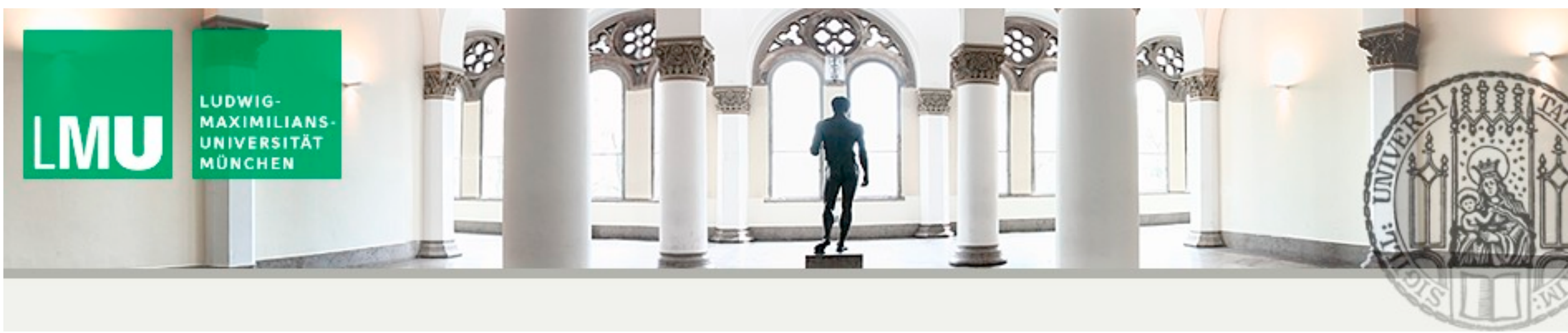
Gravity, Infinite Distance and Geometric Flow

DIETER LÜST (LMU, MPI)



Humboldt Kolleg, Corfu, September 2019

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Joint work with E. Palti, C. Vafa, arXiv:1906.05225
and with A. Kehagias and S. Lüst to appear

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Which IR consistent quantum field theories cannot be embedded into a UV complete quantum gravity theory?

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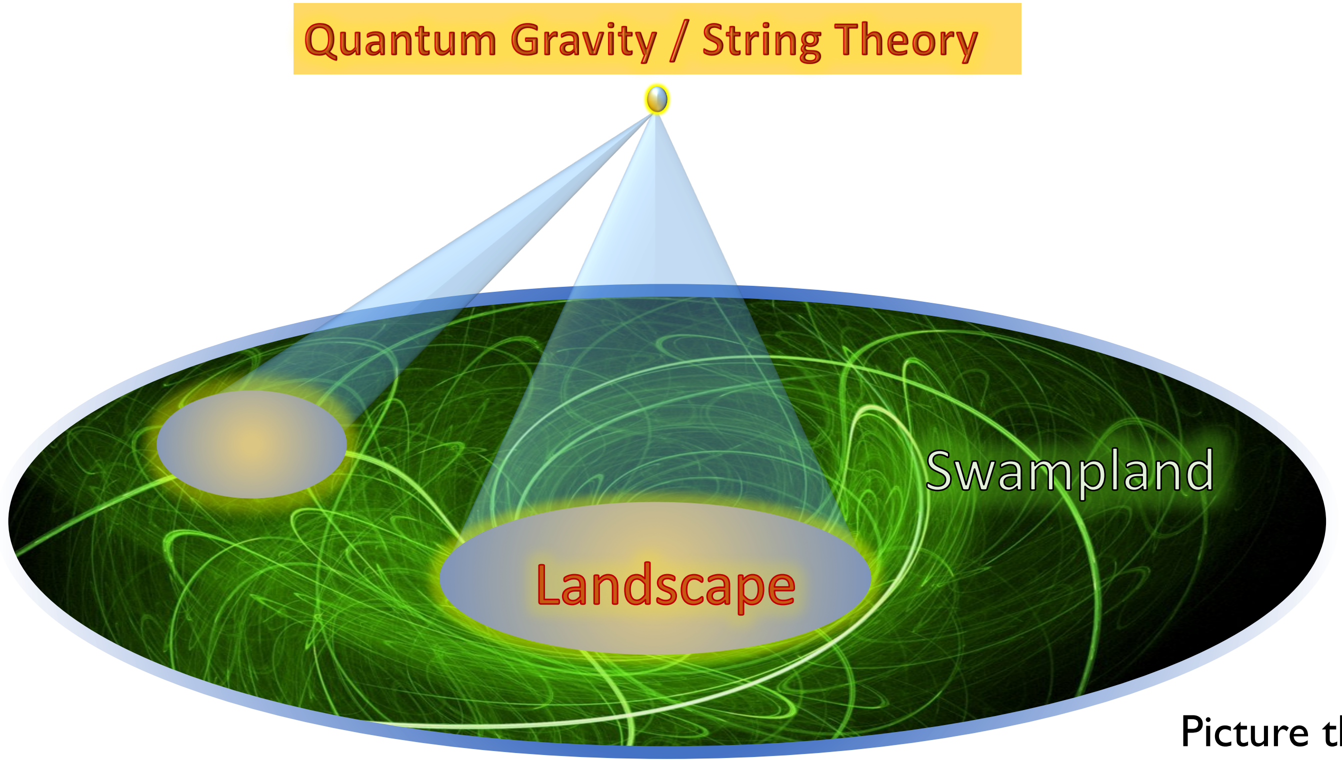

