QCD

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Summer School and Workshop on the Standard Model and Beyond

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SUMMARY

1. Basics
   - $\mathrm{SU(n_f)}$
   - Deep Inelastic Scattering
   - Asymptotic Freedom
   - The QCD Lagrangian
   - Infinities in QCD
2. Phenomenology of QCD at Hadron Colliders
   - Factorization Theorem
   - Parton Densities
   - Parton Shower and Hadronization
   - Algorithms and Tools
3. $H \to gg$ at NLO in Four Dimensions
   - Virtual Corrections
   - Real Corrections
   - Renormalization of α_s

QCD, basic concepts

- The basics concepts of QCD are reviewed

The SU(n_f) symmetry

- $|p\rangle$ and $|n\rangle$ form a SU(2) isospin doublet $(T_i = \frac{\sigma_i}{2})$

$$\begin{pmatrix} p \\ n \end{pmatrix} \quad \text{with} \quad |p\rangle = \begin{pmatrix} p \\ 0 \end{pmatrix}, \quad |n\rangle = \begin{pmatrix} 0 \\ n \end{pmatrix}$$

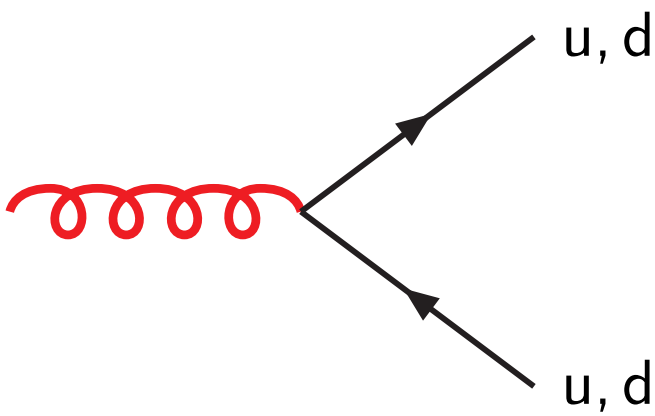
$$T_3 \begin{pmatrix} p \\ 0 \end{pmatrix} = \frac{\sigma_3}{2} \begin{pmatrix} p \\ 0 \end{pmatrix} = +\frac{1}{2} \begin{pmatrix} p \\ 0 \end{pmatrix}$$

$$T_3 \begin{pmatrix} 0 \\ n \end{pmatrix} = \frac{\sigma_3}{2} \begin{pmatrix} 0 \\ n \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} 0 \\ n \end{pmatrix}$$

- If $H_{\rm QCD}|n\rangle = E|n\rangle$ and $[H_{\rm QCD}, T_i] = 0 \Rightarrow H_{\rm QCD}|p\rangle = E|p\rangle$

