

Black Hole from **Colors**

Masanori Hanada
University of Southampton

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M.H.-Maltz, 2016

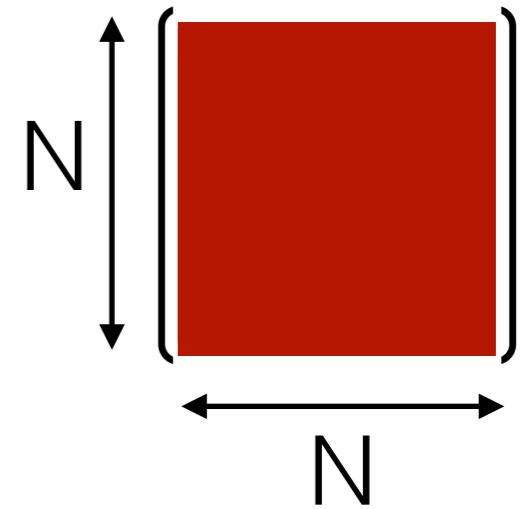
M.H.-Ishiki-Watanabe, 2018

M.H.-Jevicki-Peng-Wintergert, 2019

M.H.-O'Bannon-Robinson, in preparation

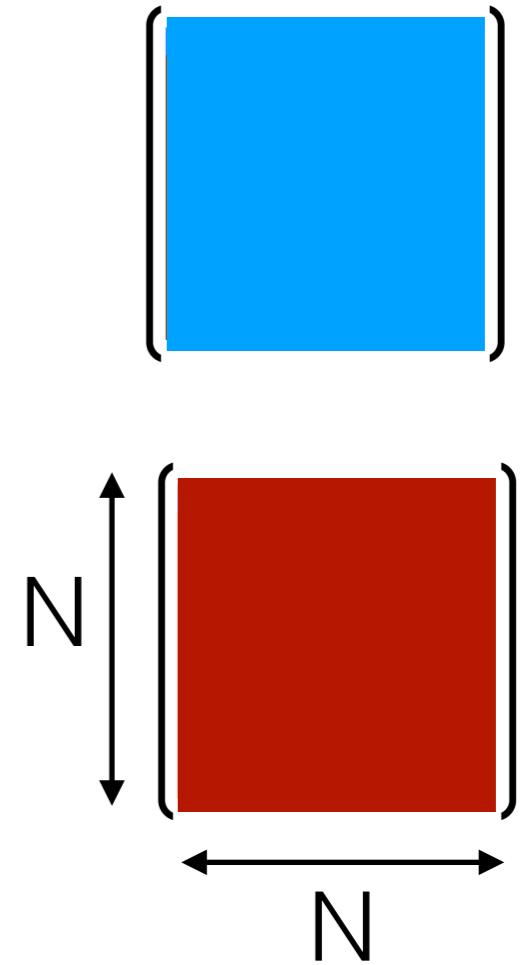
- Confinement phase: $E \sim N^0$
- Deconfinement phase: $E \sim N^2$

↔ Black Hole



- Confinement phase: $E \sim N^0$
- Deconfinement phase: $E \sim N^2$

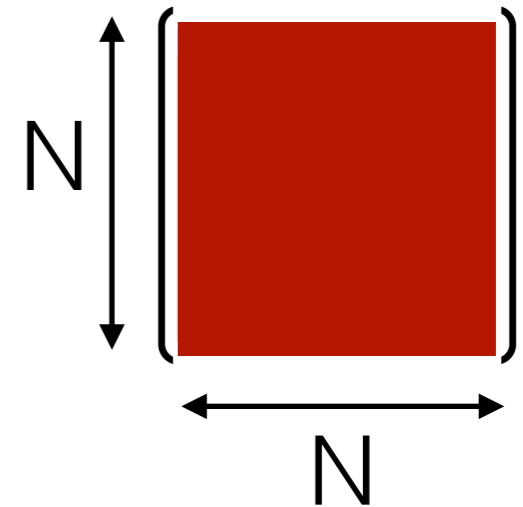
↔ Black Hole



What if $E \sim N^2/100$?

- Confinement phase: $E \sim N^0$
- Deconfinement phase: $E \sim N^2$

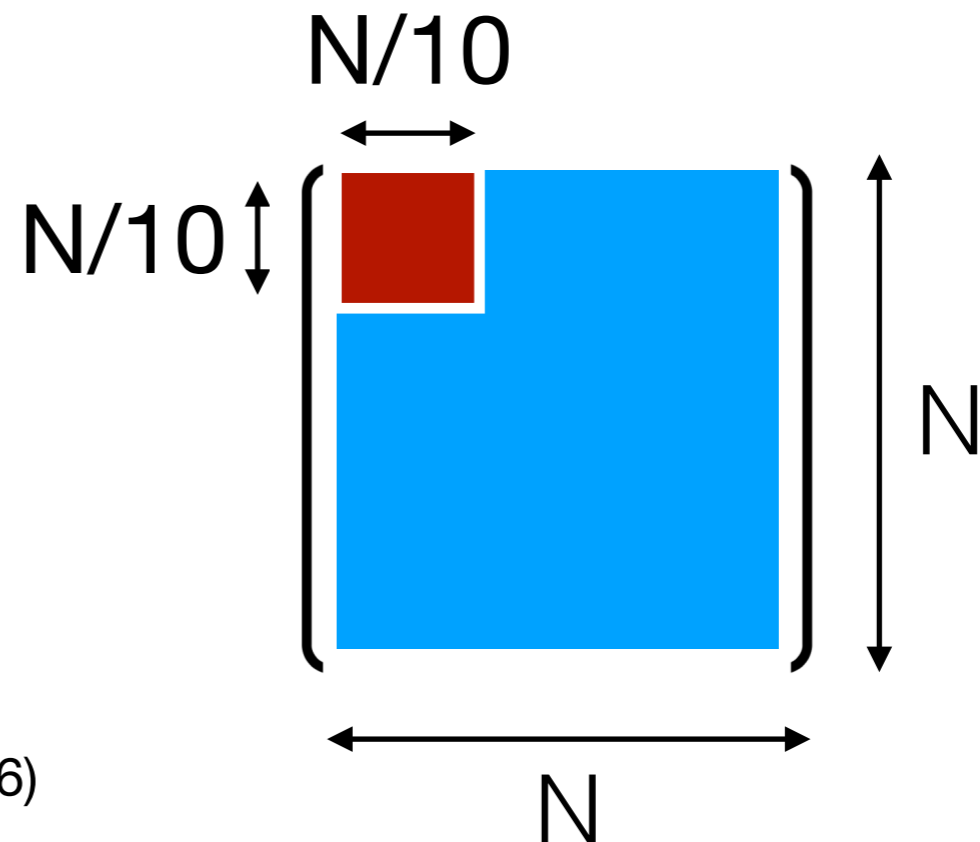
↔ Black Hole

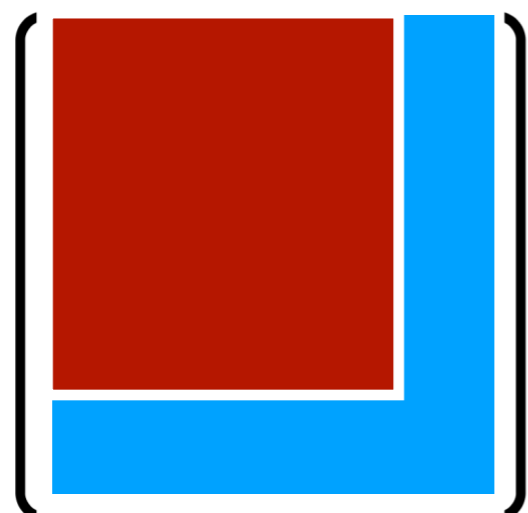
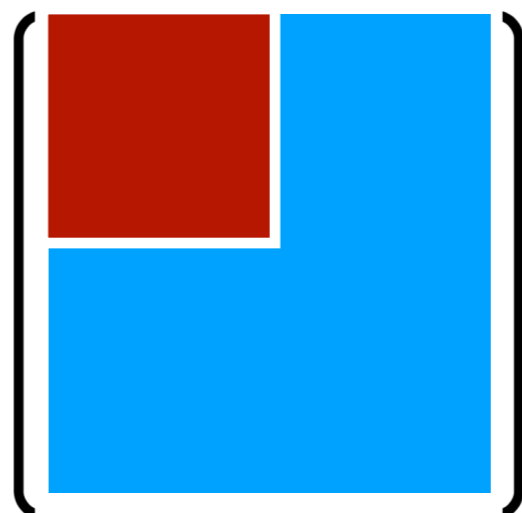
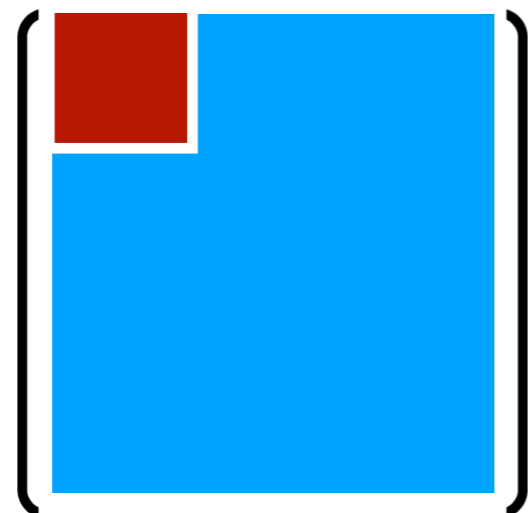
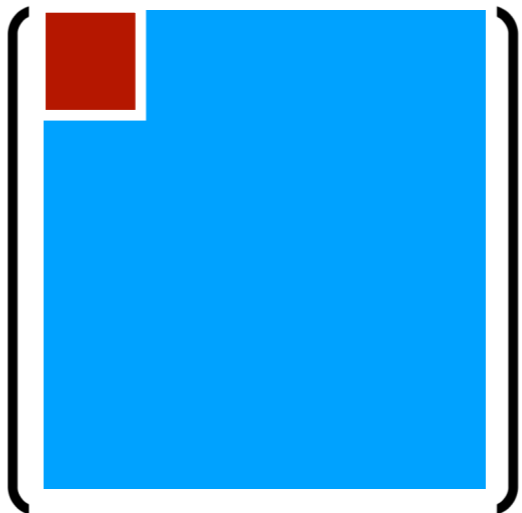


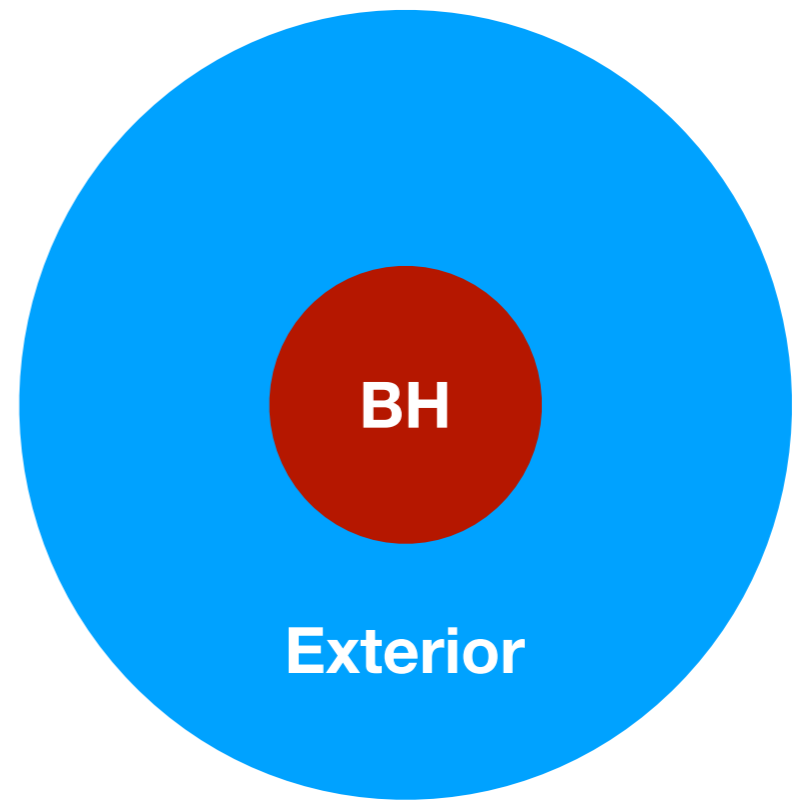
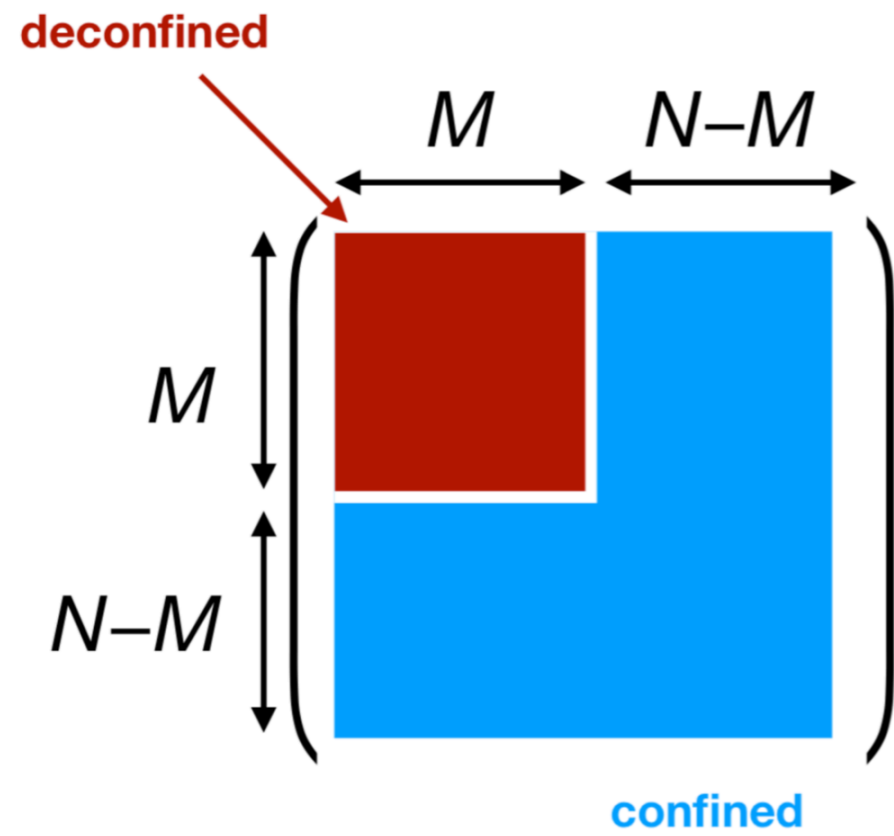
What if $E \sim N^2/100$?

'partially' deconfine

(MH-Maltz, 2016)



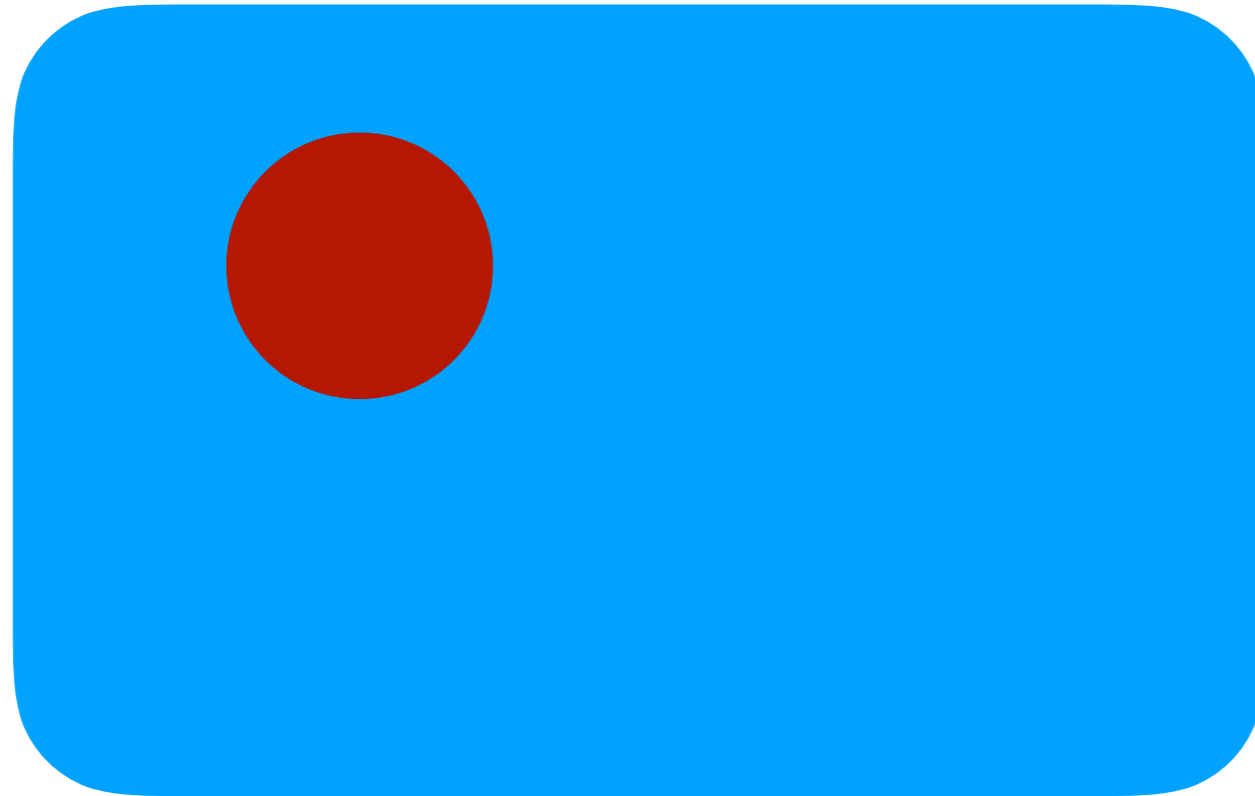




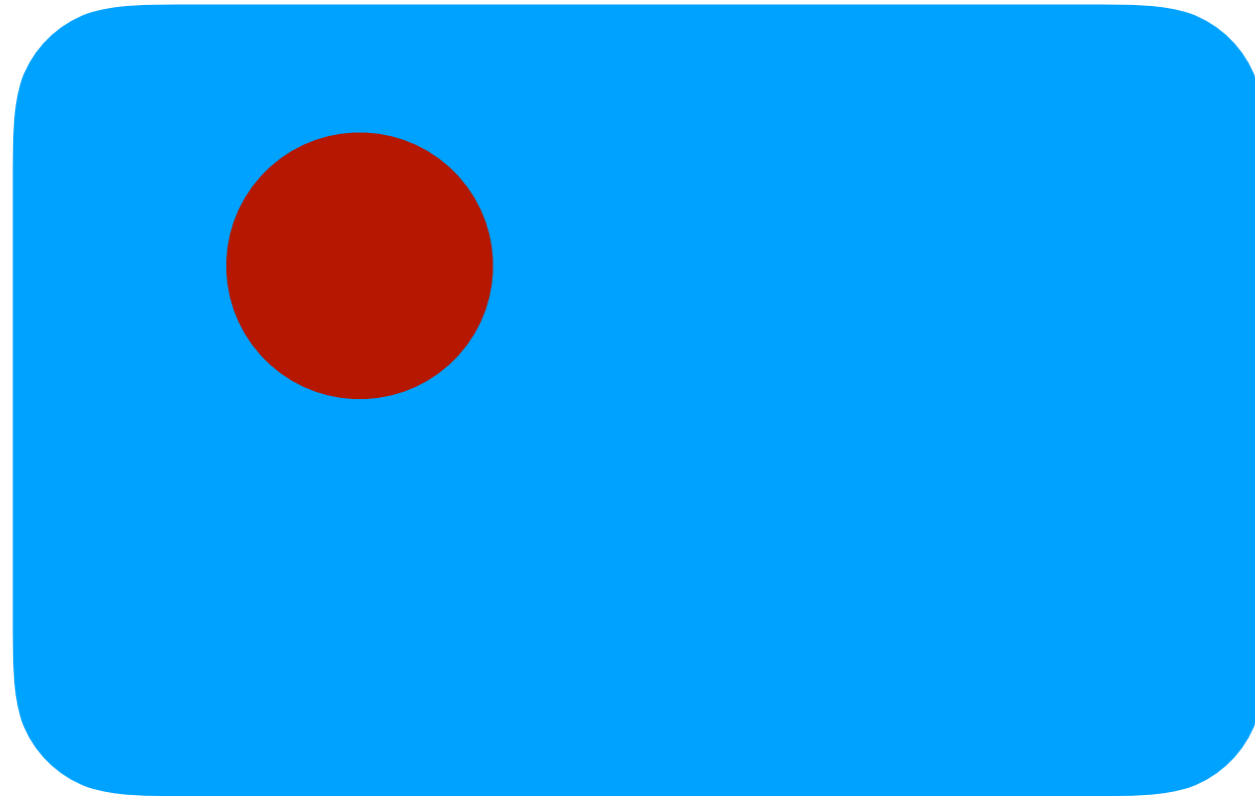
Heuristic justification

(more precise argument is given later)

Why doesn't a part of the volume deconfine?

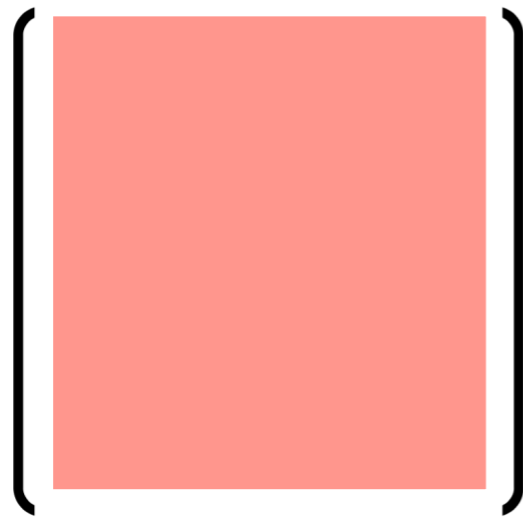


Why doesn't a part of the volume deconfine?

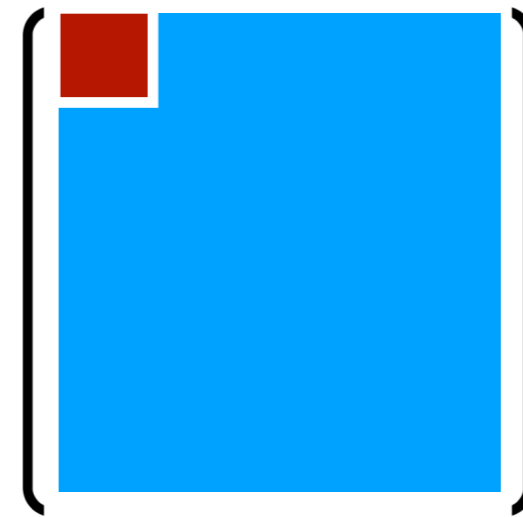


Deconfinement takes place even in matrix model,
which has no spatial dimensions.

Why don't all N^2 d.o.f. gently excited?

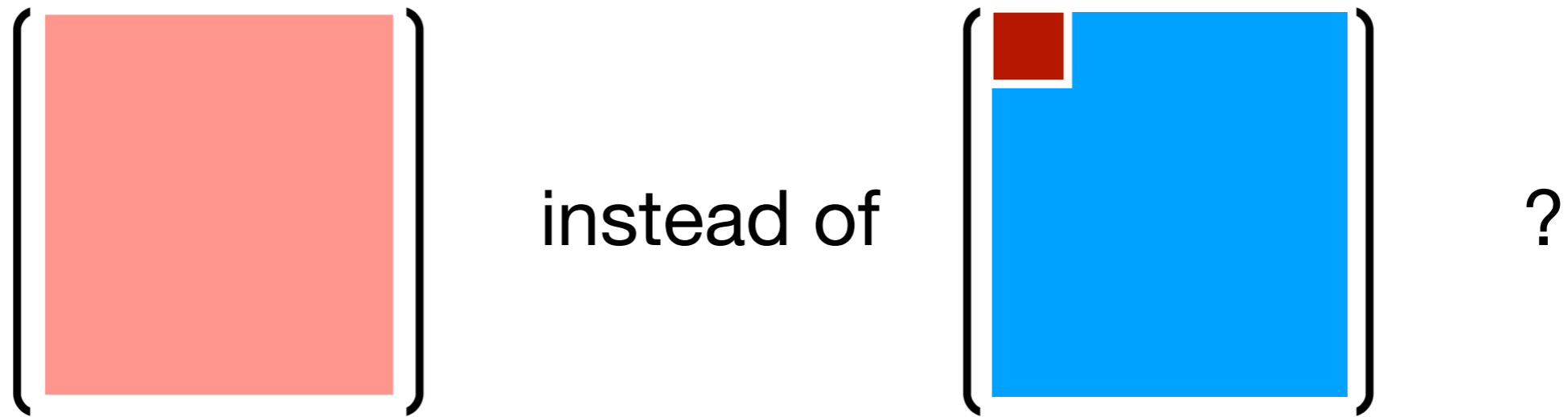


instead of



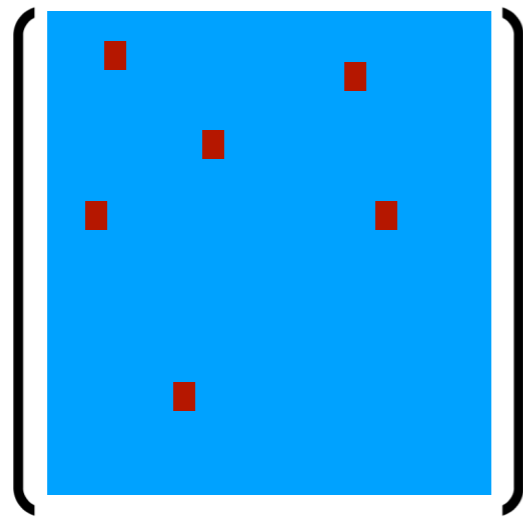
?

