LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Spontaneous conformal symmetry breaking in fishnet CFT

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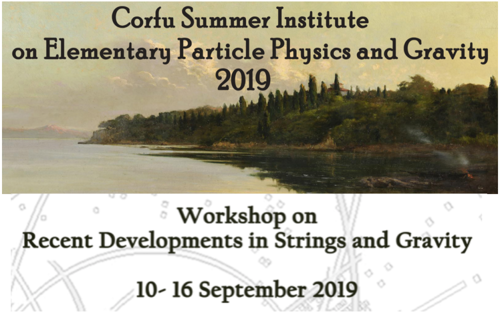
work with V. Kazakov and M. Shaposhnikov, 1908.04302 [hep-th]



Outline

Introduction and Motivation

Intoduction and (phenomenological) motivation

The Standard Model (SM) of particle physics is THE success story

- Its last missing piece, the Higgs boson was observed a few years ago
- So far no convincing deviations from the SM have been observed at particle physics experiments
- Moreover, the SM could be a self-consistent effective field theory up to very high energies ($\sim M_P$)

Intoduction and (phenomenological) motivation

Do we have in our hands the final theory of Nature!?

Intoduction and (phenomenological) motivation

Do we have in our hands the final theory of Nature!?

Compelling indications that the answer is negative!

Intoduction and (phenomenological) motivation

Theoretical point of view

The SM suffers from

- Landau Pole(s) associated with $U(1)$ & Higgs sectors, but **@** energies $\gg M_{Planck}$, so usually swept under the "quantum gravity carpet"!
- Strong-CP problem
- Cosmological Constant problem
- Hierarchy problem (smallness of Higgs mass as compared to M_{Planck})

Not a threat to its self-consistency; they reflect the (dramatic) failure of dimensional analysis
$\Rightarrow$ Indication that some pieces of the puzzle are not understood.

Various attempts

Proposals for addressing the hierarchy problem

- (low-energy) Supersymmetry [Fayet '75, '77 & Witten '81 & Dimopoulos, Georgi '81 & Ibanez, Ross '81]
- Compositeness [Weinberg '76, '79 & Susskind '79]
- Large extra dimensions [Arkani-Hamed, Dimopoulos, Dvali '98 & Randall, Sundrum '99]

Distinct experimental signatures right above the electroweak scale differentiate them from the SM

So far no convincing deviations from the SM have been observed at particle physics experiments! (Maybe they're waiting for us in the corner...)

Possible relevance of scale or conformal invariance

Could the smallness of the Higgs mass be a manifestation of some underlying symmetry?

Take the SM (at the classical level): it is invariant under scale and conformal transformations when Higgs mass$\rightarrow 0$ (in the absence of gravity).

Could it be that CFTs apart from being essential for capturing the dynamics at critical points etc play also a fundamental role in Nature?

Possible relevance of scale or conformal invariance

When this symmetry is exact it has some "peculiar" implications:

- Forbids the presence of dimensionful parameters
- No particle interpretation—the spectrum is continuous

But Nature (SM) has:

- dimensionful parameters
- particles

It certainly appears that trying to embed the SM in a conformal field theory might be a dead end...

Possible relevance of scale or conformal invariance

A way out is to require that the symmetry be anomaly free and spontaneously broken

Consequences:

- Corrections to the Higgs mass heavily suppressed if there are no particle thresholds between Fermi and Planck scales (technically natural)

- And a bonus: In such systems, along flat directions—*if at all present*[1]—vacuum energy vanishes! In other words, the spectrum comprises a massless dof—the dilaton

  Perhaps this is relevant for the cosmological constant problem?

[1] I will come back to this point in a while.

Fishnet CFT (FCFT)

(i.e. the strongly γ-deformed limit of $\mathcal{N} = 4$ SYM)

I will be working in four dimensions exclusively

Summation over all repeated indexes will be tacitly assumed

All the considerations/statements correspond to the planar limit, at the leading order in the $1/N_c$ expansion

Fishnet CFT (FCFT)

Start from the γ-deformed $\mathcal{N} = 4$ SYM

$$\mathscr{L}_\gamma = \mathscr{L} + \mathscr{L}_{ferm.} + \mathscr{L}_{gauge} \ .$$

For what follows, the relevant piece of Lagrangian is the purely scalar sector

$$\mathscr{L} = N_c \,\mathrm{tr}\left[\partial_\mu \bar{\phi}_i \partial_\mu \phi_i + g_{YM}^2 \left(\frac{1}{4}\{\bar{\phi}_i, \phi_i\}\{\bar{\phi}_j, \phi_j\} - e^{-i\epsilon_{ijk}\gamma_k} \bar{\phi}_i \bar{\phi}_j \phi_i \phi_j\right)\right]$$

Here $\phi_i,\ i = 1, 2, 3$, are 3 complex scalar traceless matrices in the adjoint of $SU(N_c)$, g_{YM} is the YM coupling and γ's are the " twists."

