Holographic Entanglement Entropy from Numerical Relativity

Based on work with Daniel Grumiller, Stefan Stricker 1506.02658 (JHEP);
& Wilke van der Schee (15XX.XXXXX)

Christian Ecker

Summer School and Workshop on
the Standard Model and Beyond
Corfu
September 11, 2015

FWF

TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

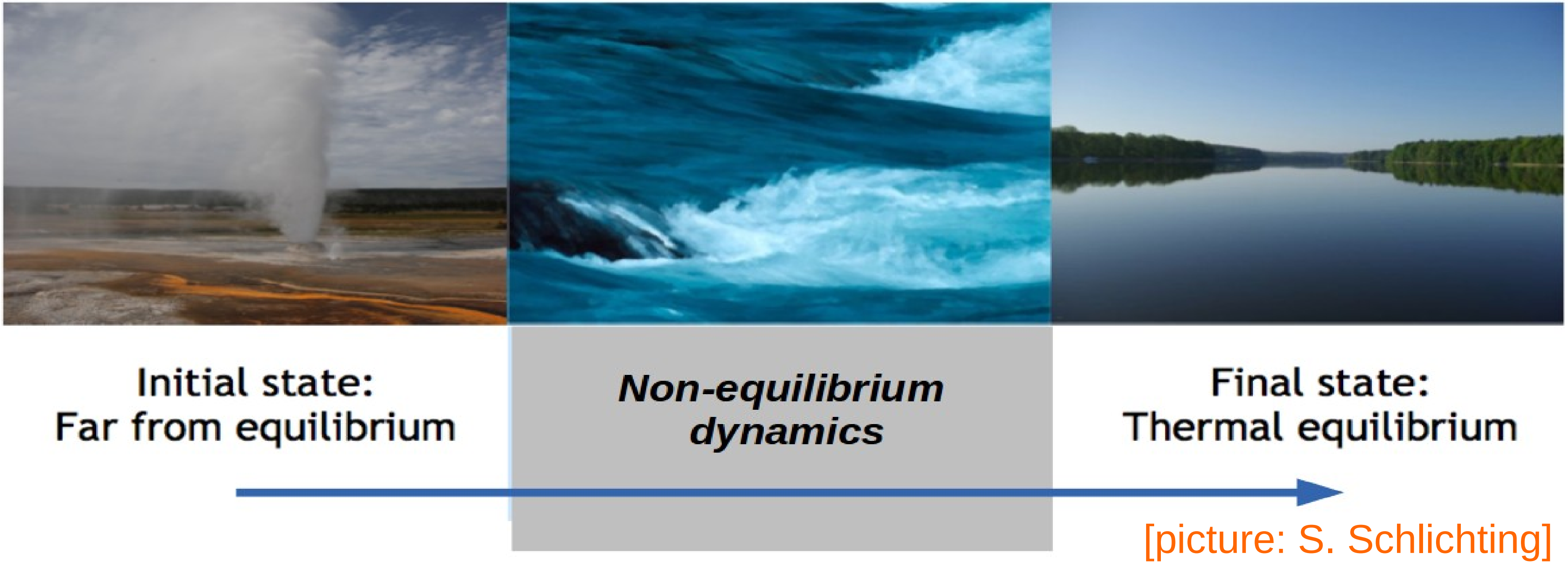
DOKTORATSKOLLEG PI
Particles and Interactions

INSTITUTE FOR THEORETICAL PHYSICS

Motivation

Central question:

How **evolves** a **strongly coupled** quantum system from a **far-from equilibrium** state to **equilibrium?**



[picture: S. Schlichting]

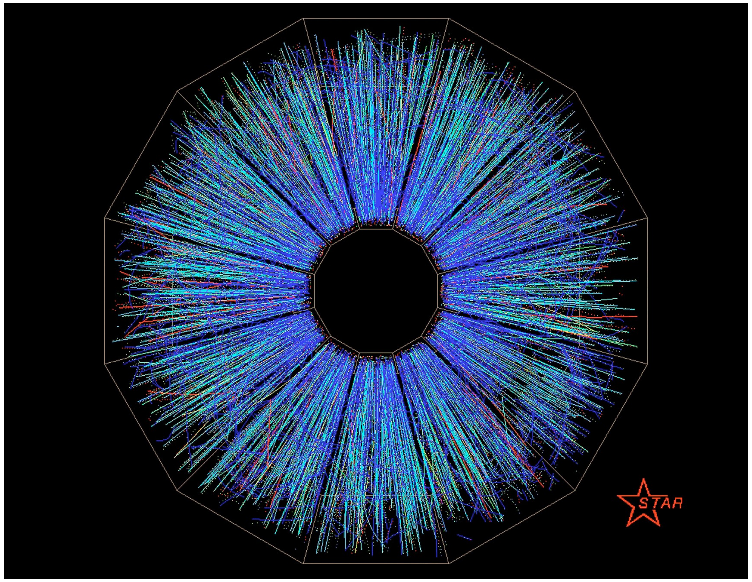
Challenges:

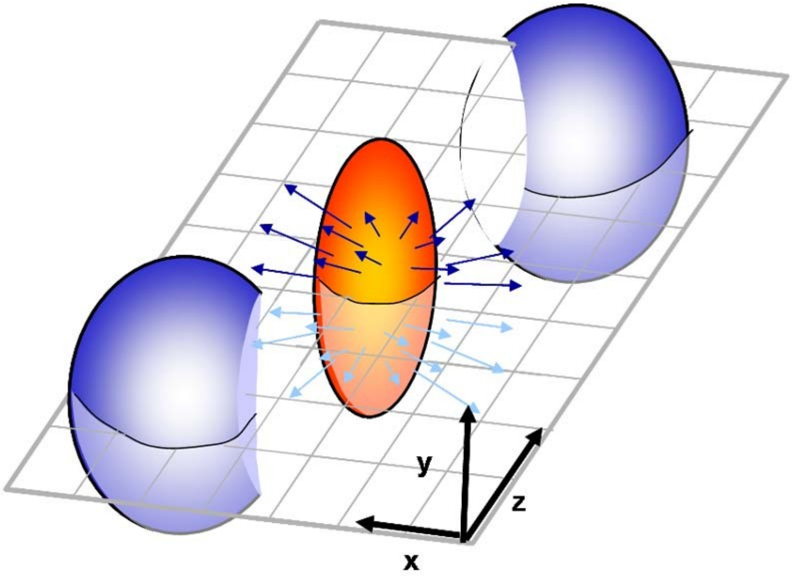
- Due to **strong coupling perturbative** methods are **not applicable**.
- **Far-from equilibrium** we are **outside the regime** of **linear response theory**.