Why don't all N^2 d.o.f. gently excited?

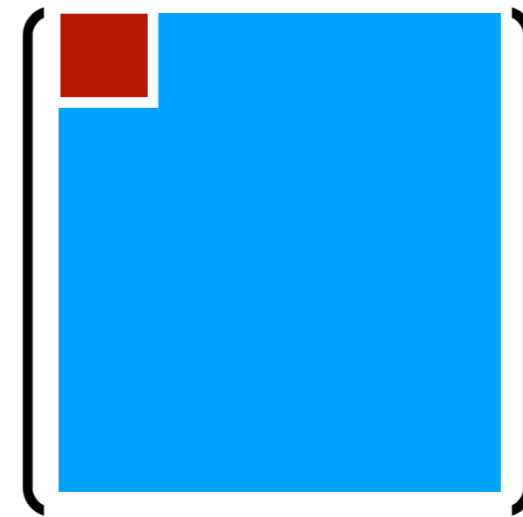


In quantum mechanics, parametrically small excitation is impossible.

Why should symmetry preserved partly?

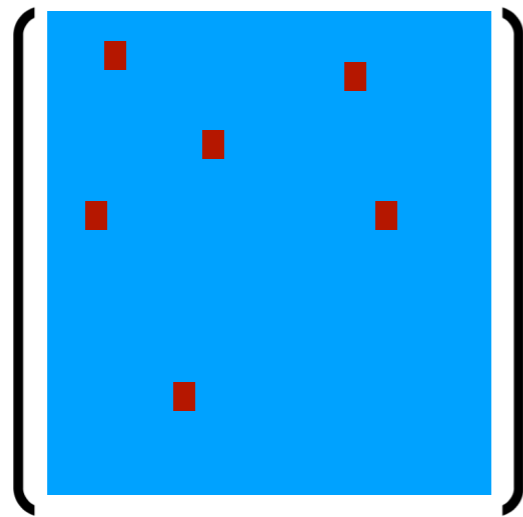


instead of

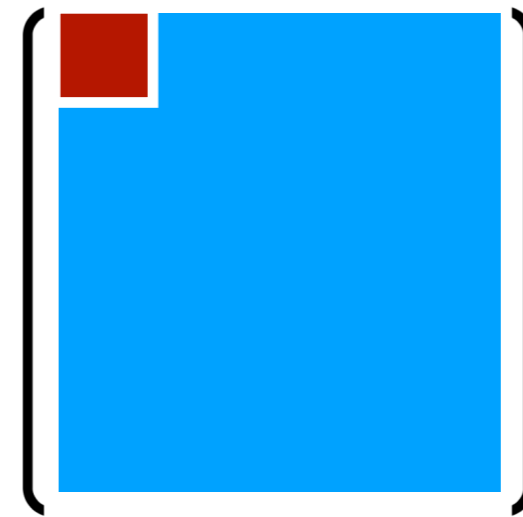


?

Why should symmetry preserved partly?



instead of



?

It is natural to expect a large symmetry at saddle point.

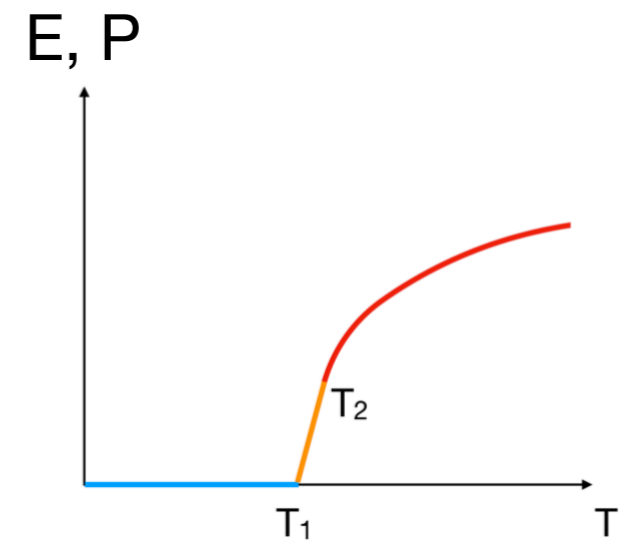
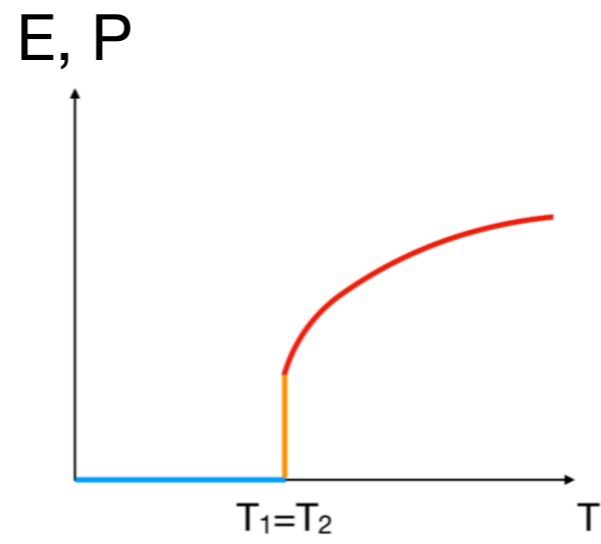
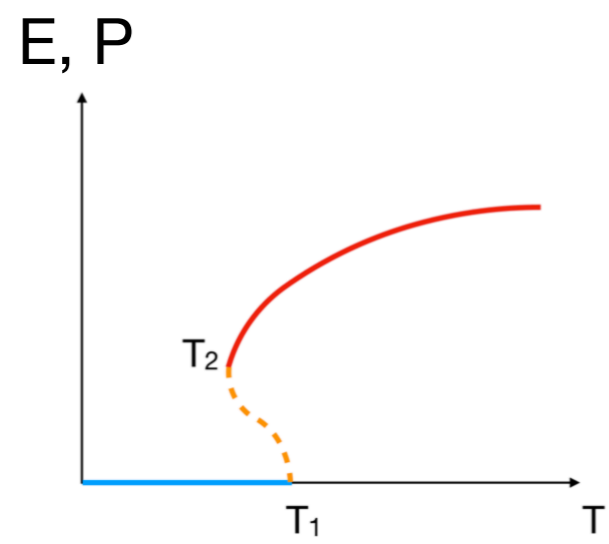
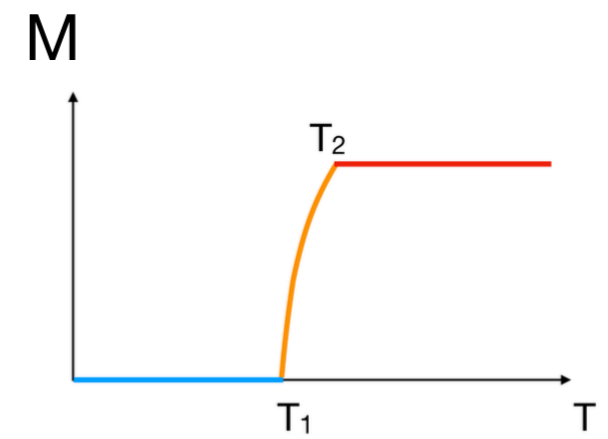
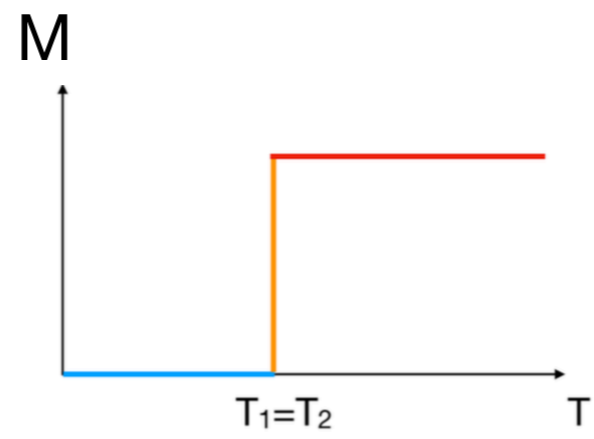
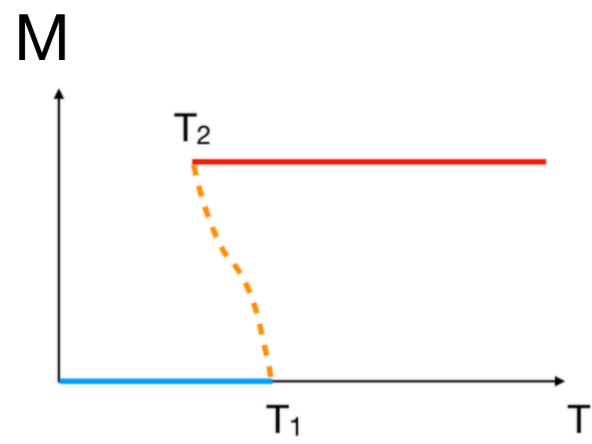
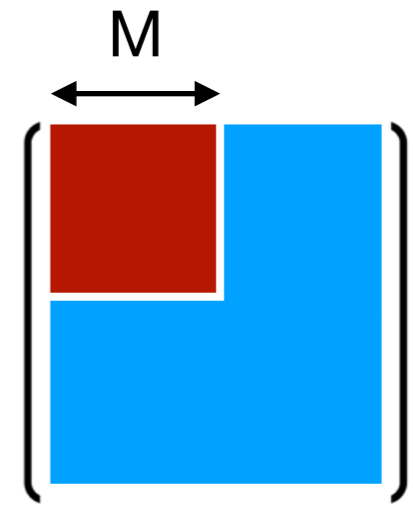
Phase Diagrams

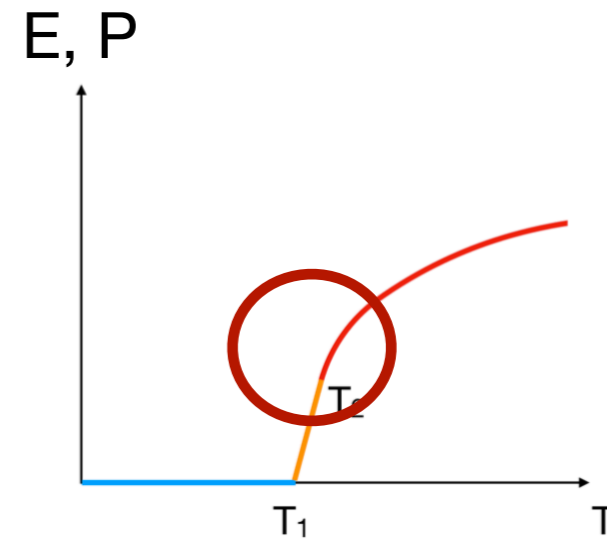
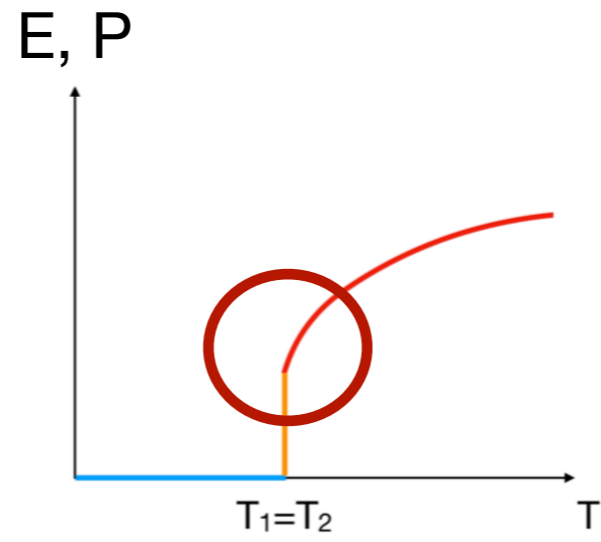
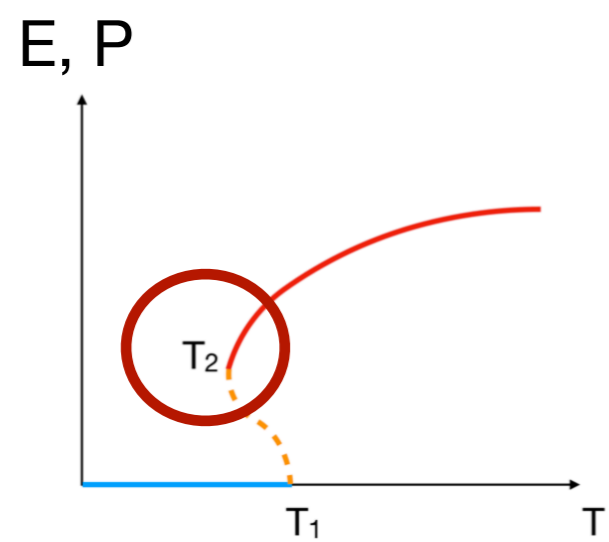
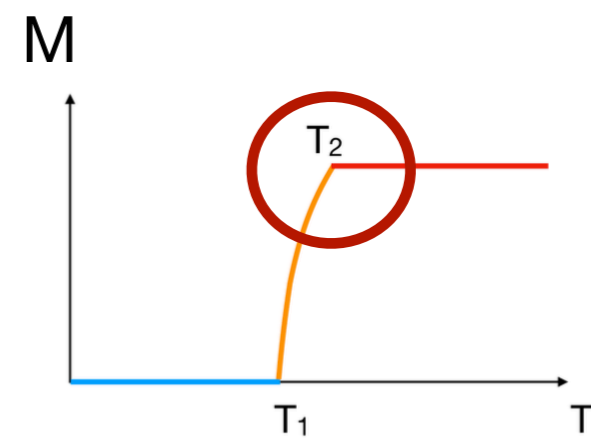
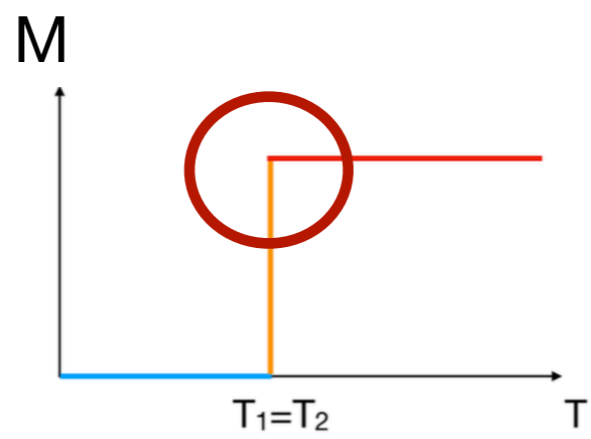
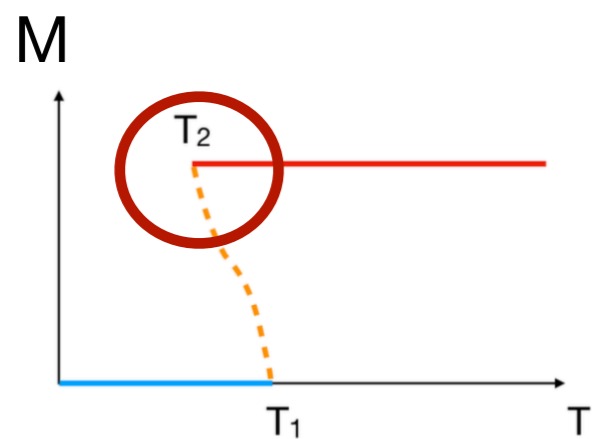
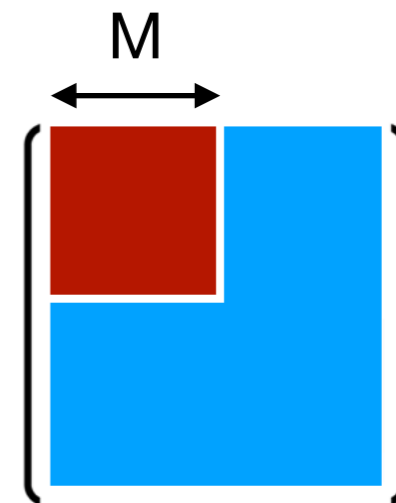
Hagedorn transition

Gross-Witten-Wadia transition

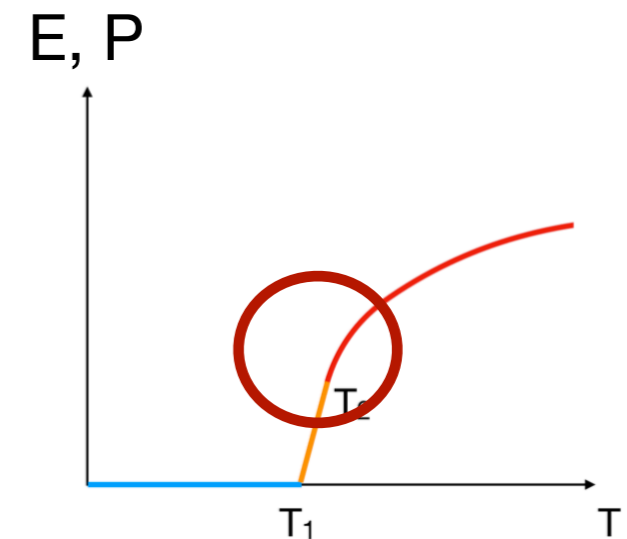
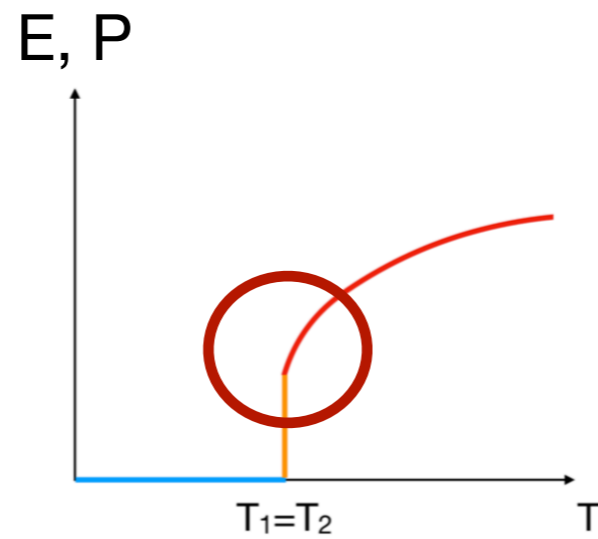
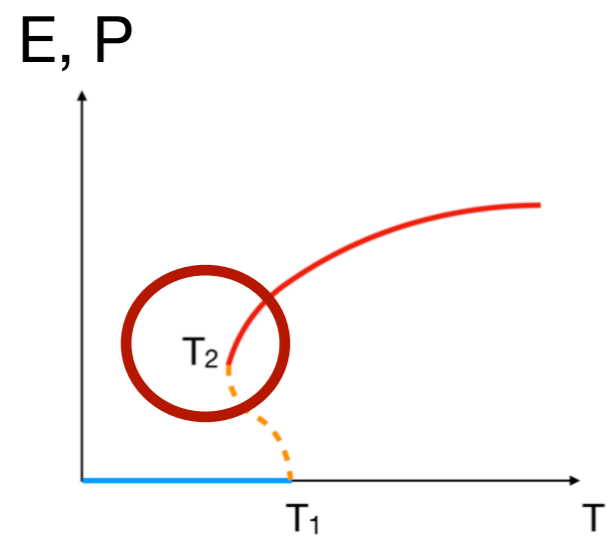
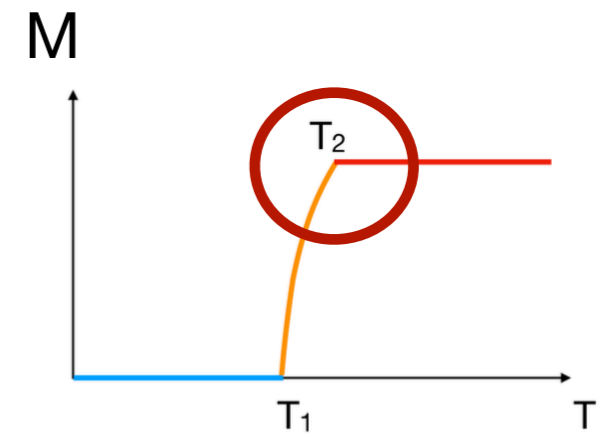
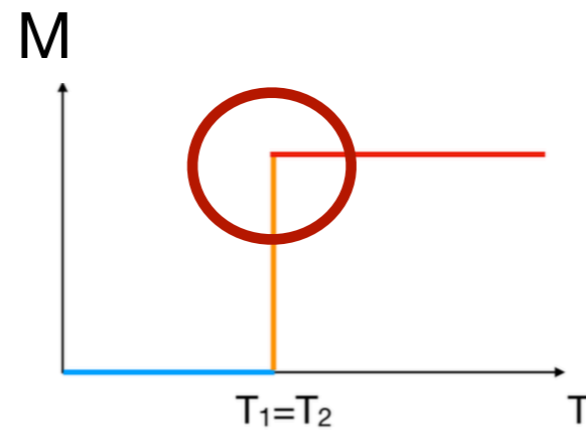
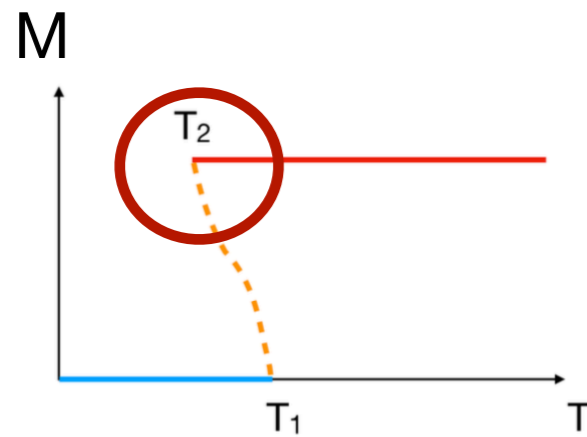
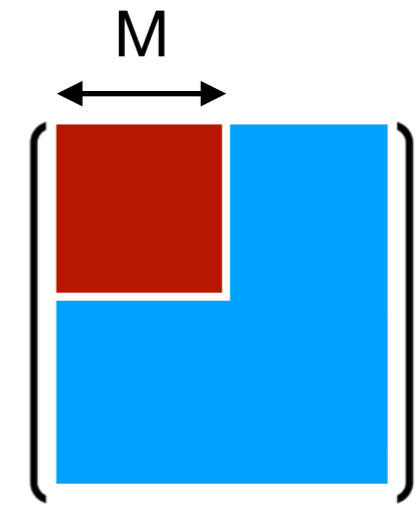
Gauge symmetry breaking

(more precise argument is given later)





Gross-Witten-Wadia transition

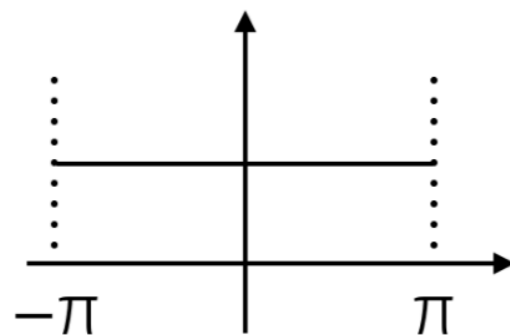


- Polyakov loop

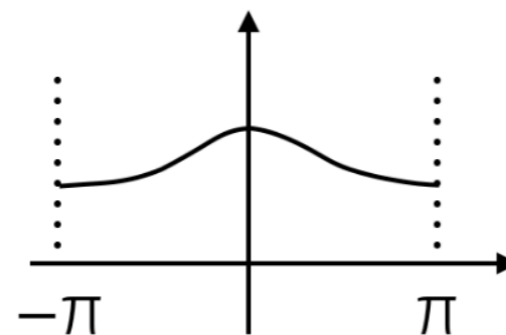
$$P = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j}$$

- Phase distribution:

