Non-relativistic String Theory

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work done in collaboration with

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Humboldt Kolleg Frontiers in Physics:

From the Electroweak to the Planck Scales

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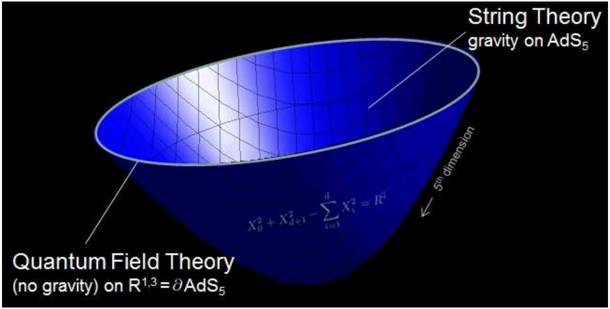
Why Non-relativistic String Theory

and (as part of it) Non-relativistic Gravity?

Holography



Gravity is not only used to describe the gravitational force!

Non-relativistic Holography

two approaches

- Keep general relativity in the bulk but take background geometry with non-relativistic isometries

Christensen, Hartong, Kiritsis, Obers and Rollier (2013-2015)

- Take non-relativistic gravity in the bulk

Gomis, Ooguri (2000); Danielsson, Guijosa, Kruczenski (2000)

Gomis, Gomis, Kamimura (2005); Gopakumar, Bagchi (2009)

- does there exist a consistent non-relativistic quantum gravity?

- The non-relativistic gravity corresponding to non-relativistic string theory is not the usual Newtonian gravity!

Outline

Newton-Cartan Gravity

Outline

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Nonrelativistic String Theory

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The 4D Galilei Algebra

- time translations: $\delta t = \xi^0$ <u>but not</u> $\delta t = \lambda^i x^i$!

- space translations: $\delta x^i = \xi^i \qquad i = 1, 2, 3$

- spatial rotations: $\delta x^i = \lambda^i{}_j\, x^j$

- Galilean boosts: $\delta x^i = \lambda^i t$

'Gauging' the 4D Galilei Algebra

symmetry	generators	gauge field	#	curvatures	#
time translations	H	τ_μ		$\tau_{\mu\nu} = \partial_{[\mu}\tau_{\nu]}$	6
space translations	P_a	$E_\mu{}^a$		$R_{\mu\nu}{}^a(P)$	18
Galilean boosts	G_a	$\Omega_\mu{}^a$	12	$R_{\mu\nu}{}^a(G)$	
spatial rotations	J_{ab}	$\Omega_\mu{}^{ab}$	12	$R_{\mu\nu}{}^{ab}(J)$	

$\mu = 0, 1, 2, 3;\ a = 1, 2, 3$

$R_{\mu\nu}{}^a(P) = 0 \quad (18)$: does only solve for part of $\Omega_\mu{}^a$, $\Omega_\mu{}^{ab}$!

From Galilei to Bargmann

Andringa, Panda, de Roo + E.B. (2011)

the zero commutator

$$[G_a, P_b] = 0$$

implies that a massive particle with non-zero spatial momentum P_B cannot by any boost transformation G_A be brought to a rest frame $\Rightarrow$

$$[G_a, P_b] = \delta_{ab} Z \quad \rightarrow \quad \text{extra gauge field } M_\mu$$

and additional 6 conventional constraints: $R_{\mu\nu}(Z) = 0$

Newton-Cartan Geometry

The independent NC fields $\{\tau_\mu, E_\mu{}^a, M_\mu\}$ transform as follows:

$$\begin{aligned}\delta\tau_\mu &= 0\,,\\ \delta E_\mu{}^a &= \lambda^a{}_b\, E_\mu{}^b + \lambda^a\tau_\mu\,,\\ \delta M_\mu &= \partial_\mu\sigma + \lambda_a\, E_\mu{}^a\end{aligned}$$

The spin-connection fields $\Omega_\mu{}^{ab}$ and $\Omega_\mu{}^a$ are functions of τ_μ , $E_\mu{}^a$ and M_μ

Metrics

general relativity has one non-degenerate metric that is invariant under Lorentz transformations

NC gravity has <u>two</u> degenerate metrics

$$\tau_{\mu\nu} = \tau_\mu \tau_\nu \qquad \text{and} \qquad h^{\mu\nu} = E^\mu{}_a E^\nu{}_b \delta^{ab}$$

that are invariant under the Bargmann transformations

$$\text{the symmetric tensor} \quad H_{\mu\nu} \equiv E_\mu{}^a E_\nu{}^b \delta_{ab} + 2M_{(\mu}\tau_{\nu)}$$

is invariant under Galilean transformations but not under central charge transformations!