$$\frac{m_n - m_p}{m_n + m_p} = 0.7 \times 10^{-3}$$

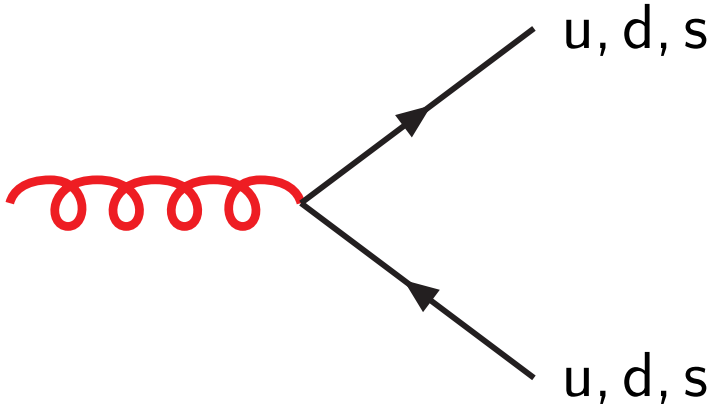
QCD is flavor blind

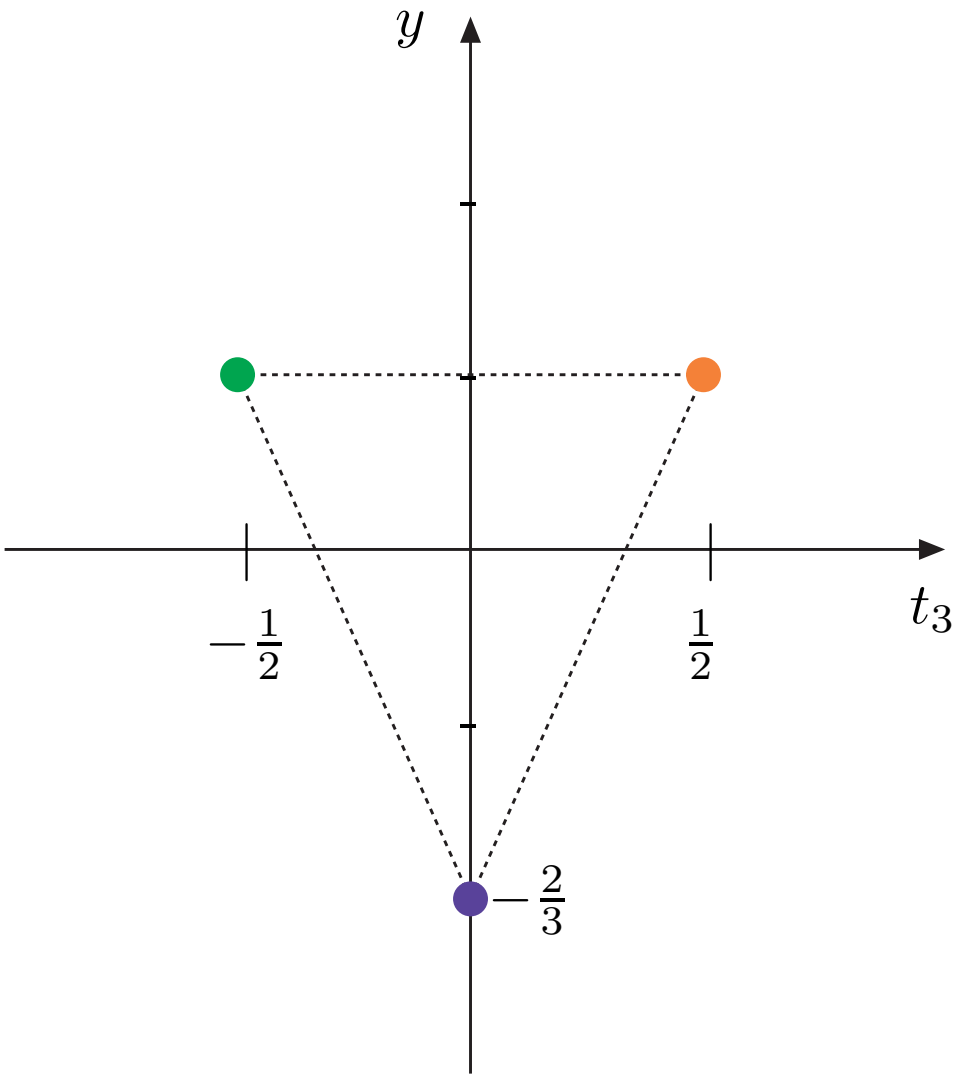


- Symmetry broken by QED/EW interactions $(m_u \sim m_d)$

$$[H_{\mathrm{QED/EW}}, T_i] \neq 0$$

- With 3 flavors $(m_u \sim m_d \sim m_s)$ SU(3) symmetry





Representation of 3
for quarks

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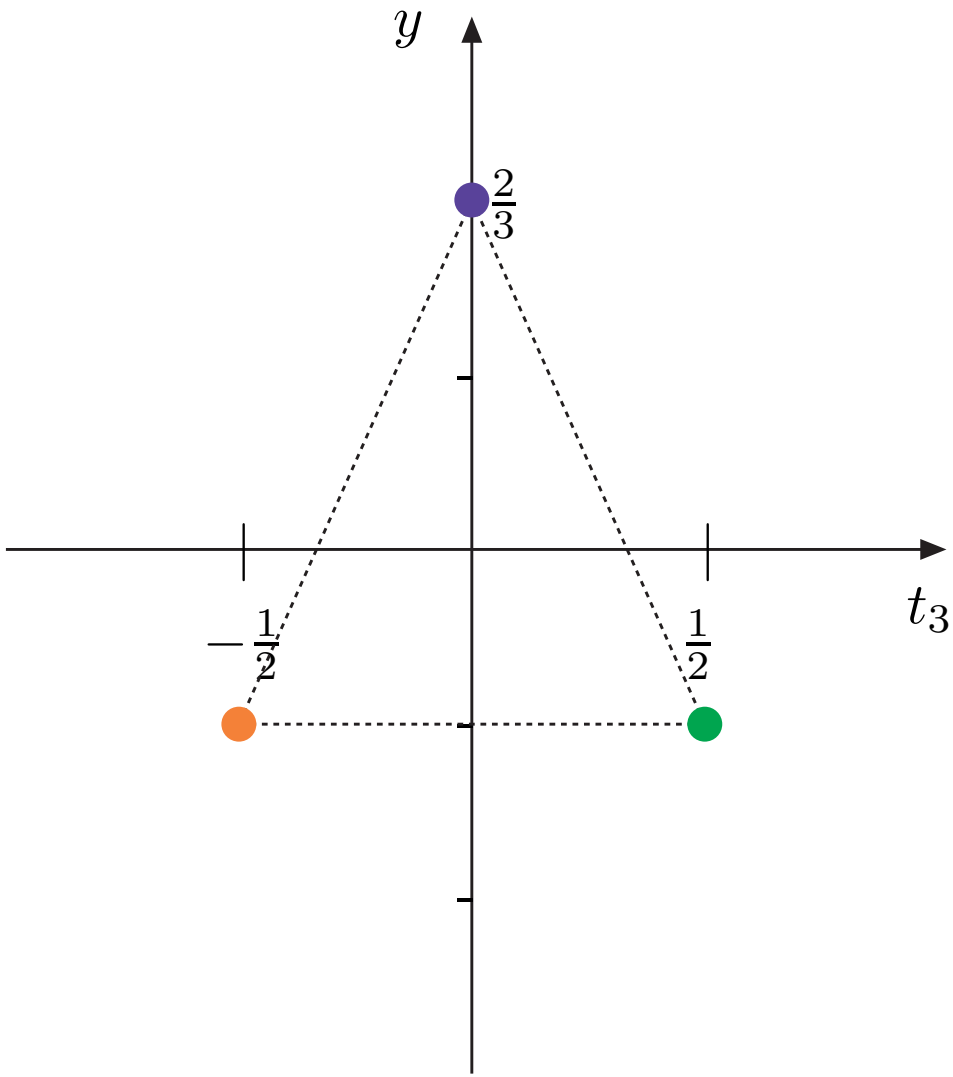
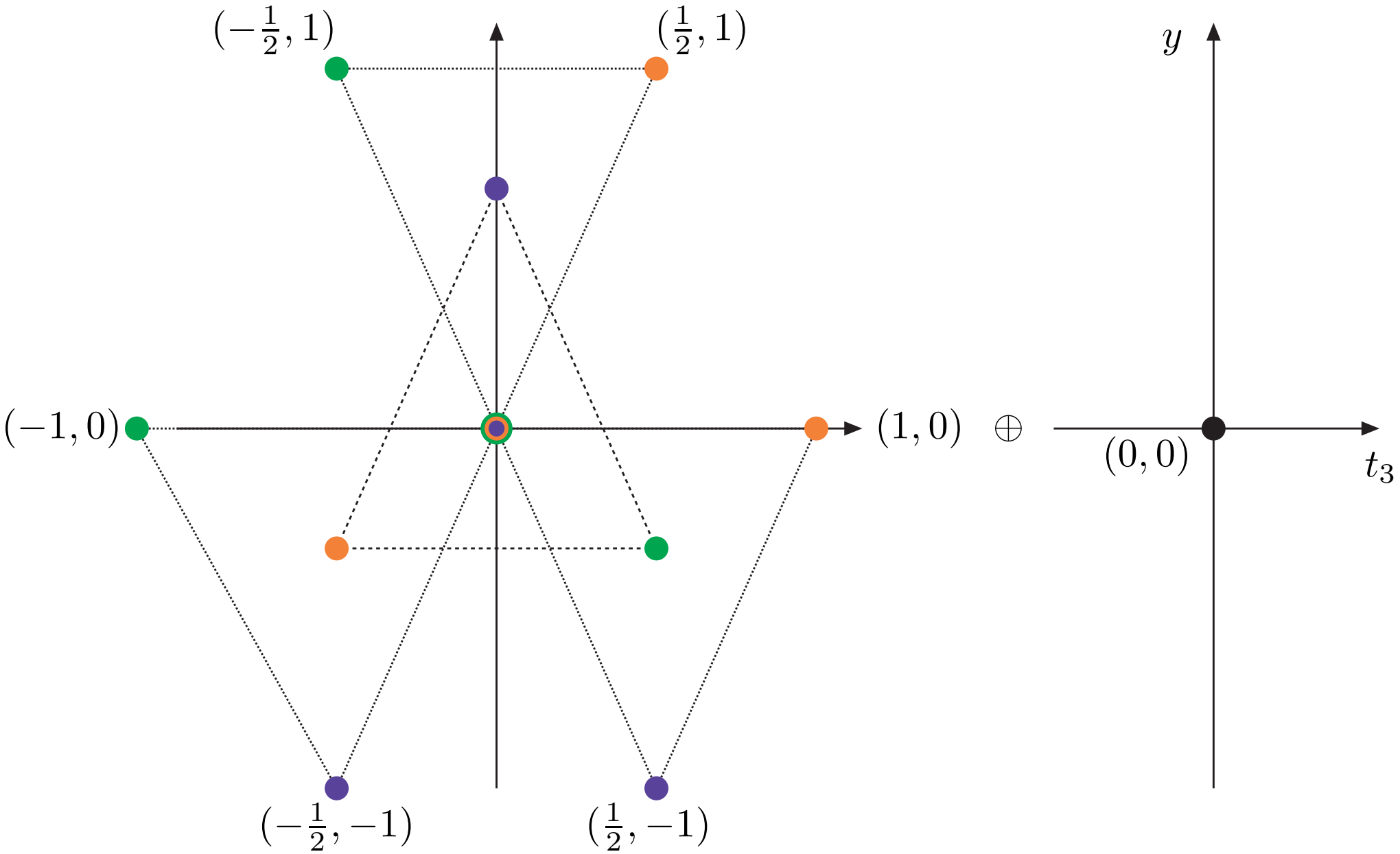
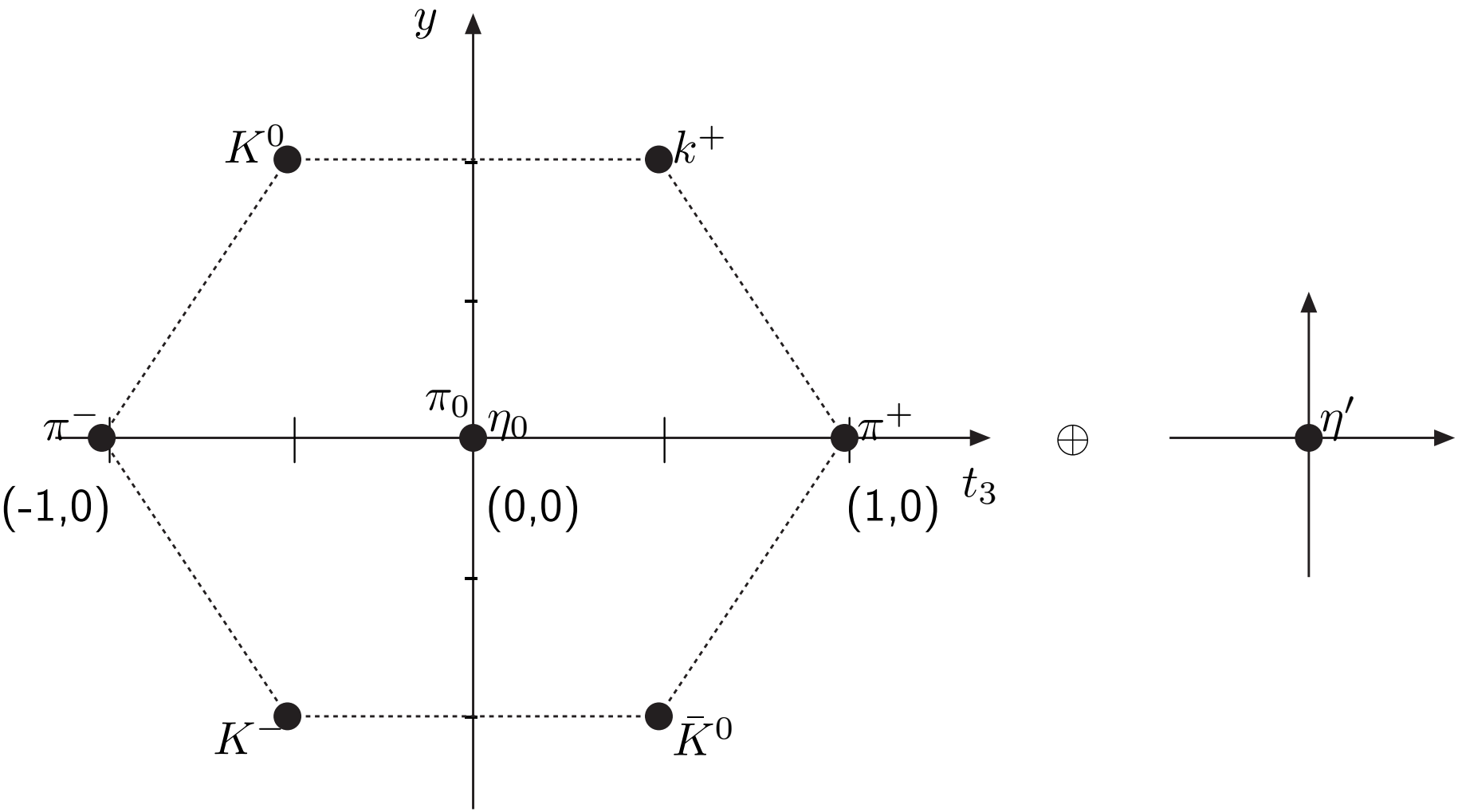


Representation of $3*$
for antiquarks

Mesons (and Barions)