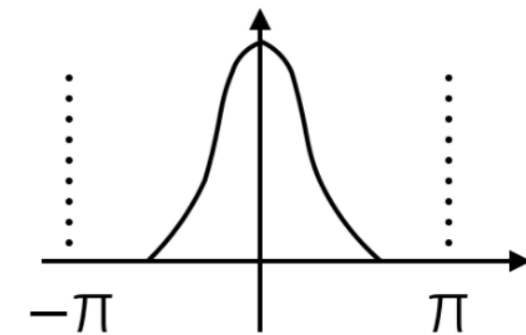
confined phase
P=0



deconfined phase
P ≠ 0



'partially' deconfined



'completely' deconfined

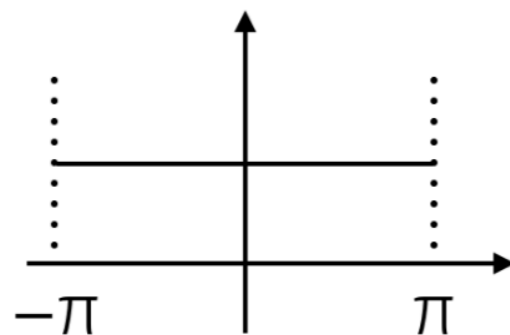
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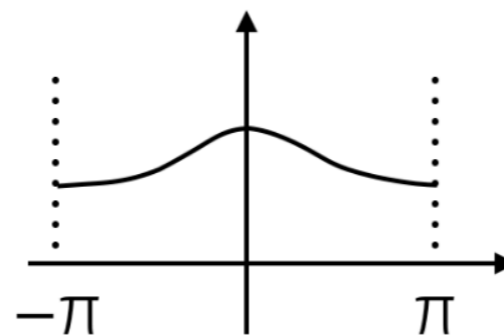
- Phase distribution:

Gross-Witten-Wadia transition (GWW)

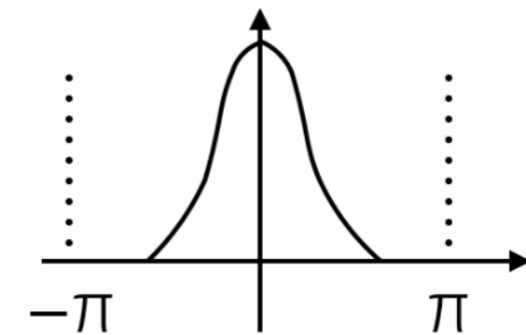
confined phase
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deconfined phase
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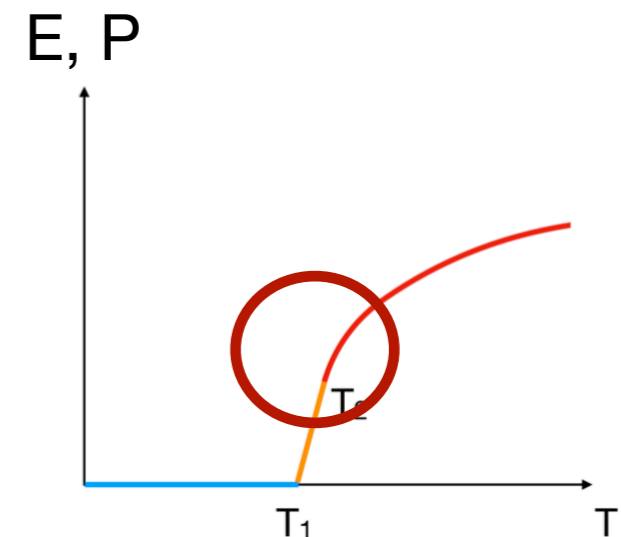
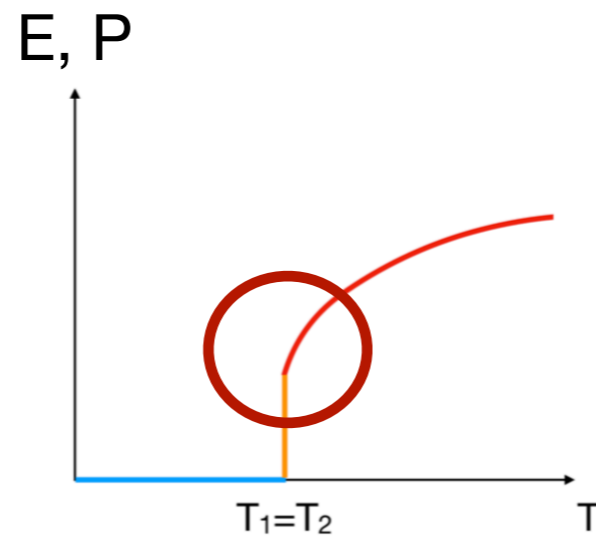
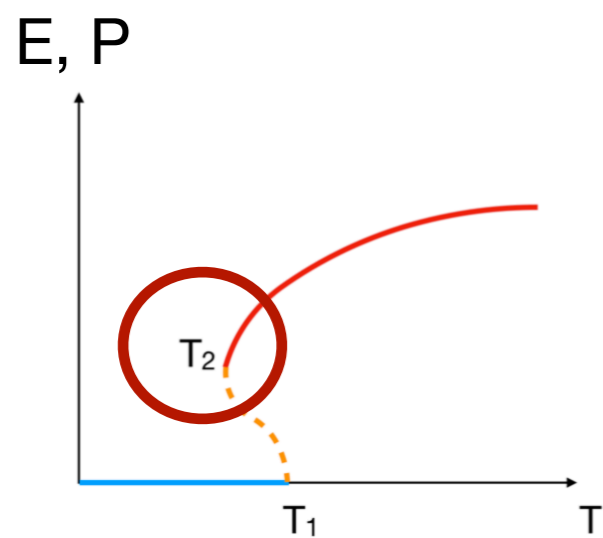
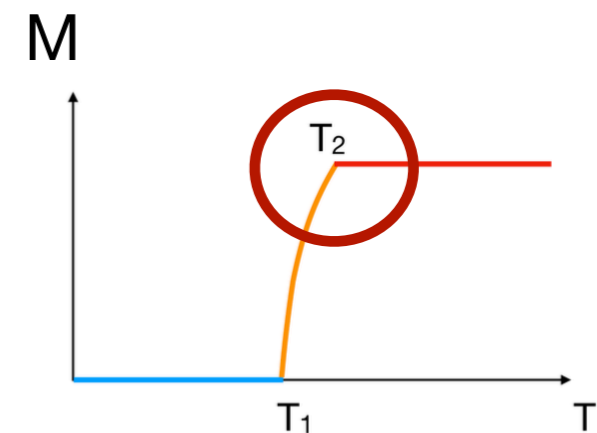
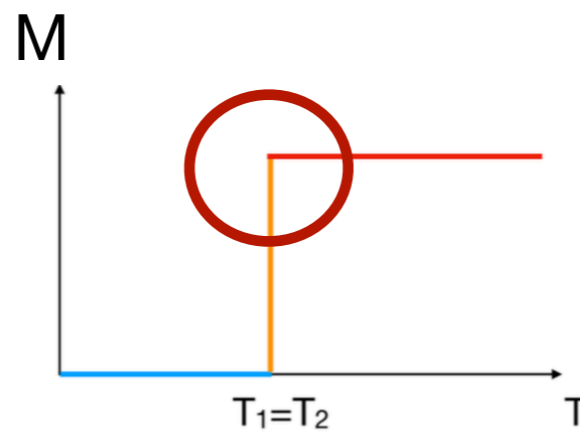
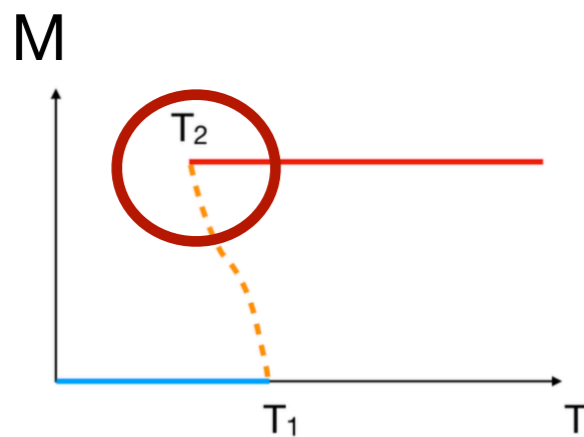


'partially' deconfined



'completely' deconfined

Gross-Witten-Wadia transition = “partial deconfinement → complete deconfinement” transition



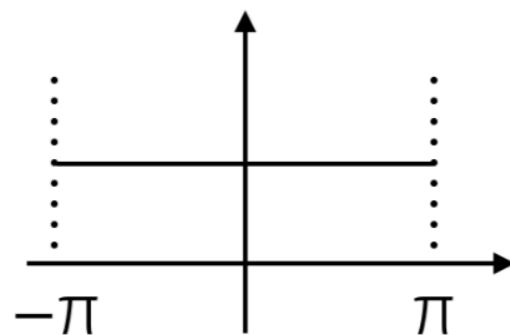
- Polyakov loop

$$P = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j}$$

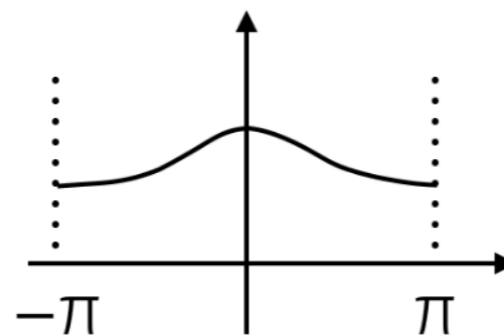
- Phase distribution:

Gross-Witten-Wadia transition (GWW)

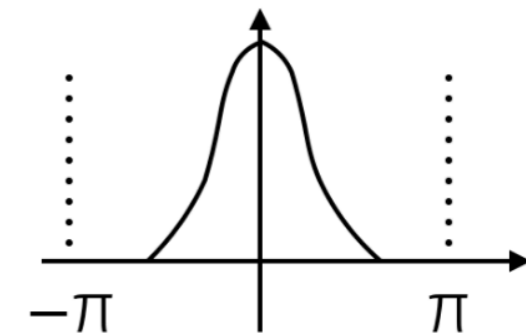
confined phase
P=0



deconfined phase
P ≠ 0



'partially' deconfined



'completely' deconfined

- Polyakov loop

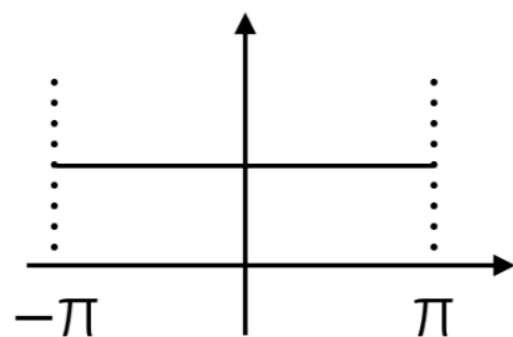
$$P = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j}$$

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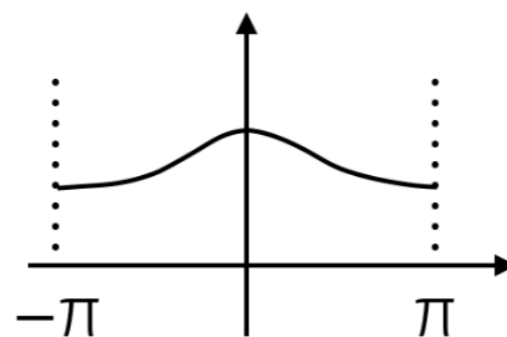
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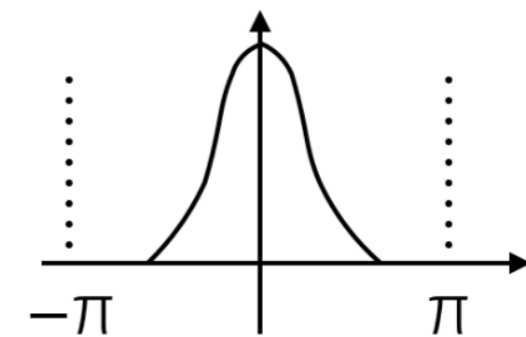
confined phase
 $P=0$



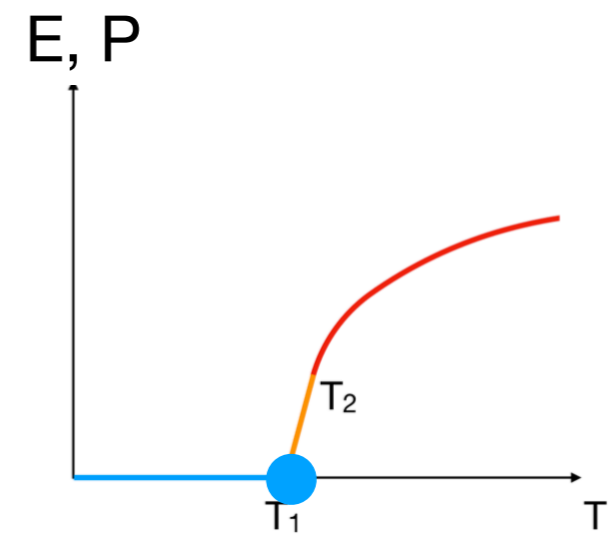
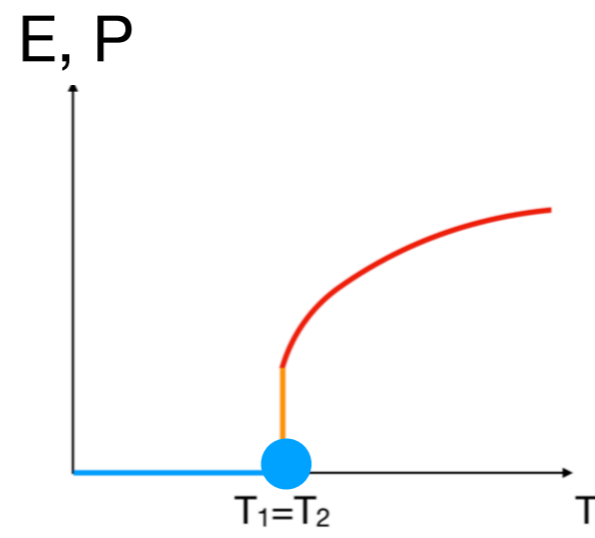
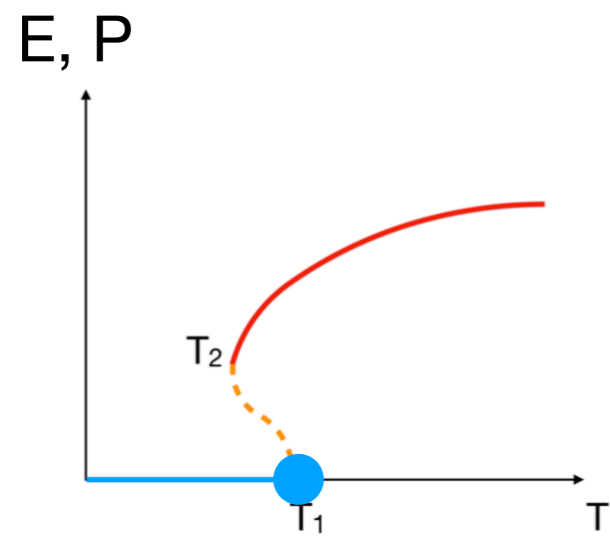
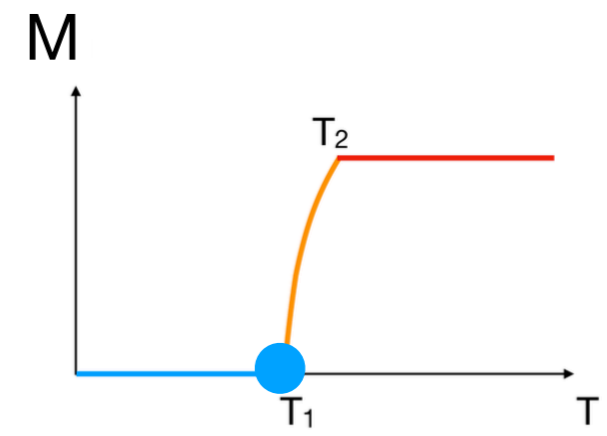
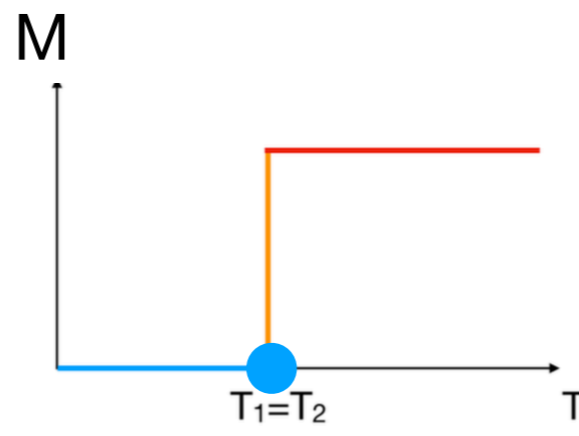
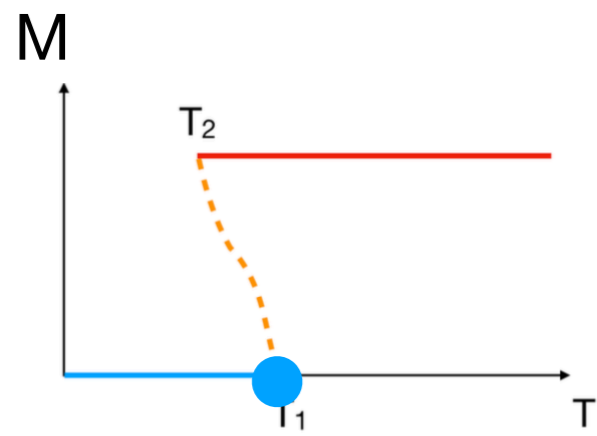
deconfined phase
 $P \neq 0$



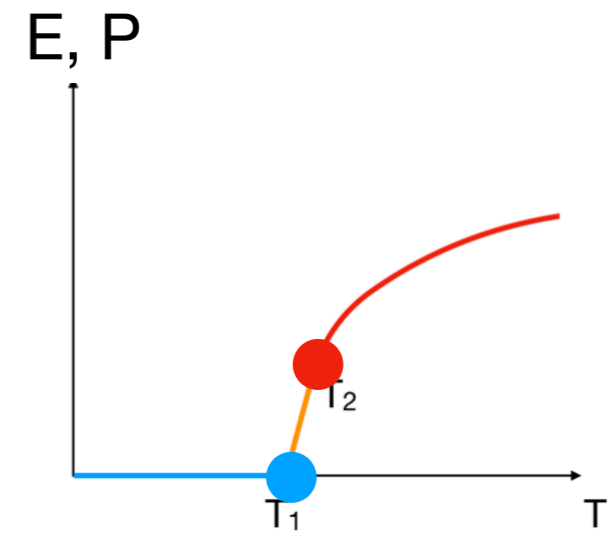
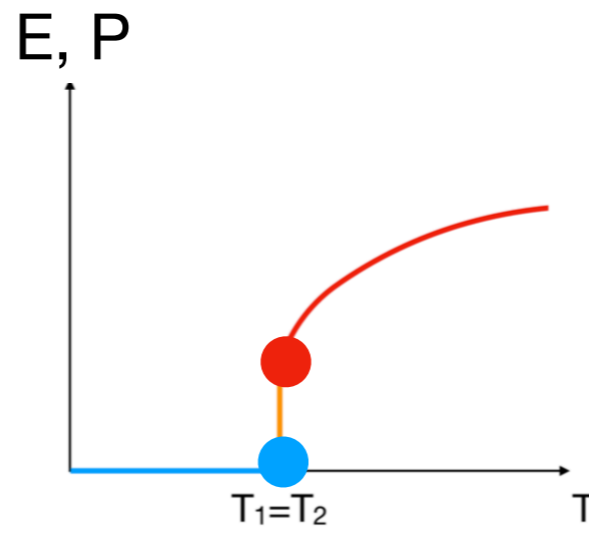
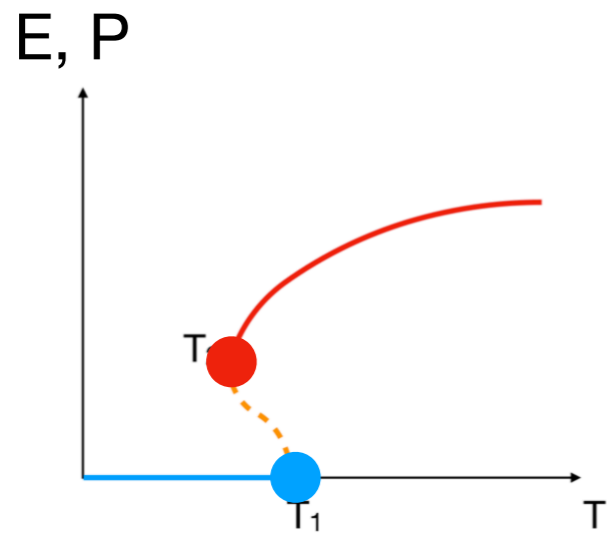
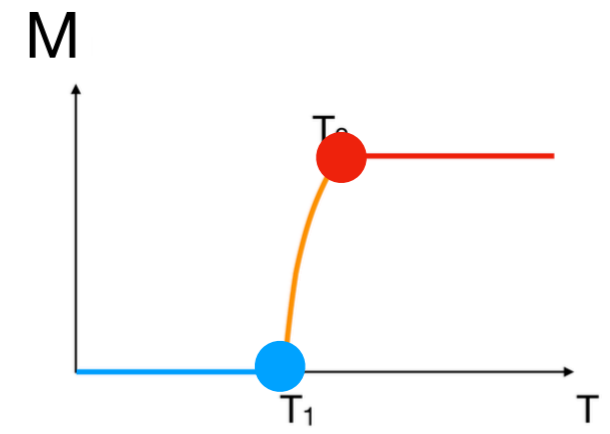
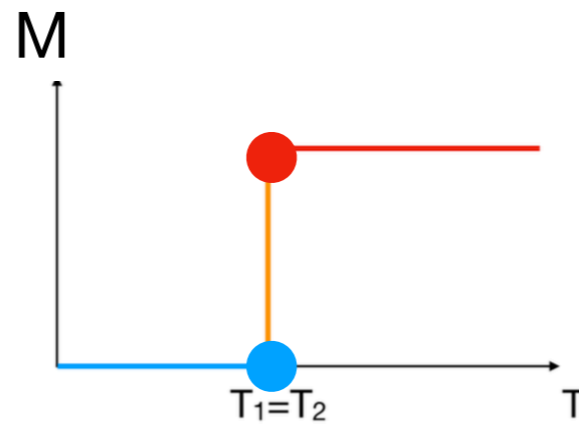
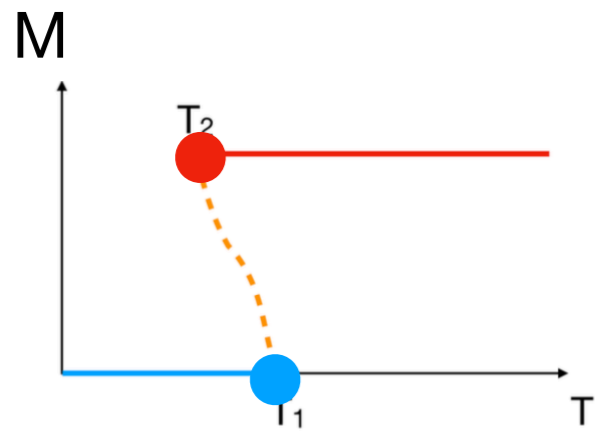
'partially' deconfined



