Quantum Noncommutative ABJM theory

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Details on

* JHEP 04 (2018) 070 by C.P.M, J. Trampetic & J. You.

PLAN

What's ABJM theory?

- In 2008 Aharony, Bergman, Jafferis & Maldacena introduced ABJM theory [JHEP 10(2008)091]
  - It's a susy $U(N) \times U(N)$ gauge theoy in 3D Minkowski: Chern-Simons terms at κ and $-\kappa$ levels + Matter.
  - It has $\mathcal{N} = 6$ supersymmetries and it is conformal invariant.
  - It's the holographic dual of M theory on $AdS_4 \times S^7/Z_\kappa$ with N units of Flux through AdS_4: a particular realization of the Gauge/gravity duality.
  - It affords the possibility of studying quantum gravity in 4D.
- Also in 2008, Bandres, Epstein and Schwarz [JHEP 0809 (2008) 027] proved by explicit computation that indeed the action of the theory is invariant under $\mathcal{N} = 6$ susy transformations.
- In 2009, Buchbinder et al. [JHEP 0910 (2009) 075] proved by using the $\mathcal{N} = 3$ harmonic superspace formalism that the ABJM is UV finite in the supergraph expansion.

Moving to noncommutative spacetime

- In 2008 Imeroni [JHEP 0810 (2008) 026] constructed the gravity dual of noncommutative of ABJM theory by deforming the ordinary dual. The noncommutative ABJM theory was not formulated.
- And yet, it was not until 2018 that noncommutative ABJM theory was considered in the scientific literature:
  - Enter "find t noncommutative and t ABJM" in Inspire hep searching engine and you get just 2 entries:
    - Noncommutative massive unquenched ABJM
      Y. Bea, N. Jokela, A. Ponni, A. V. Ramallo.
      Int.J.Mod.Phys. A33 (2018) no.14n15, 1850078
    - Quantum noncommutative ABJM theory: first steps
      C. P. Martin, J. Trampetic, J. You
      JHEP 1804 (2018) 070
  - In the first reference applications of the gravity dual of the noncommutative massive unquenched Chern-Simons matter theory using its gravity dual was studied.
  - In reference 2 the action of the NCABJM theory was displayed for the first time and their susy invariances proved.

Aim of the of the piece of research in the paper

Purpose of the paper:

- Establish the action of NC ABJM theory without auxiliary fields and its supersymmetries.
- Set up the BRST quantization of the theory and derive the Feynman rules.
- Take the first steps towards showing that the limit of vanishing noncommutativity parameters yield the ordinary theory:
  - This is nontrivial in spite of the fact that ordinary ABJM theory is UV finite, for UV finiteness is achieved by cancellation among diagrams so that remnants (Lorentz violating) with a non well defined limit may survive.
  - Notice that UV/IR mixing would imply that no divergences arise as the noncommutavity parameters go to zero.

Field content of the model

We considered the $U(1) \times U(1)$ NC theory, for it is the most different from its ordinary counterpart: NC U(1) is nonabelian.
Field content:

- Two Chern-Simons noncommutative gauge fields A_μ and $\hat{A}_\mu$.
- Four complex scalar fields X_A, $A = 1, 2, 3, 4$ carrying the **4** IRREP of $SU(4)$ (R-symmetry) & their complex conjugates X^A.
- Four Dirac spinors Ψ^A, $A = 1, 2, 3, 4$ carrying the $\bar{\mathbf{4}}$ IRREP of $SU(4)$ (R-symmetry)& their Dirac conjugates $\bar{\Psi}_A$.

The Moyal product

Being in noncommutative spacetime is tantamount to saying that functions of the coordinates are not multiplied pointwise but by means of the Moyal product –also known as $\star$-product:

$$(f \star g)(x) = f(x) \star g(x) = f(x) e^{\frac{i}{2} \overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu} g(x), \tag{1}$$

- $\theta^{\mu\nu}$ is the so-called noncommutativity tensor parameter.