Quark-gluon plasma in heavy ion collisions

[see talks by S. Kabana, A. Starinets]

Quark gluon plasma (QGP) is a **deconfined phase of quarks and gluons** which is produced in **heavy ion collision** (HIC) experiments at RHIC and LHC.





<u>Why AdS/CFT?</u>

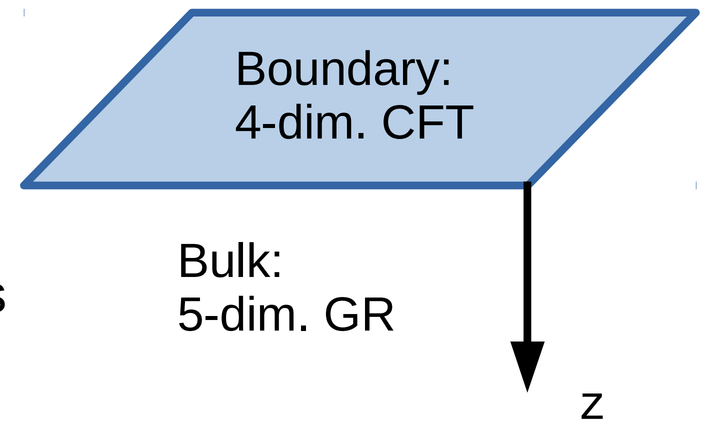
- QGP produced in HIC's behaves like a **strongly coupled liquid** rather than a **weakly coupled gas**.
- Initial fireball **thermalizes** on a very **short timescale (~1fm/c)**.
- **Fast thermalization** is a **robust feature** of **holographic HIC** models.

AdS/CFT correspondence

[see talks by A. Starinets, E. Kiritsis]

AdS/CFT correspondence: [Maldacena 97]

Type IIB string theory on $AdS_5 \times S^5$ is equivalent to $\mathcal{N}=4$ super symmetric $SU(N_c)$ **Yang-Mills theory** in 4D.



Supergravity limit:

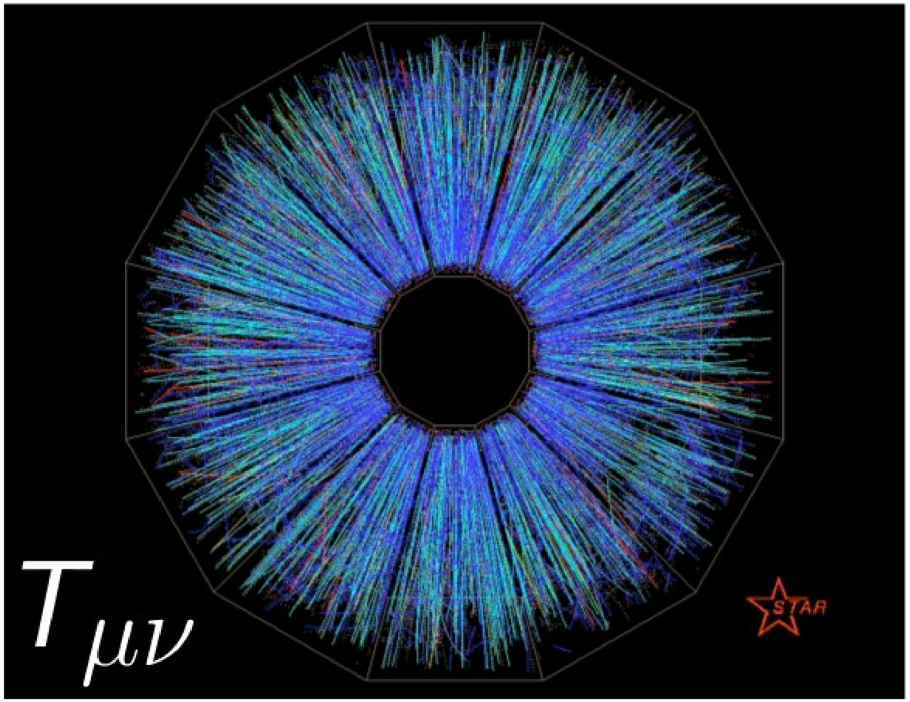
Strongly coupled large N_c $\mathcal{N}=4$ $SU(N_c)$ SYM theory is equivalent to **classical (super)gravity** on AdS_5.

Strategy:

- Use $\mathcal{N}=4$ SYM as **toymodel** for **QCD**.
- Build a **gravity model** dual to HICs, like colliding **gravitational shock waves**.
- Switch on the computer and solve the 5-dim. gravity problem **numerically**.
- Use the **holographic dictionary** to compute **observables in the 4 dim. field theory** form the gravity result.

Holographic thermalization

Thermalization = Black hole formation



- **Holographic dictionary** translates thermodynamic **properties (S,T,M)** of **black holes** to the corresponding **properties of the gauge theory**.
- Computing **black hole formation on AdS** in general requires methods from **numerical relativity**.
- The **observable** we use to **study thermalization** is **entanglment entropy**.

Entanglement entropy

Divide the system into **two parts** A,B.
The total Hilbert space factorizes:

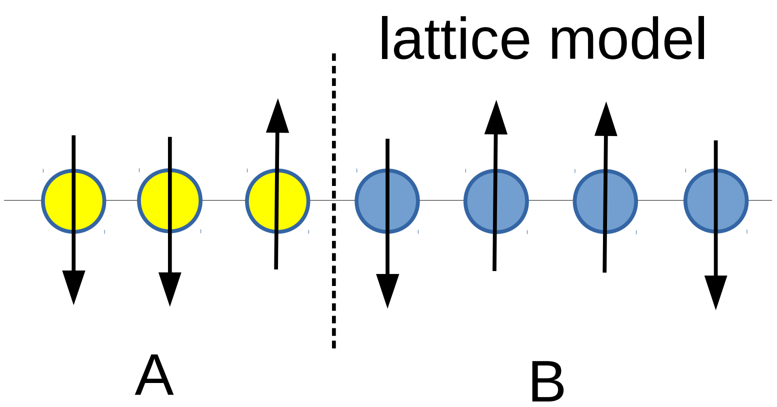
$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