'completely' deconfined

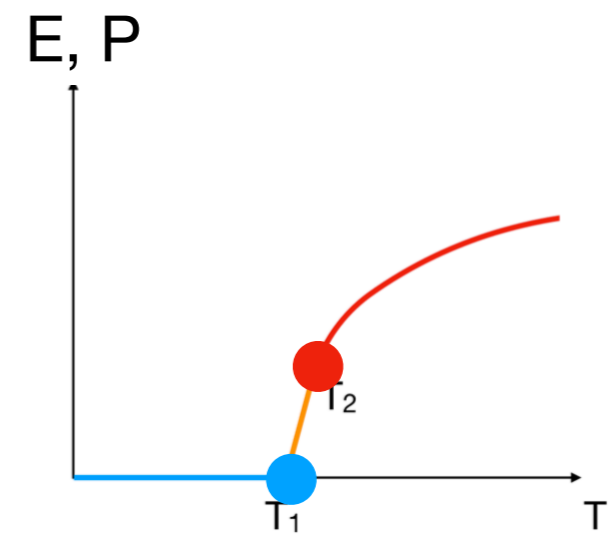
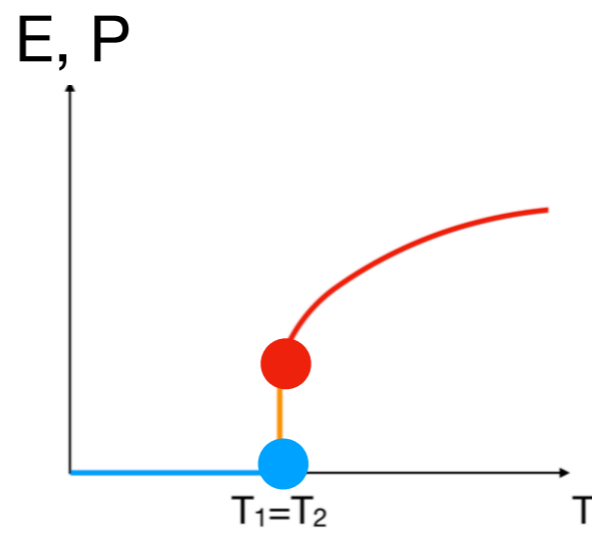
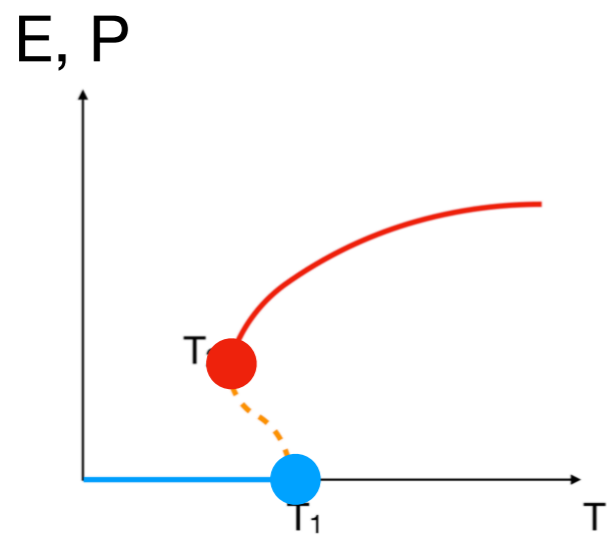
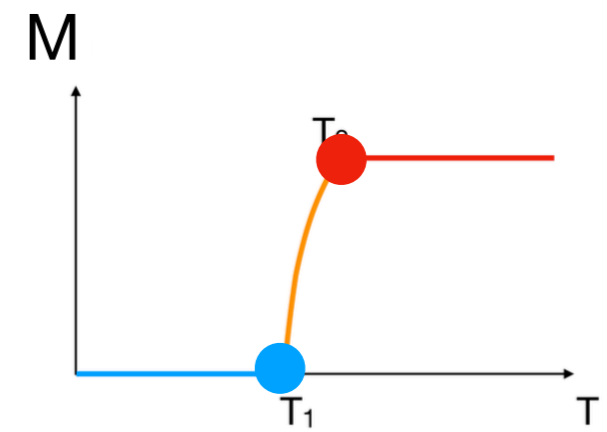
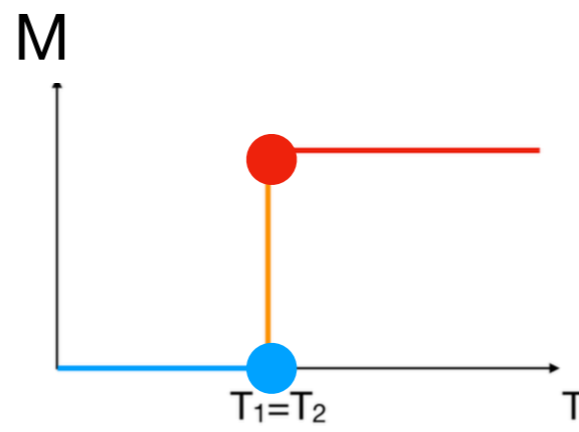
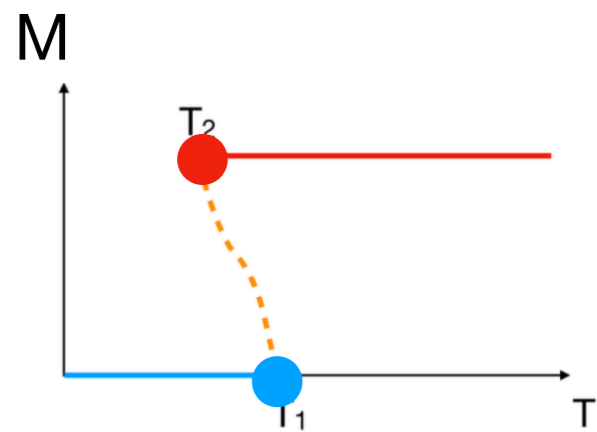


transition 1: confinement to partial deconfinement
(black hole formation begins)



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(black hole formation begins)

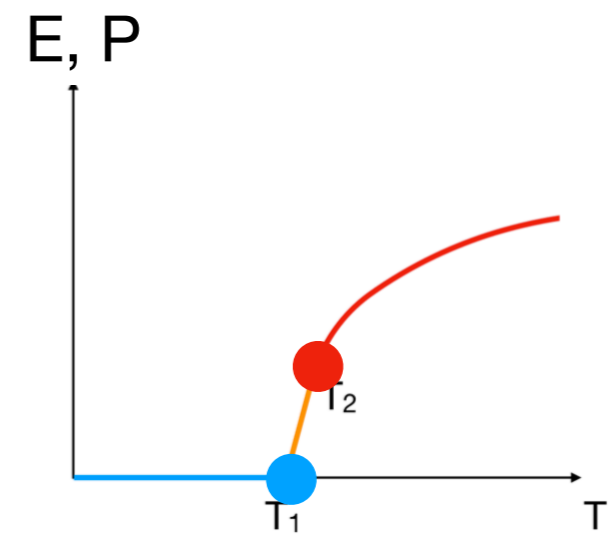
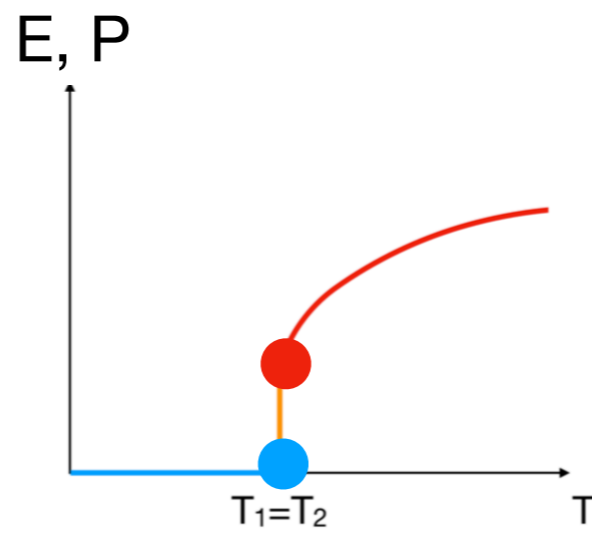
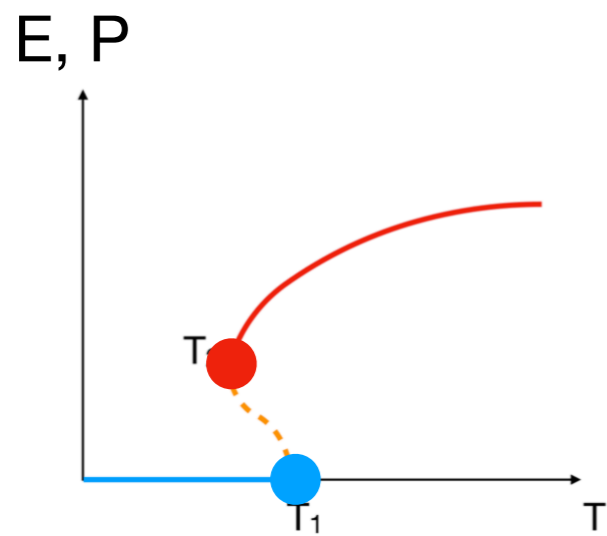
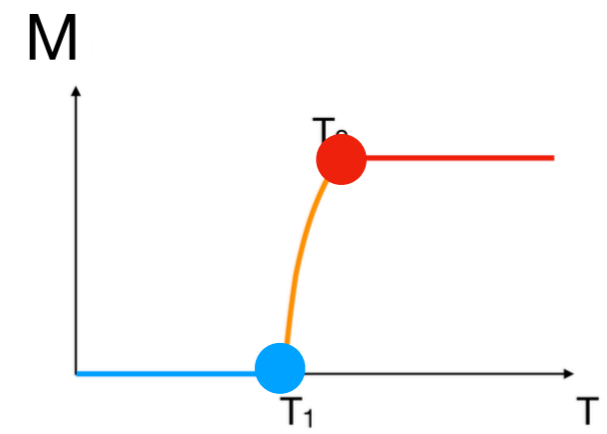
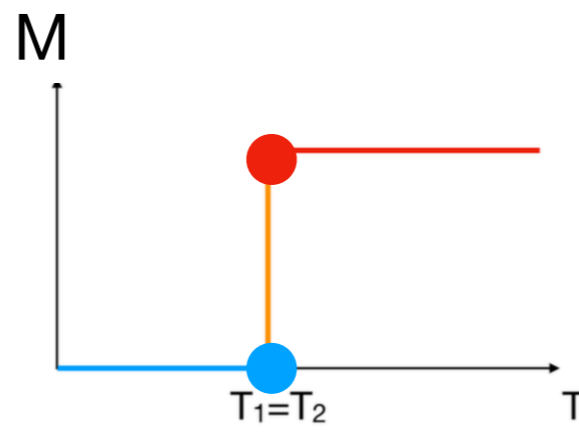
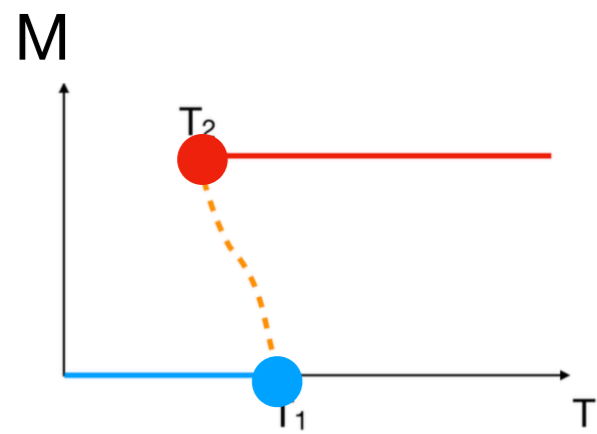
transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)



transition 1: confinement to partial deconfinement
(black hole formation begins)

transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)

$$\text{SU}(N) \rightarrow \text{SU}(M) \times \text{SU}(N-M) \times \text{U}(1) \rightarrow \text{SU}(N)$$



transition 1: confinement to partial deconfinement
(black hole formation begins)

transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)

$$\text{SU}(N) \rightarrow \text{SU}(M) \times \text{SU}(N-M) \times \text{U}(1) \rightarrow \text{SU}(N)$$

No need for center symmetry. No need for chiral symmetry. Applies to QCD.

Where did it come from?



J. Maltz

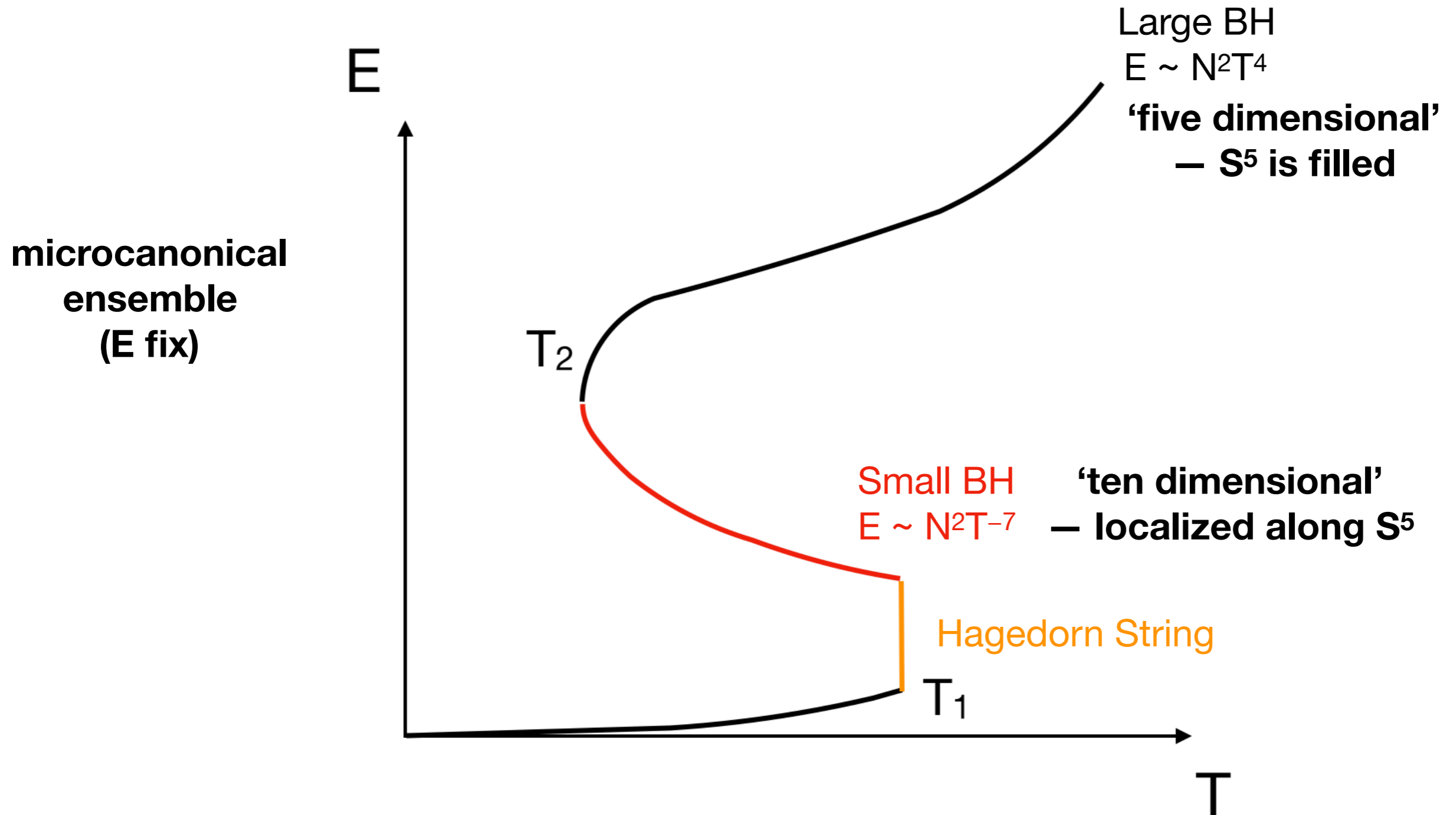


E. Berkowitz

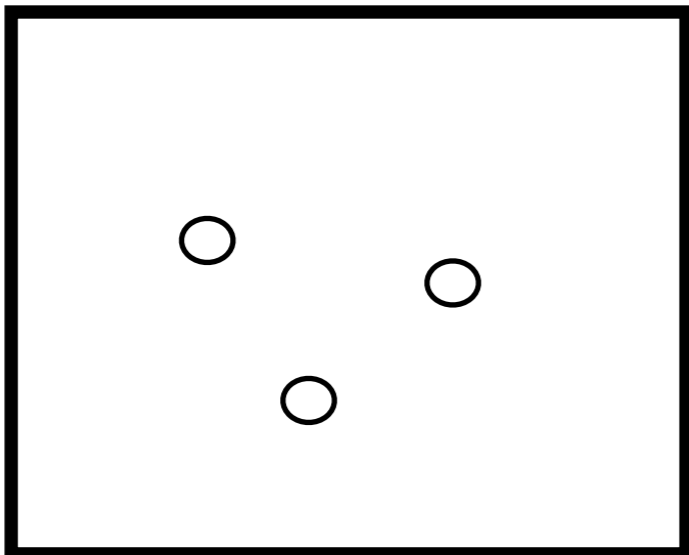
Berkowitz-M.H.-Maltz, 2016, PRD (Higgsing version)

M.H.-Maltz, 2016, JHEP; Susskind, unpublished

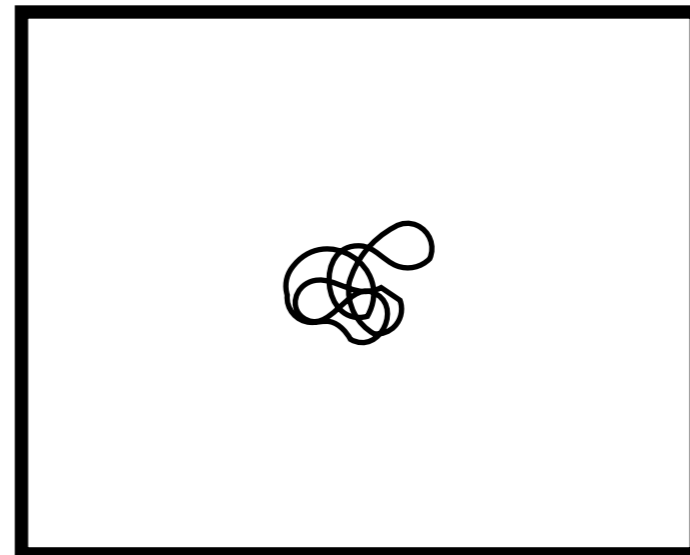
Black Hole in $AdS_5 \times S^5 = 4d$ N=4 SYM on S^3



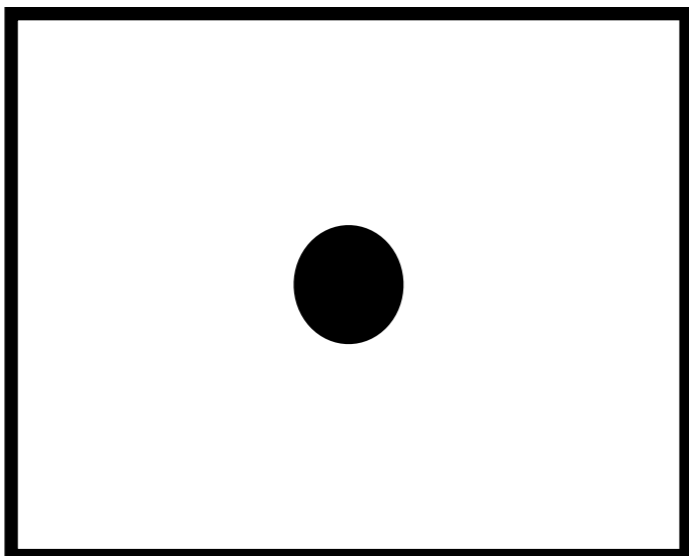
Graviton gas



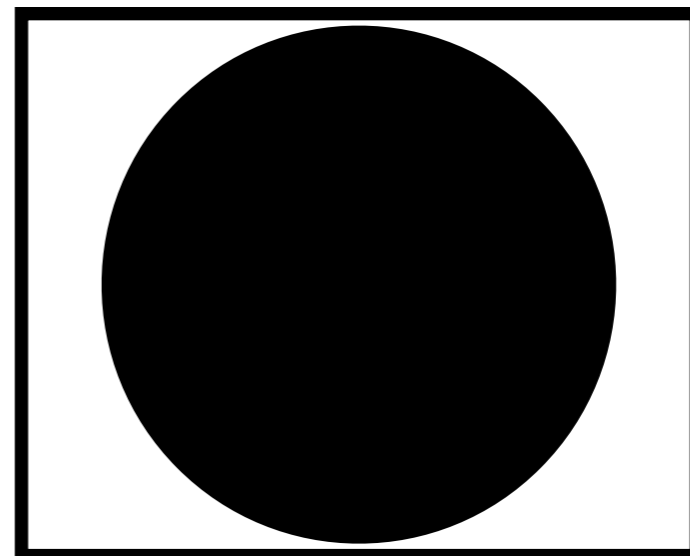
Hagedorn String

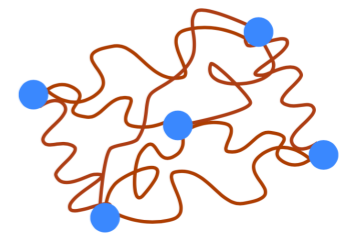
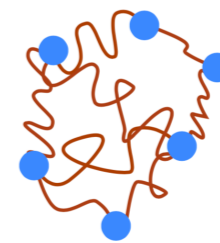
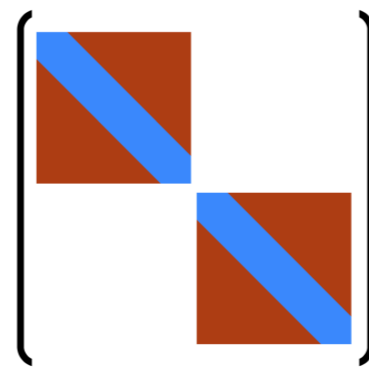
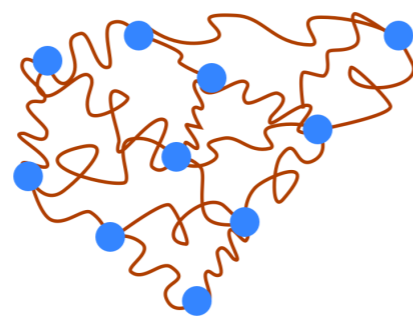
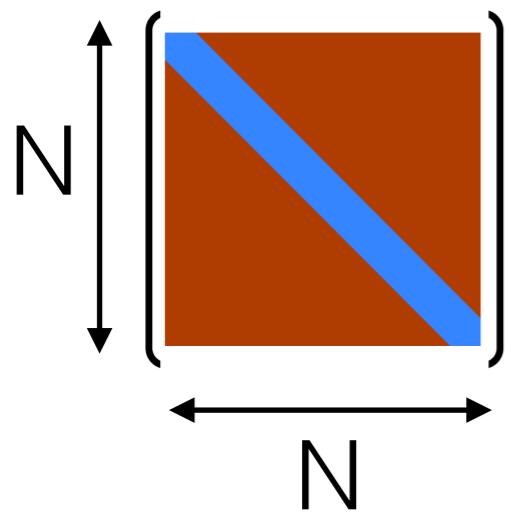


Small BH
 $E \sim N^2 T^{-7}$



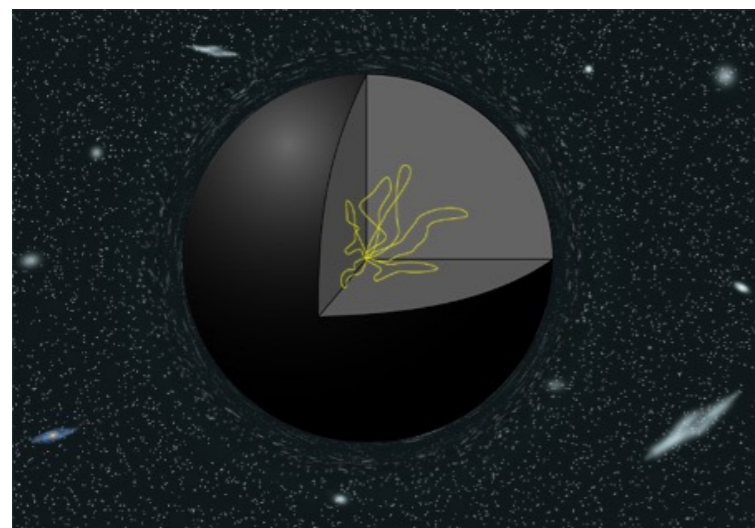
Large BH
 $E \sim N^2 T^4$



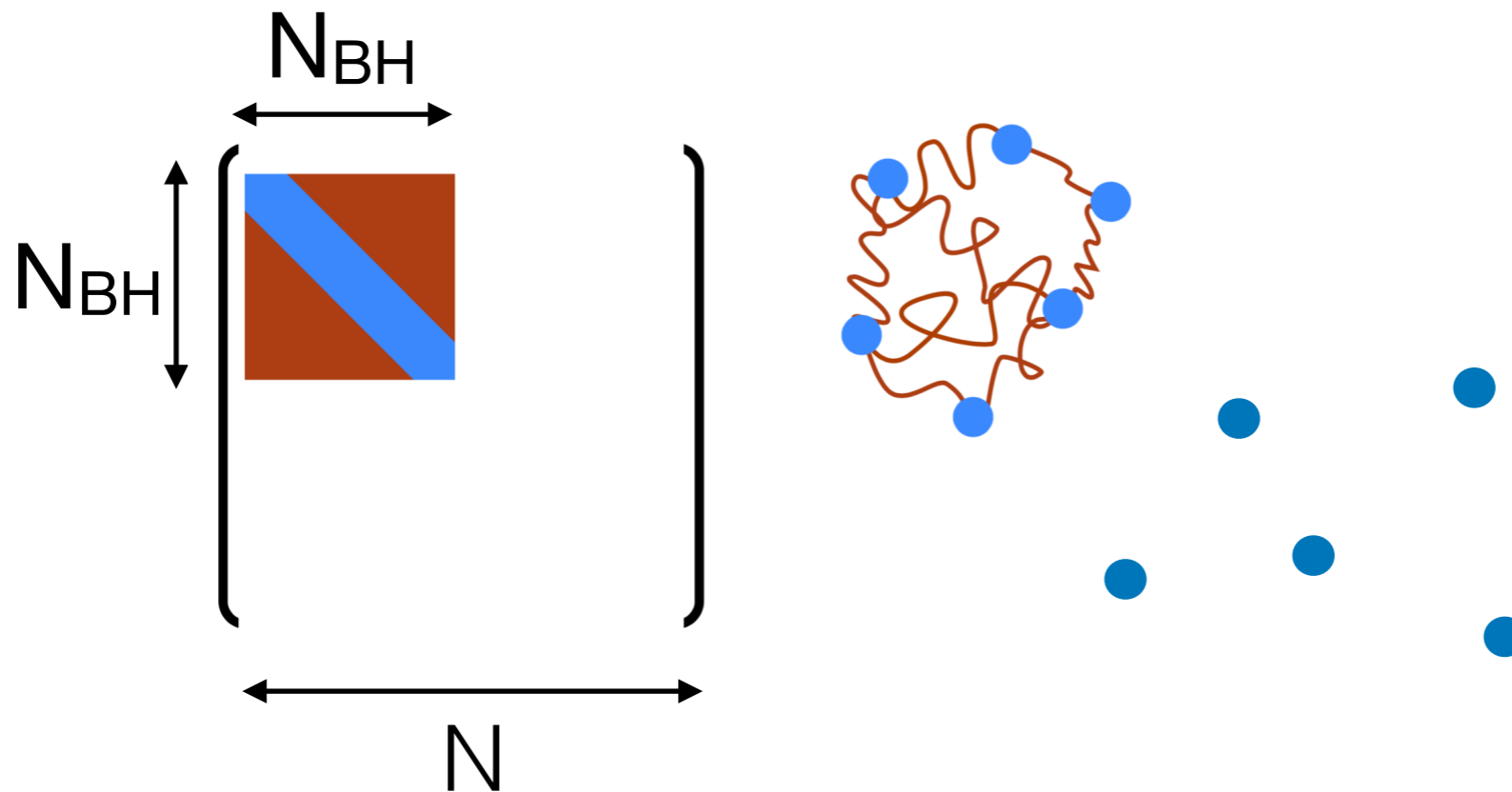


diagonal elements = particles (D-branes)
 off-diagonal elements = open strings

(Witten, 1994)



black hole = bound state of D-branes and strings



N_{BH} D-branes form the bound state

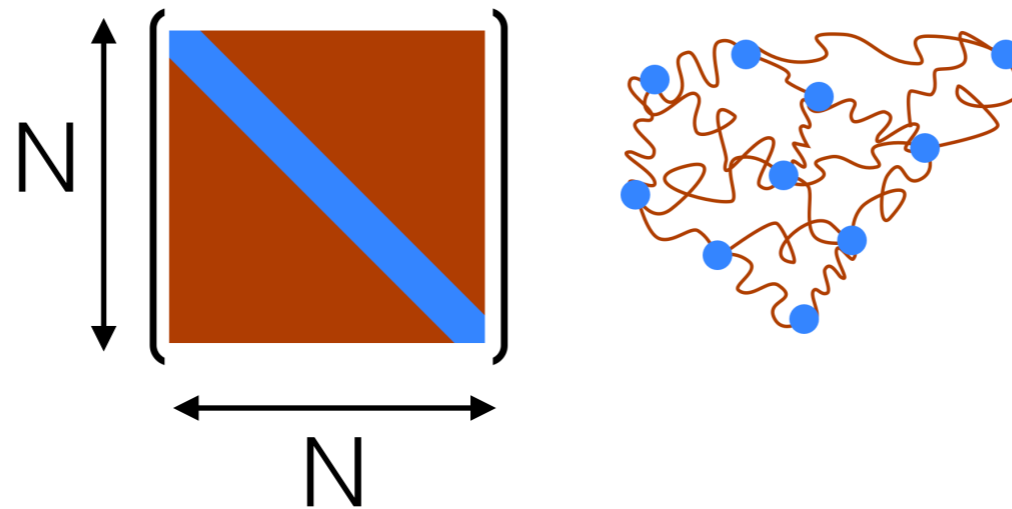
$U(N_{\text{BH}})$ is deconfined — ‘partial deconfinement’

It can explain $E \sim N^2 T^{-7}$ for 4d SYM, $N^{3/2} T^{-8}$ for ABJM

(String Theory \rightarrow 10d)

(M-Theory \rightarrow 11d)

Why is positive specific heat natural?