Gauge symmetries and covariant derivatives

NC $U(1)$ BRST transformations:

$$sA_\mu = D_\mu\Lambda = \partial_\mu\Lambda + i[A_\mu \stackrel{\star}{,} \Lambda], \quad s\hat{A}_\mu = D_\mu\hat{\Lambda} = \partial_\mu\hat{\Lambda} + i[\hat{A}_\mu \stackrel{\star}{,} \hat{\Lambda}],$$

$$sX_A = -i\Lambda\star X_A + iX_A\star\tilde{\Lambda}, \quad sX^A = iX^A\star\Lambda - i\tilde{\Lambda}\star X^A,$$

$$s\Psi^A = -i\Lambda\star\Psi^A + i\Psi^A\star\tilde{\Lambda}, \quad s\Psi_A = i\Psi_A\star\Lambda - i\tilde{\Lambda}\star\Psi_A,$$

$$s\Lambda = -i\Lambda\star\Lambda, \quad s\hat{\Lambda} = -i\hat{\Lambda}\star\hat{\Lambda},$$

with covariant derivatives

$$D_\mu X_A = \partial_\mu X_A + iA_\mu\star X_A - iX_A\star\hat{A}_\mu,$$

$$D_\mu X^A = \partial_\mu X^A + i\hat{A}_\mu\star X^A - iX^A\star A_\mu,$$

$$D_\mu\Psi^A = \partial_\mu\Psi^A + iA_\mu\star\Psi^A - i\Psi^A\star\hat{A}_\mu,$$

$$D_\mu\Psi_A = \partial_\mu\Psi_A + i\hat{A}_\mu\star\Psi_A - i\Psi_A\star A_\mu.$$

Finite gauge transformations

For finite noncommutative $U(1) \times U(1)$ gauge transformations –useful to quickly get the correct action, we have

$$A_\mu \to g \star A_\mu \star g^\dagger - ig \star \partial_\mu g^\dagger, \quad \hat{A}_\mu \to \hat{g} \star \hat{A}_\mu \star \hat{g}^\dagger - i\hat{g} \star \partial_\mu \hat{g}^\dagger$$

$$X_A \to g \star X_A \star \hat{g}^\dagger, \quad X^A \to \hat{g} \star X^A \star g^\dagger,$$

$$\Psi^A \to g \star \Psi^A \star \hat{g}^\dagger, \quad \Psi_A \to \hat{g} \star \Psi_A \star g^\dagger,$$

- $g \star g^\dagger = g^\dagger \star g = 1$, $\hat{g} \star \hat{g}^\dagger = \hat{g}^\dagger \star \hat{g} = 1$, $g(x)$, $\hat{g}(x)$ being real functions.

Guessing the action I

The action must be invariant under

- the previous noncommutative gauge transformations,
- the Rigid $SU(4) \approx SO(6)$ R-symmetry,
- $\mathcal{N} = 6$ supersymmetry transformations,

AND

- Quadratic in the fermion fields
- and made out of Lorentz invariant polynomials (Ψ and $\bar{\Psi}$ in pairs), if we also transform the $\theta^{\mu\nu}$
- One should recall that the $\star$-product defines a trace:

$$\int d^3x\,(f_1 \star f_2 \star \ldots\ldots \star f_n)(x) = \int d^3x\,(f_{\pi(1)} \star f_{\pi(2)} \star \ldots\ldots \star f_{\pi(n)})(x),$$

$\pi(1)\ldots\pi(n)$ being any cyclic permutation of $1, 2, \ldots\ldots, n$.

Guessing the action II

After some guessing and a little bit of tinkering, one comes out with the following action for $U(1)$ noncommutative ABJM theory:

$$
\begin{aligned}
S &= S_{\mathrm{CS}} + S_{\mathrm{kin}} + S_4 + S_6 , \\
S_{\mathrm{CS}} &= \tfrac{\kappa}{2\pi} \int d^3x \, \varepsilon^{\mu\nu\lambda} \Big(\tfrac{1}{2} A_\mu \star \partial_\nu A_\lambda + \tfrac{i}{3} A_\mu \star A_\nu \star A_\lambda \\
&\qquad\qquad - \tfrac{1}{2} \hat{A}_\mu \star \partial_\nu \hat{A}_\lambda - \tfrac{i}{3} \hat{A}_\mu \star \hat{A}_\nu \star \hat{A}_\lambda \Big) , \\
S_{\mathrm{kin}} &= \tfrac{\kappa}{2\pi} \int d^3x \, \left(-D^\mu X^A \star D_\mu X_A + i \bar{\Psi}_A \star \not{D} \Psi^A \right) ,
\end{aligned}
$$

Guessing the action III

$$S_4 = S_{4a} + S_{4b} + S_{4c},$$

$$S_{4a} = \frac{i\kappa}{2\pi}\int d^3x \left[\varepsilon^{ABCD}(\bar{\Psi}_A \star X_B \star \Psi_C \star X_D) - \varepsilon_{ABCD}(\bar{\Psi}^A \star X^B \star \Psi^C \star X^D)\right],$$
$$S_{4b} = \frac{i\kappa}{2\pi}\int d^3x \left[\bar{\Psi}^A \star \Psi_A \star X_B \star X^B - \bar{\Psi}_A \star \Psi^A \star X^B \star X_B\right],$$
$$S_{4c} = \frac{i\kappa}{2\pi}\int d^3x \left[2(\bar{\Psi}_A \star \Psi^B \star X^A \star X_B) - 2(\bar{\Psi}^A \star \Psi_B \star X_A \star X^B)\right],$$

