Anisotropic RG Flows and Strongly Coupled Systems

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Based on works with:
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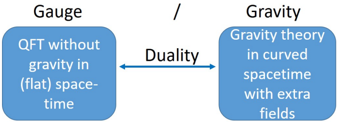
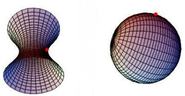
Outline

Briefly on AdS/CFT

- Gauge/Gravity duality: A way to map quantum questions to gravity geometric questions and answer them.



- The initial AdS/CFT correspondence: $\mathcal{N} = 4$ sYM on flat space $\Leftrightarrow AdS_5 \times S^5$, is the harmonic oscillator of the gauge/gravity dualities.



- It is the simple-nice model!

- Since the discovery of the initial correspondence, there is an extensive research aiming to construct more realistic gauge/gravity dualities (confinement, no susy, temperature, quarks...).

✓ This talk: Theories with Broken Rotational Symmetry in Gauge/Gravity correspondence.

Why? Existence of Natural Systems.

The existence of strongly coupled anisotropic systems.

- The expansion of the Quark-Gluon plasma at the earliest times after the collision, momentum anisotropic plasmas.
- Strong Magnetic Fields in strongly coupled theories.
- New interesting phenomena in presence on such fiels, i.e. inverse magnetic catalysis.

  eg: (Bali, Bruckmann, Endrodi, Fodor, Katz, Krieg et al. 2011)
- Anisotropic low dimensional materials in condensed matter.

Why? More:

- Weakly coupled vs strongly coupled anisotropic theories.
  (Dumitru, Strickland, Romatschke, Baier,...)
- Properties of top-down supergravity Black hole solutions that are AdS in UV **flowing** to Lifshitz-like in IR :
  ⋆ Fixed scaling parameter z for such anisotropic solutions or even isotropic flows?
  (Azeyanagi, Li, Takayanagi, 2009; Mateos, Trancanelli, 2011;...)
  ⋆ New flows to alternative IR fixed points?

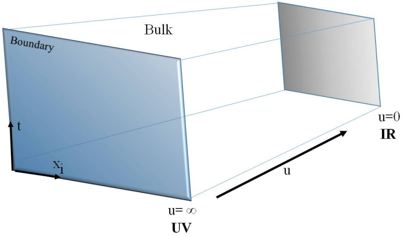
- Ⓢtriking Features! Several Universality Relations for the isotropic theories are violated in aniso!
  Shear viscosity η over entropy density s: takes parametrically low values wrt degree of anisotropy $\frac{\eta}{s} < \frac{1}{4\pi}$!
  (Rebhan, Steineder 2011; D.G. 2012; Jain, Samanta, Trivedy 2015; D.G., Gursoy, Pedraza, 2017)

Reminding Slide:

- The anisotropic hyperscaling violation metric

$$ds^2 = u^{-\frac{2\theta}{d}}\left(-u^{2z}\left(dt^2 + dy_i^2\right) + u^2 dx_i^2 + \frac{du^2}{u^2}\right)$$

exhibits a critical exponent z and a hyperscaling violation exponent θ.



- $\theta = 0,\ z = 1 \Rightarrow$ AdS.
- $\theta = 0 \Rightarrow$ scale invariant theory.
- In general no scale invariance.

$$t \to \lambda^z t, \qquad y \to \lambda^z y, \qquad x \to \lambda x, \qquad u \to \frac{u}{\lambda}\,, \qquad ds \to \lambda^{\frac{\theta}{d}} ds\ .$$

How is Anisotropy introduced? A Pictorial Representation:

- For the Lifshitz-like IIB Supergravity solutions

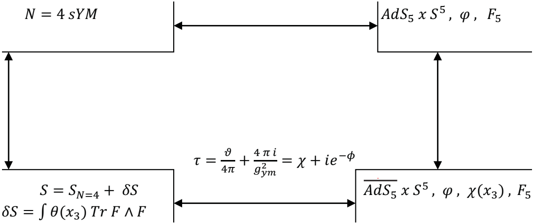
$$ds^2 = u^{2z}(dx_0^2 + dx_i^2) + u^2 dx_3^2 + \frac{du^2}{u^2} + ds^2_{S^5}\,.$$

Introduction of additional branes: *(Azeyanagi, Li, Takayanagi, 2009)*

Spacetime

	x_0	x_1	x_2	x_3	u	S^5
AdS → **D3**	x	x	x	x		
D7	x	x	x			x

- Which equivalently leads to the following AdS/CFT deformation.



- $dC_8 \sim \star d\chi$ with the non-zero component $C_{x_0 x_1 x_2 S^5}$.