The deformation breaks all SUSY

Fishnet CFT (FCFT)

Take the limit corresponding to weak YM coupling and large imaginary γ_3 [Gurdogan, Kazakov '15]

$$g_{YM} \to 0 \ , \quad \gamma_3 \to i\infty \ ,$$

such that

$$\tilde{\xi}^2 \equiv g_{YM}^2 N_c e^{-i\gamma_3} = \text{fixed} \ .$$

All gauge fields, fermions and one of the scalars decouple. The theory boils down to

$$\mathscr{L} = N_c \, \mathrm{tr} \left(\partial_\mu \overline{X} \partial_\mu X + \partial_\mu \overline{Z} \partial_\mu Z + \tilde{\xi}^2 \, \overline{X} \, \overline{Z} X Z \right)$$

where I changed notation $\phi_1 = X, \phi_2 = Z$.

The interaction term is not Hermitian - its complex conjugate counterpart did not survive the limit...

Far-reaching implications (will become clear later)

Fishnet CFT (FCFT)

Note that the above is not a UV complete theory. Correlators of composite objects give rise to divergences that require the introduction of the following double-trace terms

$$\mathscr{L}_{d.t.} = \alpha_1^2 \left[\mathrm{tr}(X^2)\mathrm{tr}(\overline{X}^2) + \mathrm{tr}(Z^2)\mathrm{tr}(\overline{Z}^2)\right] \\ -\alpha_2^2 \left[\mathrm{tr}(XZ)\mathrm{tr}(\overline{X}\,\overline{Z}) + \mathrm{tr}(X\overline{Z})\mathrm{tr}(\overline{X}Z)\right],$$

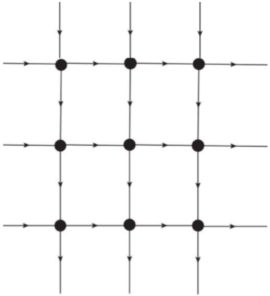
with $\alpha_1^2 = \pm\frac{i\xi^2}{2} - \frac{\xi^4}{2} \mp \frac{3i\xi^6}{4} + \mathcal{O}(\xi^8)$ and $\alpha_2^2 = \xi^2$.

Important: The theory is not unitary due to complex coupling and absence of Hermitian counterpart of the single trace term.

Fishnet CFT (FCFT)

The FCFT is is a *fully fledged finite CFT* at the large-N_c limit.

It is dominated by “fishing net” diagrams that are calculable: OPE data for a number of correlators and scaling dimensions of operators *have been computed!*



Actually, it is the only non-SUSY four dimensional theory with such properties, courtesy of the parent $\mathcal{N} = 4$ SYM.

Classical Vacua...

The spontaneous breaking of conformal symmetry

Folk theorem: Impossible in non-SUSY theories to have Poincaré-invariant ground state(s), such that the conformal symmetry is spontaneously broken without finetunings

Certainly true in $\lambda\phi^4$ and generalizations: flat directions open up only at isolated points in the corresponding couplings, e.g. one needs to fix $\lambda = 0 \quad \rightarrow \quad$ finetuning that cannot be explained

But, is this always the case?

The spontaneous breaking of conformal symmetry

Let's have a closer look at the FCFT. The EOMs for constant field configurations read

$$\begin{aligned}
\kappa\,\mathrm{tr}(\overline{X}^2)X + \mathrm{tr}(\overline{X}Z)\overline{Z} + \mathrm{tr}(\overline{X}\,\overline{Z})Z &= N_c Z\overline{X}\,\overline{Z}\ , \\
\kappa\,\mathrm{tr}(X^2)\overline{X} + \mathrm{tr}(X\overline{Z})Z + \mathrm{tr}(XZ)\overline{Z} &= N_c \overline{Z}XZ\ , \\
\kappa\,\mathrm{tr}(\overline{Z}^2)Z + \mathrm{tr}(X\overline{Z})\overline{X} + \mathrm{tr}(\overline{X}\,\overline{Z})X &= N_c \overline{X}\,\overline{Z}X\ , \\
\kappa\,\mathrm{tr}(Z^2)\overline{Z} + \mathrm{tr}(\overline{X}Z)X + \mathrm{tr}(XZ)\overline{X} &= N_c XZ\overline{X}\ ,
\end{aligned}$$

with $\kappa = -2\alpha_\pm^2/\xi^2$. We are after extrema such that at least one of the field's vev is nonvanishing.