- $\bar{\Psi}_A \to [\bar{\Psi}_A]_i \equiv [\Psi_A]_j \gamma^0_{ji} = ([\Psi^A]_j)^* \gamma^0_{ji} \Longrightarrow \bar{\Psi}_A$ carries the $\bar{\mathbf{4}}$

Guessing the action IV

$$S_6 = -\frac{1}{6}\frac{\kappa}{2\pi}\int d^3x\, N^{IA}\star N^I_A$$
$$= \frac{1}{3}\frac{\kappa}{2\pi}\int d^3x \Big[X^A\star X_A\star X^B\star X_B\star X^C\star X_C + X_A\star X^A\star X_B\star X^B\star X_C\star X^C$$
$$+4X_A\star X^B\star X_C\star X^A\star X_B\star X^C - 6X^A\star X_B\star X^B\star X_A\star X^C\star X_C\Big],$$

where

$$N^{IA} = \tilde{\Gamma}^{IAB}\Big(X_C\star X^C\star X_B - X_B\star X^C\star X_C\Big) - 2\tilde{\Gamma}^{IBC}X_B\star X^A\star X_C,$$
$$N^I_A = \Gamma^I_{AB}\Big(X^C\star X_C\star X^B - X^B\star X_C\star X^C\Big) - 2\Gamma^{IBC}X^B\star X_A\star X_C,$$

Γ^I_{AB} being 4×4 matrices, the generators of the $\mathrm{SO}(6)$ group:

$$\Gamma^I_{AB} = -\Gamma^I_{BA},\ \forall I = 1,\dots,6;\ \Gamma^I\Gamma^J + \Gamma^J\Gamma^I = 2\delta^{IJ}$$
$$\tilde{\Gamma}^I = (\Gamma^I)^\dagger \iff \tilde{\Gamma}^{IAB} = (\Gamma^I_{BA})^* = -(\Gamma^I_{AB})^* = \frac{1}{2}\varepsilon^{ABCD}\Gamma^I_{CD},$$
$$N^I_A = \left(N^{IA}\right)^\dagger.$$

How gauge invariance comes about

- Gauge invariance occurs because of the trace property of the $\star$-product:
- It is the action which is gauge invariant not the Lagrangian, unlike in the ordinary ABJM theory.

Examples:

$$\int d^3x\, \bar{\Psi}'_A \star \not{D}'\Psi'^A = \int d^3x\, \hat{g} \star \bar{\Psi}_A \star g^\dagger \star g \star \not{D}\Psi_A \star \hat{g}^\dagger =$$
$$\int d^3x\, \bar{\Psi}_A \star g^\dagger \star g \star \not{D}\Psi_A \star \hat{g}^\dagger \star \hat{g} = \int d^3x\, \bar{\Psi}_A \star \not{D}\Psi_A$$

$$\int d^3x\, \varepsilon^{ABCD} \bar{\Psi}'_A \star X'_B \star \Psi'_C \star X'_D =$$
$$\int d^3x\, \varepsilon^{ABCD} \hat{g} \star \bar{\Psi}_A \star g^\dagger \star g \star X_B \star \hat{g}^\dagger \star \hat{g} \star \Psi_C \star g^\dagger \star g \star X_D \star \hat{g}^\dagger =$$
$$\int d^3x\, \varepsilon^{ABCD} \bar{\Psi}_A \star X_B \star \Psi_C \star X_D \star \hat{g}^\dagger \star \hat{g} = \int d^3x\, \varepsilon^{ABCD} \bar{\Psi}_A \star X_B \star \Psi_C \star X_D$$

Setting $\theta^{\mu\nu} = 0$ in the action

S_4 and S_6 VANISH, when $\theta^{\mu\nu} = 0$.