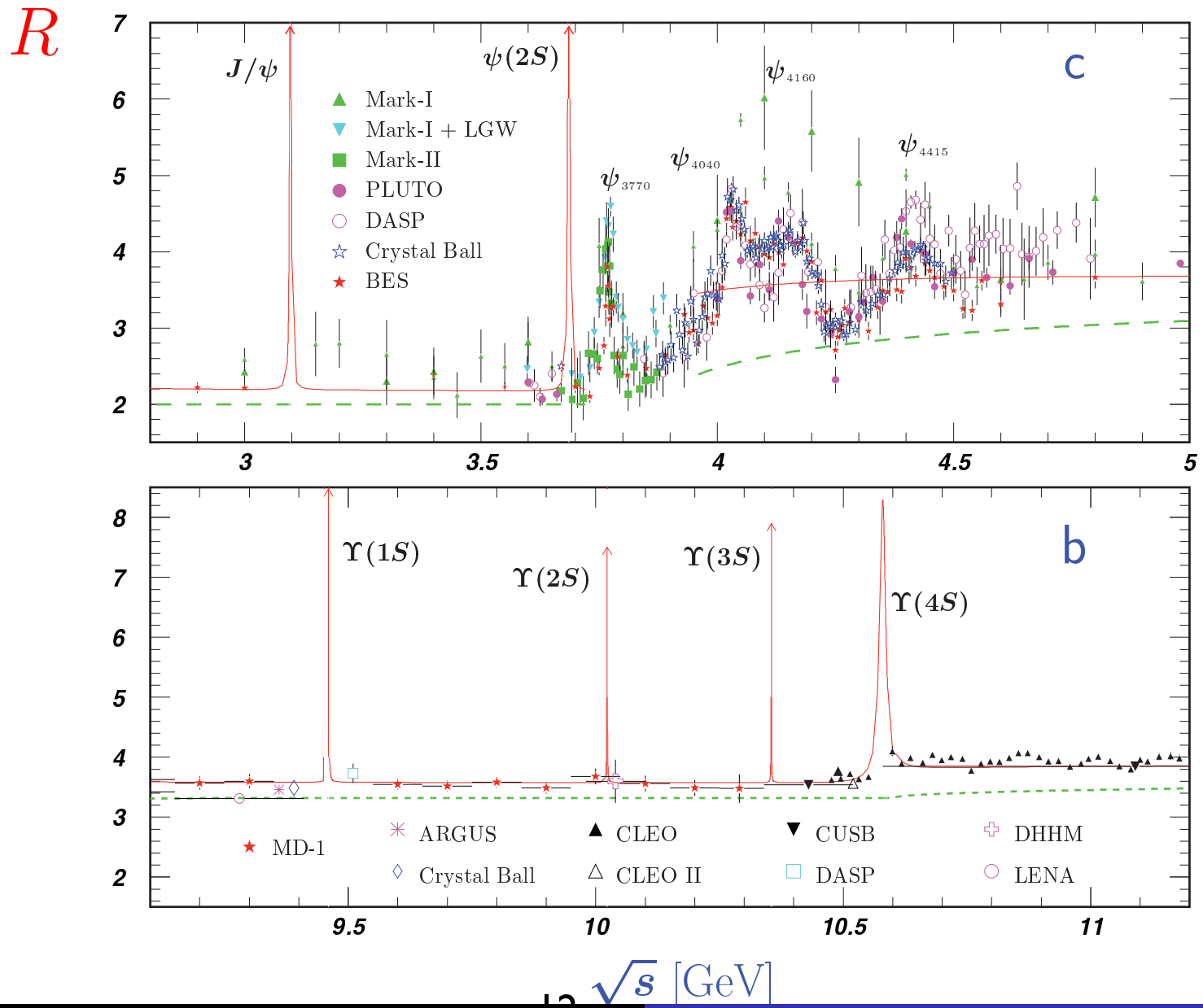
The 0^- meson nonet



- But $m_{k^+} = 490\,\frac{MeV}{c^2}$ and $m_{\pi^+} = 140\,\frac{MeV}{c^2}$

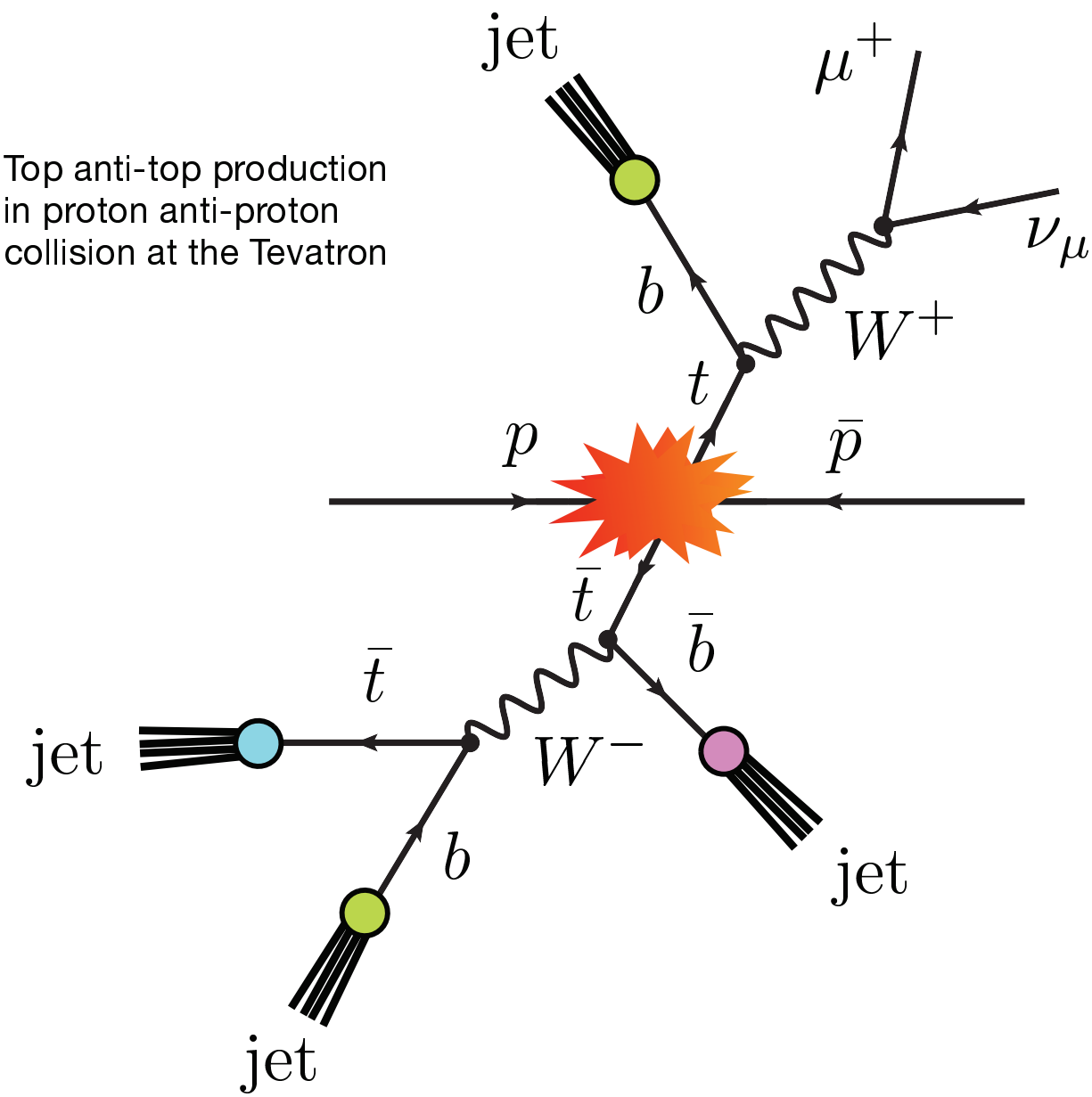
symmetry broken by quark masses

Heavier quarks (c and b)



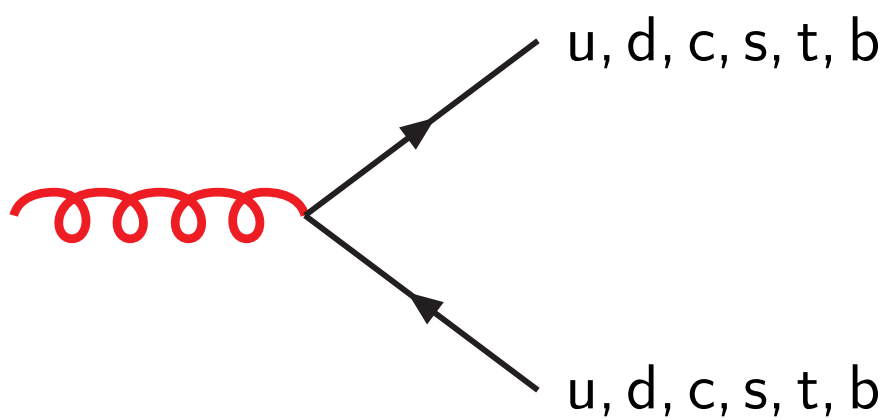
The heaviest quark (top)