Picture thanks to Eran Palti

Can gravity be in the swampland?

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New swampland conjectures:

Higher-spin (spin-2) theories.

Massive gravity. [D. Kläwer, D.L., E. Palti, arXiv:1811.07908]

Pure AdS vacua and the swampland?

- Weak gravity conjecture

[N. Arkani-Hamed, L. Motl, A. Nicolis, C. Vafa (2007)]

[B. Heidenreich, M. Reece, T. Rudelius (2015/17);
D. Klaewer, E. Palti (2016);
M. Montero, G. Shiu, P. Soler (2016);
S. Andriolo, D. Junghans, T. Noumi, G. Shiu (2018)
T. Grimm, E. Palti, I. Valenzuela (2018)];

$$\Updownarrow$$

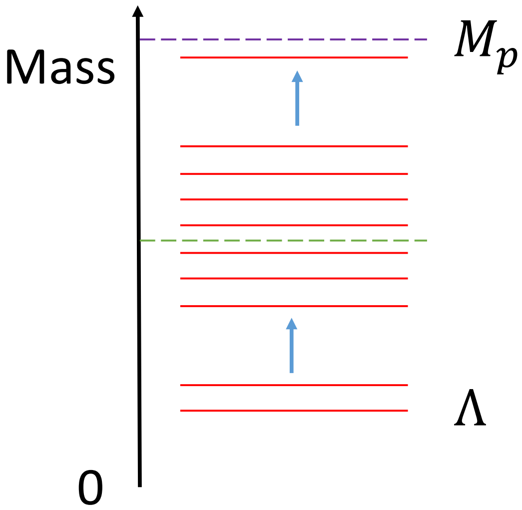
- Infinite distance conjecture

[H. Ooguri, C. Vafa (2006)]

At the boundary of the moduli space there is an infinite tower of (almost) massless states with scale:

$$m \sim M_p e^{-c\Delta\phi} \quad , \quad m \to 0 \quad \text{for} \quad \Delta\phi \to \infty$$

U(1) gauge theory: $g_{\mathrm{U}(1)} \to 0$ is at infinite distance.



$$m = g_{U(1)} M_p$$ (magnetic WGC)

Infinite tower of states above m.

EFT breaks down above the cut-off m .