- By removing the $\star$ from the previous eqs.

$$S_{4a} = \tfrac{i\kappa}{2\pi}\int d^3x \left[\varepsilon^{ABCD}(\bar\Psi_A X_B \Psi_C X_D) - \varepsilon_{ABCD}(\bar\Psi^A X^B \Psi^C X^D)\right] = 0,$$
$$S_{4b} = \tfrac{i\kappa}{2\pi}\int d^3x \left[\bar\Psi^A \Psi_A X_B X^B - \bar\Psi_A \Psi^A X^B X_B\right] = 0,$$
$$S_{4c} = \tfrac{i\kappa}{2\pi}\int d^3x \left[2(\bar\Psi_A \Psi^B X^A X_B) - 2(\bar\Psi^A \Psi_B X_A X^B)\right] = 0,$$

$$N^{IA} = \tilde\Gamma^{IAB}\left(X_C X^C X_B - X_B X^C X_C\right) - 2\tilde\Gamma^{IBC} X_B X^A X_C = 0,$$
$$N^I_A = \Gamma^I_{AB}\left(X^C X_C X^B - X^B X_C X^C\right) - 2\Gamma^{IBC} X^B X_A X_C = 0,$$

Hence

$$S_6 = -\frac{1}{6}\frac{\kappa}{2\pi}\int d^3x\, N^{IA} N^I_A = 0.$$

- Huge difference between ordinary and NC $U(1)$ ABJM theory: Not clear what the limit $\lim_{\theta\to 0}$ of the quantum theory will be.
- NC $U(1)$ theory closer to ordinary $U(N)$ ABJM theory, at least in the planar limit: Replace in the action the gauge fields with $U(N)$ fields and the matter fields with fields transforming thus

$$Field \to U_A\, FIELD\, U^\dagger_{\hat A}.$$

The $\mathcal{N}=6$ supersymmetry transformations

After inspection of the susy transformations of the ordinary $U(N)$, we came up with the following proposal for the susy transformations of the NC U(1) ABJM theory:

$$
\begin{aligned}
&\delta A_\mu = \Gamma^I_{AB}\bar{\varepsilon}^I\gamma_\mu\Psi^A\star X^B - \tilde{\Gamma}^{IAB}X_B\star\bar{\Psi}_A\gamma_\mu\varepsilon^I,\\
&\delta\hat{A}_\mu = \Gamma^I_{AB}X^B\star\bar{\varepsilon}^I\gamma_\mu\Psi^A - \tilde{\Gamma}^{IAB}\bar{\Psi}_A\gamma_\mu\varepsilon^I\star X_B,\\
&\delta X_A = i\Gamma^I_{AB}\bar{\varepsilon}^I\Psi^B\\
&\delta X^A = -i\tilde{\Gamma}^{IAB}\bar{\Psi}_B\varepsilon^I,\\
&\delta\Psi^A = -\tilde{\Gamma}^{IAB}\not{D}X_B\varepsilon^I + N^{IA}\varepsilon^I,\\
&\delta\Psi_A = \Gamma^I_{AB}\not{D}X^B\varepsilon^I + N^I_A\varepsilon^I,\\
&\delta\bar{\Psi}_A = \delta\Psi^T_A\gamma^0 = -\Gamma^I_{AB}\bar{\varepsilon}^I\not{D}X^B + N^I_A\bar{\varepsilon}^I,\\
&N^{IA} = \tilde{\Gamma}^{IAB}\Big(X_C\star X^C\star X_B - X_B\star X^C\star X_C\Big) - 2\tilde{\Gamma}^{IBC}X_B\star X^A\star X_C
\end{aligned}
$$

$$
\bar{\varepsilon}^I = (\varepsilon^I)^T\gamma^0,\quad (N^I_A)^T = N^I_A,\quad (N^{IA})^\dagger = N^I_A.
$$

- The summands in RED do not occur in the ordinary $U(1)$ theory.
- Now, is $\delta S = 0$?

$\mathcal{N} = 6$ susy invariances

After a lengthy algebra, we obtained that indeed

$$\delta S = 0.$$

- I will not torture you with the details (please, read the appropriate 6 pages of the paper if you are interested in them).
- In hindsight all the efforts were succesfull because
  - 1) the trace property of the $\star$-product and
  - 2) the "noncommutative" Fierz identity for 2D spinors $\int d^3x \left(\Psi_{1i} \star \bar{\Psi}_2 \star \chi_3 + \Psi_{2i} \star \bar{\chi}_3 \star \Psi_1 + \chi_{3i} \star \bar{\Psi}_1 \star \Psi_2 \right) = 0,\ \forall i = 1,2.$
  - We were lucky that the Fierz identity holds in the integrated sense, otherwise the susy invariance under the previous transformations would not have hold.