For simplicity take the following ansatz

$$\langle X\rangle_{\text{tree}} = 0 \ \text{ and }\ \langle Z\rangle_{\text{tree}} = v\,\mathrm{diag}\left(z_1,\ldots,z_{N_c}\right)\ ,\ \sum_{k=1}^{N_c} z_k = \sum_{k=1}^{N_c} \bar{z}_k = 0\ ,$$

with v a (complex) parameter with dimension of mass.

The spontaneous breaking of conformal symmetry

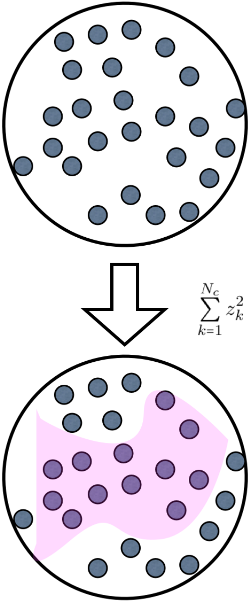
Plugging the ansatz into the EOMs, it is easy to see that as long as the matrices are such that

$$\sum_{k=1}^{N_c} z_k^2 = \sum_{k=1}^{N_c} \bar{z}_k^2 = 0 \ ,$$

they are satisfied $\forall \xi$!

It is remarkable and very unusual for a non-SUSY CFT to support flat vacua without finetunings.

It is not a mystery though: the parent $\mathcal{N} = 4$ SYM has a plethora of flat directions along which conformal symmetry is nonlinearly realized. The FCFT has inherited many of them although it has been heavily deformed!



$$\sum_{k=1}^{N_c} z_k^2 = \sum_{k=1}^{N_c} \bar{z}_k^2 = 0$$

...and their quantum fate

It is well known that conformal symmetry—although present at the classical level—may well be badly broken by quantum corrections.

This would be quite problematic, since the flat directions are uplifted and the vacuum energy of the theory is not zero anymore. Put differently, the dilaton acquires a mass.

Whether or not this is the case, becomes apparent in the Coleman-Weinberg effective potential.

Coleman-Weinberg effective potential

Textbook exercise

$$V_{eff} = V_{tree} + V_{1-loop} \ ,$$

with

$$V_{1-loop} \propto \mathrm{tr}\left(M^4 \log \frac{M^2}{\mu^2} \right) \ ,$$

M is the "mass" matrix that comprises the non-derivative part of the Lagrangian quadratic in the excitations; μ is the 't Hooft-Veltman renormalization point.

We now look at extrema of the full theory including the one-loop correction

$$\frac{\partial}{\partial\langle X\rangle}V_{eff} = 0\ldots$$

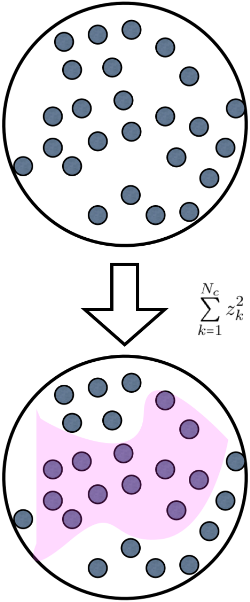
A straightforward computation reveals that a subclass of the classical "asymmetric" flat directions survive quantum corrections at the large-N_c limit! These are the solutions to the following set of transcendental eqs.

$$\sum_{k=1}^{N_c} z_k = \sum_{k=1}^{N_c} \bar{z}_k = \sum_{k=1}^{N_c} z_k^2 = \sum_{k=1}^{N_c} \bar{z}_k^2 = \sum_{k=1}^{N_c} z_k^2 \log z_k = \sum_{k=1}^{N_c} \bar{z}_k^2 \log \bar{z}_k = 0$$