A Theory with Phase Transitions in One Page:

- How the Field Theory looks like?
  - ✓ 4d $SU(N)$ Strongly coupled anisotropic gauge theory.
  - ✓ Its dynamics are affected by a scalar operator $\mathcal{O}_\Delta$.
  - ✓ Anisotropy is introduced by another operator $\tilde{\mathcal{O}} \sim \theta(x_3)\,TrF \wedge F$ with a space dependent coupling.

- The gravity dual theory is an Einstein-Axion-Dilaton theory in 5 dimensions with a non-trivial potential.
  - ✓ A "backreacting" scalar field depending on spatial directions, the axion; and a non-trivial dilaton.
  - ✓ Solutions are non-trivial RG flows:
    Conformal fixed point in the UV $\Rightarrow$ Anisotropic (Hyperscaling Lifshitz-like) in IR.

- The vacuum state confines color and there exists a phase transition at finite T_c above which a deconfined plasma state arises.

(D.G., Gursoy, Pedraza, 2017)

An Anisotropic Theory

The generalized Einstein-Axion-Dilaton action with a potential for the dilaton and an arbitrary coupling between the axion and the dilaton:

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{1}{2}Z(\phi)(\partial\chi)^2\right].$$

The eoms read

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}Z(\phi)\partial_\mu\chi\partial_\nu\chi - \frac{1}{4}g_{\mu\nu}(\partial\phi)^2 - \frac{1}{4}g_{\mu\nu}Z(\partial\chi)^2 + \frac{1}{2}g_{\mu\nu}V(\phi)\ ,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) = \frac{1}{2}\partial_\phi Z(\phi)(\partial\chi)^2 - V'(\phi)\ ,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\chi\right) = 0\ .$$

Where

$$V(\phi) = 12\cosh(\sigma\phi) + \left(\frac{m(\Delta)^2}{2} - 6\sigma^2\right)\phi^2, \qquad Z(\phi) = e^{2\gamma\phi}\ .$$

((Gubser, Nellore), Pufu, Rocha 2008a,b)

Remark: For $\sigma = 0, \gamma = 1, m(\Delta) = 0$ the action and the solution of eoms, are reduced of IIB supergravity.

A Solution : The RG Flow

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}(u)\mathcal{B}(u)\,dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}(u)dx_3^2 + \frac{du^2}{\mathcal{F}(u)}\right),$$

$$\chi = \alpha x_3\,, \qquad \phi = \phi(u)\,, \qquad \mathcal{F}(u_h) = 0\ .$$

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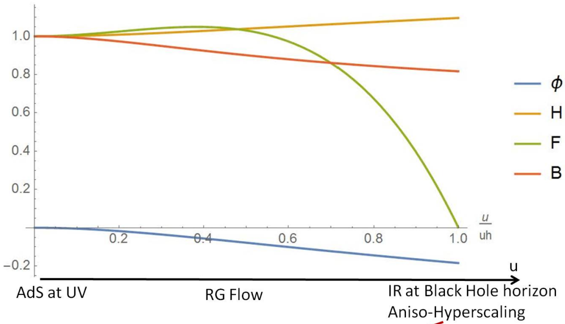


$$ds^2 = u^{-\frac{2\theta}{3}}\left(-u^{2z}\left(f(u)dt^2 + dx_{1,2}^2\right) + \tilde{\alpha}u^2 dx_3^2 + \frac{du^2}{f(u)u^2}\right)$$

We have obtained the theories, are they physical and stable?

$$\Downarrow$$
$$\Downarrow$$

✓ Energy Conditions Analysis: $T_{\mu\nu} N^{\mu} N^{\nu} \geq 0 \ , \quad N^{\mu} N_{\mu} = 0 \ .$

AND

✓ Local Thermodynamical Stability Analysis: Specific Heat...

$$\Downarrow$$
$$\Downarrow$$

YES!

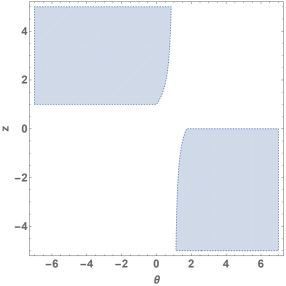
Three conditions that constrain (z,θ) and as a result (γ,σ).