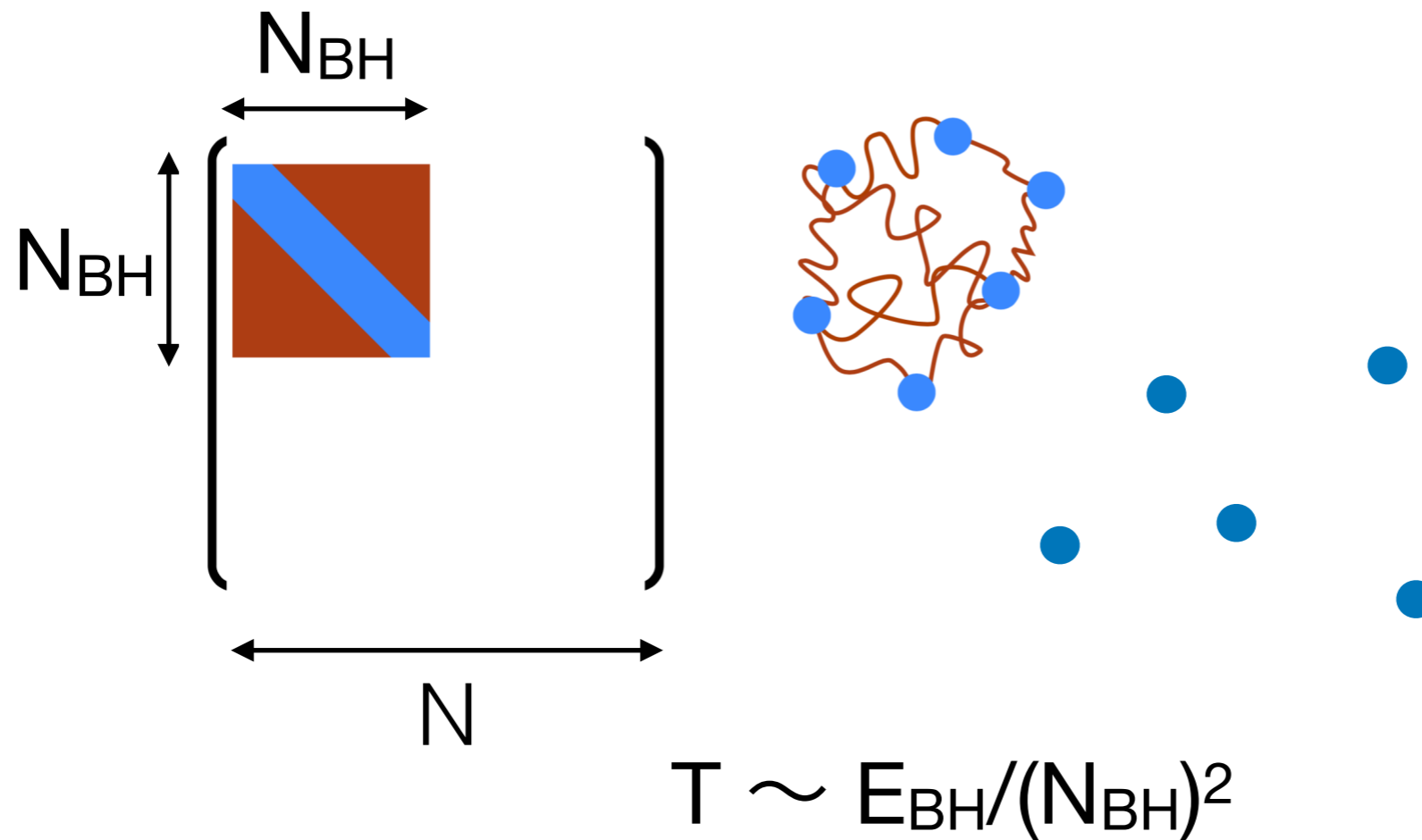


$$T \sim E/N^2$$

$$T' \sim E'/N^2$$

N^2 is fixed $\rightarrow T' > T$ if $E' > E$

Why can negative specific heat appear?



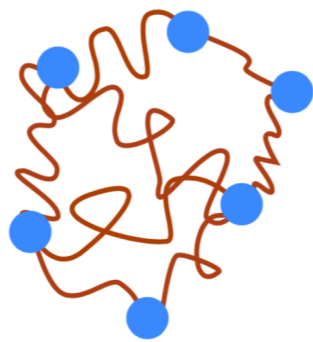
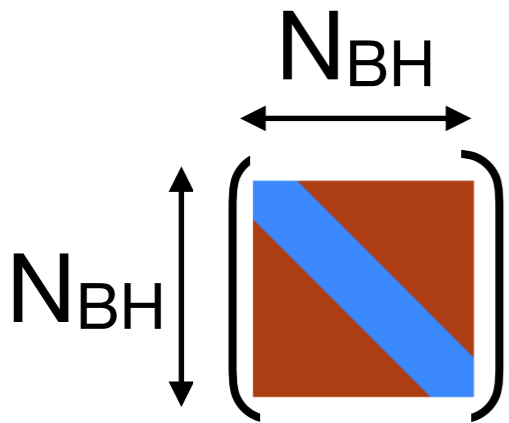
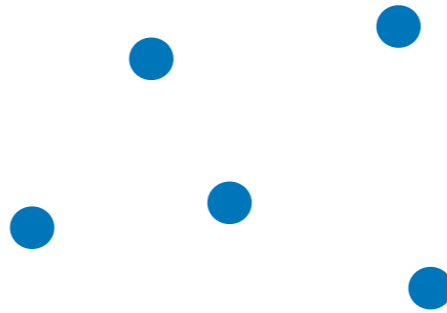
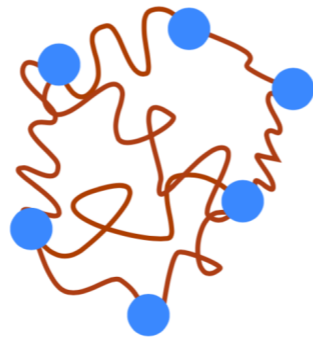
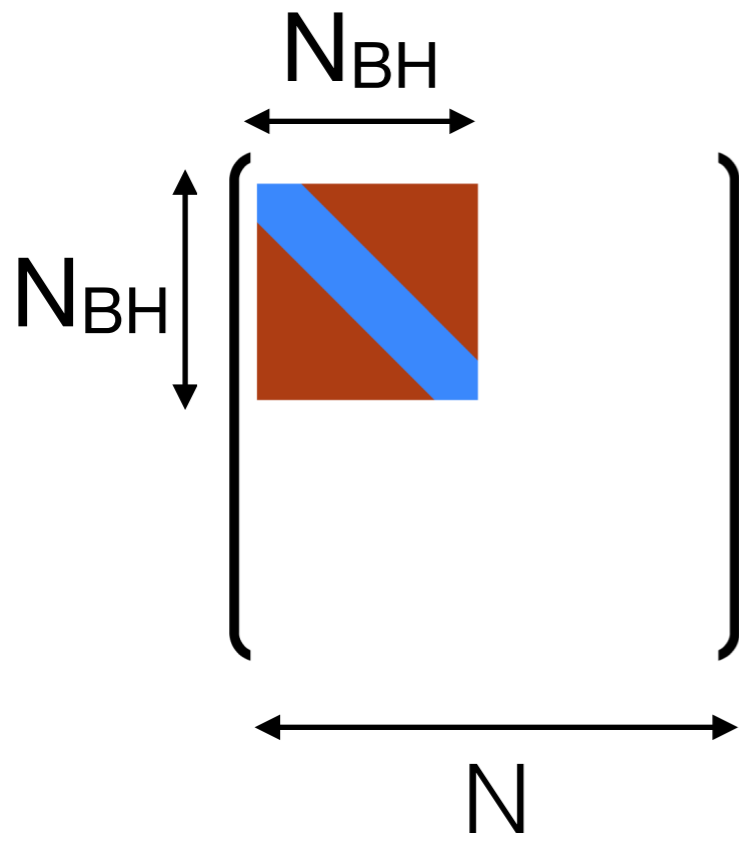
N_{BH} is a function of E_{BH}

Berkowitz-M.H.-Maltz, 2016, PRD (Higgsing version)

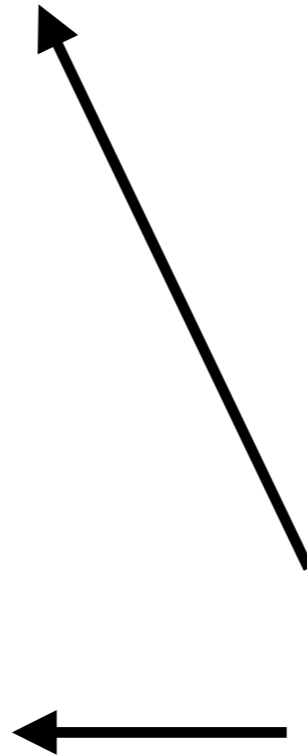
M.H.-Maltz, 2016, JHEP

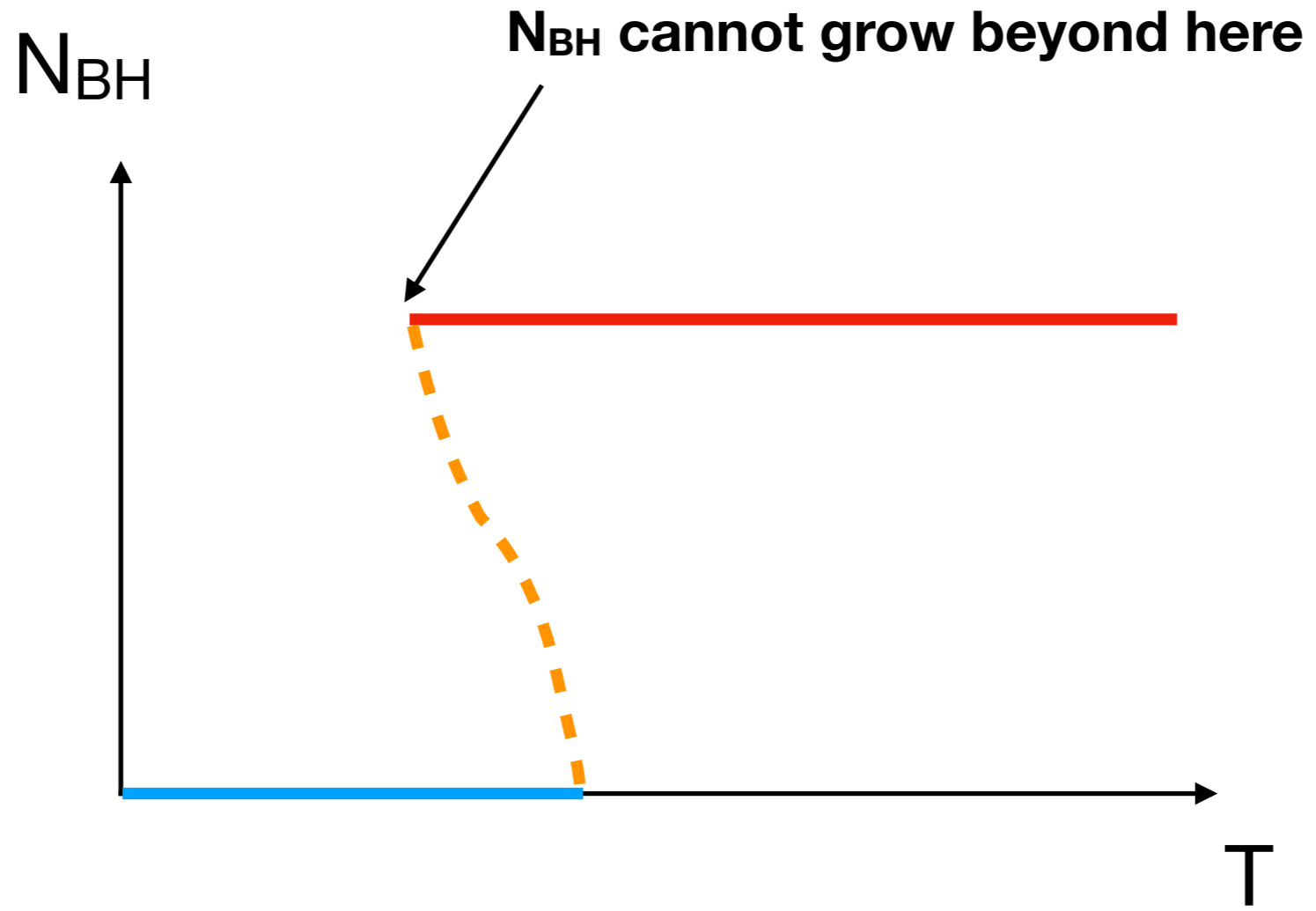
Explicit demonstration in simple theories

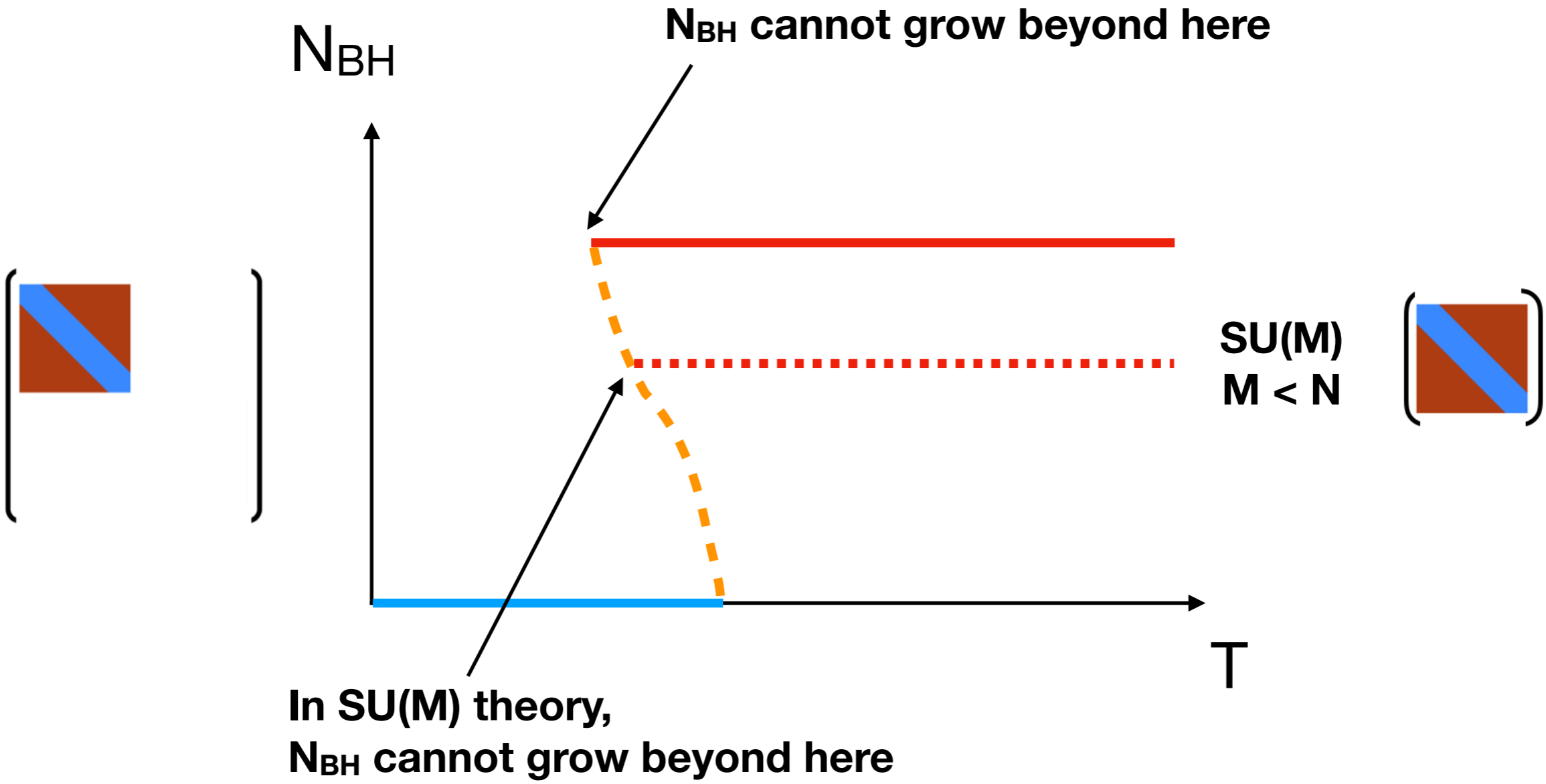
M.H., Jevicki, Peng, Wintergerst, 1909.09118 [hep-th]

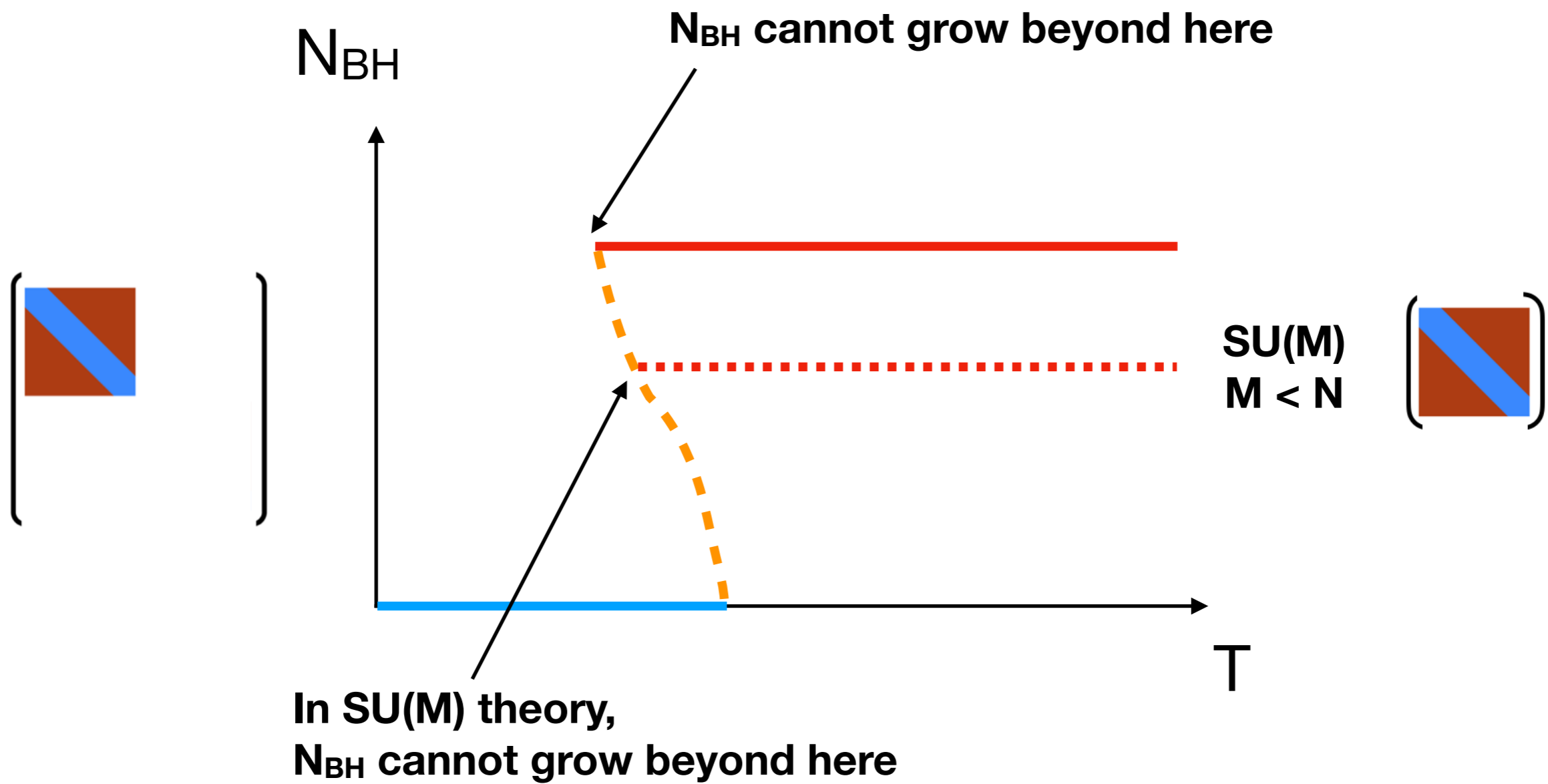


**Suppose the same result
is obtained from them.**









“Partial deconfinement \rightarrow complete deconfinement” in $SU(M)$ theory.