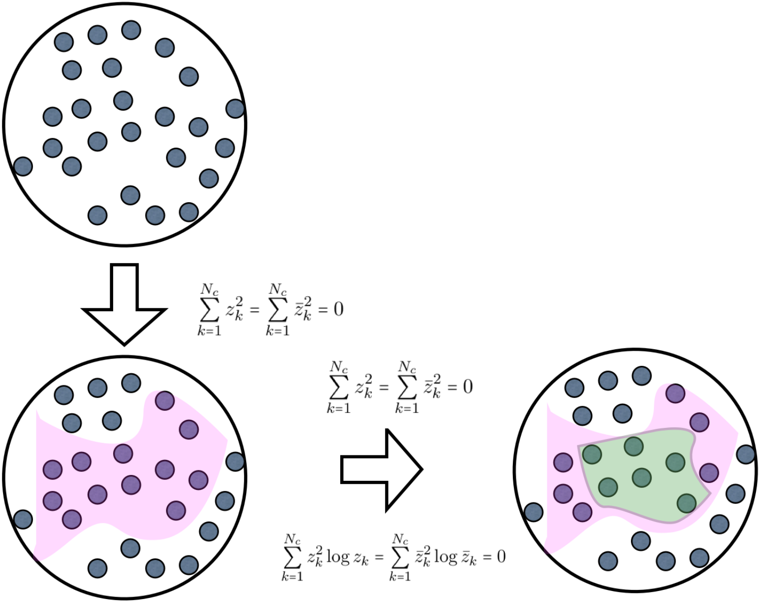
Consequently, along these flat directions,

$$V_{eff} = 0\ ,$$

i.e. the loop corrected theory has nonlinearly realized conformal symmetry!



$$\sum_{k=1}^{N_c} z_k^2 = \sum_{k=1}^{N_c} \bar{z}_k^2 = 0$$



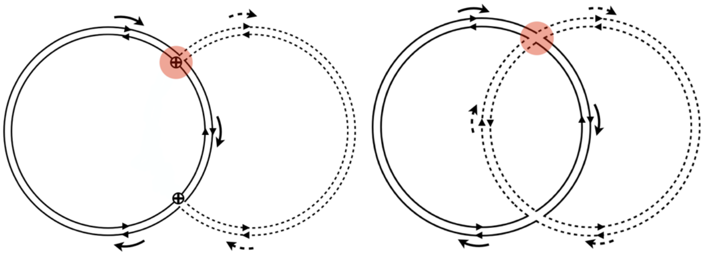
$$\sum_{k=1}^{N_c} z_k^2 = \sum_{k=1}^{N_c} \bar{z}_k^2 = 0$$

$$\sum_{k=1}^{N_c} z_k^2 = \sum_{k=1}^{N_c} \bar{z}_k^2 = 0$$

$$\sum_{k=1}^{N_c} z_k^2 \log z_k = \sum_{k=1}^{N_c} \bar{z}_k^2 \log \bar{z}_k = 0$$

Once again, the parent theory has been quite generous to its descendant.

The SUSY stabilization mechanism in the $\mathcal{N} = 4$ SYM has been replaced by the absence of dangerous graphs in the planar limit, that would normally lift the classical flat directions à la Coleman-Weinberg.
Another bonus of "chirality": the effective action seems to be one-loop exact due to the absence of multi-loop diagrams...



A proof of concept

A simple example of a flat vacuum. Take $\langle X \rangle = 0$ and $\langle Z \rangle$ to be a block-diagonal matrix comprising $N_c/4$ diagonal sub-blocks each with dimensions 4×4

$$\langle Z \rangle = v\,\mathrm{diag}(\underbrace{z_1, z_2, z_3, z_4}, \underbrace{z_1, z_2, z_3, z_4}, \ldots)\ ,$$

with

$$\begin{aligned} z_1 &= -0.587849 - 0.808971\, i\,, \;\; z_2 = 0.260305 + 1.45187\, i\,, \\ z_3 &= 1.32754 - 0.642903\, i\,, \qquad z_4 = -1\ . \end{aligned}$$

Generalizations are straightforward

Conclusions

- non-SUSY theories were believed to not allow for spontaneous conformal symmetry breaking to take place without finetunings
- Fishnet CFT is the first example of a non-SUSY theory that allows for SSB without tuning the corresponding coupling
- At the large-N_c limit, a subclass of the flat directions is not lifted by quantum corrections $\rightarrow$ nonlinearly realized conformal symmetry persists at the quantum level
- One-loop exactness due to the chirality of the theory

Open questions

- What happens at finite N_c (in general for the FCFT)?
- Consistency conditions [Karananas, Shaposhnikov '17]

$$\langle \mathcal{O}_I \rangle \langle \mathcal{O}_J \rangle \sim \lim_{x \to \infty} \sum_K \frac{c_{IJK}}{|x|^{\Delta_I + \Delta_J - \Delta_K}} \langle \mathcal{O}_K \rangle$$

- Are CFTs in the heart of the solutions to the SM finetuning issues?

Thank you!