Top anti-top production in proton anti-proton collision at the Tevatron

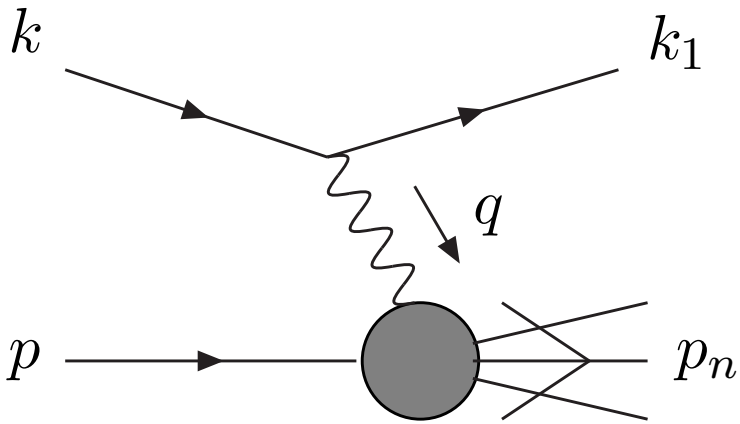


Flavor	Mass(GeV/c²)	Elect. Charge
u up	.005	+2/3
d down	.01	-1/3
c charm	1.5	+2/3
s strange	0.2	-1/3
t top	180	+2/3
b bottom	4.7	-1/3

http://pdg.web.cern.ch/



Deep inelastic scattering



$$k = (E, \bar{k}) \qquad k_1 = (E_1, \bar{k}_1)$$

$$q = (k - k_1)$$

$$Q^2 = -q^2$$

- If $p = (M, \vec{0})$

$$E - E_1 = \frac{q \cdot p}{M} = \nu \quad \text{and} \quad x = \frac{Q^2}{2M\nu} = \frac{EE_1(1 - \cos\theta)}{M(E - E_1)}$$

$$d\sigma = \frac{(2\pi)^4}{4EM} \sum_n \prod_{i=1}^{n} \left[\frac{d^3 p_i}{(2\pi)^3 2\omega_i} \right] |\bar{M}|^2 \delta^4(p + k - k_1 - p_n) \frac{d^3 k_1}{(2\pi)^3 2E_1}$$

$$M = -\frac{e^2}{q^2} \bar{u}_\lambda(k_1) \gamma_\mu u_\lambda(k) T^\mu(\sigma)$$

$$|\bar{M}|^2 = \frac{e^4}{q^4} \left\{ k_1^\mu k^\nu + k_1^\nu k^\mu + \frac{q^2}{2} g^{\mu\nu} \right\} \sum_\sigma T_\mu(\sigma) T_\nu^*(\sigma)$$

$T^\mu(\sigma)$ is the *unknown* current

$$\frac{d\sigma^2}{dE_1 d\Omega} = \frac{\alpha^2}{q^4}\left(\frac{E_1}{E}\right) L_{\mu\nu} W^{\mu\nu}$$

$$\begin{aligned} L^{\mu\nu} &= \frac{1}{2} Tr[\not{k}_1 \gamma_\mu \not{k} \gamma_\nu] \\ W^{\mu\nu} &= \frac{1}{4M} \sum_{n,\sigma} \int \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2\omega_i} (2\pi)^3 \delta^4(p+q-p_n)\, T_\mu(\sigma) T^*_\nu(\sigma) \end{aligned}$$

- Charge conservation $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$ gives

$$W_{\mu\nu} = \frac{1}{M}\left\{ -F_1 \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \frac{F_2}{M\nu}\left(p^\nu - \frac{(p\cdot q)}{q^2} q^\nu\right)\left(p^\mu - \frac{(p\cdot q)}{q^2} q^\mu\right)\right\}$$

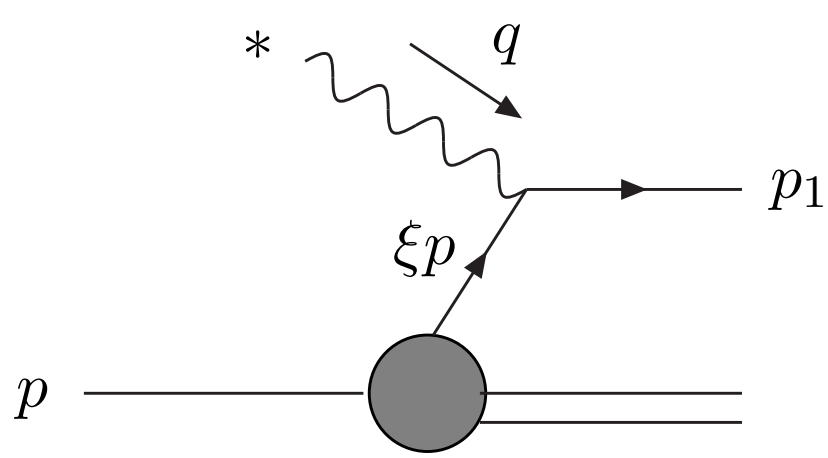
$$\frac{d^2\sigma}{d\Omega dE_1} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \frac{1}{\nu}\left[\cos^2\frac{\theta}{2} F_2 + \sin^2\frac{\theta}{2}\frac{Q^2}{xM^2} F_1\right]$$

Scaling and *Callan-Gross* relation

- Experimental facts

$$\underbrace{\lim_{Q^2\to\infty}}_{at\ \ x\ \ fixed} F_j(x, \frac{Q^2}{M^2}) = F_j(x) \quad \text{and} \quad F_2 = 2xF_1$$

- Explained by



$$T_\mu(\sigma) = \bar{u}(p_1)\gamma_\mu u(\xi p)$$

$$W^{\mu\nu} = \int_0^1 d\xi\, f(\xi)\, W_\xi^{\mu\nu}$$

$$W_\xi^{\mu\nu} = \frac{1}{8M^2\nu}\delta(x-\xi)\, Tr[(\not{q}+\xi\not{p})\gamma^\mu\not{p}\gamma^\nu]$$

- $\delta(x-\xi)$ from on-shellness final state parton

$$W^{\mu\nu} = f(x)\left\{p^\mu p^\nu\left(\frac{x}{M^2\nu}\right) - g^{\mu\nu}\frac{(q\cdot p)}{2M^2\nu}\right\}$$

- As promised:

$$F_2(x) = x f(x) \quad \text{and} \quad F_1(x) = \frac{f(x)}{2}$$

Parton densities $f(x)$

- In a proton

$$f(x) = \frac{4}{9}[u(x) + \bar{u}(x)] + \frac{1}{9}[d(x) + \bar{d}(x)] + \frac{1}{9}[s(x) + \bar{s}(x)]$$

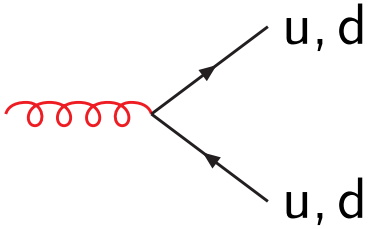
- Sum rules ($Q_{\text{proton}} = 1$)

$$\int_0^1 dx \left\{ \frac{2}{3}[u(x) - \bar{u}(x)] - \frac{1}{3}[d(x) - \bar{d}(x)] - \frac{1}{3}[s(x) - \bar{s}(x)] \right\} = 1$$

- **Measured** momentum sum rule for the proton

$$\int_0^1 dx \sum_i x\, q_i(x) = 0.546$$

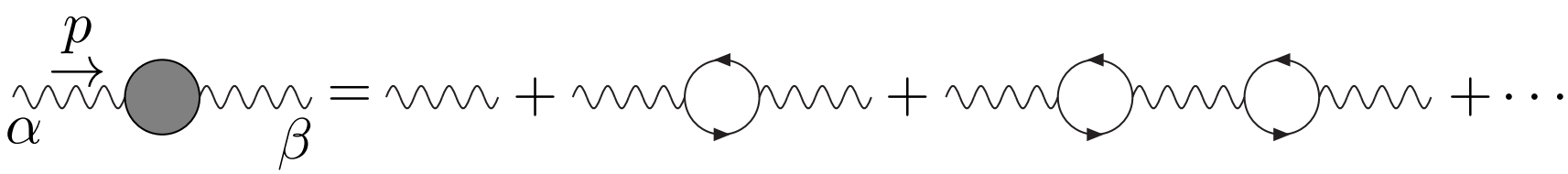
$\Rightarrow$ **gluons** inside proton



Asymptotic freedom

- Quarks **strongly bounded by gluons** in hadrons but behaving as **free partons in DIS**
- Paradox explained by the **running** of strong coupling constant

QED (Dimensional Regularization $n = 4 + \epsilon$)



$$\overset{p\to\infty}{\sim} -ig_{\alpha\beta}\frac{e_0^2}{p^2\left(1-\frac{e_0^2}{12\pi^2}\left\{\ln(p^2/\mu^2)-\Delta_{\rm UV}\right\}\right)}$$

$\Delta_{\rm UV} = -\dfrac{2}{\epsilon} - \gamma_E - \ln\pi$ and $\ln(\mu^2)$ (arbitrary) **same coefficient**

- When renormalizing e_0, $\Delta_{\rm UV}$ disappears and $\ln(\mu^2)$ reabsorbed in renormalized coupling constant $e \Rightarrow$ (depends on μ^2)

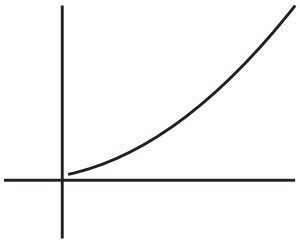
$$e^2(p^2) = \frac{e^2(\mu^2)}{1 - \frac{e^2(\mu^2)}{12\pi^2}\ln(p^2/\mu^2)}$$

- Namely

$$e^2(\mu^2) = \frac{e^2(\mu_0^2)}{1 - \frac{e^2(\mu_0^2)}{12\pi^2}\ln(\mu^2/\mu_0^2)}$$

giving the $\beta_{\rm QED}$ function

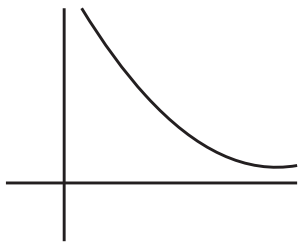
$$\beta_{\rm QED} \equiv \mu\frac{\partial e(\mu^2)}{\partial\mu} = \frac{e^3(\mu^2)}{12\pi^2} > 0$$