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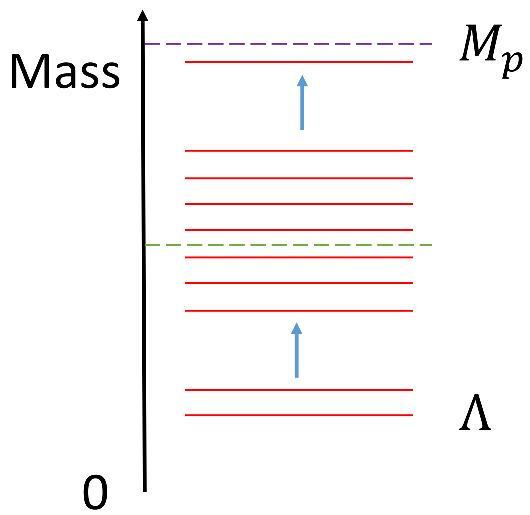

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Tower of states is given by internal KK modes:

$$g_{U(1)} \sim \frac{1}{R} \quad , \quad m_n \sim \frac{n}{R} = n e^{-\phi}$$

$\phi = \log R$: internal modulus of compact space.

AdS - distance conjecture:

[D.L. , E. Palti, C. Vafa (2019)]

Consider AdS vacua in quantum gravity with varying negative cosmological constant Λ .

What is happening in the limit $\Lambda \rightarrow 0$?

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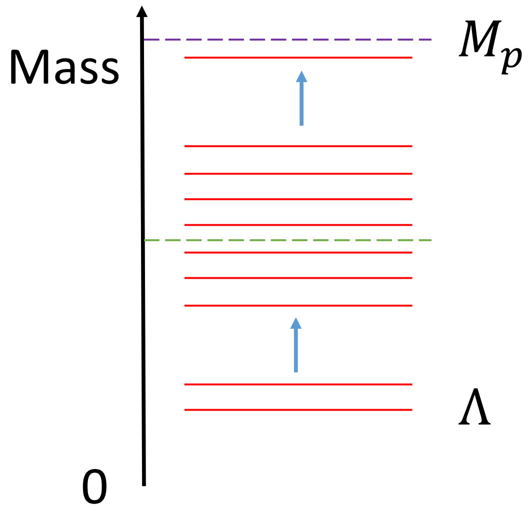
What is happening in the limit $\Lambda \to 0$?

AdS Distance conjecture (ADC):

There exist an infinite tower of states with mass scale m, which behaves as

$$m \sim |\Lambda|^{\alpha} \quad \text{with} \quad \alpha > 0$$

$\Lambda \to 0$ is at infinite distance !



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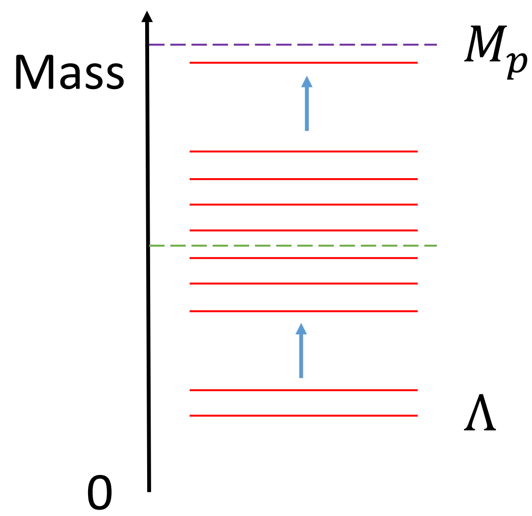

Infinite tower of states above Λ

Strong AdS distance conjecture (SADC):

The bound is satisfied for supersymmetric AdS vacua with $\alpha = 1/2$: $m \sim |\Lambda|^{1/2}$

The conjecture is satisfied for many know backgrounds of string and M - theory like $AdS_5 \times S^5$ via the tower of KK modes.