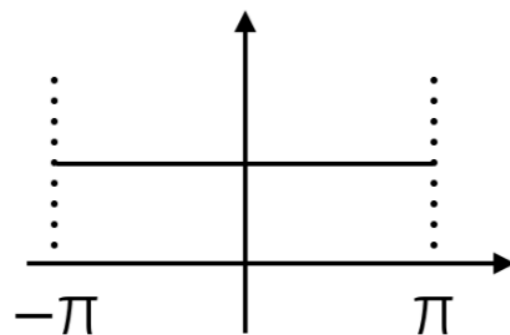
- Polyakov loop

$$P = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j}$$

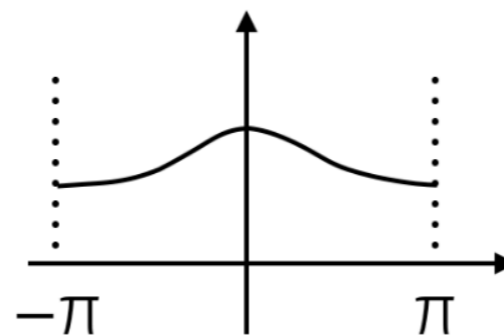
- Phase distribution:

Gross-Witten-Wadia transition (GWW)

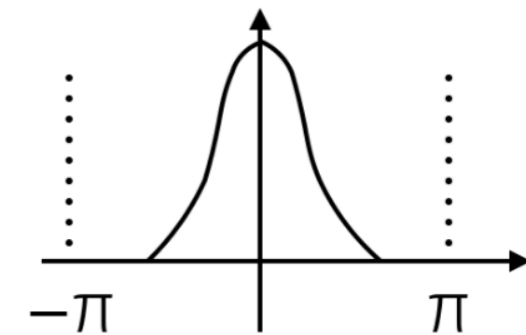
confined phase
P=0



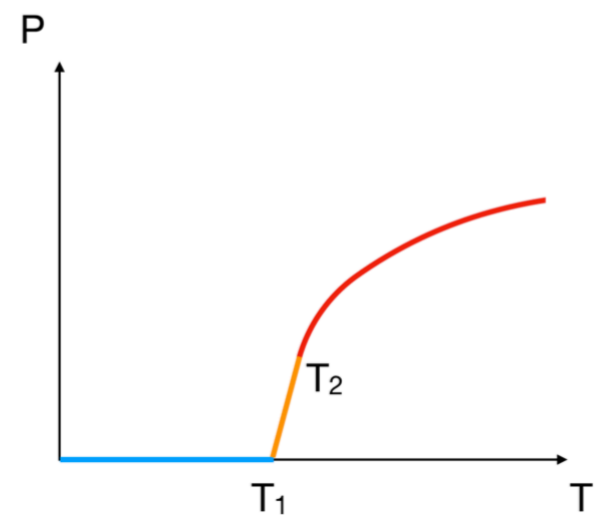
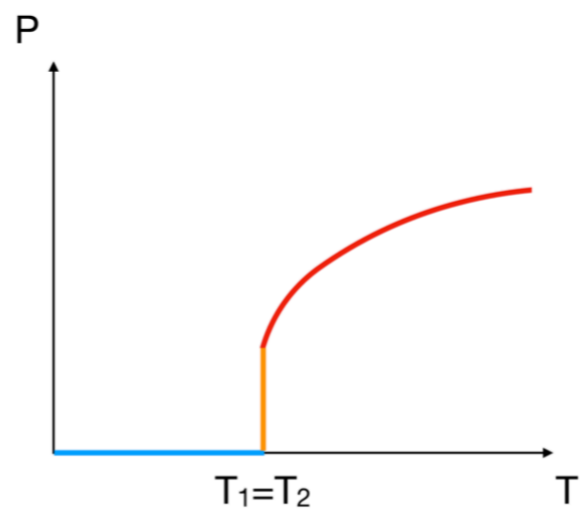
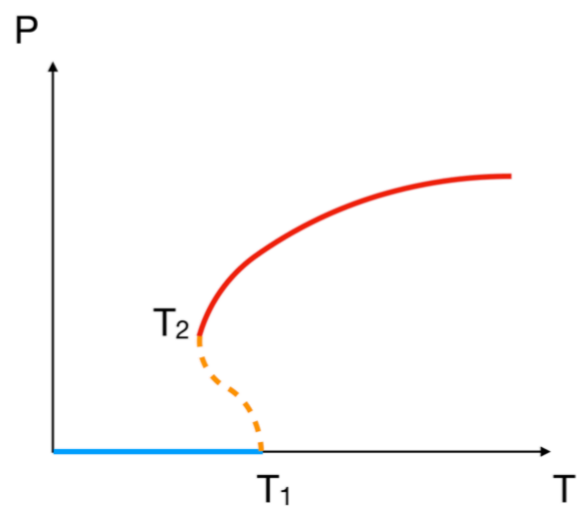
deconfined phase
P ≠ 0

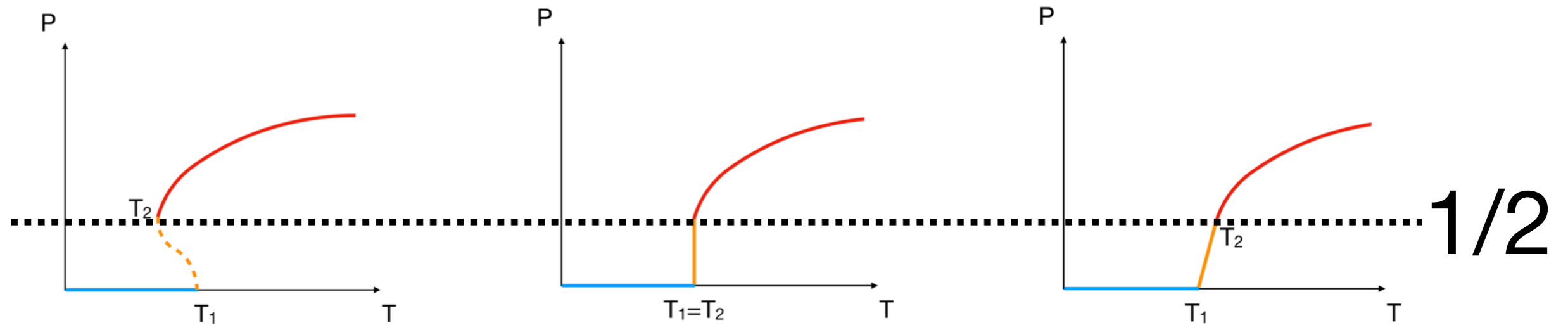


'partially' deconfined



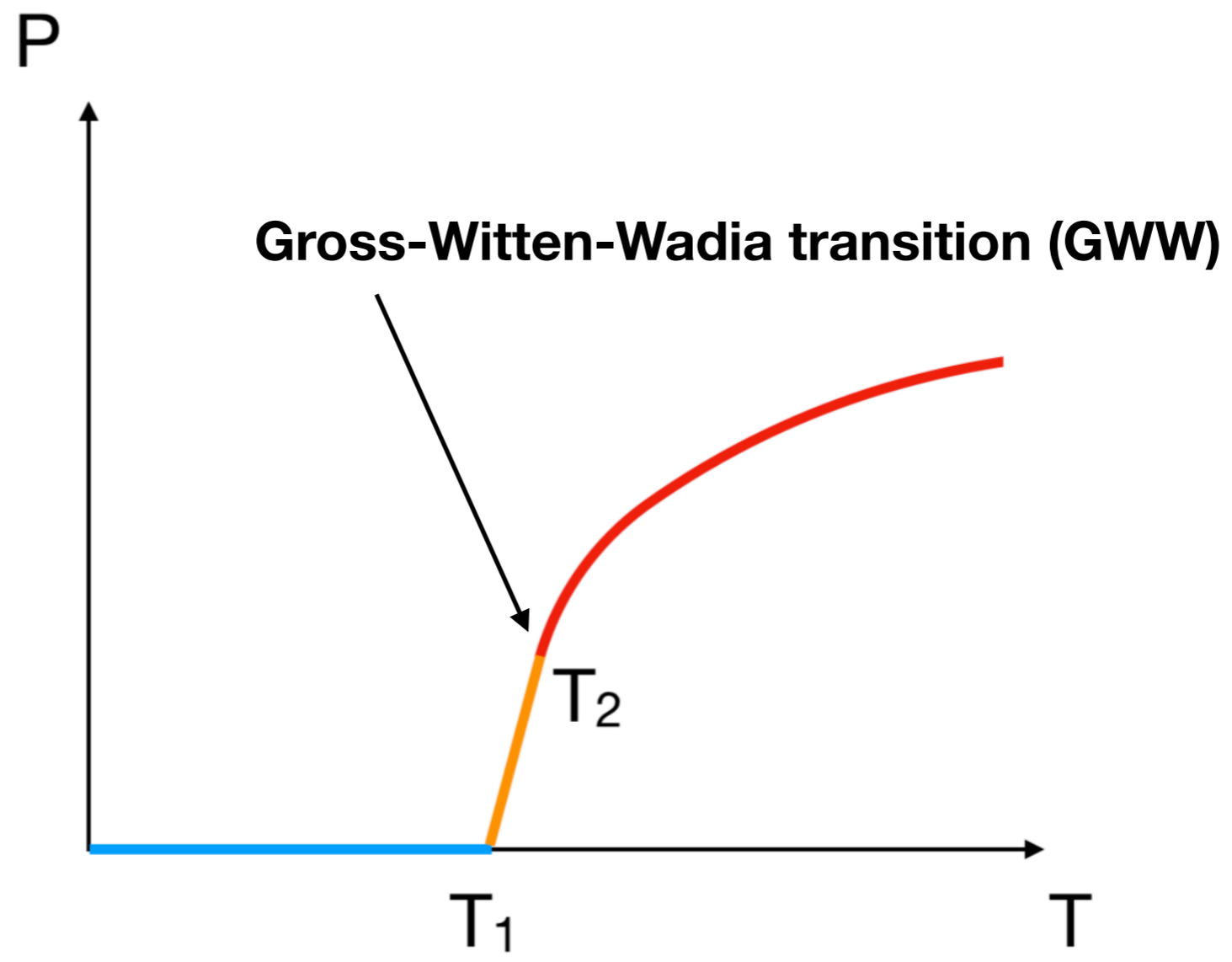
'completely' deconfined

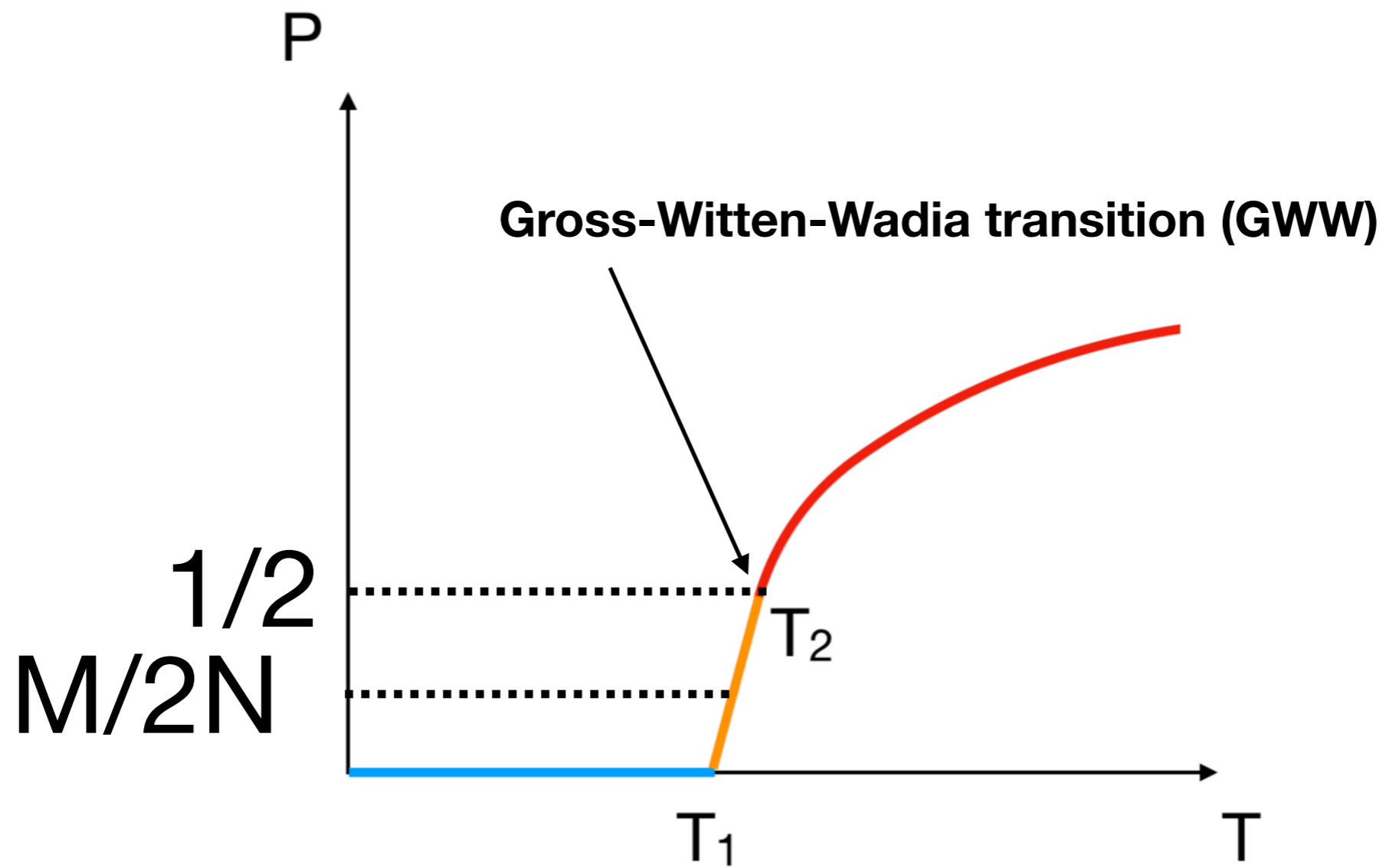


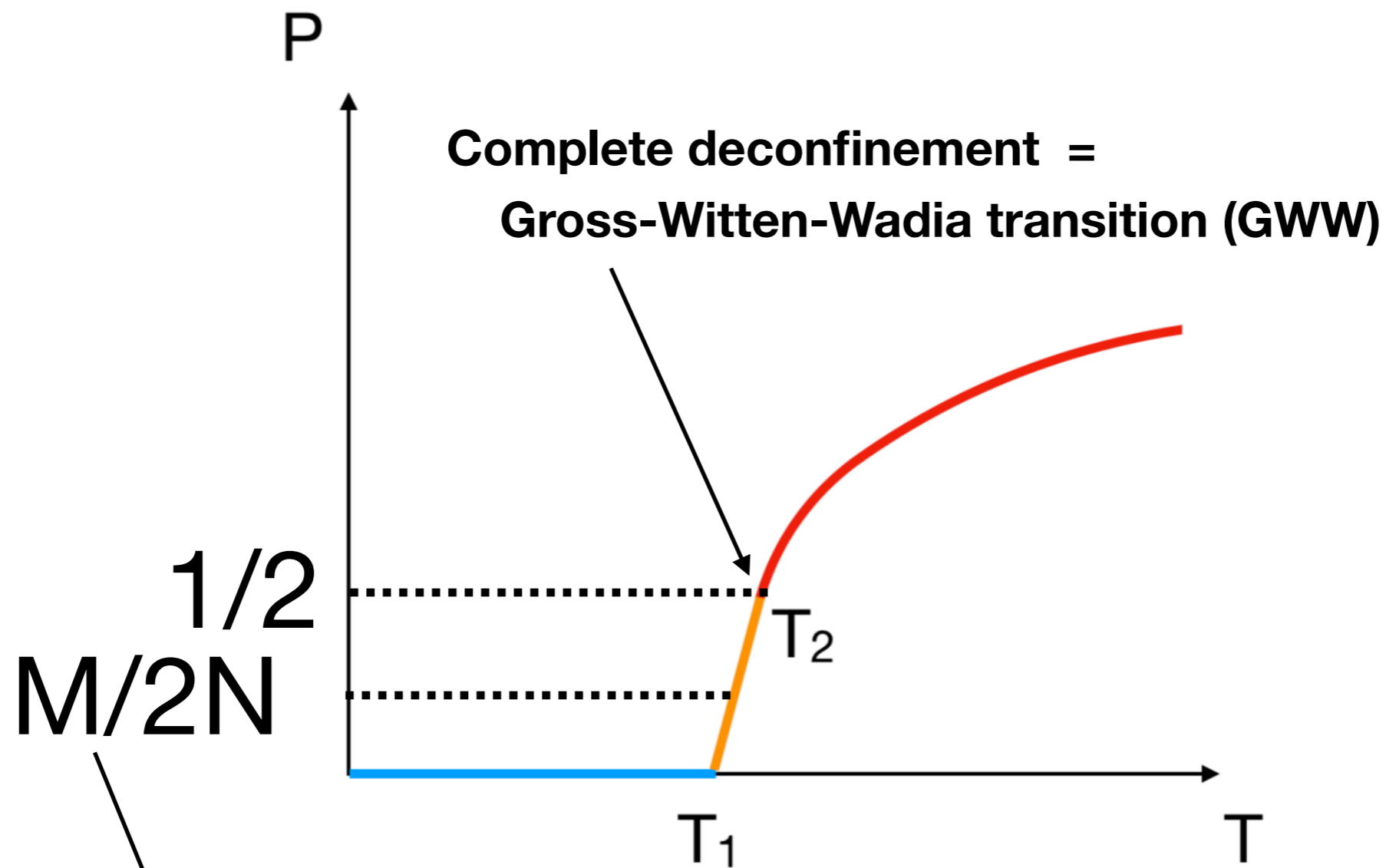


$$\begin{aligned} \rho(\theta) &= \left(1 - \frac{M}{N}\right) \rho_{\text{confine}}(\theta) + \frac{M}{N} \cdot \rho_{\text{GWW}}(\theta; M) \\ &= \frac{1}{2\pi} \left(1 - \frac{M}{N}\right) + \frac{M}{N} \cdot \rho_{\text{GWW}}(\theta; M). \end{aligned}$$

Holds in all examples we have studied.

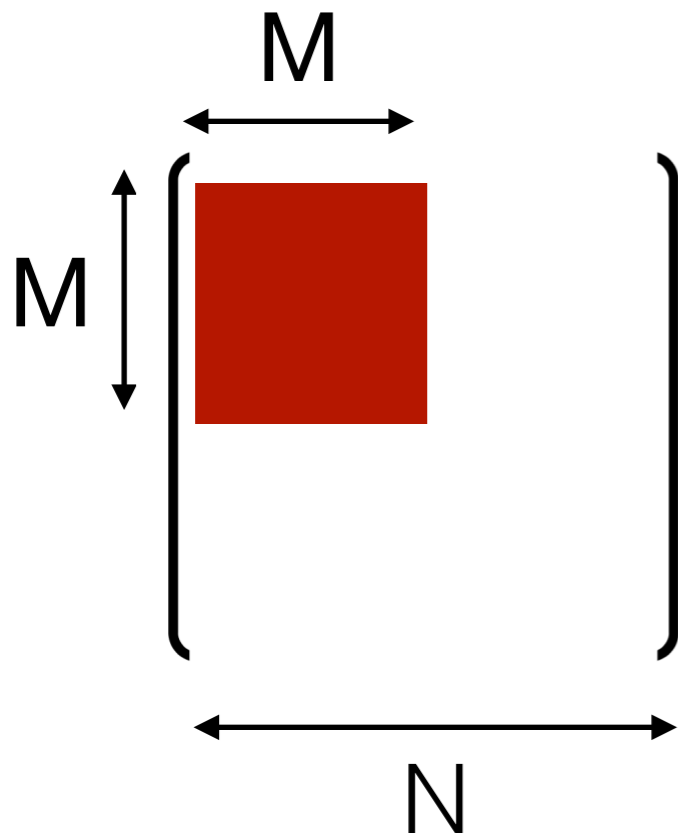






$$E = E_{\text{GWW}}(M)$$

$$S = S_{\text{GWW}}(M)$$

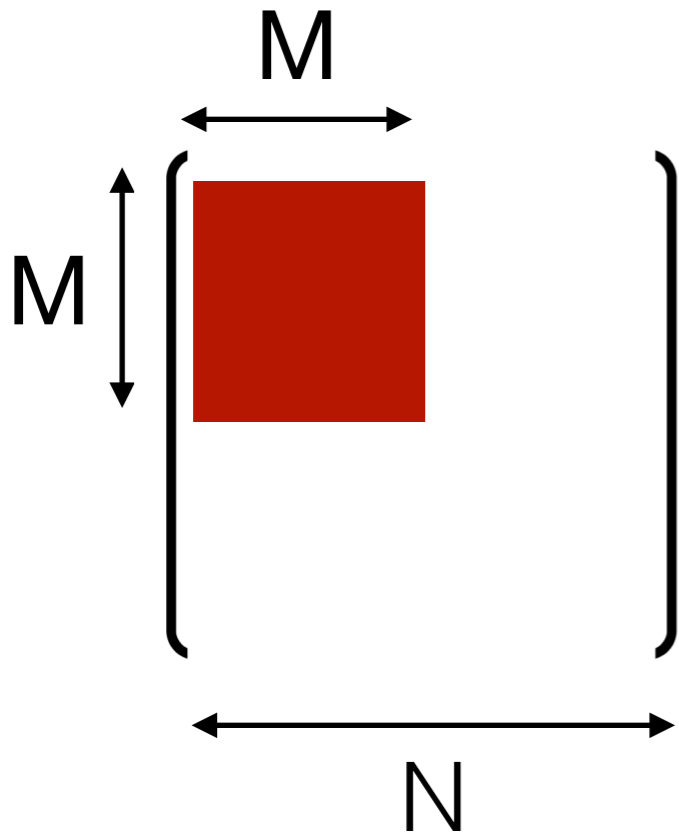


not $SU(N)$ -invariant

$$|E; SU(M)\rangle$$

At weak coupling, this is an energy eigenstate.

$$S = S_{\text{GWW}}(M)$$



not $SU(N)$ -invariant

$$|E; SU(M)\rangle$$

At weak coupling, this is an energy eigenstate.

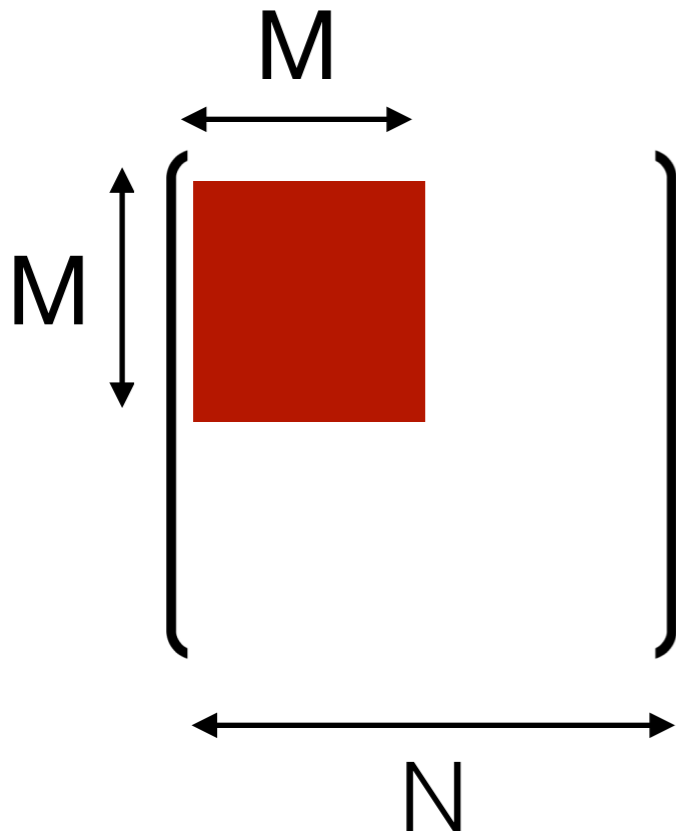
$$S = S_{\text{GWW}}(M)$$

one-to-one correspondence

$SU(N)$ -invariant

$$|E\rangle_{\text{inv}} \equiv \mathcal{N}^{-1/2} \int dU \mathcal{U} (|E; SU(M)\rangle)$$

This is also an energy eigenstate.



not $SU(N)$ -invariant

$$|E; SU(M)\rangle$$

At weak coupling, this is an energy eigenstate.

$$S = S_{\text{GWW}}(M)$$

one-to-one correspondence

$SU(N)$ -invariant

$$|E\rangle_{\text{inv}} \equiv \mathcal{N}^{-1/2} \int dU \mathcal{U} (|E; SU(M)\rangle)$$

This is also an energy eigenstate.

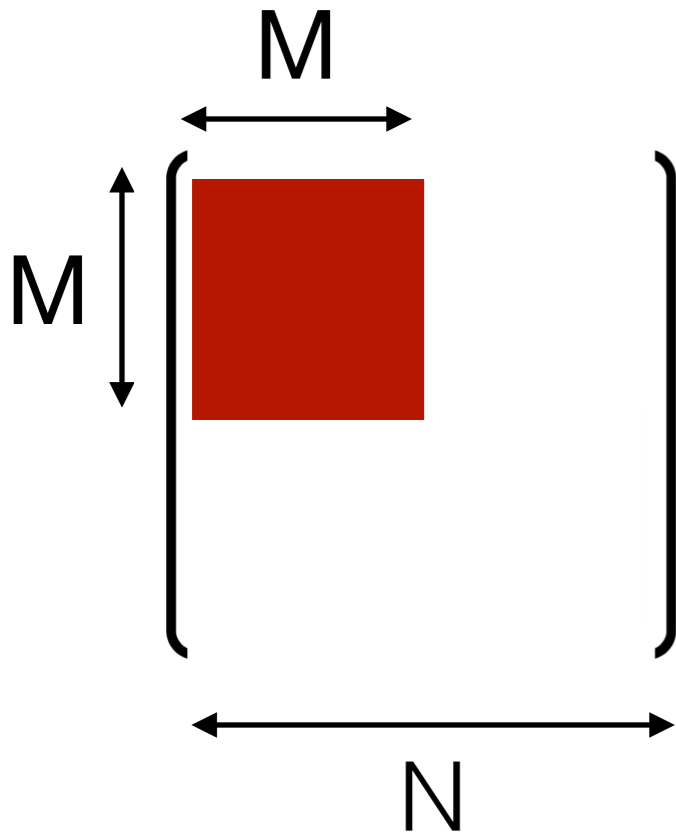
These states explain the entropy precisely.