QCD

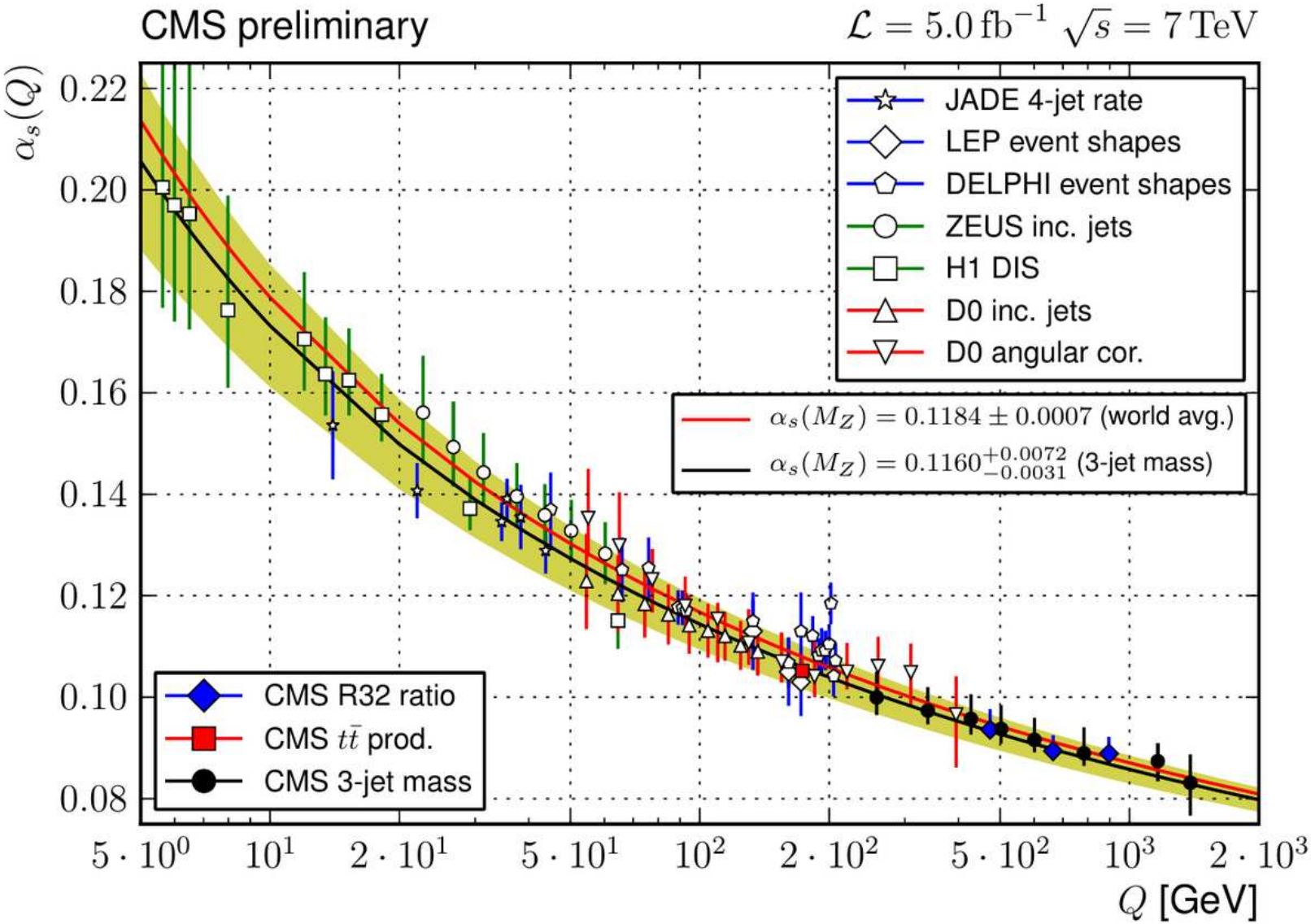
- By computing the coefficient of $\Delta_{\rm UV}$ contributing to the g_s renormalization

$$\beta_{\rm QCD} \equiv \mu \frac{\partial g_s(\mu^2)}{\partial \mu} = -\frac{g_s^3(\mu^2)}{48\pi^2}\,(11\,N_C - 2\,n_f) < 0$$



$$g_s^2(\mu^2) = \frac{g_s^2(\mu_0^2)}{1 + \frac{g_s^2(\mu_0^2)}{48\pi^2}\,(11\,N_C - 2\,n_f)\ln(\mu^2/\mu_0^2)}$$

The running of $\alpha_s = \frac{g_s^2}{4\pi}$



$\Lambda_{\rm QCD}$

- Given

$$\alpha_s(\mu^2) = \frac{1}{\frac{1}{\alpha(\mu_0^2)} + b_0 \ln(\mu^2/\mu_0^2)} \qquad b_0 \equiv \frac{11 N_c - 2\, n_f}{12\pi}$$

define

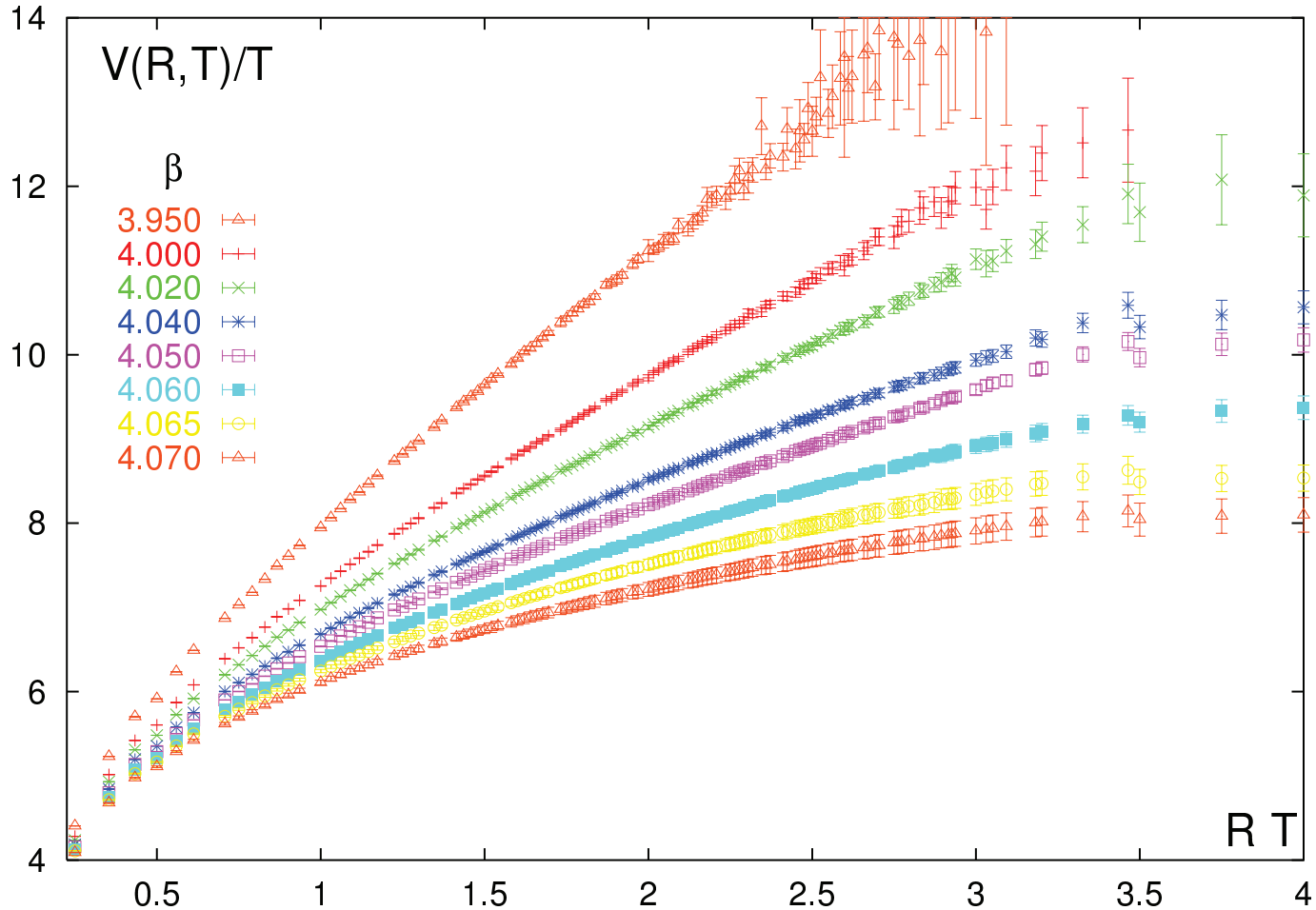
$$\frac{1}{\alpha(\mu_0^2)} + b_0 \ln(\mu^2/\mu_0^2) = b_0 \ln(\mu^2/\Lambda_{\rm QCD}^2)$$

$$\alpha_s(\mu^2) = \frac{1}{b_0 \ln(\mu^2/\Lambda_{\rm QCD}^2)}$$

- $\Lambda_{\rm QCD}^2 \sim$ 200 MeV sets the scale at which α_s becomes strong/non-perturbative (quarks confined in hadrons)