Generalized distance conjecture:

Recall: the swampland distance conjecture states that given scalars with kinetic terms

$$\mathcal{L}_{kin} = -p_{ij}(\phi)\partial\phi^i\partial\phi^j$$

one can associate a distance Δ in field space to a path with parameter τ:

$$\Delta = \int_{\tau_i}^{\tau^f} \left(p_{ij}(\phi)\frac{\partial\phi^i}{\partial\tau}\frac{\partial\phi^j}{\partial\tau} \right)^{1/2} d\tau$$

Then the conjecture says that there is an infinite tower of states with mass m:

$$m(\tau_f) \sim m_(\tau_i)e^{-\alpha\Delta}\,, \quad \alpha > 0\,, \alpha \sim \mathcal{O}(1)$$

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Apply the distance conjecture to
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Apply the distance conjecture to
the space-time metric without any reference to the
internal space :

Family of metrics with a distance on the space of metrics:

$$g_{MN} = g^0_{MN} + \delta g_{MN}$$

Associated distance:

$$\Delta = c \int_{\tau_i}^{\tau_f} \left(\frac{1}{V_M} \int_M \sqrt{g} g^{MN} g^{OP} \frac{\partial g_{MO}}{\partial \tau} \frac{\partial g_{NP}}{\partial \tau} \right)^{\frac{1}{2}} d\tau$$

Distance with respect to Weyl rescalings of the metric:

$$\tilde{g}_{MN} = e^{2\tau} g_{MN}$$

Associated distance:

$$\Delta = \sqrt{(d-2)\,(d-1)}(\tau_f - \tau_i)$$

Now consider AdS space with metric

$$ds^2 = e^{2\tau}\left(-\left(\cosh\rho\right)^2 dt^2 + d\rho^2 + \left(\sinh\rho\right)^2 d\Omega_{d-2}^2\right)$$

$$\Lambda = -\frac{1}{2}\left(d-1\right)\left(d-2\right)e^{-2\tau}$$

Distance under Weyl rescaling:

$$\Delta = -\frac{1}{2}\int_{\Lambda_i}^{\Lambda_f}\sqrt{\left(d-2\right)\left(d-1\right)}d\log\Lambda$$

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This then immediately leads to the ADC:

$$m\left(\Lambda_f\right) = m\left(\Lambda_i\right)\left(\frac{\Lambda_f}{\Lambda_i}\right)^{\alpha} \quad \text{resp.} \quad m\left(\Lambda\right) = M_p\left(\frac{\Lambda}{M_p^2}\right)^{\alpha}$$

ADC in string theory:

- There is no separation of scales in AdS string vacua.

[M. Duff, B. Nilsson, C. Pope (1986);
M. Douglas, S. Kachru (2006);
F. Gautason, M. Schillo, T. Van Riet, M. Williams (2015);
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- In the flat limit $\Lambda \to 0$ always a infinite tower of massless states is opening up - there always exists an extra space factor like

$$AdS_d \times K^{d'}$$

Pure AdS space cannot exist alone in quantum gravity.

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Holography: AdS X $\longleftrightarrow$ CFT duality.

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However M-theory on $AdS_7 \times S^4/Z_k$ gives $SU(k) \times SU(k)$ gauge theory on AdS_7 with arbitrarily high k.

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However M-theory on $AdS_7 \times S^4/Z_k$ gives $SU(k) \times SU(k)$ gauge theory on AdS_7 with arbitrarily high k.

But the „correct" perspective is to view the gauge theory to live on a lower dimensional defect within an higher dimensional space.

The generalized distance conjecture should also hold for de Sitter vacua (if they exist):

There should exist a light tower of states like

$$m \sim 10^{-120\alpha} M_p$$

Infinite distance and Ricci flow:

Consider pure Einstein gravity

$$S = \frac{M_p^2}{2}\int_{M_d} d^d x\sqrt{-g}R$$

Consider a family of metrics $g_{\mu\nu}(t)$

Ricci flow: is smoothing out irregularities in the metric:

[R.S. Hamilton (1982)]

$$\frac{\partial}{\partial t}g_{\mu\nu}(t) = -2R_{\mu\nu}(t)$$

Ricci flow shrinks regions of positive curvature.