Path integral BRST quantisation

- Quantisation is carried out in a standard BRST fashion by adding to the classical action the Landau gauge BRST exact bit

$$S_{\mathrm{gf+ghost}} = -\tfrac{\kappa}{2\pi}\int d^3x\, s\Big[\bar{\Lambda}\star\partial_\mu A^\mu - \bar{\hat{\Lambda}}\star\partial_\mu \hat{A}^\mu\Big],$$
$$s\bar{\Lambda} = B, \quad sB = 0; s\bar{\hat{\Lambda}} = \hat{B}, s\hat{B} = 0.$$

- We have chosen the Landau gauge so that the gauge propagators have the softest possible IR behaviour:

$$\frac{\varepsilon_{\mu\rho\nu}p^\rho}{p^2} \quad \text{instead of} \quad \xi\frac{p_\mu p_\nu}{(p^2)^2},$$

and thus one does not have to face integrals that are not IR finite by power-counting at nonexceptional momenta, e.g,

$$\int \frac{d^3\ell}{(2\pi)^3}\, e^{i\ell_\mu\theta^{\mu\nu}P_\nu}\frac{1}{(\ell^2)^2}$$

- The Feynman rules are obtained in a standard way

UV/IR mixing and the limit $\theta^{\mu\nu} \to 0$, I

- In NC field theories the phenomenon of UV/IR mixing occurs: In the "nonplanar" part of a 1PI Green function, one meets, when $\theta^{\mu\nu}$ is set to 0 in the integrand, non UV finite integrals. These are integrals of the type

$$\int \frac{d^n\ell}{(2\pi)^n}\, e^{-i\ell\theta P}\, \frac{N_{\mu_1\cdots\mu_m}(q,p_1,\cdots,p_n)}{\ell^2(\ell+Q_1)^2\cdots(\ell+Q_s)^2},$$

$$\ell\theta P \equiv \ell_\mu \theta^{\mu\nu} P_\nu .$$

- A non zero $\theta^{\mu\nu} P_\nu$ makes $e^{i\ell\theta P}$ oscillate very rapidly in the UV so that it renders the integral UV finite.
  - $\theta^{\mu\nu}$ provides a built-in partial UV cut-off, although non Lorentz invariant
- The divergence reappears when $\theta^{\mu\nu} P_\nu \to 0$, ie, in the IR.
- Hence the limit $\theta^{\mu\nu} \to 0$ is problematic, unless the theory is finite by power-counting. See next$\to$

- An integral that occurs in the Feynman diagram expansion of our NC ABJM theory is

$$I^{\mu}(p,\tilde{q}) = \int \frac{d^3\ell}{(2\pi)^3}\, e^{-i\ell\theta q}\, \frac{\ell^{\mu}}{\ell^2(\ell-p)^2} = \hat{I}_5\, p^{\mu} + \hat{I}_6\, \tilde{q}^{\mu},\; \tilde{q}^{\mu} \equiv \theta^{\mu\nu} q_{\nu}$$

- If we set $\theta^{\mu\nu} = 0$ in the integrand –ie, remove the phase factor from the integrand, the resulting integral is UV divergent –logarithmically– by power-counting
- When $\theta^{\mu\nu} \neq 0$, $\hat{I}_5$ and $\hat{I}_6$ read

$$\hat{I}_5 = \frac{i}{2(2\pi)^{3/2}} \int_0^1 dx\; x e^{-ixp\theta q} \left(\frac{p^2 x(1-x)}{\tilde{q}^2}\right)^{-1/4} K_{-1/2}\left(\sqrt{\tilde{q}^2 p^2 x(1-x)}\right),$$
$$\hat{I}_6 = \frac{1}{2(2\pi)^{3/2}} \int_0^1 dx e^{-ixp\theta q} \left(\frac{p^2 x(1-x)}{\tilde{q}^2}\right)^{1/4} K_{1/2}\left(\sqrt{\tilde{q}^2 p^2 x(1-x)}\right),$$

- For small $\tilde{q}^{\mu}$, we have

$$\hat{I}^{\mu}(p,\tilde{q}) = \frac{1}{8\pi} \frac{\tilde{q}^{\mu}}{\sqrt{\tilde{q}^2}} + \frac{i}{16} \frac{p^{\mu}}{\sqrt{p^2}} + \hat{f}^{\mu}(p,\tilde{q}),$$
$$\hat{f}^{\mu}(p,\tilde{q}) \to 0 \quad \text{as} \quad \tilde{q}^{\mu} \to 0.$$

- $\frac{\tilde{q}^{\mu}}{\sqrt{\tilde{q}^2}}$ bounded but NO LIMIT as $\theta^{\mu\nu} \to 0$.