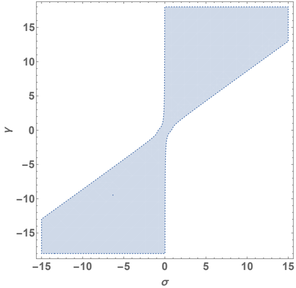
$$(z-1)(1-\theta+3z)\geq 0\ , \qquad z=\frac{2+4\gamma^2-3\sigma^2}{2\gamma(2\gamma-3\sigma)}\ ,$$

$$\theta^2-3+3z(1-\theta)\geq 0\ , \qquad \theta=\frac{3\sigma}{2\gamma}\ ,$$

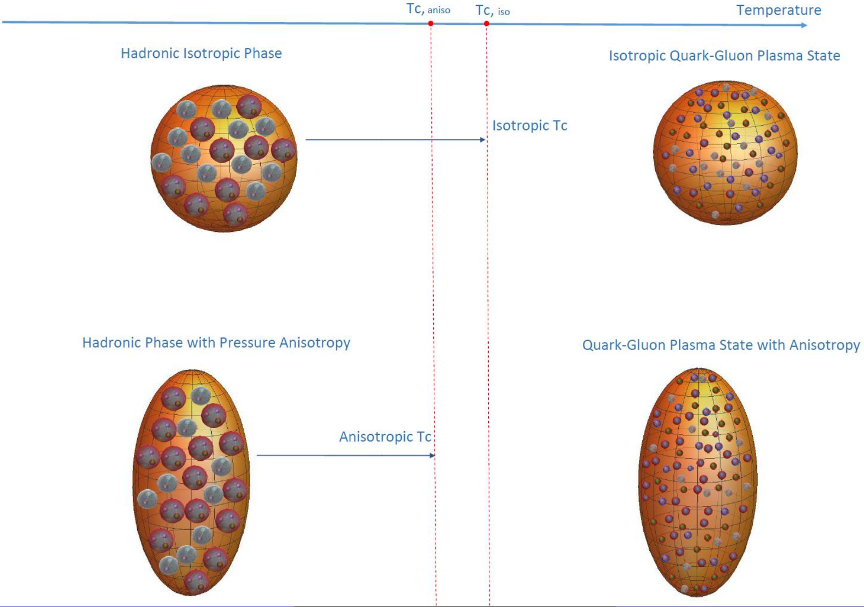
$$1-\theta+2z\geq 0\ .$$

The blue region is the acceptable for the theory parameters.



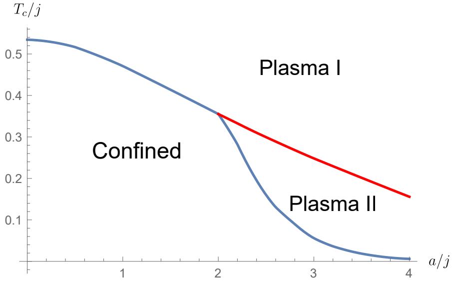


Catalysis: Lower Phase Transition Temperature
due to any Source of Anisotropy.



Phase Transitions

- Competition for dominance between different gravitational backgrounds.
- The Critical Temperature of the theories vs the anisotropy gives:



- The T_c is reduced in presence of anisotropies of the theory.

(D.G., Gursoy, Pedraza, 2017)

A Proposal

- The $Tc(\alpha)$ decrease with anisotropy α.
- No charged fermionic degrees of freedom in our case; our plasma is neutral.
- Anisotropy causes lower T_c= "Inverse Anisotropic Catalysis".

Universal Results: η/s in Theories with Broken Symmetry

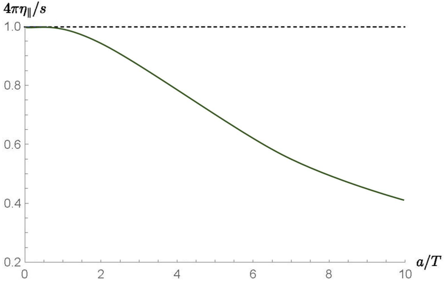
Consider a finite T theory in the deconfined phase:

$$ds^2 = g_{tt}(u)dt^2 + g_{11}(u)(dx_1^2 + dx_2^2) + g_{33}(u)dx_3^2 + g_{uu}(u)du^2$$

- The anisotropic shear viscosity violates the isotropic "bound" of $1/4\pi$:

$$\eta_{ij,kl} = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\int dtdxe^{i\omega t}\langle T_{ij}(t,x), T_{kl}(0,0)\rangle$$

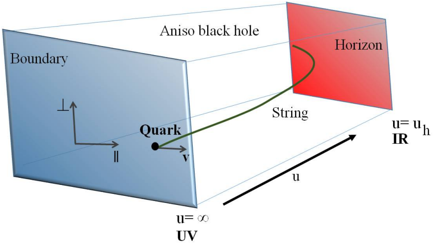
"Frictionless" Anisotropic Plasma.