(Analytic calculation doable for weakly coupled QCD on S^3 , $O(N)$ vector model, matrix model)

'Spontaneous gauge symmetry breaking'

SU(N)-invariant

$$|E\rangle_{\text{inv}} \equiv \mathcal{N}^{-1/2} \int dU \mathcal{U} (|E; \text{SU}(M)\rangle)$$

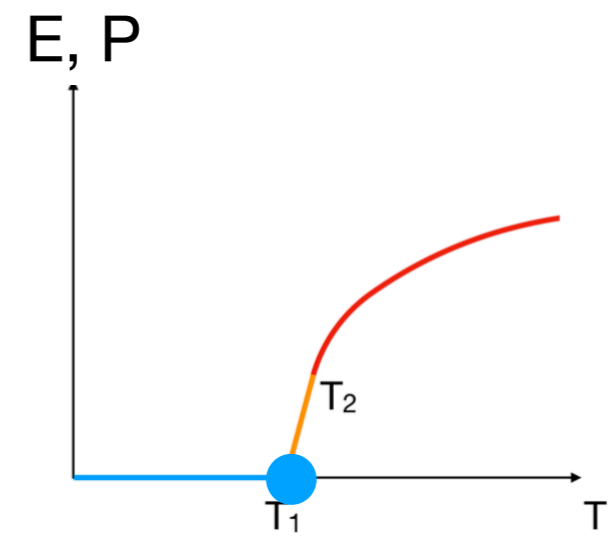
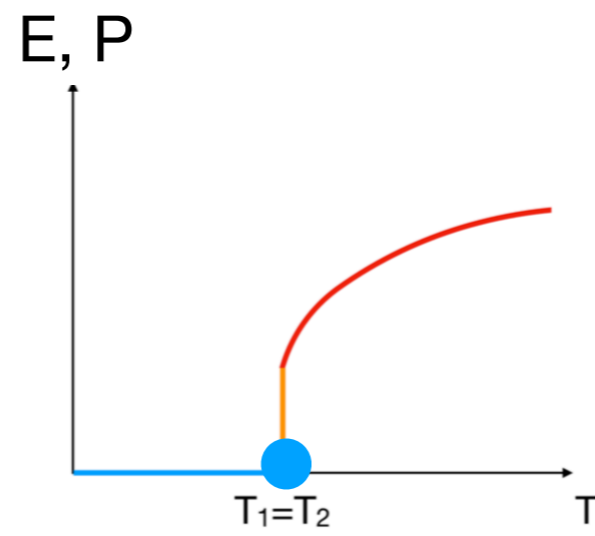
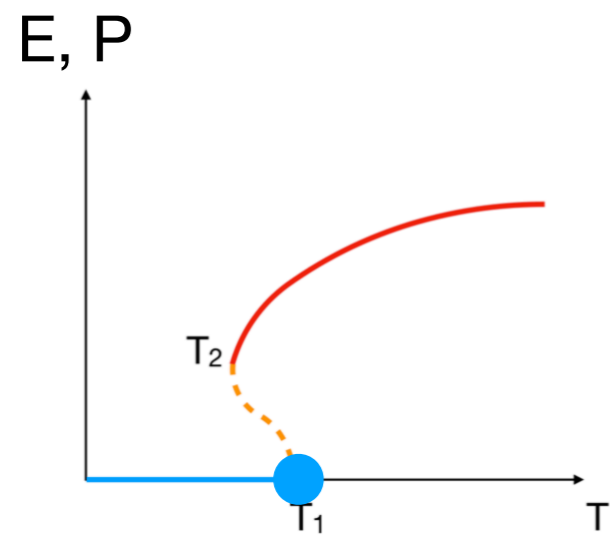
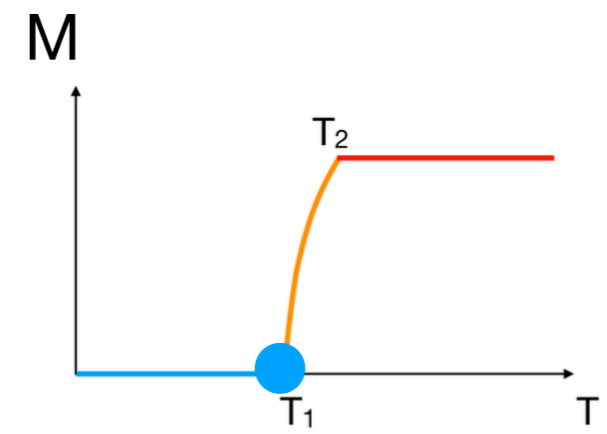
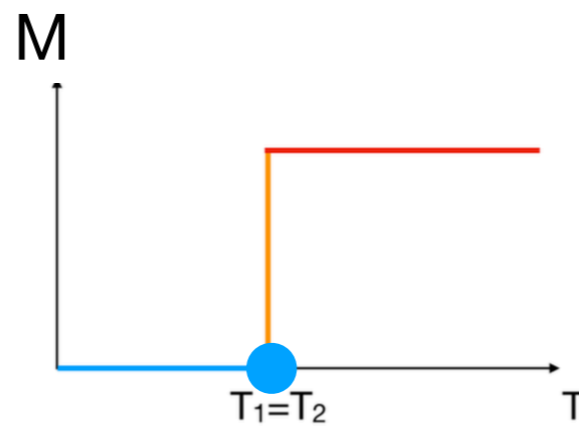
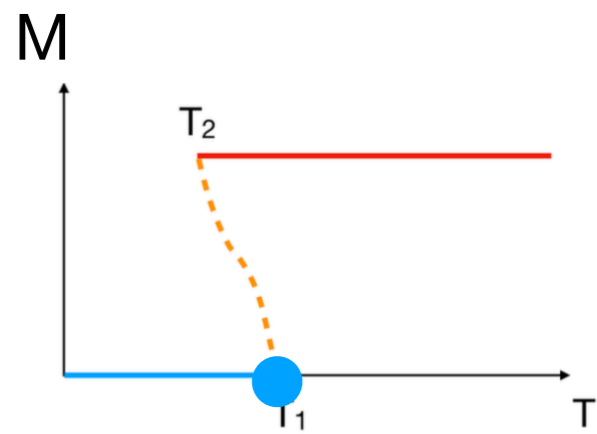


gauge fixing

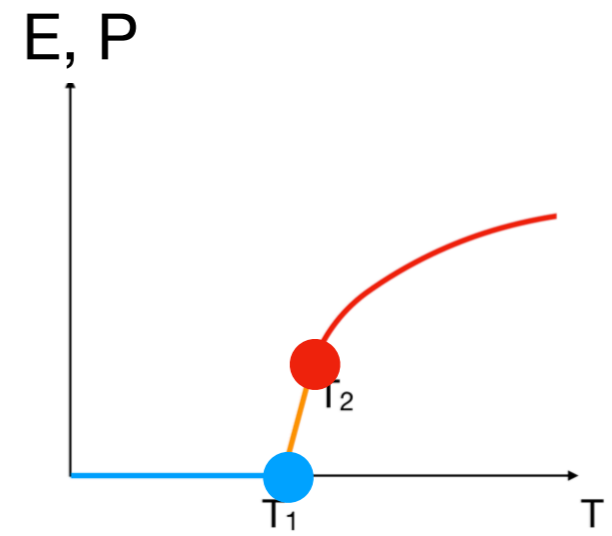
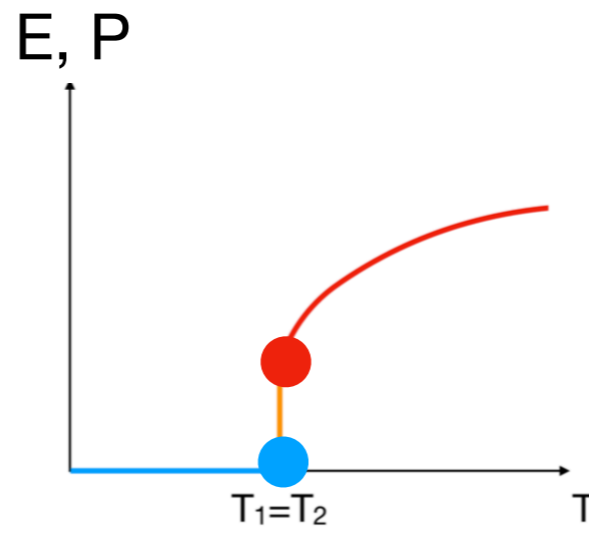
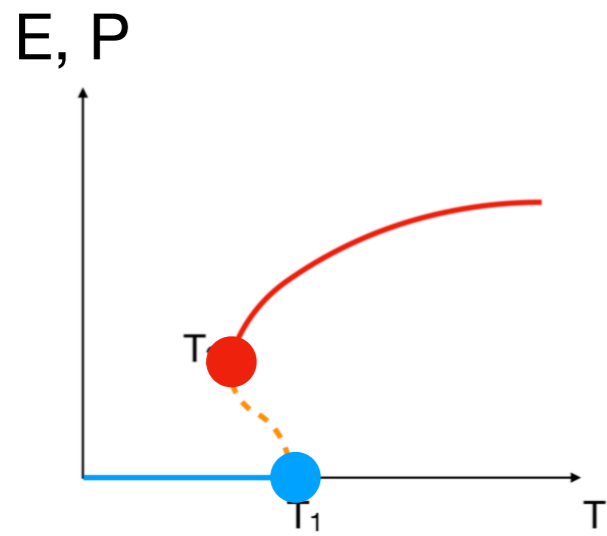
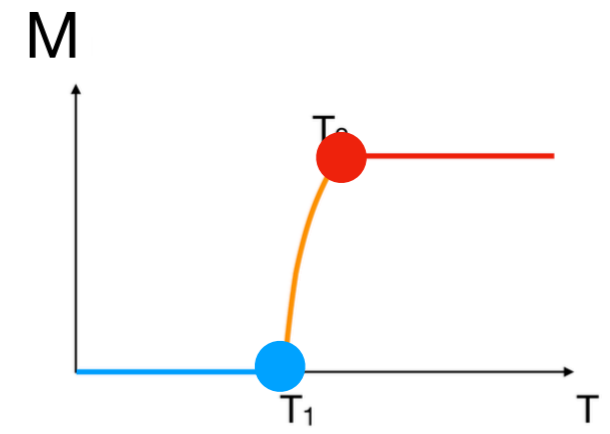
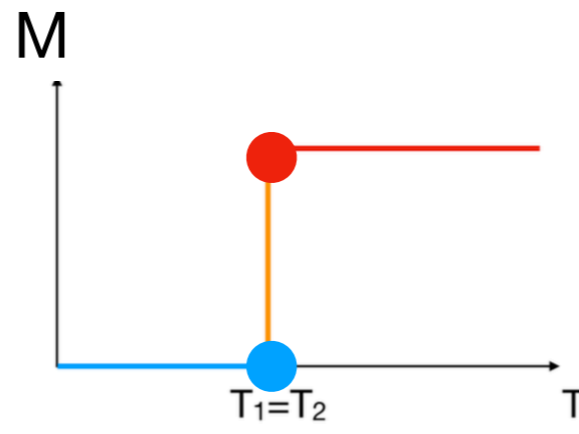
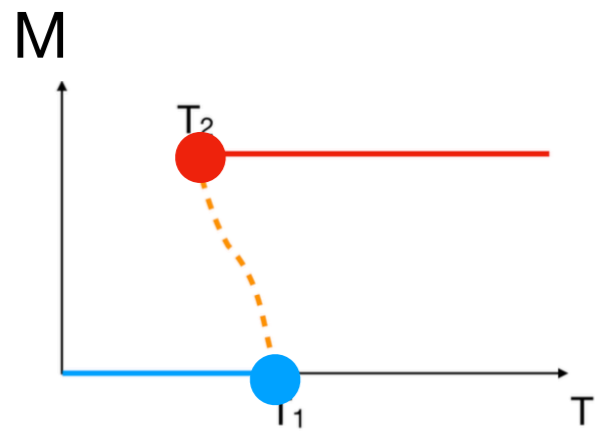
not SU(N)-invariant

$$|E; \text{SU}(M)\rangle$$

- Global part of gauge symmetry breaks spontaneously.
- It is convenient to fix the local part, like usual Higgsing.
- ‘Gauge symmetry breaking’ provides us with a ‘useful fiction’ which makes physics understandable.

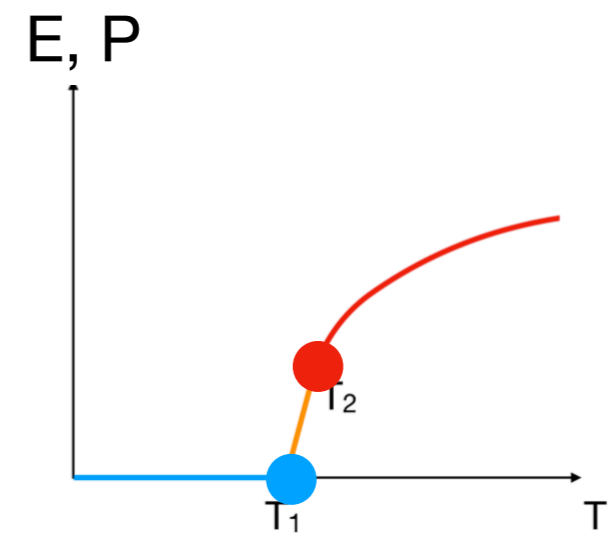
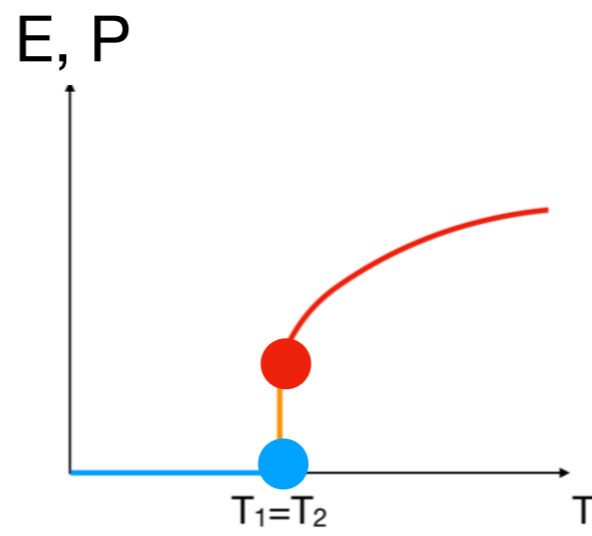
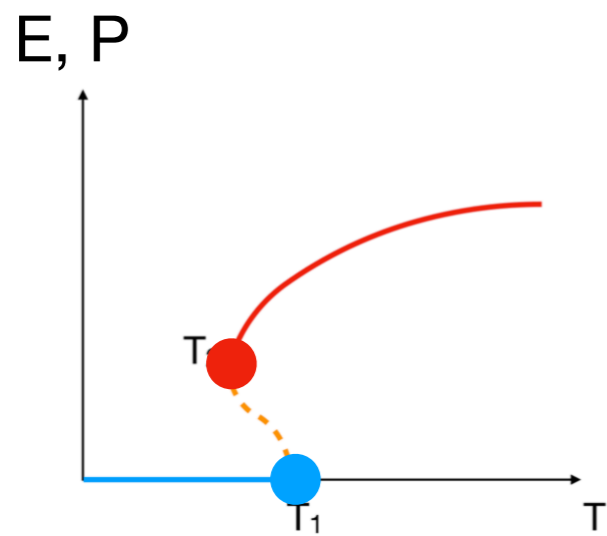
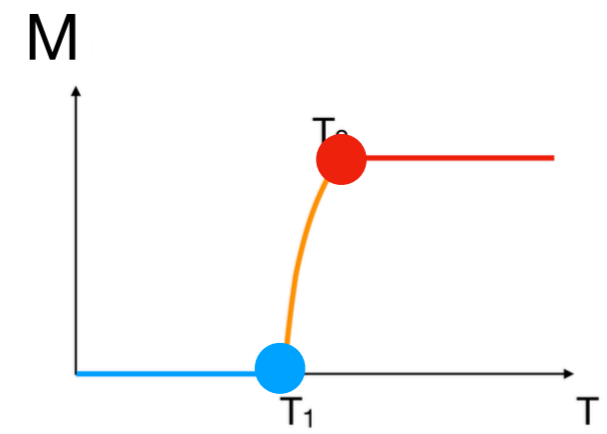
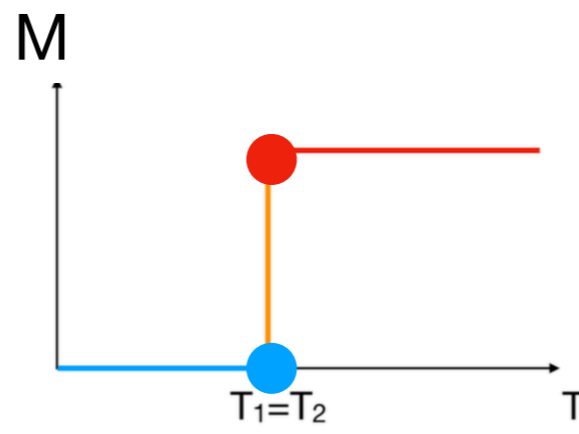
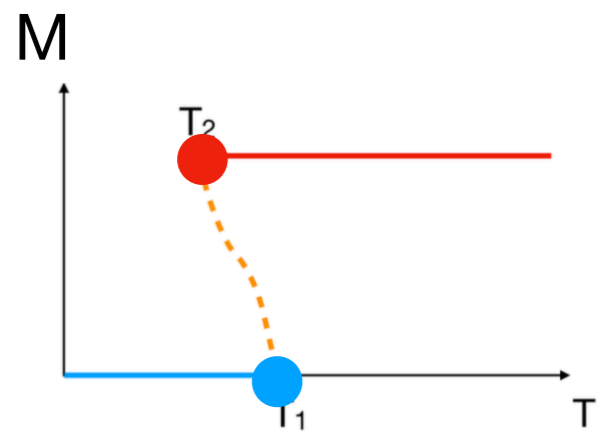


transition 1: confinement to partial deconfinement
(black hole formation begins)



transition 1: confinement to partial deconfinement
(black hole formation begins)

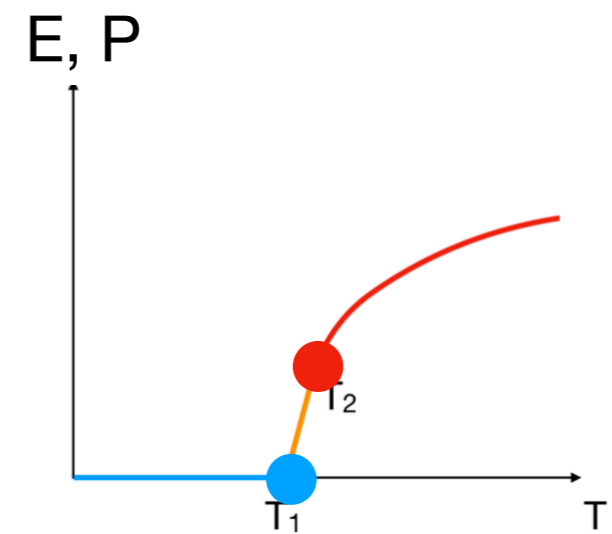
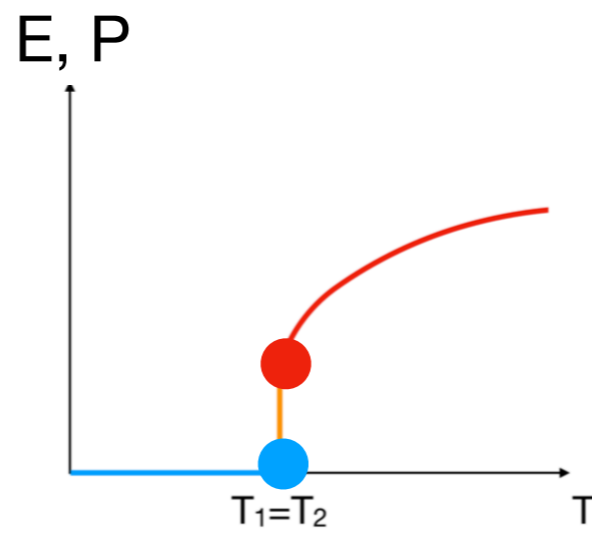
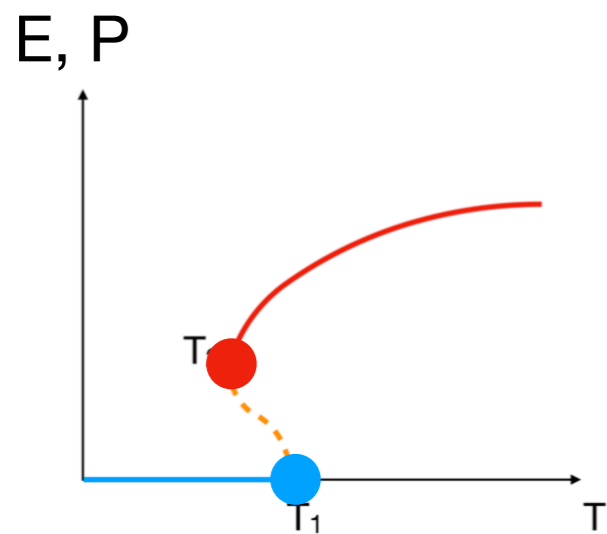
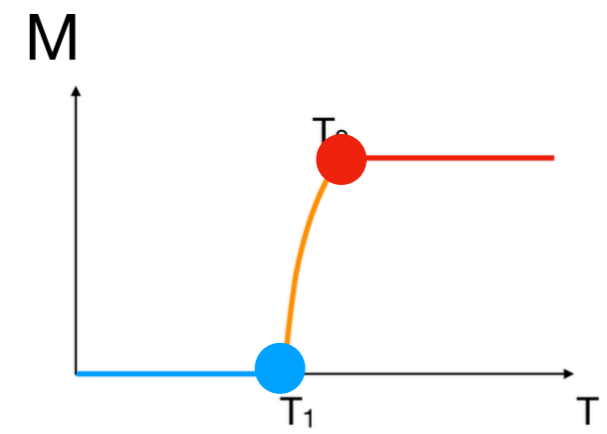
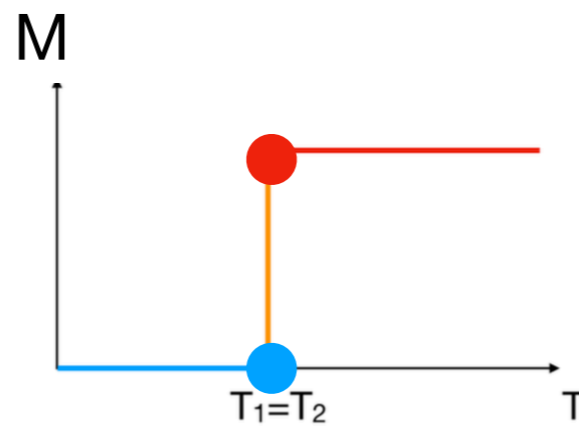
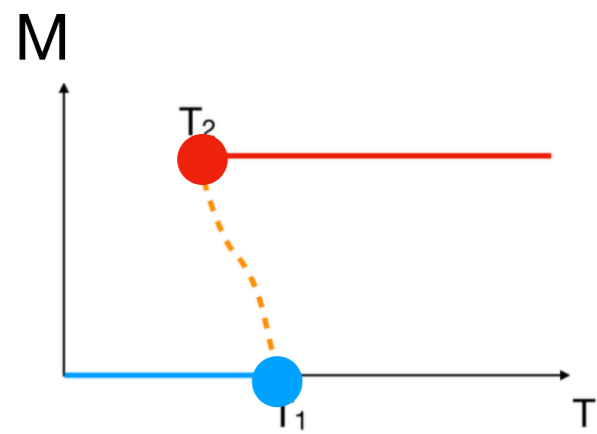
transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)



transition 1: confinement to partial deconfinement
(black hole formation begins)

transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)

$$\text{SU}(N) \rightarrow \text{SU}(M) \times \text{SU}(N-M) \times \text{U}(1) \rightarrow \text{SU}(N)$$



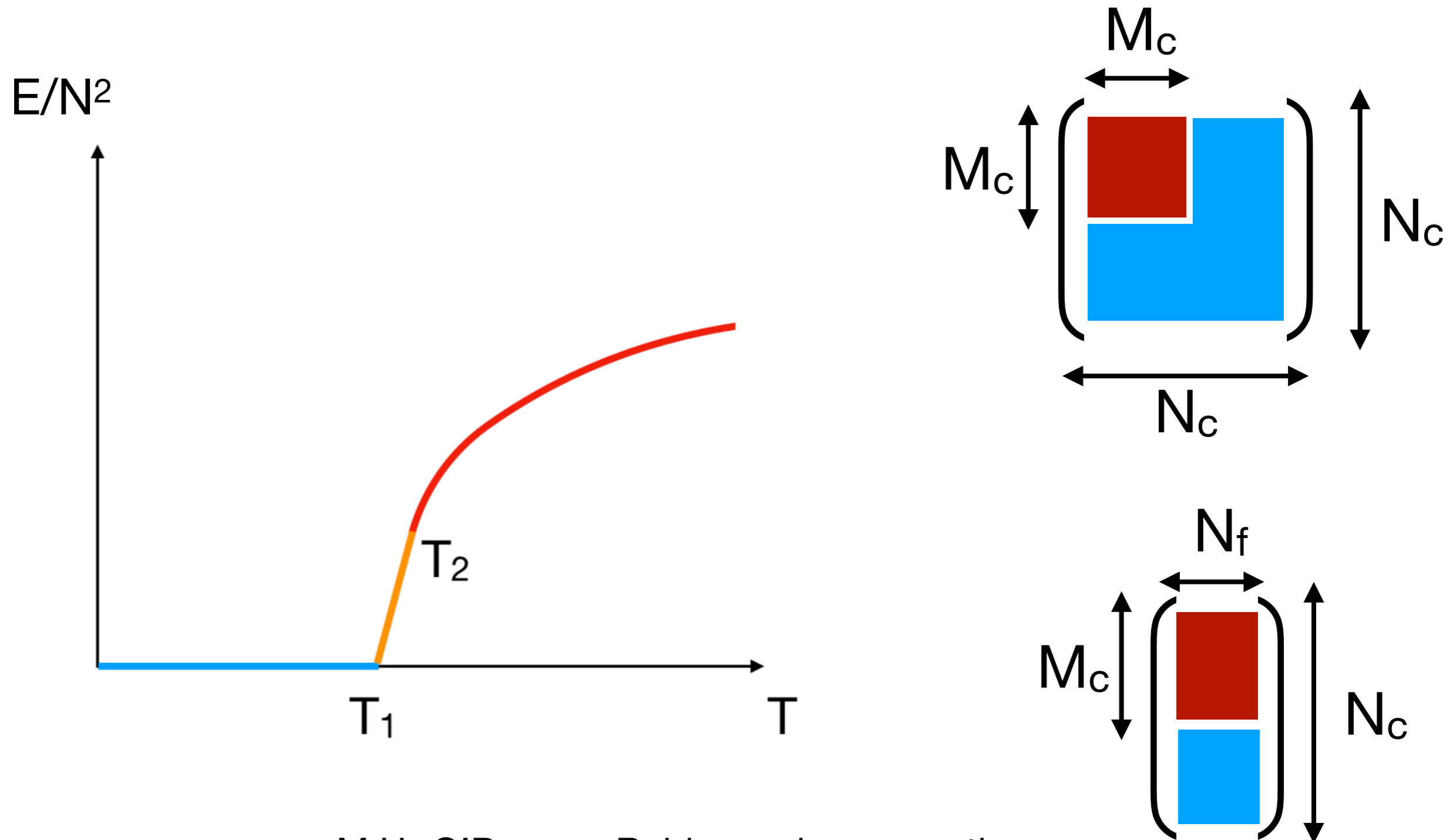
transition 1: confinement to partial deconfinement
(black hole formation begins)

transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)

$$\text{SU}(N) \rightarrow \text{SU}(M) \times \text{SU}(N-M) \times \text{U}(1) \rightarrow \text{SU}(N)$$

No need for center symmetry. No need for chiral symmetry. Applies to QCD.