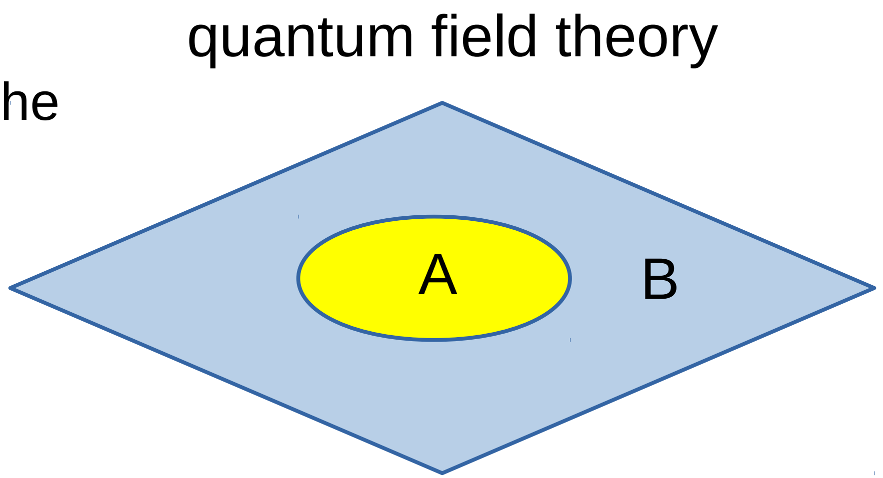
The **reduced density matrix** of A is obtained by the trace over $\mathcal{H}_\mathcal{B}$

$$\rho_A = \mathrm{Tr}_B \rho$$

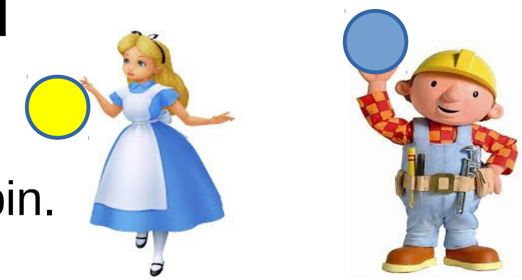
Entanglement entropy is defined as the **von Neumann entropy** of ρ_A:

$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$





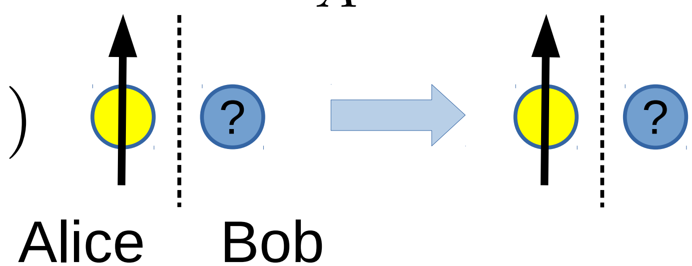
Entanglement entropy in a two quantum bit system



Consider a quantum system of two spin 1/2 dof's.
Observer Alice has only access to one spin and Bob to the other spin.

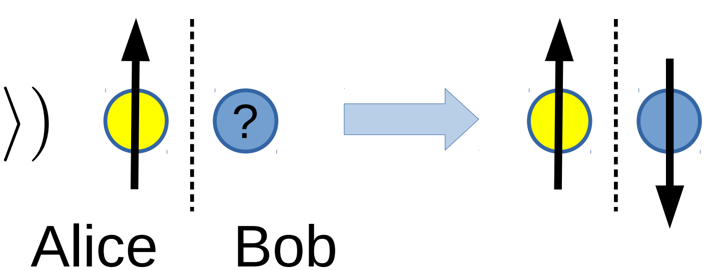
A **product state (not entangled)** in a two spin 1/2 system: $S_A = 0$

$$|\psi\rangle = \frac{1}{2}(|\uparrow_A\rangle + |\downarrow_A\rangle) \otimes (|\uparrow_B\rangle + |\downarrow_B\rangle)$$



A (maximally) **entangled state** in a two spin 1/2 system: $S_A = \log 2$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A\rangle \otimes |\downarrow_B\rangle - |\downarrow_A\rangle \otimes |\uparrow_B\rangle)$$



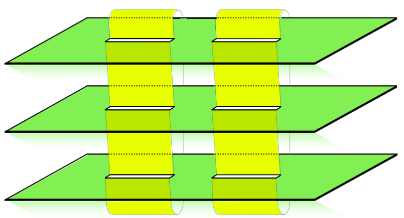
Entanglement entropy is a **measure** for how much a given quantum state is **entangled**.

Entanglement entropy in quantum field theories

The Basic Method to compute entanglement entropy in quantum field theories is the **replica method.**

Involves path integrals over n-sheeted Riemann surfaces ~ it's **complicated!**

With the **replica method** one gets **analytic results** for **1+1 dim. CFTs**. [Holzhey-Larsen-Wilczek 94]



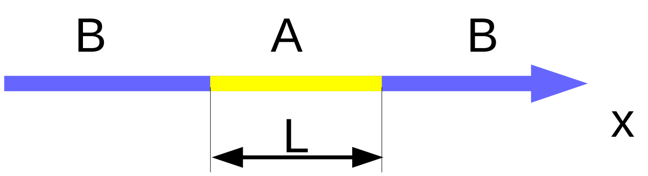
3-sheeted Riemann surface

One finds **universal scaling** with interval size:

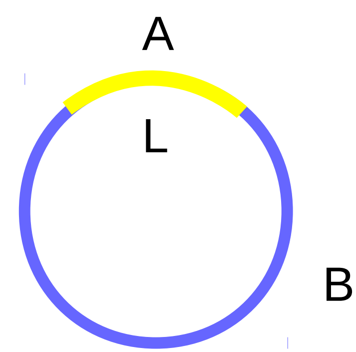
central charge of the CFT

$$S_A = \frac{c}{3}\log\frac{L}{a} + finite$$

UV cut off



1+1 dim. CFTs



Message: Computing entanglement entropy in interacting QFTs is complicated and analytically only possible in 1+1 dim. CFTs.