The $q\bar{q}$ potential from Lattice QCD (confinement)

$$V(R) \sim 1/R + \sigma R$$



O. Kaczmarek *et al.*, Phys.Rev. D62 (2000) 034021

The QCD Lagrangian

$$\mathcal{L}_{QCD} = \mathcal{L}_{inv} + \mathcal{L}_{g.f.} + \mathcal{L}_{ghost}$$

COLOR indices

$$\begin{aligned} a, b, c &= 1, 2, \ldots, 8 \\ j, k, l &= 1, 2, 3 \end{aligned}$$

- $$\mathcal{L}_{inv} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \bar{\Psi}_j \left[i \not{D}_{jk} - m\, \delta_{jk} \right] \Psi_k$$

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\nu G^c_\mu \\ D_\mu &= \partial_\mu + i g t^a G^a_\mu \end{aligned}$$

The SU(3) color algebra

$$t_a = \frac{\lambda_a}{2}\,, \quad [t_a, t_b] = i f^{abc} t_c$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

f^{abc} is totally antisymmetric and only takes the following values

$$0, 1, \tfrac{1}{2}, -\tfrac{1}{2}, \sqrt{\tfrac{3}{2}}$$

Color singlets

- Two diagonal color matrices t_3 and t_8 with eigenvalues F_3 and F_8

- Color state characterizes by **two additive quantum numbers**

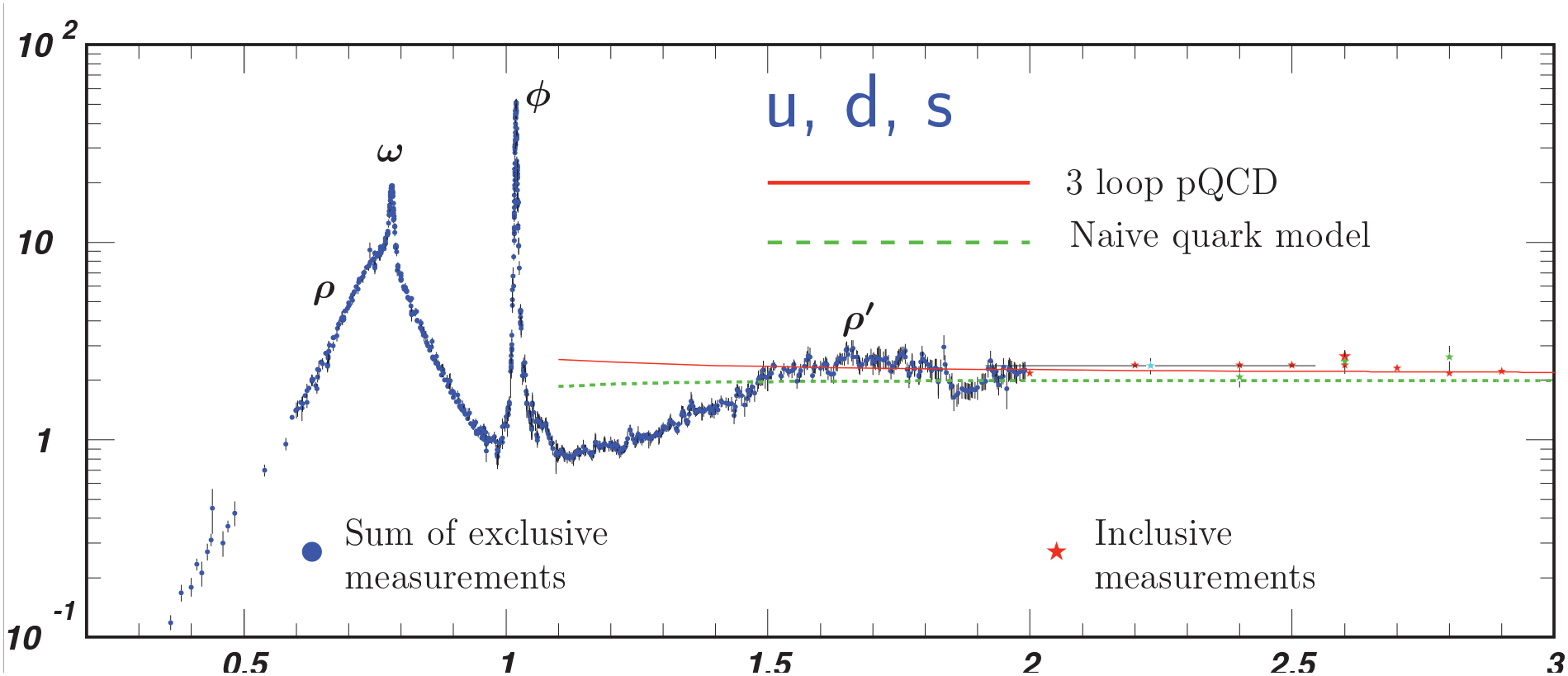
$$I_3^c = F_3 \quad \text{and} \quad Y^c = \frac{2}{\sqrt{3}} F_8$$

- **Experimentally observed states are color singlets** with 0 color charges

$$(I_3^c)_{\text{tot}} = (Y^c)_{\text{tot}} = 0$$

Experimental evidence of color

$$R = \frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-} \propto N_c \sum_f Q_f^2$$



Gauge Invariance

$\mathcal{L}_{inv}$ is invariant under the LOCAL infinitesimal transformations

$$\begin{aligned} G^a_\mu &\rightarrow G^a_\mu - \partial_\mu \Lambda^a(x) + g f^{abc} G^b_\mu \Lambda^c(x) \\ \Psi_j &\rightarrow \left[\delta_{jk} + i g \Lambda^a(x) (t^a)_{jk}\right] \Psi_k \end{aligned}$$

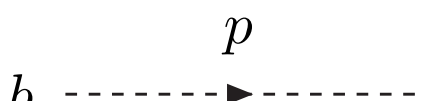
- $\mathcal{L}_{g.f.} = -\frac{1}{2}(\partial_\mu G^\mu_a)(\partial_\nu G^\nu_a)$

breaks Gauge Invariance (needed to quantize QCD)

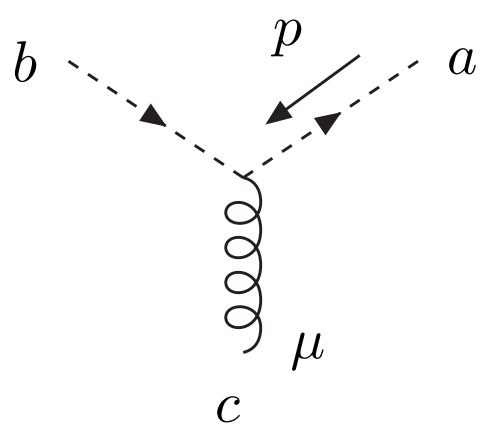
- $\mathcal{L}_{ghost} = \bar{\eta}^a \left[-\partial^2 \delta_{ab} - g f^{abc} \partial_\mu G^{c,\mu}\right] \eta^b$

ensures that physical quantities **do not depend** on $\mathcal{L}_{g.f.}$

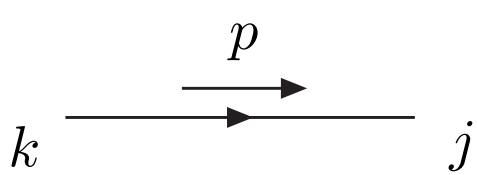
QCD Feynman Rules I (Feynman-'t Hooft gauge)