QCD at zero chemical potential



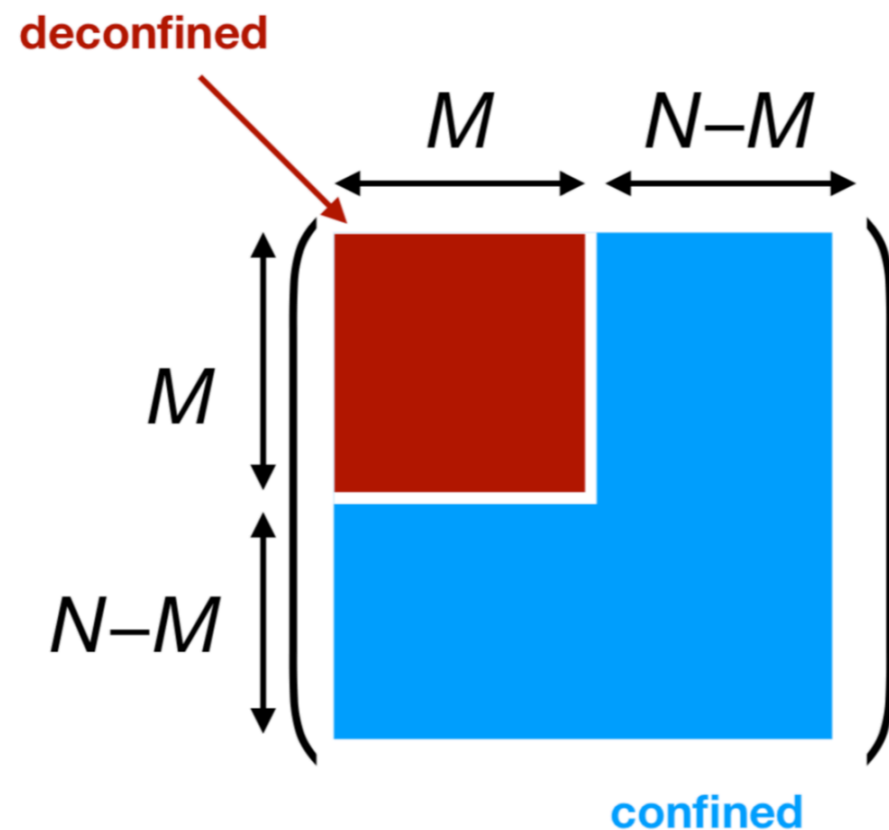
M.H.-O'Bannon-Robinson, in preparation

Quantum Entanglement

between **color** d.o.f.

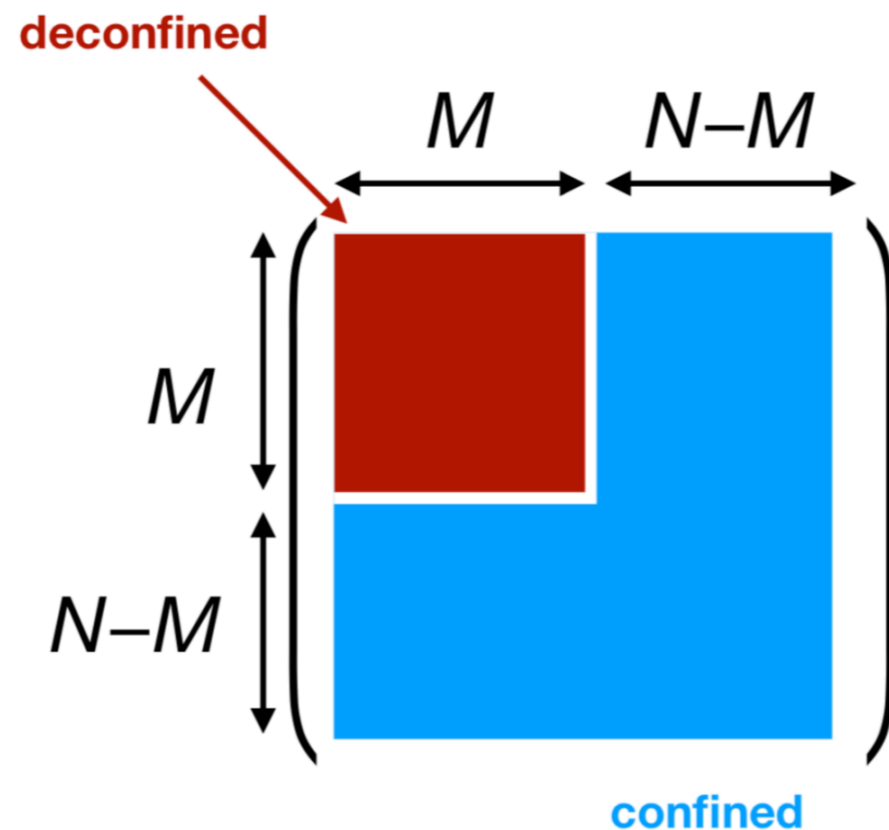
- Typically, ground state of interacting system is highly entangled.
- Thermal excitations can destroy the entanglement.

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- Thermal excitations can destroy the entanglement.



Confined \rightarrow ground state up to $1/N$ corrections

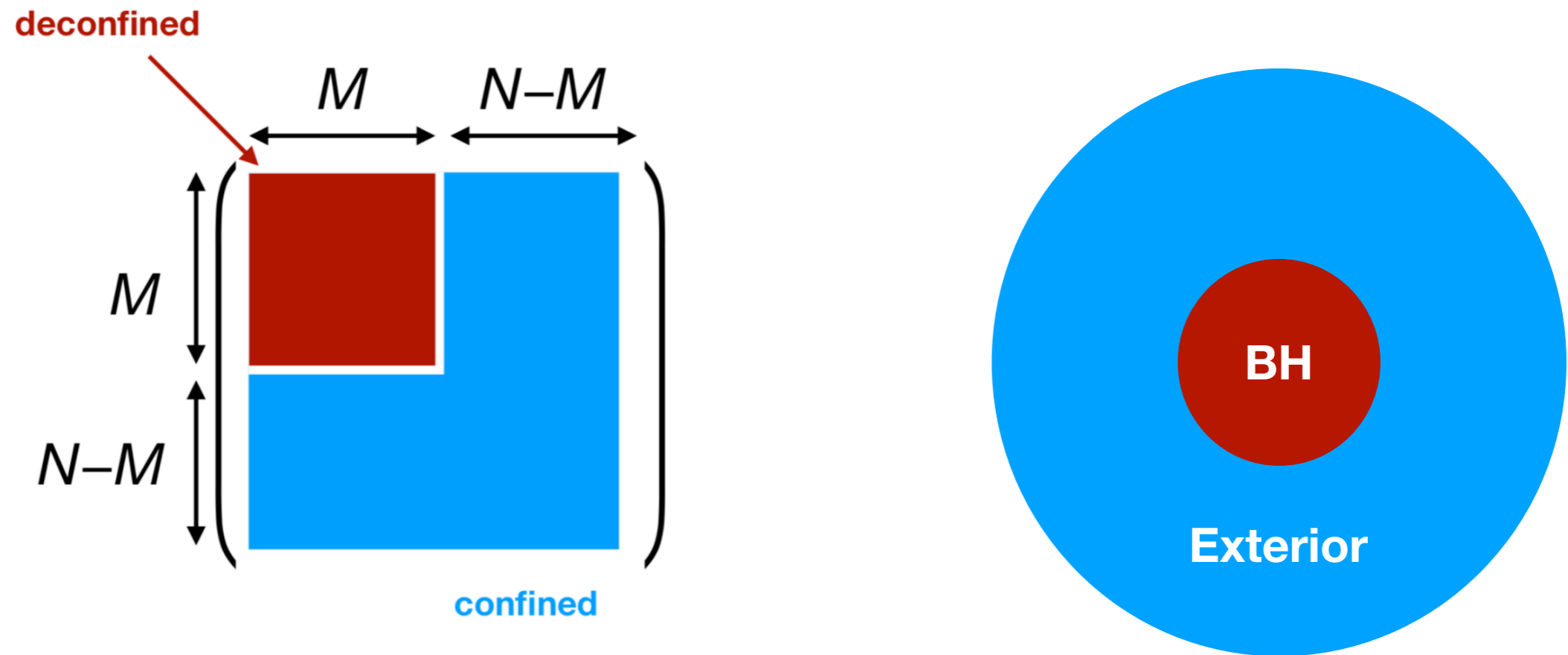
- Typically, ground state of interacting system is highly entangled.
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Confined \rightarrow ground state up to $1/N$ corrections

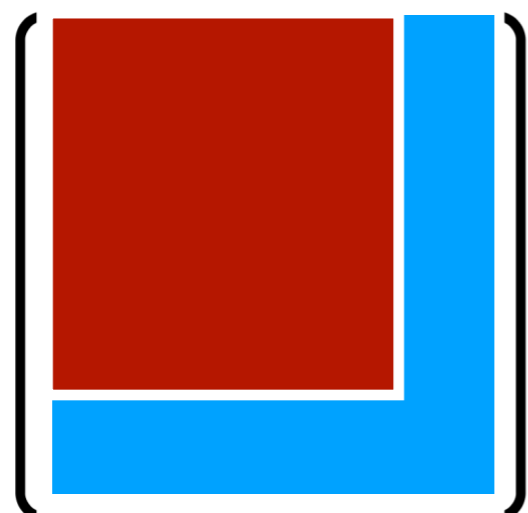
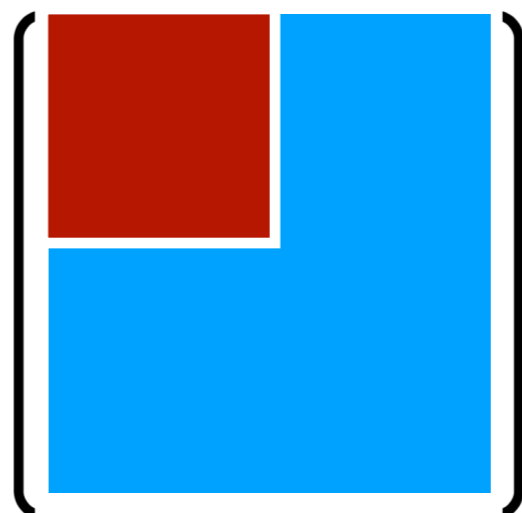
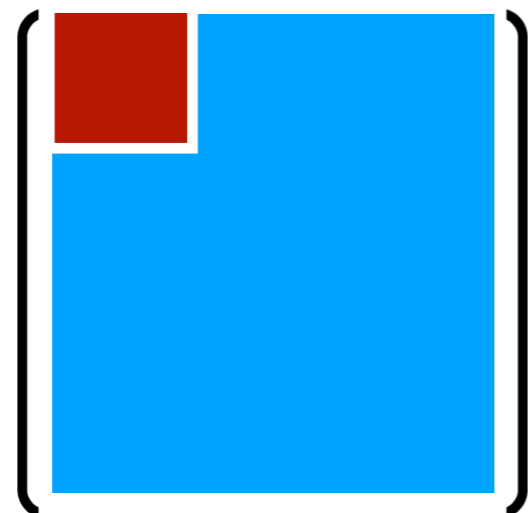
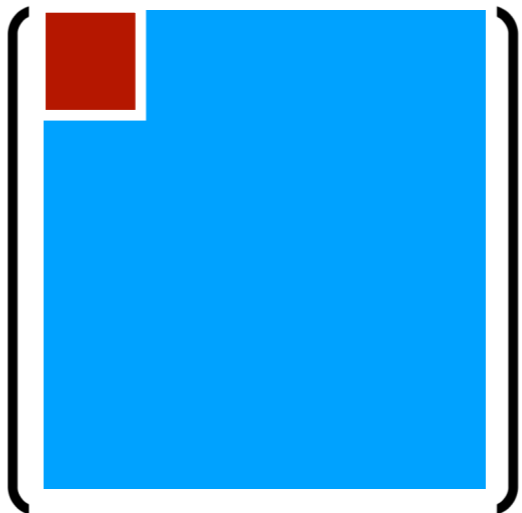
Large entanglement can survive even at finite temperature.

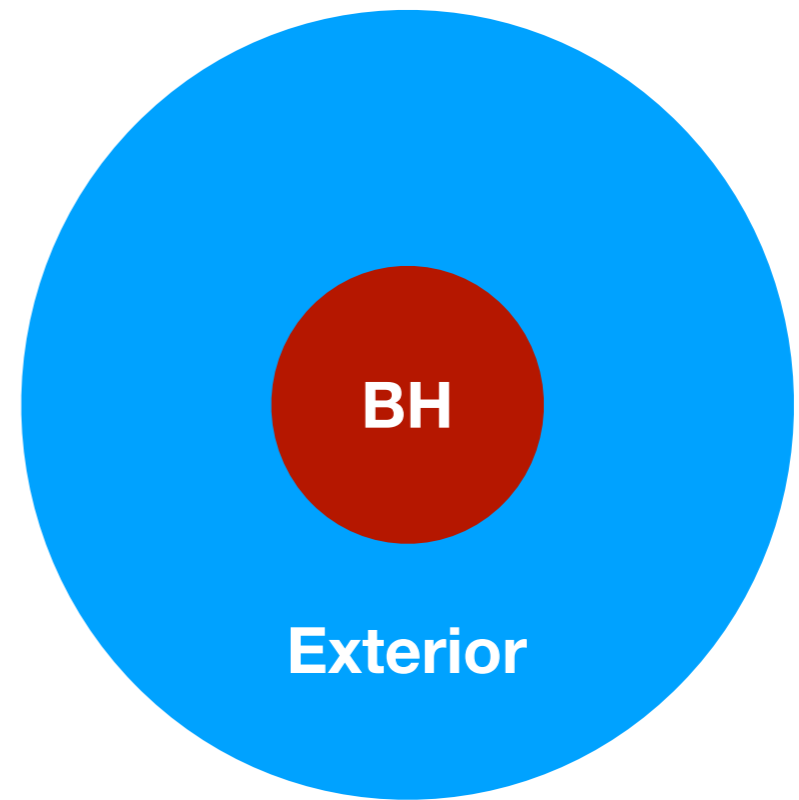
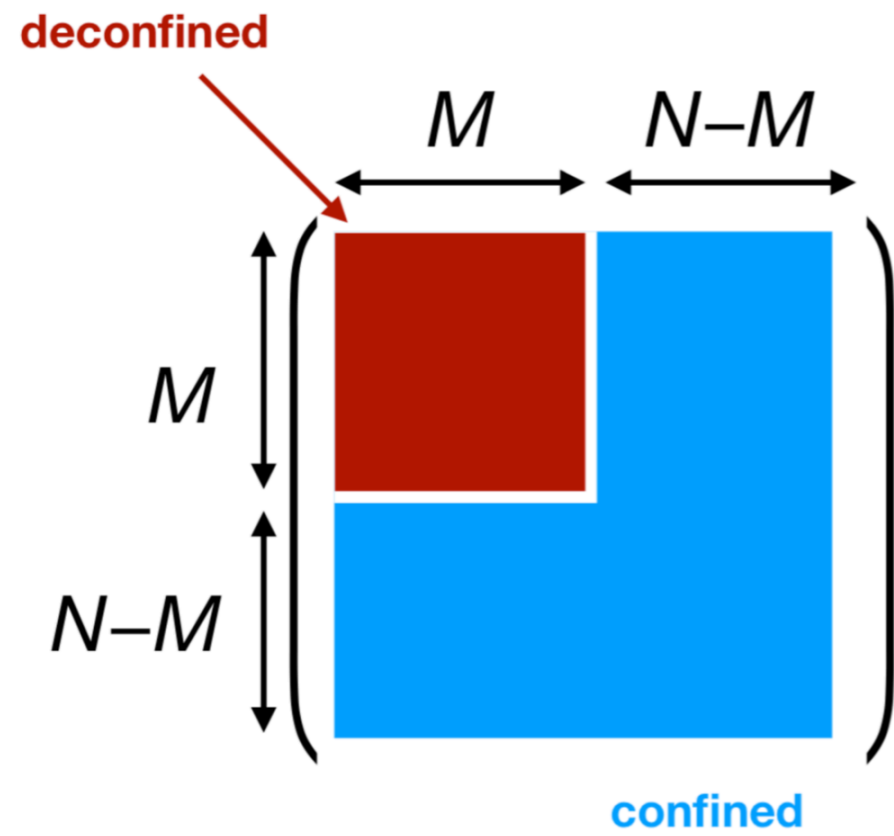
gauge/gravity duality

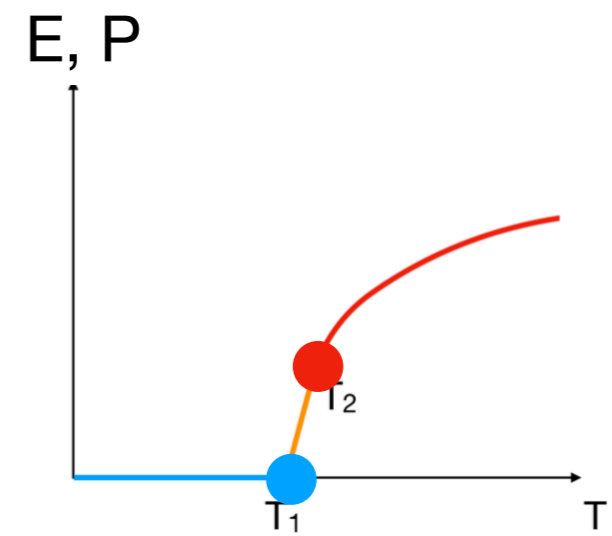
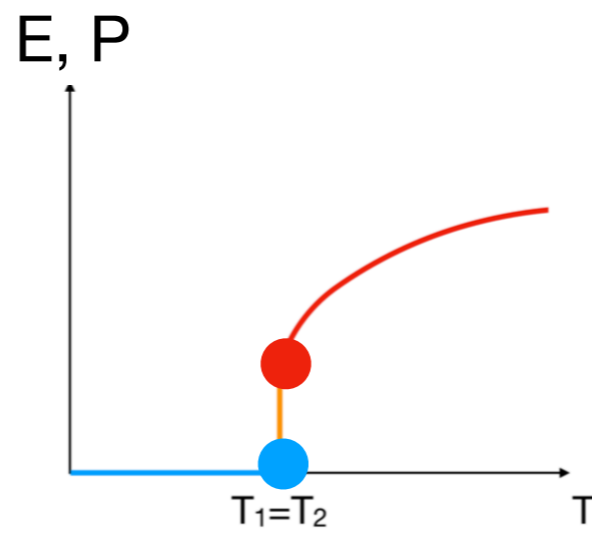
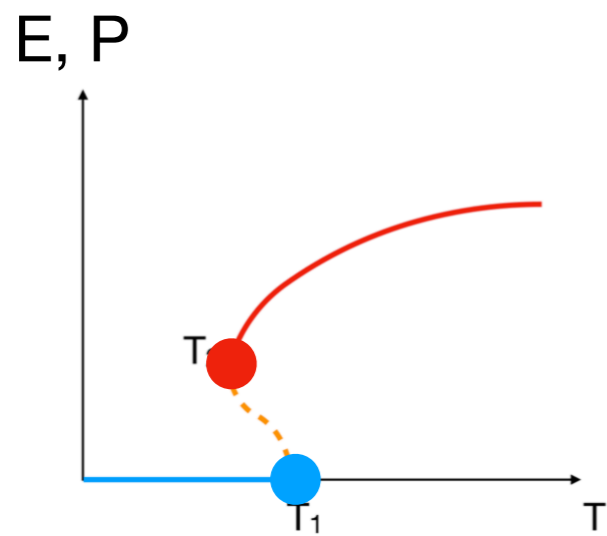
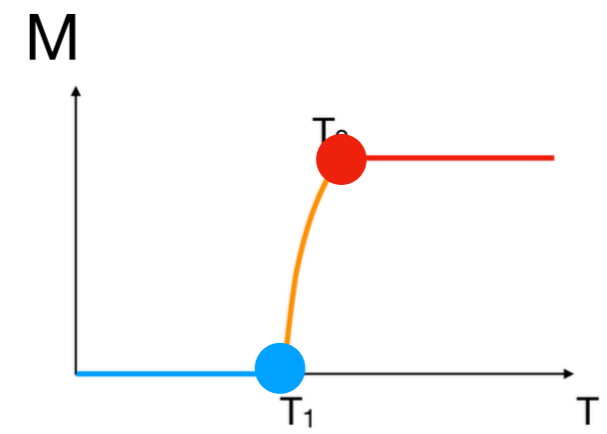
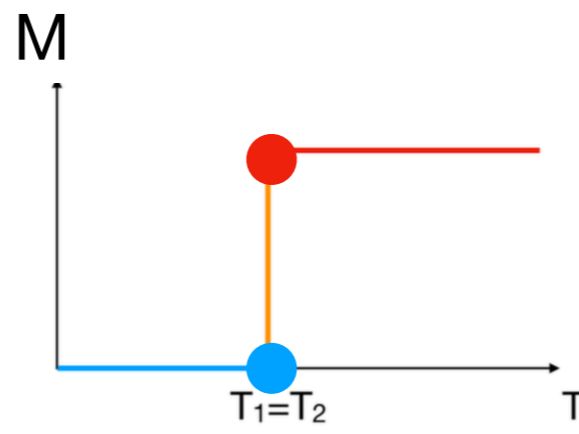
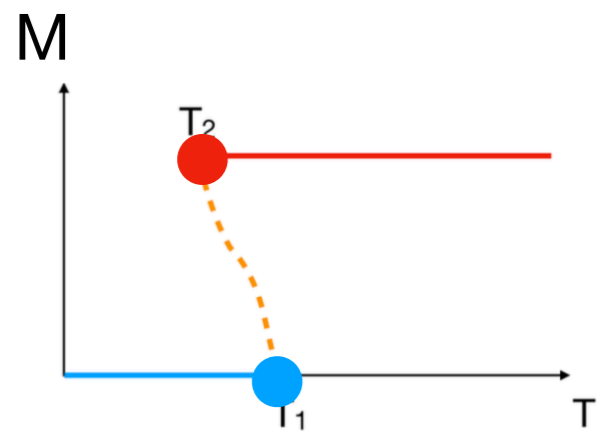


Entanglement between color d.o.f.
→ geometry outside the horizon?

Summary







transition 1: confinement to partial deconfinement
(black hole formation begins)

transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)

$$\text{SU}(N) \rightarrow \text{SU}(M) \times \text{SU}(N-M) \times \text{U}(1) \rightarrow \text{SU}(N)$$

No need for center symmetry. No need for chiral symmetry. Applies to QCD.