The Lagrangian describing a particle coupled to NC gravity includes a Wess-Zumino term coupling of the form $M_\mu \dot{x}^\mu$

Particle Limits and NC Gravity

The NR limit of a Nambu-Goto particle coupled to general relativity is finite provided that the particle is coupled to a (zero flux) gauge field $\hat{M}_\mu$ which gives

$$\hat{M}_\mu = \omega \tau_\mu$$

with $\omega \to \infty$ in the NR limit

Galilei $\Rightarrow$ Bargmann

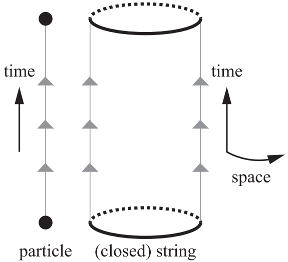
Outline

String NC Geometry

The string should be coupled to a (zero flux) 2-form gauge field $\hat{M}_{\mu\nu}$ with

$$\hat{M}_{\mu\nu} = \omega^2 \epsilon_{AB} \tau_\mu{}^A \tau_\nu{}^B \,, \qquad A = 0, 1$$

defining a string NC geometry with $\tau_\mu \;\rightarrow\; \tau_\mu{}^A$



The string Galilei algebra

$$D \text{ flat indices} \rightarrow \begin{cases} 2 \ \text{longitudinal indices } A \\ D-2 \ \text{transverse indices } a \end{cases}$$

longitudinal translations	H_A
transverse translations	P_a
string Galilei boosts	G_{Ab}
longitudinal Lorentz rotations	M_{AB}
transverse spatial rotations	J_{ab}

String Galilei $\Rightarrow$ String Bargmann

Gomis, Ooguri (2001); Brugues, Curtright, Gomis, Mezincescu (2004); Andringa, Gomis, de Roo + E.B. (2012)

$$[G_{Aa}, P_b] = 0 \quad \rightarrow \quad [G_{Aa}, P_b] = \delta_{ab} Z_A \qquad \text{plus} \quad Z_{[AB]}$$

The independent string NC fields are

$$\{\tau_\mu{}^A, E_\mu{}^a, M_\mu{}^A\}$$

longitudinal metric: $\tau_{\mu\nu} \equiv \tau_\mu{}^A \tau_\nu{}^B \eta_{AB}$

transverse 'metric': $H_{\mu\nu} \equiv E_\mu{}^a E_\nu{}^b \delta_{ab} + \left(\tau_\mu{}^A M_\nu{}^B + \tau_\nu{}^A M_\mu{}^B\right) \eta_{AB}$

Non-relativistic string theory in flat spacetime

$$S_{\text{flat}} = -\frac{1}{4\pi\alpha'} \int d^2\sigma \left(\partial x^a \,\overline{\partial} x^b \delta_{ab} + \lambda \,\overline{\partial} X + \overline{\lambda} \,\partial \overline{X} \right)$$

$$X = x^0 + x^1 \,, \qquad\qquad \overline{X} = x^0 - x^1$$

the x^1 direction is compactified over a circle of radius R

Gomis, Ooguri (2000); Danielsson, Guijosa, Kruczenski (2000)

- The spectrum consists of winding strings

NR Limit 'Polyakov' Particle

$$S_{\text{Pol.}} = -\tfrac{1}{2}\int d\tau \Big\{ -\frac{1}{e}\hat{E}_\mu{}^{\hat{A}}\dot{x}^\mu \hat{E}_\nu{}^{\hat{B}}\dot{x}^\nu \eta_{\hat{A}\hat{B}} + M^2 e - 2M\hat{M}_\mu \dot{x}^\mu \Big\}$$

$$\begin{aligned} S_{\text{Pol.}}(\omega^2) &= -\tfrac{1}{2}\int d\tau \frac{1}{e}\omega^2 \big[\tau_\mu \dot{x}^\mu - me\big]^2 \\ &= -\tfrac{1}{2}\int d\tau \frac{1}{e}\Big\{\lambda(\tau_\mu \dot{x}^\mu - me) - \frac{1}{4\omega^2}\lambda^2\Big\} \quad \Rightarrow \end{aligned}$$

$$S_{\text{Pol.}}(\text{N.R.}) = -\tfrac{1}{2}\int d\tau \frac{1}{e}\Big\{\dot{x}^\mu \dot{x}^\nu H_{\mu\nu} + \lambda\big(\tau_\mu \dot{x}^\mu - me\big)\Big\}$$