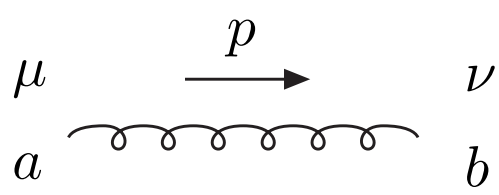
$$= \; i\frac{\delta_{ab}}{p^2}$$



$$= \; g f^{abc} p_\mu$$

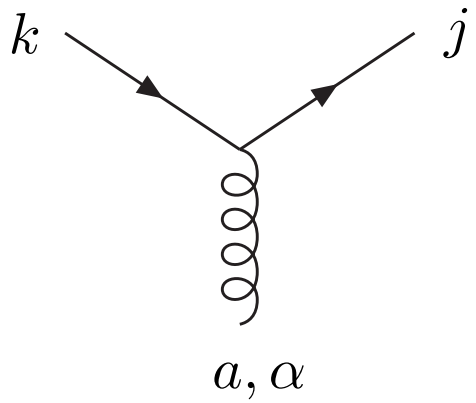


$$= \; i\frac{1}{\not{p} - m}\delta_{kj}$$

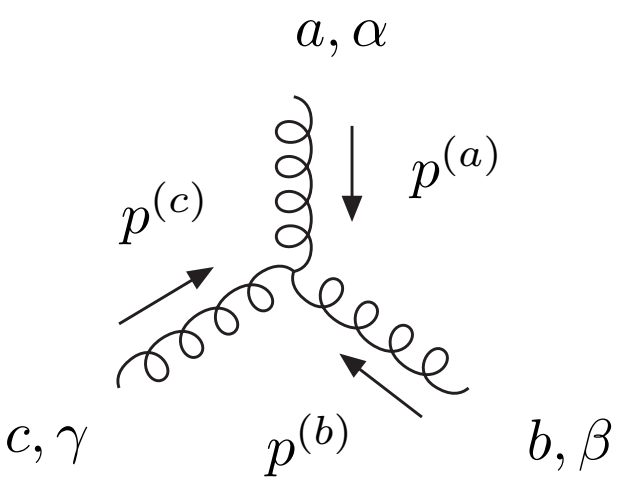


$$= \; -i\frac{g^{\mu\nu}}{p^2}\delta^{ab}$$

QCD Feynman Rules II

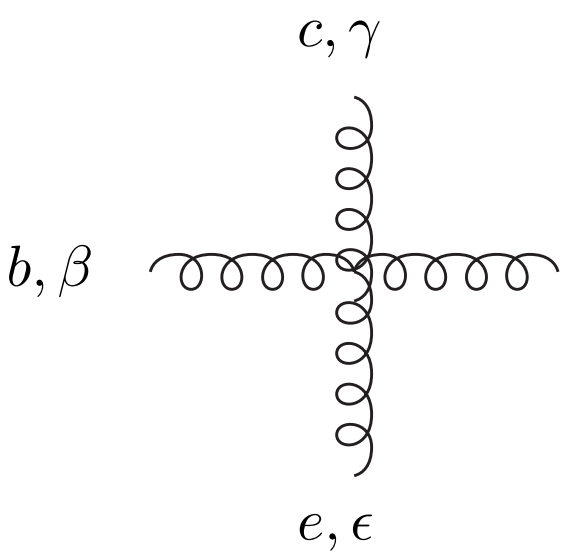


$$= \quad -ig\gamma_\alpha t^a_{jk}$$



$$= \quad gf^{abc}\Big\{g_{\alpha\beta}(p^{(b)} - p^{(a)})_\gamma \quad + \quad g_{\beta\gamma}(p^{(c)} - p^{(b)})_\alpha \quad + \quad g_{\gamma\alpha}(p^{(a)} - p^{(c)})_\beta\Big\} \ ,$$

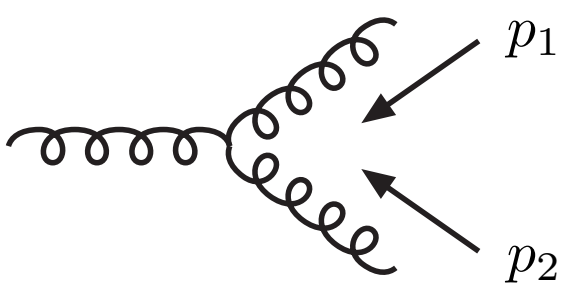
QCD Feynman Rules III



$$= -ig^2\{f^{abc}f_{ade}(g_{\beta\delta}g_{\gamma\epsilon} - g_{\beta\epsilon}g_{\gamma\delta}) + f^{abd}f_{aec}(g_{\beta\epsilon}g_{\delta\gamma} - g_{\beta\gamma}g_{\delta\epsilon}) + f^{abe}f_{acd}(g_{\beta\gamma}g_{\epsilon\delta} - g_{\beta\delta}g_{\epsilon\gamma})\}$$

Infinities

1. **Ultraviolet** $\Rightarrow$ Renormalization of g_s (and m_q)
2. **Infrared** $\Rightarrow$ Bloch-Nordsieck theorem *
3. **Collinear** $\Rightarrow$ Kinoshita-Lee-Nauenberg theorem **



$$\frac{1}{2(p_1 \cdot p_2)} \rightarrow \frac{1}{2 \underbrace{E_1 E_2}_{\textbf{Infrared}} \underbrace{(1-\cos\theta)}_{\textbf{Collinear}}}$$

* Disappear when adding up indistinguishable final states: **summing Real (R) and Virtuals (V) contributions**

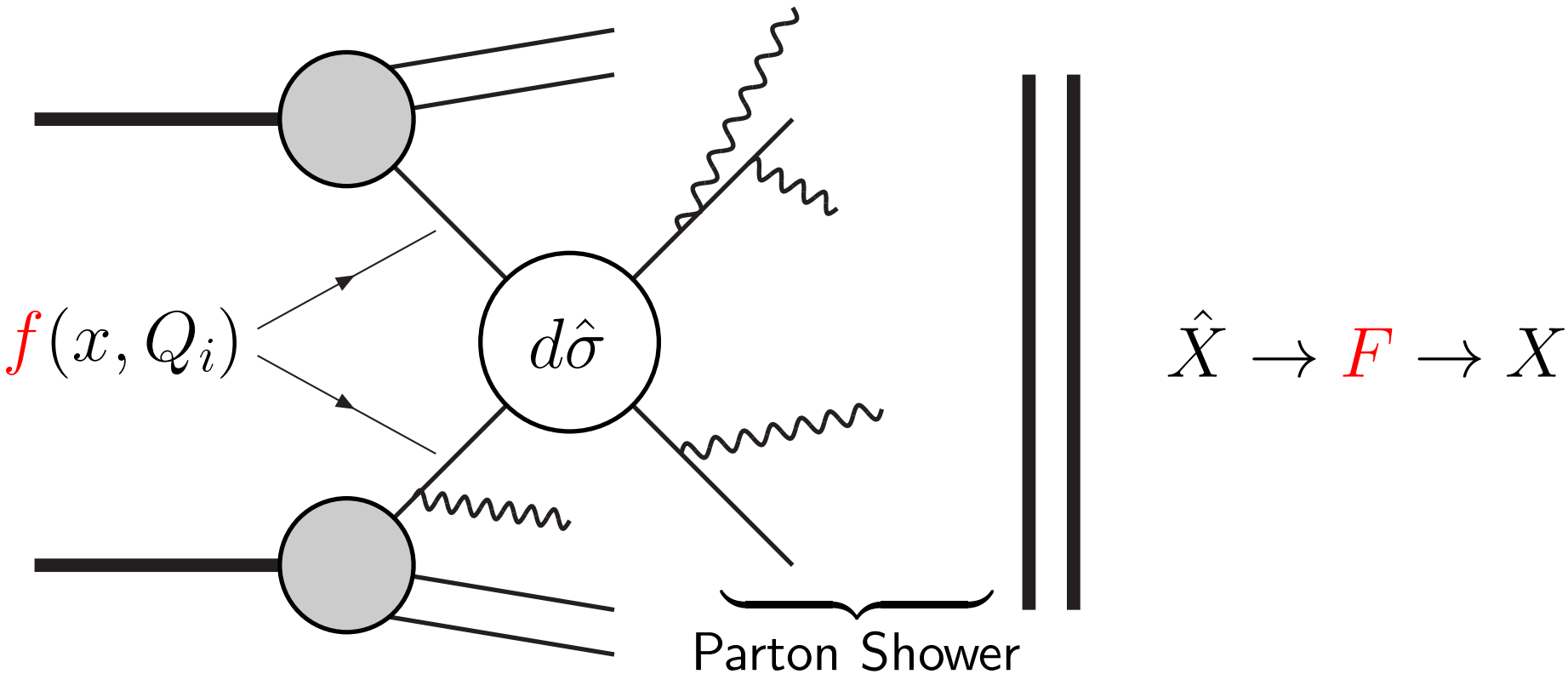
** Disappear when **adding up R and V** in **sufficiently inclusive** quantities. Initial state ones **absorbed inside parton densities**

QCD at hadron colliders

- The phenomenology of QCD at hadron colliders is discussed
- Current algorithms and tools are also reviewed