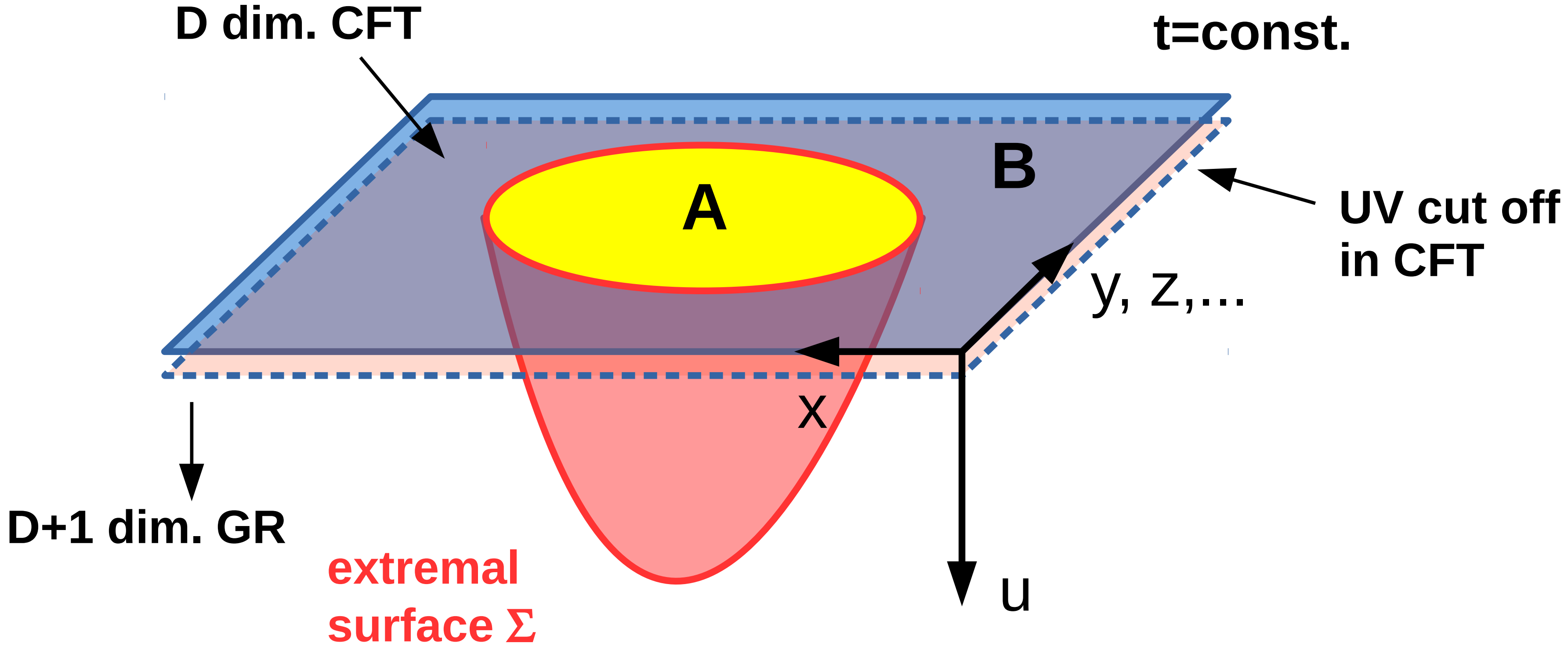
The **AdS/CFT** provides a **simpler method** that works also in **higher dimensions**.

Holographic entanglement entropy

Within **AdS/CFT** entanglement entropy can be computed form the **area of minimal (extremal) surfaces** in the gravity theory.

$$S_A = \frac{\mathrm{Area}(\Sigma)}{4G_N}$$

[Ryu-Takayanagi 06, Hubeny-Rangamani-Takayanagi 07]



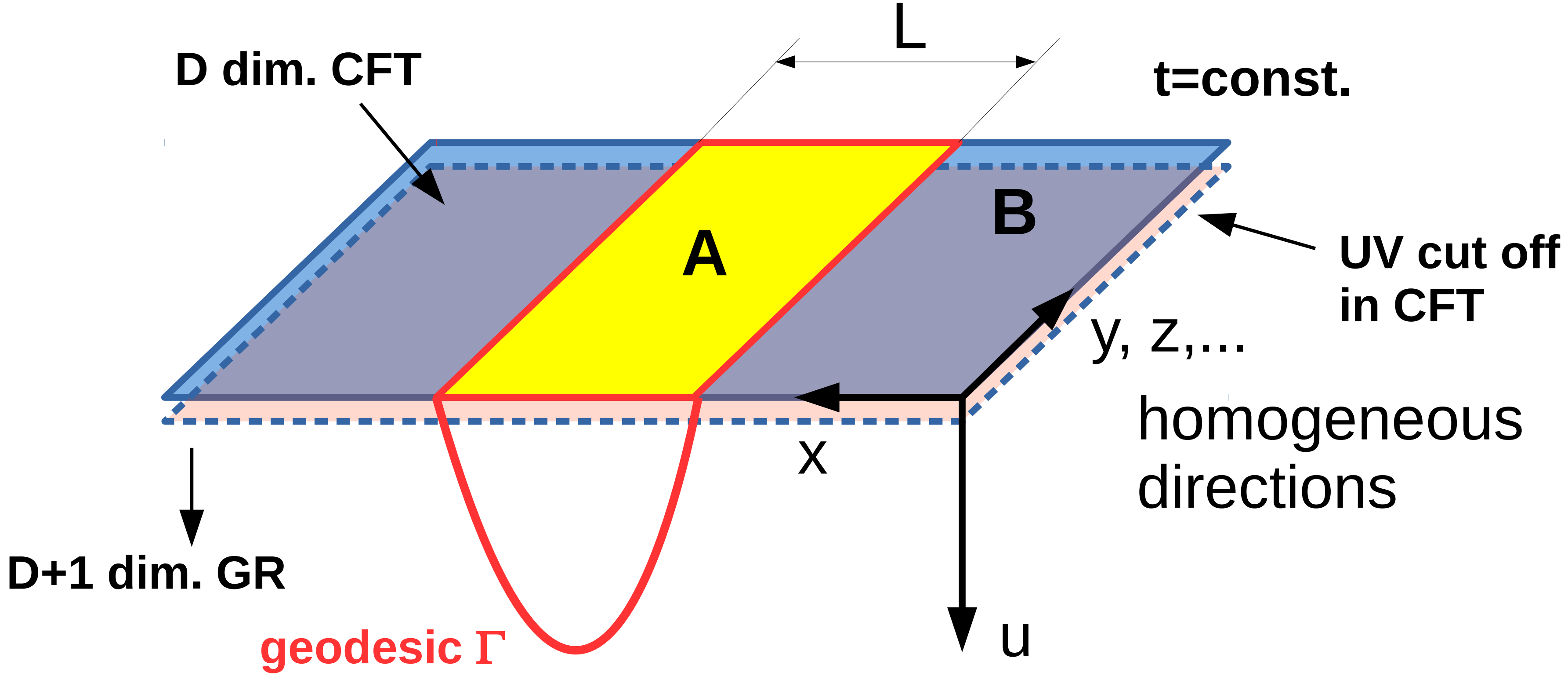
Holographic entanglement entropy

- In practice computing extremal co-dim. 2 hyper-surfaces is numerically involved.
- Can we somehow simplify our lives?