The strategy to obtain the limit $\theta^{\mu\nu} \to 0$

- In view of what we've discussed in the previous slide, for the limit $\theta^{\mu\nu} \to 0$ to exist, we need that the $\mathcal{N} = 6$ susy of the theory be strong enough as to give rise to a deep cancellation among the several diagrams contributing to a given 1PI function.
- The strategy we followed was
  - to add up the integrands of classes of diagrams, simplify the numerators and, then, end up with a Feynman integral that is UV finite by POWER-COUNTING when $\theta^{\mu\nu} = 0$ in the integrand and, then,
  - apply Zimmermann's power-counting th. and Lebesgue's dominated converge th.

The theorems

- Zimmermann's power-counting th.
  - Make the replacement $\frac{1}{\ell^2 - i\varepsilon} \to \frac{1}{\ell^2 - i\vec{\ell}^2\varepsilon}$. Then,
  - Power-counting guarantees that the integral is ABSOLUTELY convergent.
- Lebesgue's dominated convergence theorem:

If $\int \frac{d^n\ell}{(2\pi)^n} |g(\ell)| < +\infty, \quad |f(\ell,\theta)| \leq |g(\ell)|$ almost everywhere $\forall \theta \in [-a, a]$

Then $\lim_{\theta\to 0} \int \frac{d^n\ell}{(2\pi)^n} f(\ell,\theta) = \int \frac{d^n\ell}{(2\pi)^n} \lim_{\theta\to 0} f(\ell,\theta)$

- In our case

$$|\sum e^{i\ell\theta P_i} \frac{\text{N(Polinomial)}}{\text{D(Polinomial)}}| \leq \sum |\frac{\text{N(Polinomial)}}{\text{D(Polinomial)}}|$$

- Apply Zimmermann's th to

$$\sum \frac{\text{N(Polinomial)}}{\text{D(Polinomial)}}$$

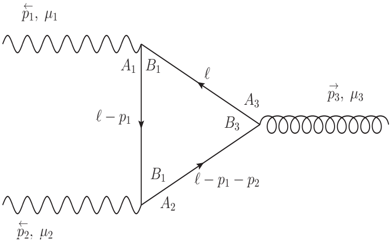
- The one-loop 1PI, $\hat{\Pi}^{\mu_1\mu_2\mu_3}_{AA\hat{A}}$, contribution to $\langle A^{\mu_1} A^{\mu_2} \hat{A}^{\mu_3} \rangle$ is given by

$$\hat{\Pi}^{\mu_1\mu_2\mu_3}_{AA\hat{A}} = \hat{\Pi}^{\mu_1\mu_2\mu_3}_{AA\hat{A}+} + \hat{\Pi}^{\mu_1\mu_2\mu_3}_{AA\hat{A}-}$$

$$\hat{\Pi}^{\mu_1\mu_2\mu_3}_{AA\hat{A}+} = \hat{S}^{\mu_1\mu_2\mu_3}_{\rm tria1} + \hat{F}^{\mu_1\mu_2\mu_3}_{\rm tria1} + \hat{S}^{\mu_1\mu_2\mu_3}_{\rm bub1} + \hat{S}^{\mu_1\mu_2\mu_3}_{\rm bub3+}$$

$$\hat{\Pi}^{\mu_1\mu_2\mu_3}_{AA\hat{A}-} = \hat{S}^{\mu_1\mu_2\mu_3}_{\rm tria2} + \hat{F}^{\mu_1\mu_2\mu_3}_{\rm tria2} + \hat{S}^{\mu_1\mu_2\mu_3}_{\rm bub2} + \hat{S}^{\mu_1\mu_2\mu_3}_{\rm bub3-}.$$

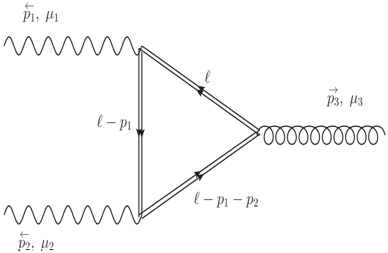
Scalar Triangle 1



$$\hat{S}^{\mu_1\mu_2\mu_3}_{\text{trial}} = \sum_A e^{\frac{i}{2}p_1\theta p_2} \int \frac{d^D\ell}{(2\pi)^D} e^{-i\ell\theta(p_1+p_2)} \cdot \frac{(2\ell-p_1)^{\mu_1}(2\ell-2p_1-p_2)^{\mu_3}(2\ell-p_1-p_2)^{\mu_2}}{\ell^2(\ell-p_1)^2(\ell-p_1-p_2)^2}.$$

- IN RED, integrals which give rise to an ill-defined $\theta^{\mu\nu} \to 0$ limit.