Ricci flow expands regions of negative curvature.

Fixed point of Ricci flow

$$\frac{\partial}{\partial t} g_{\mu\nu}(t)|_{t=t_0} = 0 \quad \Longleftrightarrow \quad R_{\mu\nu}(t_0) = 0$$

Conjecture A:

For a Riemannian manifold, the fixed points of the Ricci flow at infinite distance in the field space of the background metrics are accompanied by an infinite tower of additional massless states in quantum gravity.

Einstein manifold $$R_{\mu\nu} = \Lambda g_{\mu\nu}$$

$$\frac{\partial R}{\partial t} = \nabla^2 R + 2R^{\mu\nu} R_{\mu\nu}$$

Cosmological constant

$$\frac{d\Lambda}{dt} = 2\,\Lambda^2 \quad \Rightarrow \quad \Lambda(t) = \frac{\Lambda_0}{1 - 2\Lambda_0(t - t_0)}$$

Fixed point $\hat{\Lambda} = 0$

AdS $\Lambda_0 < 0\,, \quad t \to \infty$ (immortal Ricci soliton)

dS $\Lambda_0 > 0\,, \quad t \to -\infty$ (ancient Ricci soliton)

Compare with Weyl rescaling: $\Lambda(\tau) = e^{-2\tau}\Lambda_0$

$$\tau = \frac{1}{2}\ln\Big(1 - 2\Lambda_0(t - t_0)\Big)$$

Distance along the Ricci-flow

$$\begin{aligned}\Delta_R =& 2\sqrt{d}|\Lambda_0|c\int_{t_0}^{t_f}\frac{dt}{1 - 2\Lambda_0(t - t_0)} \\ =& c\sqrt{d}\,\ln\Big(1 - 2\Lambda_0(t - t_0)\Big)\Big|_{t_0}^{t} \simeq \tau \\ \equiv& \Delta_W\end{aligned}$$

This agrees with the Weyl distance.

The fixed point of Ricci flow is indeed at infinite distance:

$$t, \tau \longrightarrow \pm\infty$$

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Go off shell in string theory:

$\Longrightarrow$ Fixed point of the Ricci flow is a CFT background (zero of 2d beta-function) . [Iioannis Bakas]

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[Iioannis Bakas]

Off shell background $M_t \Longrightarrow$ on shell background M_{t_0}

Flow $t \to t_0$

If M_{t_0} is at infinite distance away from M_t , then one gets an infinite tower of states in the limit $t \to t_0$.

One possible origin of the massless tower of states:

In addition to M_t one has an extra space factor $K_{t'}$:

Total string theory (quantum gravity) background :

$$M_t \times K_{t'}$$

$$\Big\downarrow \text{ Flow } \quad t \to t_0 \ , \ t' \to t'_0$$

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The massless tower corresponds to states of $K_{t'}$, which become massless at the fixed point of the flow.

E.g. KK-modes, wrapped branes or strings.

Flow in fibred CY „moduli“ space:

A simple example is the Nilmanifold (twisted torus):

Circle fibration with negative curvature.

$$ds^2 = a^2(t)(dx_1^2 + dx_2^2) + b^2(t)(dx_3 + x_1 dx_2)^2$$

$$R = -\frac{y}{2x^2} \qquad x = a(t)^2, \quad y = b(t)^2$$

Ricci-flow equations: $$x' = \frac{y}{x}, \quad y' = -\frac{y^2}{x^2}$$

Solution $$R(t) = \frac{R_0}{1 - 6R_0 t}$$

Limit $t \longrightarrow \infty$ is indeed at infinite distance.

What is happening at this limit:

The radii are very large or, respectively, very small:

$$t \to \infty : \qquad a(t) \to \infty, \quad b(t) \to 0$$

Infinite tower of massless KK or winding states.