Entanglement entropy from geodesics

For **infinitely extended stripe regions respecting the symmetries** of the geometry, computing entanglement entropy essentially reduces to computing the **geodesics length**.

$$S_A = const. \frac{\text{Length}(\Gamma)}{4G_N}$$

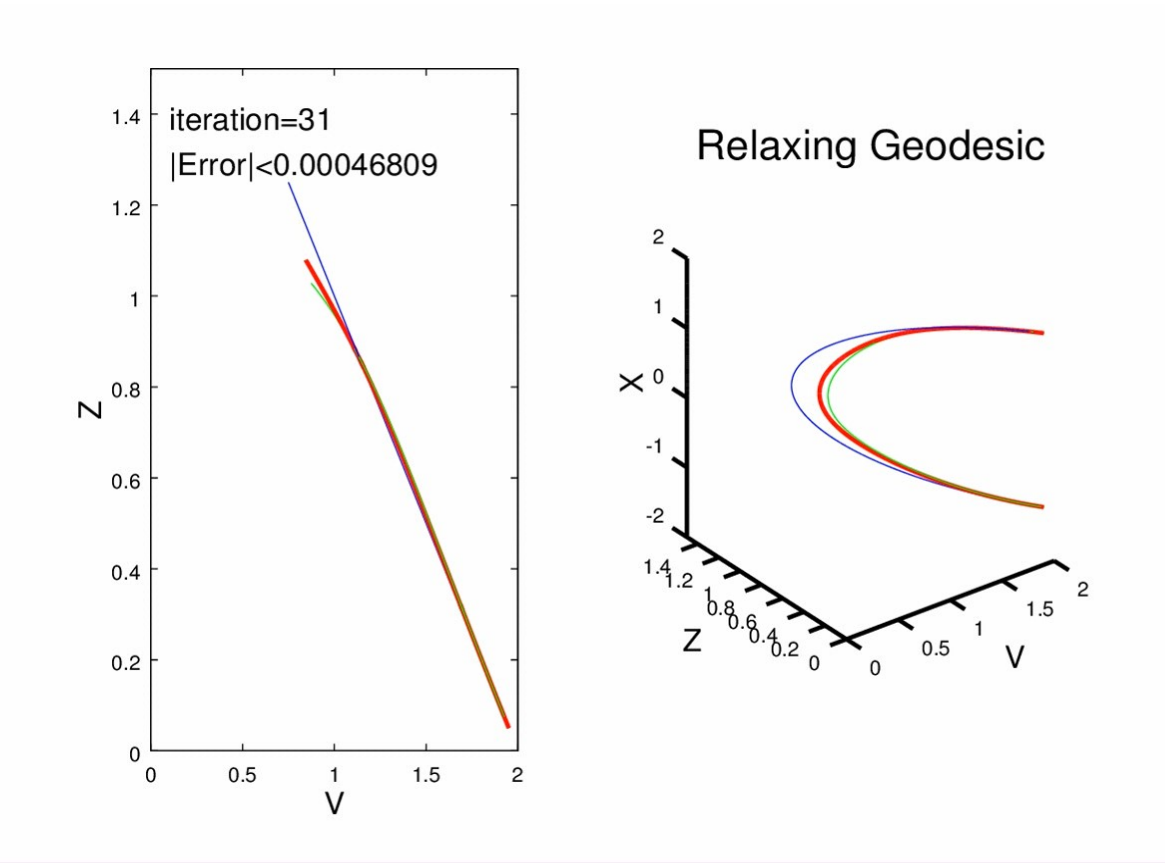


Numerics: Relax, don't shoot!

Geodesic equation as **two point boundary value problem**.

$$\ddot{X}^{\mu}(\tau) + \Gamma^{\mu}_{\alpha\beta}\dot{X}^{\alpha}(\tau)\dot{X}^{\beta}(\tau) = 0$$

$$BCs: \quad (V(\pm1), Z(\pm1), X(\pm1)) = (t_0, 0, L/2)$$



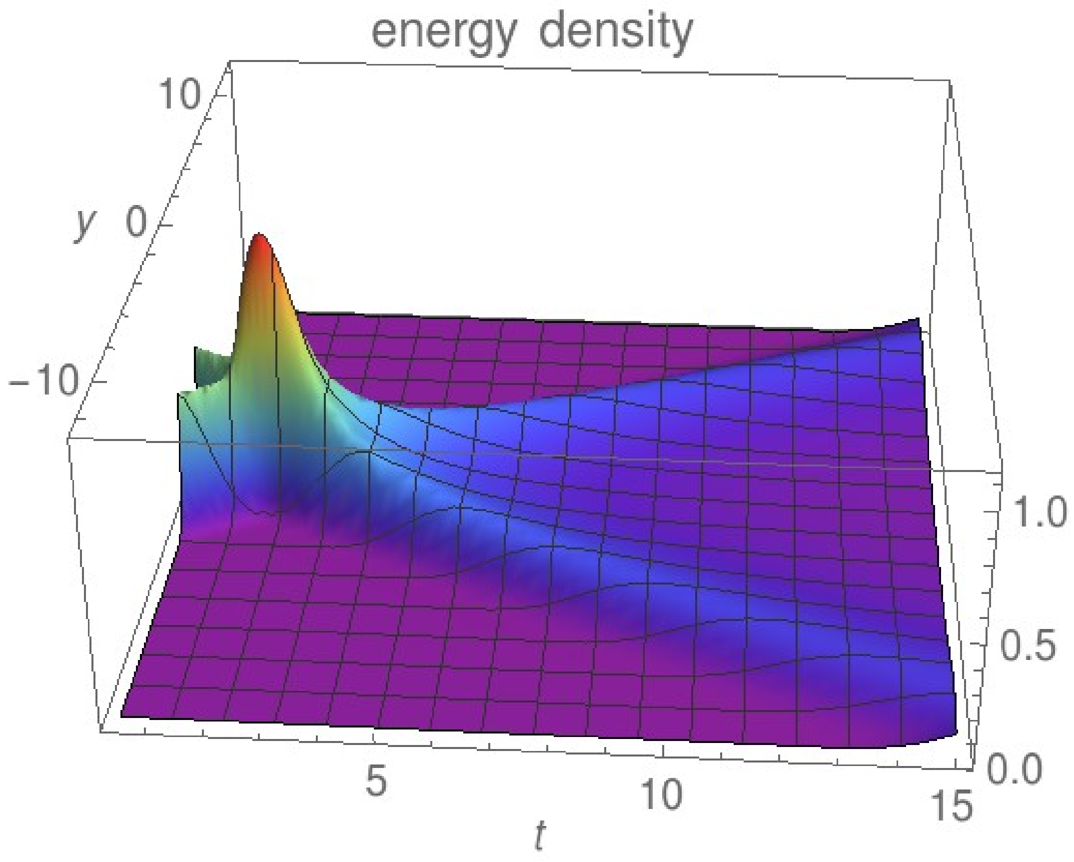
- There are two **standard numerical methods** for solving two point boundary value problems.