The Nonrelativistic Polyakov String

cp. to Harmark, Hartong, Menculini, Obers (2019)

$$h_{\alpha\beta} = e_\alpha{}^a e_\beta{}^b \eta_{ab}$$

$$e_\alpha{}^a \,, \tau_\mu{}^A : \quad \text{light-cone notation}$$

$$S_{\text{Pol.}} = -\frac{T}{2}\int d^2\sigma \left[\sqrt{-h}\, h^{\alpha\beta}\, \partial_\alpha x^\mu \partial_\beta x^\nu H_{\mu\nu} + \epsilon^{\alpha\beta}\left(\lambda\, e_\alpha \tau_\mu + \bar\lambda\, \bar e_\alpha \bar\tau_\mu\right) \partial_\beta x^\mu\right]$$

$$-\frac{T}{2}\int d^2\sigma\, \epsilon^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x^\nu B_{\mu\nu} + \frac{1}{4\pi}\int d^2\sigma \sqrt{-h} R(h) \Phi$$

This is the generalization of flat spacetime to a string NC background

J. Gomis, Z. Yan + E.B. (2018); J. Rosseel, C. Şimşek, Z. Yan + E.B. (to appear)

Outline

Newton-Cartan Gravity

Nonrelativistic String Theory

Properties

Comments

Ambiguity in Background Fields

Solving for the Lagrange multiplier E.O.M. gives the

Nambu-Goto particle

$$S_{\mathrm{NG}}(\mathrm{N.R.}) = -\frac{1}{2}\int d\tau\, m\left\{\frac{\dot{x}^\mu \dot{x}^\nu}{\tau_\mu \dot{x}^\mu} H_{\mu\nu} - B_\mu \dot{x}^\mu\right\}$$

$$H_{\mu\nu} \to H_{\mu\nu} + \tau_{(\mu} C_{\nu)}\,, \qquad B_\mu \to C_\mu$$

similarly, the background fields of non-relativistic string theory

$H_{\mu\nu}, B_{\mu\nu}, \Phi$ are defined by equivalence classes

Non-relativistic T-duality

The transverse T-dual of a non-relativistic string gives the usual dual $R \leftrightarrow 1/R$ string NC gravity

The spatial longitudinal T-dual of a nonrelativistic string in a string NC background is a relativistic Polyakov string in a general relativity (plus Kalb-Ramond plus dilaton) background with a null Killing vector

- Discrete Lightcone Quantization

Banks, Fischler, Shenker, Susskind (1996)

β-functions

quantum Weyl invariance at linearized level requires setting the linearized beta-functions equal to zero.

Gomis, Oh and Yan (2019); see also Callegos, Gürsoy, Zinnato (2019)

The nonlinear beta functions can be obtained as a nonrelativistic limit of the relativistic beta functions

The full nonlinear equations of motion of string NC Gravity are known

Gomis, Şimşek, Rosseel, Yan + E.B.

Solutions

work in progress

within relativistic string theory the pp-wave solution of general relativity is T-dual to the fundamental relativistic string solution

under non-relativistic T-duality in the null isometry direction the same pp-wave solution maps to a fundamental non-relativistic string solution

both the pp-wave and the non-relativistic fundamental string reduce to a 0-brane solution of NC gravity in one dimension lower

what about the other branes?

Actions

- NC gravity and String NC Gravity have no action

see, however, Hansen, Hartong, Obers (2019)

- Using the Lie algebra expansion technique, one can construct actions for extended versions of the (string) Bargmann algebra

Şimşek, Grosvenor, Yan + E.B. (2018); Izquierdo, Ortín, Romano + E.B. (2019)

Example

3D Poincare $\rightarrow$ 3D Galilei $+$ 3D extended Bargmann

cp. to Duval, Horvathy (2000); Jackiw, Nair (2000)

Outline

Summary

- String NC Geometry is to NR string theory what Riemannian geometry is to relativistic string theory

  - Polyakov action, equivalence classes
  - non-relativistic T-duality and DLCQ
  - β-functions and string NC equations of motion
  - actions

Open Issues

- Nonrelativistic holography?
- non-relativistic branes
- Double Field Theory?

S. M. Ko, C. Melby-Thompson, R. Meyer and J.-H. Park (2015); C. Blair (2019)

Take Home Message

nonrelativistic string theory can be studied on its own!