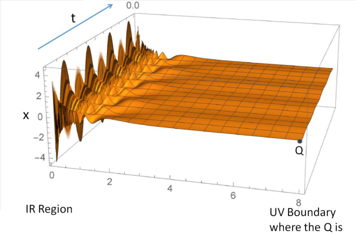


- The Ratio:

$$4\pi\frac{\eta_\parallel}{s} = \frac{g_{11}}{g_{33}}\Big|_{u=u_h} \sim \left(\frac{T}{\alpha}\right)^p, \qquad p = 2 - \frac{2}{z} \sim [0,\infty)\ , \quad \alpha \gg T.$$

Langevin Dynamics and Brownian Motion





$$\frac{\kappa_{\parallel}}{\kappa_{\perp}} = \left.\frac{\left(g_{00}g_{\parallel\parallel}\right)'}{g_{\perp\perp}g_{\parallel\parallel}\left(\frac{g_{00}}{g_{\parallel\parallel}}\right)'}\right|_{u=u_{wh}}, \qquad \left\langle p^2_{\parallel,\perp}\right\rangle \sim \kappa_{\parallel,\perp}\mathcal{T}$$

A Universal Inequality for Isotropic Theory:

$\kappa_{\parallel} \geq \kappa_{\perp}$ for any isotropic strongly coupled plasma!

Can be inverted in the anisotropic theories: $\kappa_{\parallel} \geq < \kappa_{\perp}$.

(Gursoy, Kiritsis, Mazzanti, Nitti, 2010; D.G, Soltanpanahi, 2013a,b; D.G. 2018)

Anisotropic candidate of c-function

- A proposed the c-function is
  (Chu, Giataganas, 2019;(2d) Casini, Huerta 2006; (iso 2d+) Ryu, Takayanagi 2006; Myers, Singh 2012; (nrcft) Cremonini, Dong 2014)

$$c_x := \beta_x \frac{l_x^{d_x - 1}}{H_x^{d_1 - 1} H_y^{d_2}} \frac{\partial S_x}{\partial \ln l_x} \,, \qquad d_x := d_1 + d_2 \frac{n_2}{n_1}$$

  where H is the infrared regulator, the dimensions n_1, n_2 are defined at the fixed point

$$[t] = L^{n_t}, \quad [x_i] = L^{n_1}, \quad [y_j] = L^{n_2} \ .$$

- A relativistic "c-theorem" is guaranteed as long as the NEC: $T_0^0 - T_r^r \leq 0$ is satisfied!

$$\frac{dc}{dr} \propto - \int_0^l dx A'^{-2} \left(T_0^0 - T_r^r \right) \geq 0 \ .$$

- How about the Anisotropic theories?

- Not a one-to-one correspondence between NEC ($g_i'(r) > 0$) and c-function monotonicity, but not surprising!
- Interesting observation: For an anisotropic theory with $d_1 = d_2$, the boundary condition

$$g_{i\ UV} \leq 0 \ ,$$

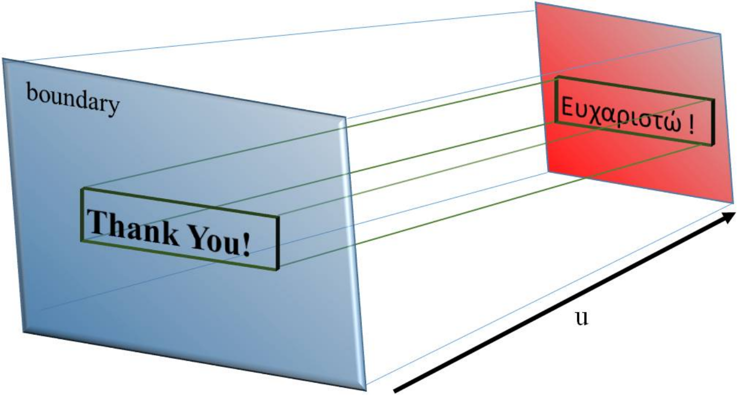
with a conformal UV fixed point guarantees the right monotonicity the c-functions along the RG flow.

Are there any other observables that form functions, to have monotonic behavior along the RG flow?

(Chu, Derendinger,Giataganas, in progress)

Conclusions

- ✓ Observation: In strongly coupled theories many phenomena are more sensitive to the presence of the anisotropy than the source that triggers it.
- ✓ Strongly Coupled Confining Anisotropic theories with confinement /deconfinement phase transition.
- ✓ The phase transitions occur at lower critical Temperature as the anisotropy is increased = Inverse Anisotropic Catalysis!
- ✓ Several Universal Isotropic relations are anisotropically violated.
- ✓ Holographic monotonic functions and conditions of monotonicity for (anisotropic) RG flows.


boundary
Thank You!
Ευχαριστώ !
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