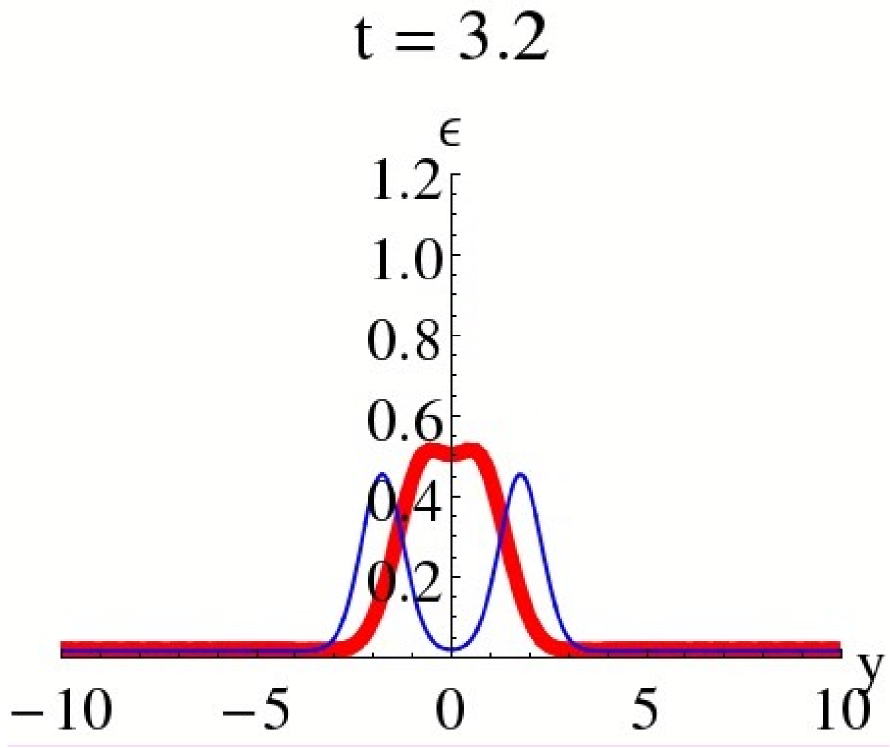
Shooting:
Very **sensitive to initialization** on **asymptotic AdS** spacetimes.

Relaxation:
Converges very **fast** if **good initial geodesic** is provided.

Shock wave collisions

- HIC is modeled by **two colliding sheets of energy** with **infinite extend in transverse direction** and **Gaussian profile** in **beam direction**.

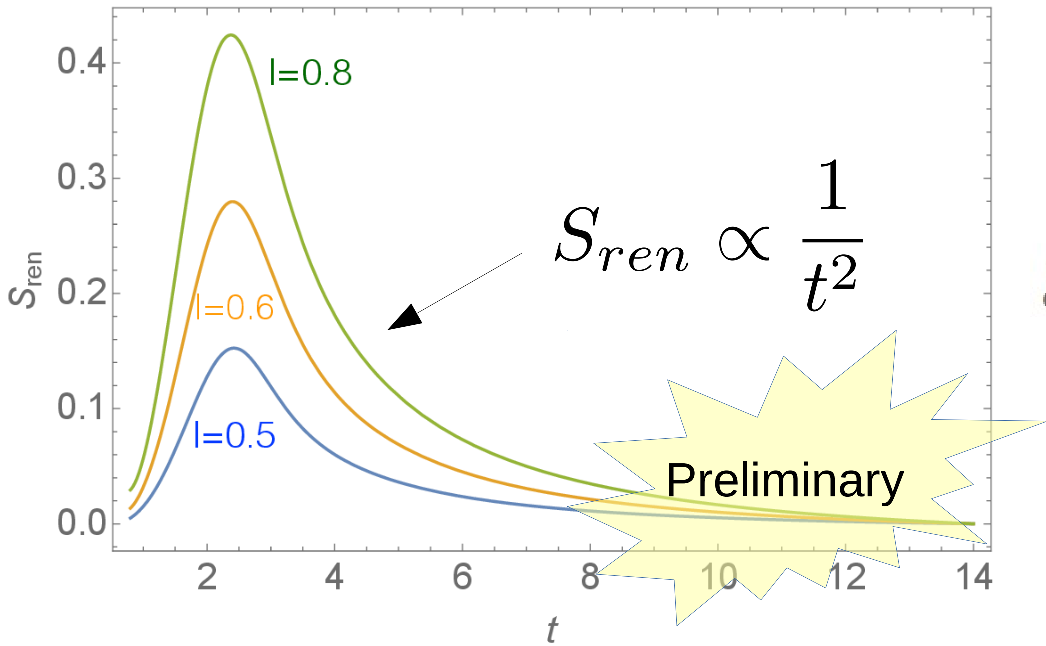


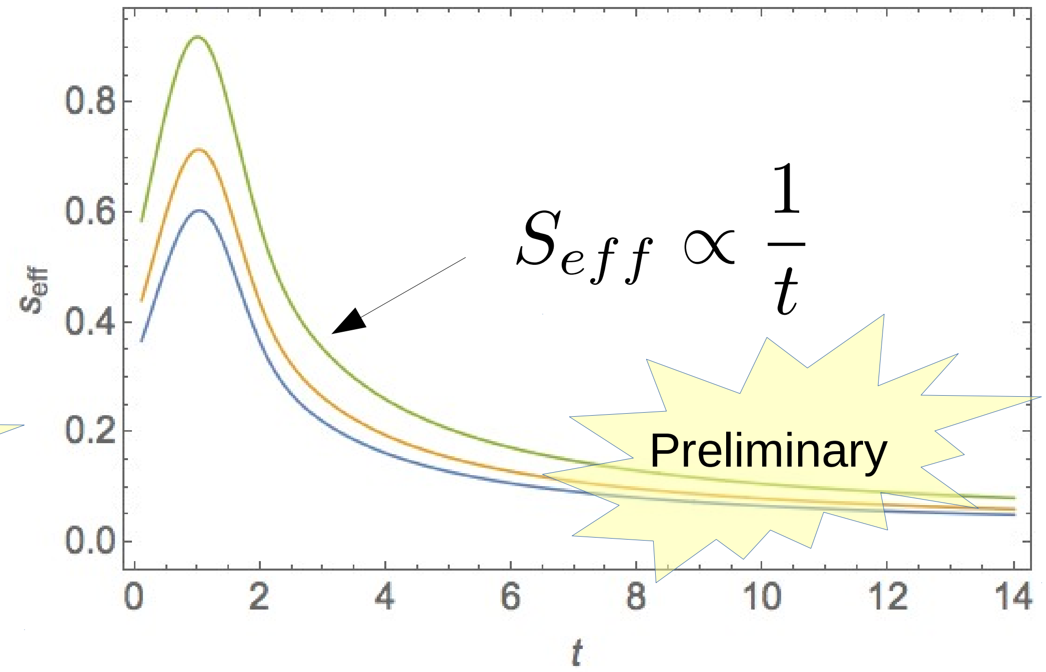


Entanglement entropy for shock wave collisions

S_{ren} ... renormalized entanglement entropy

$S_{eff} \propto A_{ah}$



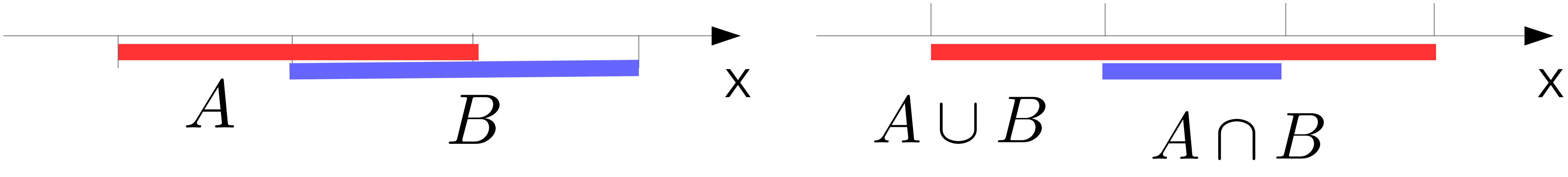


- Entanglement entropy and effective horizon entropy **grow linearly**.
- The **fall off** behavior is however **different!**

Strong subadditivity

- A **fundamental property** of entanglement entropy is strong subadditivity.
- Hard to prove within QFT, **intuitive** in the **dual gravity picture**.

