ mode $\ni$ (lin. Schwarzschild !)

$$\boxed{\mathcal{R}^{\mu\nu}_{(\mathrm{lin})}[h[\mathcal{A}^{(-)}]] \approx 0}$$ on-shell (up to cosm. scales)

(linearized) Schwarzschild solution

HS 1905.07255

Ricci-flat "scalar" metric perturbation from $\mathcal{A}^{(-)}[D^+ D^+ \phi]$

$$ds^2 = (G_{\mu\nu} - h_{\mu\nu})dy^\mu dy^\nu = -dt^2 + a(t)^2 d\Sigma^2 + \phi'(dt^2 + a(t)^2 d\Sigma^2)$$

$$\phi' \sim \frac{e^{-\chi}}{\sinh(\chi)} \frac{1}{a(t)^2} \sim \frac{1}{\rho} \frac{1}{a(t)^2}$$

$\approx$ lin. Schwarzschild (Vittie) solution on FRW, mass $m(t) \sim \frac{1}{a(t)}$

linearized approx. valid only in quasi-static case $\tau = -2$,
otherwise large pure gauge contribution (cf. massive graviton)

(cf. Vainshtein)

(similar for vector modes)

discussion

- "would-be massive" modes: lin. approx. breaks down
  long wavelengths: extra metric mode(s), not Ricci-flat

  (cf. dark matter ?)
- induced Einstein-Hilbert action S_{EH},
  expect $\approx$ inhomog. Einstein eq. $G_{\mu\nu} \propto T_{\mu\nu}$
- non-linear sector to be understood
- cosm. const. $\int d^4y \sqrt{g}$ replaced by YM-action, stabilizes $\mathcal{M}^{3,1}$

  (no c. c. problem ?)
- reasonable (coasting) cosmology without fine-tuning !
- Schwarzschild is slowly decreasing during cosmic evolution ... ?

summary

- IKKT matrix model as quantum theory of space-time & matter
- 3+1D covariant quantum cosmological space-time

  finite density of microstates
- $\rightarrow$ higher spin theory, no ghosts or tachyons
- quantized as Yang-Mills, good UV behavior

  (cf. $\mathcal{N} = 4$ SYM, 1-loop finite, ...)
- emergent gravity rather than GR

  lin. gravity seems ok

  extra ”scalar“ mode (dark matter/energy?)

... consistent theory, work it out !

coupling to matter & eom:

for physical transverse traceless spin 2 modes $h_{\mu\nu}[\phi^{(2,0)}]$:

$$S_2[\mathcal{A}^{(-)}[\phi^{(2,0)}]] \propto -\int h^{\mu\nu}[\phi^{(2,0)}](\Box - R^{-2})(\Box_H - 2r^2)^{-1} h_{\mu\nu}[\phi^{(2,0)}]$$

leads to eom

$$(\Box - 2R^{-2})h_{\mu\nu} \sim -(\Box_H - 2r^2)T_{\mu\nu}$$

upon adding S_{EH} expect

$$\Big(\Box - \frac{\Box - R^{-2}}{\Box_H - 2r^2}\Big)h_{\mu\nu} \sim T_{\mu\nu}$$

... Einstein eq., with slight modifications, still no ghosts (?)

2 points of view for $\phi^{(s)} \in \mathcal{C}^s$:

- functions on H_n^4: full $SO(4,1)$ covariance

$$\begin{aligned} \phi^H_{a_1 \ldots a_s} &\propto \{x^{a_1}, \ldots \{x^{a_s}, \phi^{(s)}\} \ldots \}_0 \\ \phi^{(s)} &= \{x^{a_1}, \ldots \{x^{a_s}, \phi^H_{a_1 \ldots a_s}\} \ldots \} \end{aligned}$$

- functions on $\mathcal{M}_n^{3,1}$: reduced $SO(3,1)$ covariance

$$\phi^{(s)} = \phi_{\mu_1 \ldots \mu_s}(y) t^{\mu_1} \ldots t^{\mu_s}$$

$t_\mu y^\mu = 0 \Rightarrow$ "space-like gauge" $\boxed{y^{\mu_i} \phi_{\mu_1 \ldots \mu_s} = 0}$

($\rightarrow$ no ghosts!)