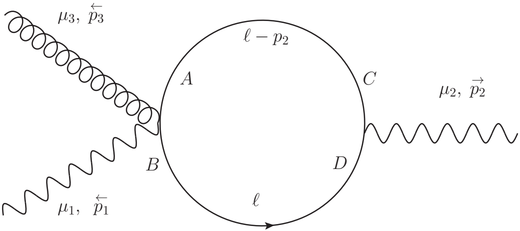
Fermion Triangle 1



$$\hat{F}^{\mu_1\mu_2\mu_3}_{\text{tria1}} = -\sum_A e^{\frac{i}{2}p_1\theta p_2}\int \frac{d^D\ell}{(2\pi)^D}e^{-i\ell\theta(p_1+p_2)}\frac{\mathrm{tr}\gamma^{\mu_1}\not{\ell}\gamma^{\mu_3}(\not{\ell}-\not{p}_1-\not{p}_2)\gamma^{\mu_2}(\not{\ell}-\not{p}_1)}{\ell^2(\ell-p_1)^2(\ell-p_1-p_2)^2}.$$

- IN RED, integrals which give rise to an ill-defined $\theta^{\mu\nu} \to 0$ limit.

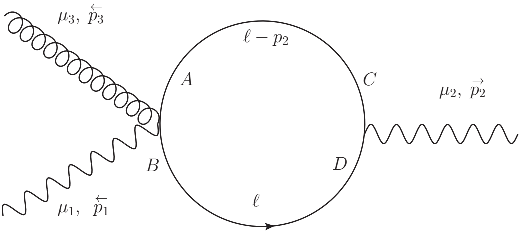
Scalar Bubble 1



$$\hat{S}^{\mu_1\mu_2\mu_3}_{\text{bub1}} = -\sum_A e^{\frac{i}{2}p_1\theta p_2} \int \frac{d^D\ell}{(2\pi)^D} e^{-i\ell\theta(p_1+p_2)} \frac{\eta^{\mu_2\mu_3}(2\ell-p_1)^{\mu_1}}{\ell^2(\ell-p_1)^2}.$$

- IN RED, integrals which give rise to an ill-defined $\theta^{\mu\nu} \to 0$ limit.

Scalar Bubble 3



$$\hat{S}^{\mu_1\mu_2\mu_3}_{\text{bub3}} = \hat{S}^{\mu_1\mu_2\mu_3}_{\text{bub3}+} + \hat{S}^{\mu_1\mu_2\mu_3}_{\text{bub3}-}$$

$$\hat{S}^{\mu_1\mu_2\mu_3}_{\text{bub3}+} = -e^{\frac{i}{2}p_1\theta p_2}\sum_A\int\frac{d^D\ell}{(2\pi)^D}e^{-i\ell\theta(p_1+p_2)}\frac{\eta^{\mu_1\mu_2}(2\ell-p_1-p_2)^{\mu_3}}{\ell^2(\ell-p_1-p_2)^2}$$

$$\hat{S}^{\mu_1\mu_2\mu_3}_{\text{bub3}-} = -e^{-\frac{i}{2}p_1\theta p_2}\sum_A\int\frac{d^D\ell}{(2\pi)^D}e^{-i\ell\theta(p_1+p_2)}\frac{\eta^{\mu_1\mu_2}(2\ell-p_1-p_2)^{\mu_3}}{\ell^2(\ell-p_1-p_2)^2}$$

- IN RED, integrals which give rise to an ill-defined $\theta^{\mu\nu} \to 0$ limit.

All dangerous summands add up to zero

- Upon adding the INTEGRANDS and doing a little algebra, one shows that all the dangerous contributions IN RED cancel at the integrand level:

$$
\begin{aligned}
&\hat{\Pi}^{\mu_1\mu_2\mu_3}_{AA\hat{A}+} = \hat{S}^{\mu_1\mu_2\mu_3}_{\text{tria1}} + \hat{F}^{\mu_1\mu_2\mu_3}_{\text{tria1}} + \hat{S}^{\mu_1\mu_2\mu_3}_{\text{bub1}} + \hat{S}^{\mu_1\mu_2\mu_3}_{\text{bub3}+},\\
&\hat{\Pi}^{\mu_1\mu_2\mu_3}_{AA\hat{A}+} = -e^{\frac{i}{2}p_1\theta p_2}\Big(\Pi_1^{\mu_1\mu_2\mu_3}\cdot I(p_1+p_2) + \Pi_2^{\mu_1\mu_2\mu_3}\cdot\hat{I}(p_1) + \Pi_3^{\mu_1\mu_2\mu_3}\cdot I_+\\
&\qquad + \Pi_4^{\mu_1\mu_2}(p_1,p_2)\cdot I_+^{\mu_3} + \Pi_4^{\mu_2\mu_3}(p_2,p_3)\cdot I_+^{\mu_1} + \Pi_4^{\mu_1\mu_3}(p_1,p_3)\cdot I_+^{\mu_2}\Big),\\
&\hat{I}(p_1) = \int \frac{d^D\ell}{(2\pi)^D}\,\frac{e^{-i\ell\theta(p_1+p_2)}}{\ell^2(\ell-p_1)^2},\\
&I_+ = \int \frac{d^D\ell}{(2\pi)^D}\,\frac{e^{-i\ell\theta(p_1+p_2)}}{\ell^2(\ell-p_1)^2(\ell-p_1-p_2)^2},\quad I_+^{\mu} = \int \frac{d^D\ell}{(2\pi)^D}\,\frac{\ell^\mu\, e^{-i\ell\theta(p_1+p_2)}}{\ell^2(\ell-p_1)^2(\ell-p_1-p_2)^2}\\
&\Pi_1^{\mu_1\mu_2\mu_3} = 2\big(\eta^{\mu_2\mu_3}(p_1+p_2)^{\mu_1} - \eta^{\mu_1\mu_3}(p_1+p_2)^{\mu_2}\big),\ \Pi_2^{\mu_1\mu_2\mu_3} = 2\eta^{\mu_2\mu_3}p_2^{\mu_1},\\
&\Pi_3^{\mu_1\mu_2\mu_3} = p_1^{\mu_1}(p_1+p_2)^{\mu_3}(2p_1+p_2)^{\mu_2} + \eta^{\mu_1\mu_2}\big(p_1^{\mu_3}(2p_1\cdot p_2+p_2^2) - p_2^{\mu_3}p_1^2\big)\\
&\qquad - \eta^{\mu_1\mu_3}\big(p_2^{\mu_2}p_1^2 + p_1^{\mu_2}(2p_1\cdot(p_1+p_2)+p_2^2)\big) + \eta^{\mu_2\mu_3}\big(p_2^{\mu_1}p_1^2 - p_1^{\mu_1}(2p_1\cdot p_2+p_2^2)\big),\\
&\Pi_4^{\mu_1\mu_2} = 2(\eta^{\mu_1\mu_2}p_1\cdot p_2 - p_1^{\mu_2}p_2^{\mu_1}).
\end{aligned}
\tag{2}
$$

- The integrals IN RED are finite by power-counting when $\theta^{\mu\nu}=0$ in the integrand. Hence Zimmermann's th. and Lebesgue's dominated convergence th. imply that the limit $\theta^{\mu\nu}\to 0$ can be taken inside the integral safely.
- Likewise for $\hat{\Pi}^{\mu_1\mu_2\mu_3}_{AA\hat{A}-}$.

Superficial UV degree and the limit $\theta^{\mu\nu} \to 0$

- The superficial degree, $\mathscr{D}$ of divergence of a 1PI Feynman diagram with, E_G, external gauge legs, E_F, external fermion legs and E_x, external scalar legs, reads

$$\mathscr{D} = 3 - E_G - E_F - \frac{1}{2} E_X$$

- Hence, all diagrams with $E_G + E_F > 3$ are UV finite by power-counting when all the Moyal phases are removed and, therefore, they have a well-defined limit when $\theta^{\mu\nu} \to 0$, which agrees with the ordinary –ie, in Minkowski– value: just apply Zimmermann's th. and Lebesgue's dominated convergence limit.
- There remains to be analysed the diagrams corresponding to the following triplets,(E_G, E_F, E_X):

$$(2,0,0),(0,2,0),(0,0,2),(3,0,0,),(1,0,2),(1,2,0),(0,0,4),(1,0,4),(0,0,6)$$

- In the paper we have shown that all 1PI functions corresponding to the triplets in RED have a well-defined $\theta^{\mu\nu} \to 0$ limit and that this limit is the value of the ordinary 1PI function. It remains to see what happens with (0,0,4),(1,0,4),(0,0,6).

Conclusions

- Noncommutative ABJM theory as defined above seems to be a perfectly (at least in the perturbative regime) well-defined finite theory with $\mathscr{N} = 6$ supersymmetries
- Seems to flow to ordinary ABJM theory in the IR.
- Can be put to work to further study the gauge/gravity duality and, perhaps, give a definition of noncommutative quantum gravity in 4D.

THANK YOU FOR LISTENING!