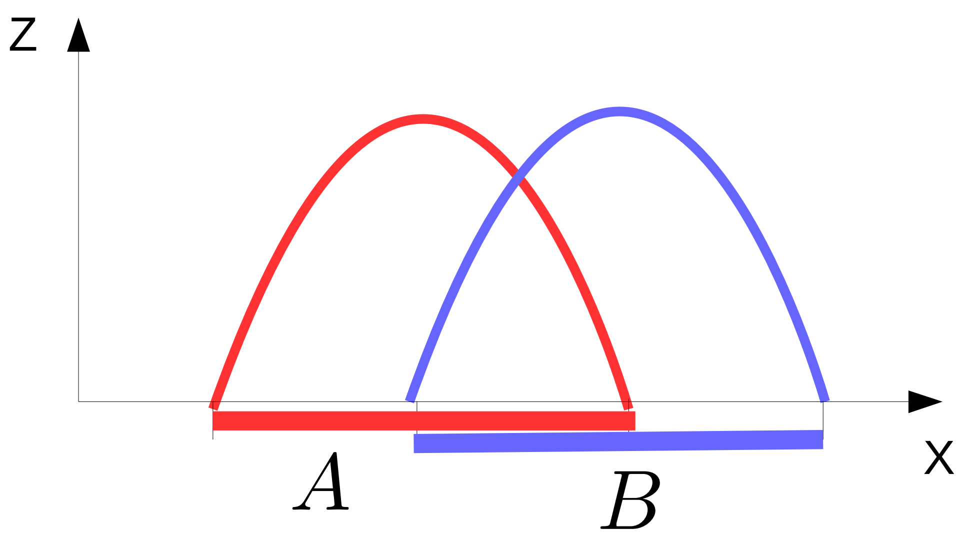
$$S_A + S_B \geq S_{A\cup B} + S_{A\cap B}$$

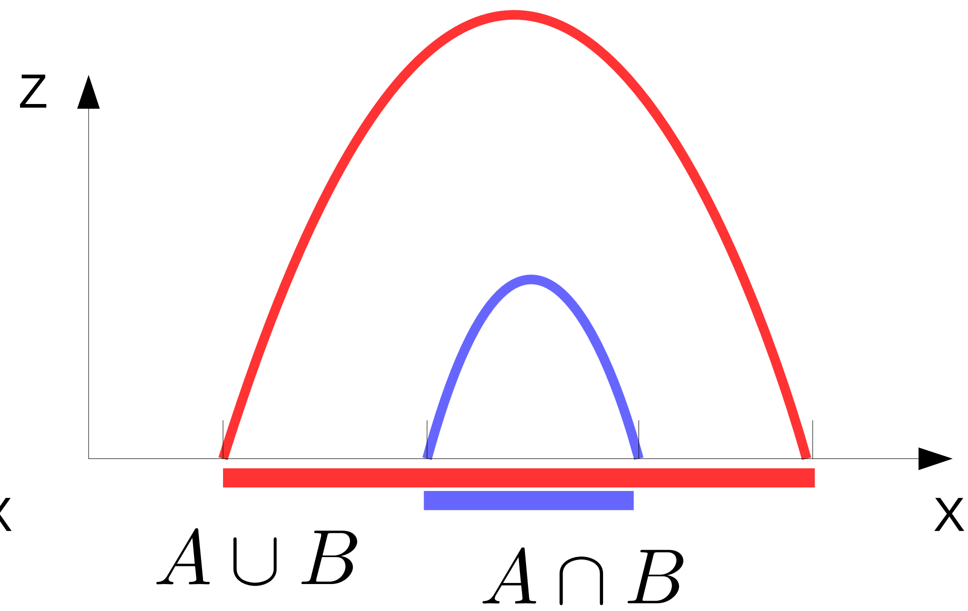


Strong subadditivity

- A **fundamental property** of entanglement entropy is strong subadditivity.
- Hard to prove within QFT, **intuitive** in the **dual gravity picture**.