In M-theory/II A theory:

Infinite tower of massless D0-branes

Dual limit: light D6-branes

Distance and entropy functionals:

(A)dS distance $$\Delta = -\frac{1}{\alpha}\log|\Lambda|$$

Positive cosmological constant and Gibbons-Hawking entropy

$$\mathcal{S}_{GH} = 1/\Lambda \Longrightarrow \Delta = \frac{1}{\alpha}\log\mathcal{S}_{GH} \qquad \text{for positive } \Lambda \ .$$

This formula makes sense, since dS space has an horizon and a temperature.

What about more general spaces without horizon and temperature?

We now try to relate the distance to the entropy functionals of the (generalized) Ricci-flow.

These are basically given in terms of the effective string action.

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Pure Einstein gravity and Ricci flow:

Conjecture B0:

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$$\Delta_R \simeq -\log|R| \qquad m \simeq e^{-\alpha\Delta_R} \simeq R^{\alpha}$$

Generalized flows and Perelman entropy functionals:

[G. Perelman(2002)]

Combined metric - dilaton flow:

$$\mathcal{F}(g,f) = \int d^d x \sqrt{-g} \Big((R_g + (\nabla f)_g^2 \Big) e^{-f}$$

Generalizd gradient flow equations:

$$\frac{\partial}{\partial t} g_{\mu\nu}(t) = -\,2\Big(R_{\mu\nu}(t) + \nabla_\mu \nabla_\nu f(t) \Big)\,,$$

$$\frac{\partial}{\partial t} f(t) = -\,R(t) - \Delta f(t)$$

Conjecture B1:

For a Riemannian manifold plus a dilaton field, the distance along the combined dilaton-metric flow is determined by the entropy functional $\mathcal{F}$.
At $\mathcal{F} = 0\,(\mathrm{or}\,\infty)$ there is an infinite tower of additional massless states in quantum gravity.

Conjecture B1:

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Generalized distance: $\Delta_{\mathcal{F}} = -\log|\mathcal{F}|$

$$m \simeq e^{-\alpha\Delta_{\mathcal{F}}} \simeq \mathcal{F}^{\alpha}$$

String example:
Flow of a 2D metric-dilaton background to the 2D Witten cigar.

$\mathcal{W}$ - functionial:

was introduced also for pinched cycles and the proof of the Poincare conjecture.

[G. Perelman(2002)]

$$\mathcal{W}(g,f,\lambda) = \int d^d x \sqrt{-g} \left(\lambda (R_g + (\nabla f)^2_g + f - d \right) \frac{e^{-f}}{(4\pi\lambda)^{d/2}}$$

$$\frac{\partial}{\partial t} g_{\mu\nu}(t) = -\,2R_{\mu\nu}(t)\,,$$

$$\frac{\partial}{\partial t} f(t) = -\,R(t) - \Delta f(t) + |\nabla f(t)|^2 + \frac{d}{2\lambda(t)}\,,$$

$$\frac{\partial}{\partial t} \lambda(t) = -\,1$$

Conjecture B2:

The distance along the Perelman flow
is determined by the entropy functional $\mathcal{W}$.
At $\mathcal{W} = 0 \, (\mathrm{or}\ \infty)$ there is an infinite tower of additional massless states in quantum gravity.

Generalized distance: $\Delta_{\mathcal{W}} = -\log|\mathcal{W}|$

$$m \simeq e^{-\alpha \Delta_{\mathcal{W}}} \simeq \mathcal{W}^{\alpha}$$

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  The AdS (dS) cosmological constant is accompanied by a tower of states.

  Their masses goes to zero in the flat limit $\Lambda \to 0$

- No known counter example in string theory, except possibly KKLT.

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For a recent interesting paper on the relation between swampland, infinite distance and RG energy flow see [C. Gomez, arXiv:1907:13386]

Conjecture, asymptotic safety is at infinite distance and therefore belongs to the swampland.

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Conjecture, asymptotic safety is at infinite distance and therefore belongs to the swampland.

- The distance conjecture possibly has important implications in cosmology - tower of states in quintessence models …..

Thank you !