$$S_A + S_B \geq S_{A\cup B} + S_{A\cap B}$$

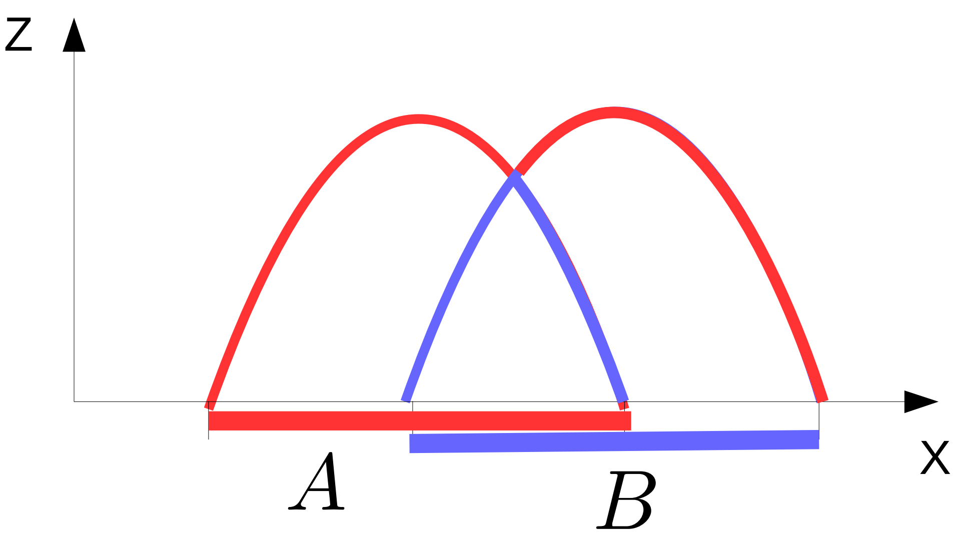


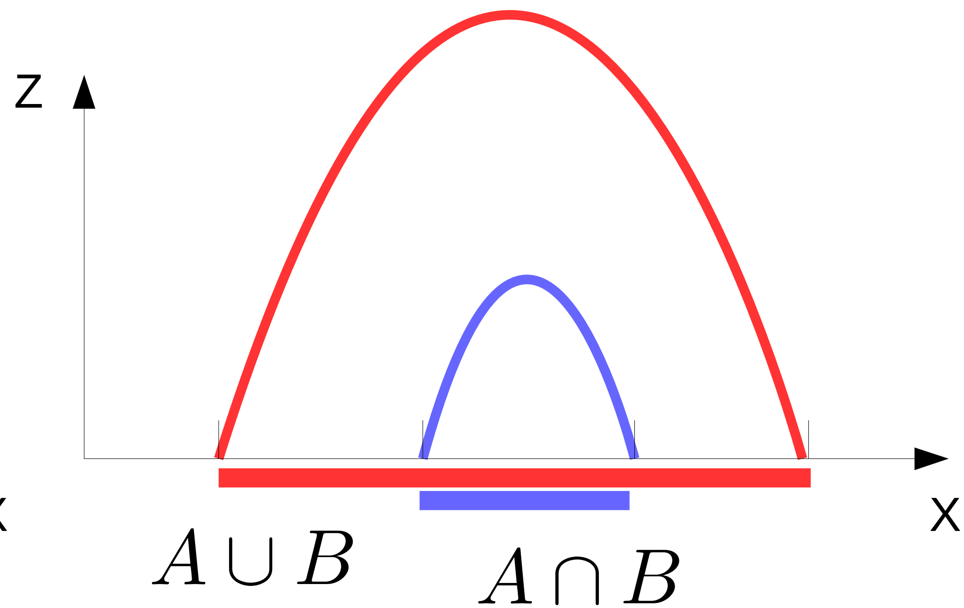


Strong subadditivity

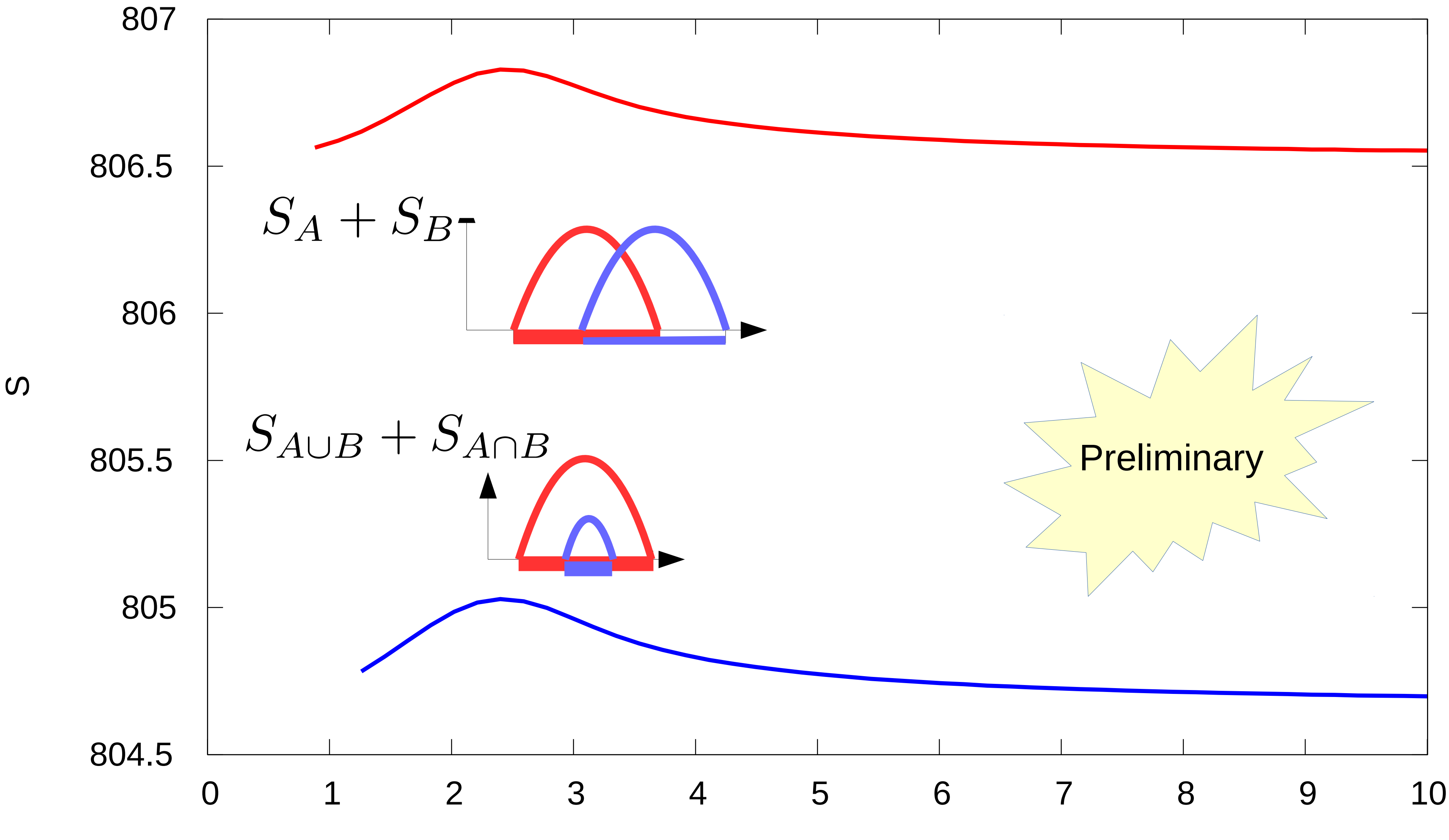
- A **fundamental property** of entanglement entropy is strong subadditivity.
- Hard to prove within QFT, **intuitive** in the **dual gravity picture**.

$$\boxed{S_A} + \boxed{S_B} \geq \boxed{S_{A \cup B}} + \boxed{S_{A \cap B}}$$





Numerical check of strong subadditivity



Summary & Outlook

Summary

- I have shown you the **first entanglement entropy simulations** for holographic HIC models. (15XX.XXXXX)
- In the shock wave geometry the **entanglement entropy** and horizon entropy **grow in the same way but show different fall off behavior**
- Successfully checked the **strong subadditivity** condition.
- **Take home message**: Complicated stuff in QFT often has a very intuitive geometric interpretation on the gravity side.

Outlook

- Effective horizon entropy is gauge dependent – is **entanglement entropy**, which is gauge independent, an alternative **measure for entropy production in HIC's**?
- Go **beyond** the **supergravity** approximation: study the influence of **string corrections** on thermalization patterns.