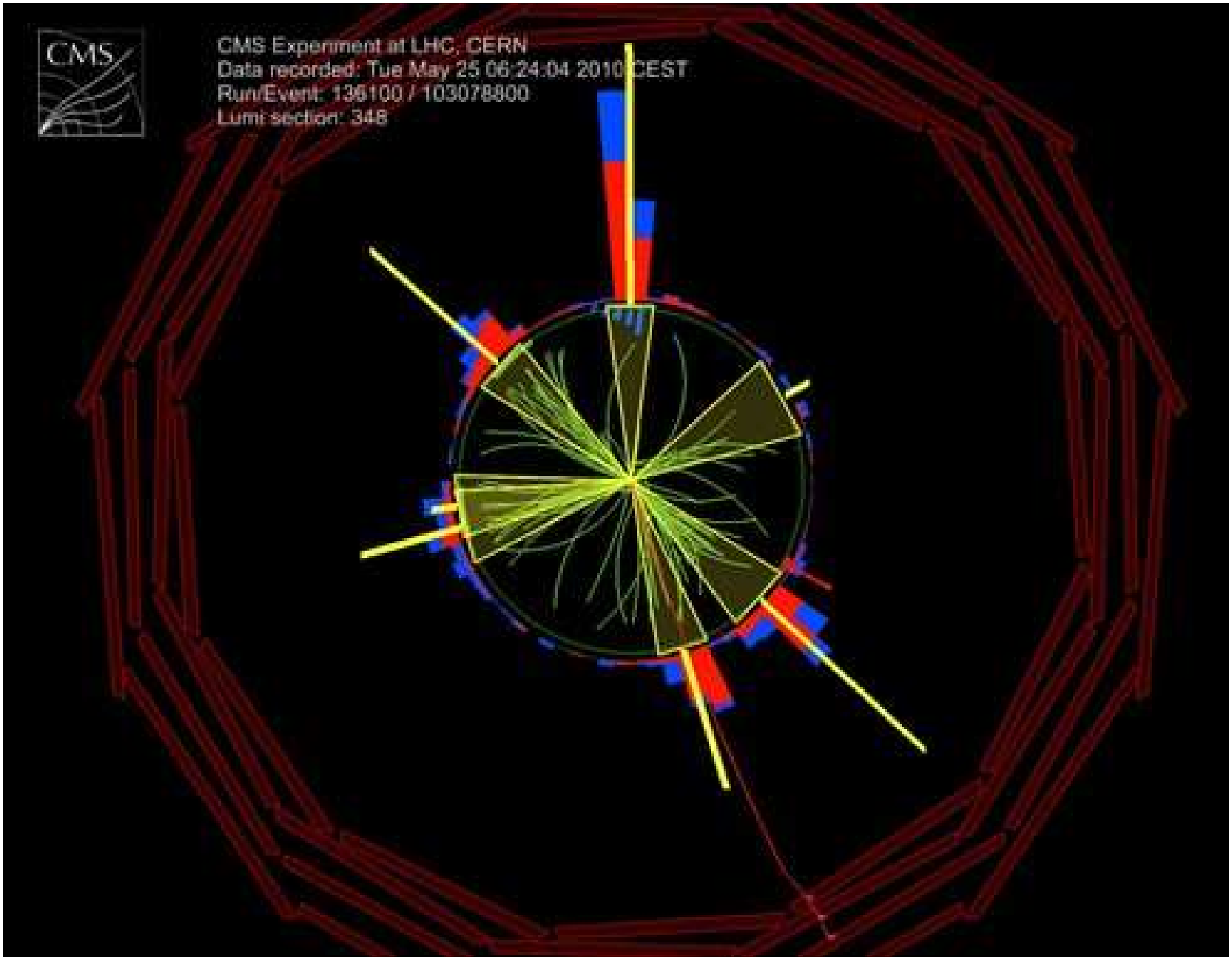
The Factorization Theorem

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1, Q_i) f_k(x_2, Q_i) \frac{d\hat{\sigma}_{j,k}(Q_i, Q_f)}{d\hat{X}} F(\hat{X} \to X; Q_i, Q_f)$$



$f \Rightarrow$ Sum over all initial state histories leading to $p_j = xP_{proton}$

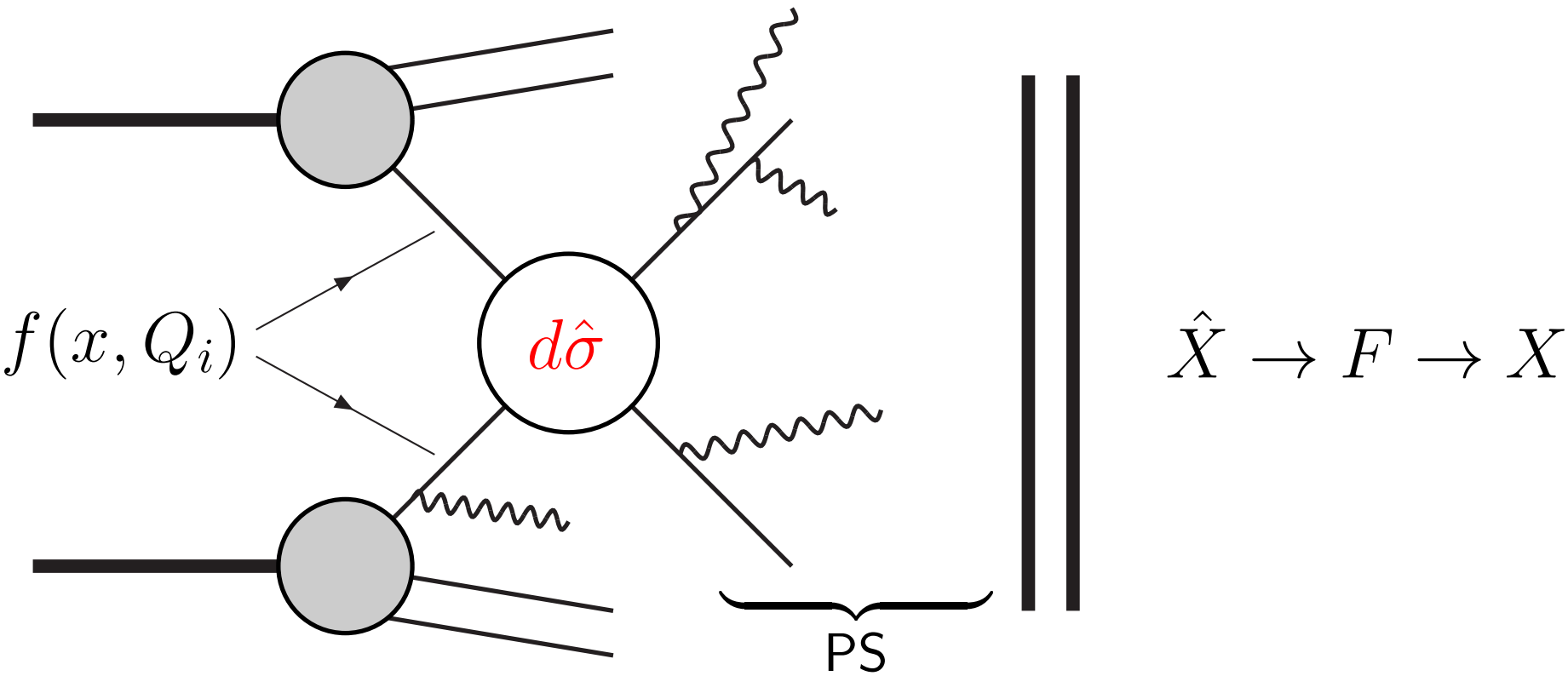
$F \Rightarrow$ Transition from partons to hadronic observable



Multijet event at CMS

The Hard Process

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1, Q_i) f_k(x_2, Q_i) \frac{d\hat{\sigma}_{j,k}(Q_i, Q_f)}{d\hat{X}} F(\hat{X} \to X; Q_i, Q_f)$$



1. Matrix element at LO
2. Matrix element at NLO

The ME computation (Tree Level)

1. Helicity Methods
2. The Dyson Schwinger approach

Helicity Methods

One evaluates directly the amplitudes as complex numbers (instead of squaring them)

Explicit representations of the wave functions of the external particles (ϵ_μ, u, v) are needed

- Massless vector:

$$\epsilon_\mu^{(\lambda)}(k) = \frac{1}{\sqrt{2}} \frac{\bar{u}_\lambda(k)\gamma_\mu u_\lambda(n)}{\bar{u}_{-\lambda}(n)u_\lambda(k)}\,, \qquad n^2 = 0\,, \quad \lambda = \pm\,.$$

- Massless (Weyl) spinors:

$$v_-(1) = u_+(1) = 1_{\mathsf{A}} \qquad \bar{v}_-(1) = \bar{u}_+(1) = 1_{\dot{\mathsf{A}}}$$
$$v_+(1) = u_-(1) = 1^{\dot{\mathsf{A}}} \qquad \bar{v}_+(1) = \bar{u}_-(1) = 1^{\mathsf{A}}$$

$\left(\frac{1}{2},0\right) + \left(0,\frac{1}{2}\right)$ $\mathrm{SL}(2,\mathrm{C})$ representation of the Lorentz group

- γ matrices (Weyl representation):

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_{\mu\dot{\mathsf{B}}\mathsf{A}} \\ \sigma_\mu^{\dot{\mathsf{A}}\mathsf{B}} & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix}.$$

see e.g. Berends *et al.*, Nucl.Phys. B321 (1989) 39

- Basic rules and spinorial inner products:

$$\sigma_\mu^{\dot{A}B}\sigma^{\mu\dot{C}D} = 2\,\epsilon^{\dot{A}\dot{C}}\epsilon^{BD}$$

$$\epsilon^{AB} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \epsilon_{AB} = \epsilon^{\dot{A}\dot{B}} = \epsilon_{\dot{A}\dot{B}}$$

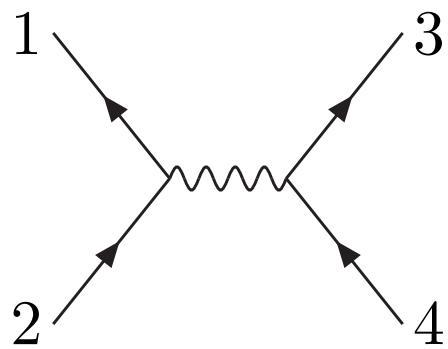
$$\epsilon_{AB} \equiv -\epsilon_{BA}$$

$$1^B = 1_A\epsilon^{AB}$$

$$< 12 > \equiv 1_A 2^A\,, \quad [12] \equiv 1_{\dot{A}} 2^{\dot{A}}$$

$$< 12 > [12] = 2\,(p_1 \cdot p_2)$$

- Example: $e^+e^- \to \mu^+\mu^-$ in (massless) QED



$$
\begin{aligned}
A(++) &\propto \bar{v}_+(1)\gamma_\mu u_-(2)\,\bar{v}_+(3)\gamma^\mu u_-(4) \\
&= 1^{\mathsf{A}}\,2^{\dot{\mathsf{B}}}\,3^{\mathsf{C}}\,4^{\dot{\mathsf{D}}}\;\sigma_{\mu\dot{\mathsf{B}}\mathsf{A}}\sigma^{\mu}_{\dot{\mathsf{D}}\mathsf{C}} \\
&= 2\;1^{\mathsf{A}}\,2^{\dot{\mathsf{B}}}\,3^{\mathsf{C}}\,4^{\dot{\mathsf{D}}}\epsilon_{\dot{\mathsf{B}}\dot{\mathsf{D}}}\epsilon_{\mathsf{AC}} = 2\;[42] < 31 >
\end{aligned}
$$

- Helicity techniques based on spinorial inner products are successfully used also in one-loop calculations
  (care necessary with dimensional regularization)

MadGraph 5: J. Alwall *et al.*, JHEP 1106 (2011) 128

- Although each diagram can be computed efficiently multi-particle amplitudes involve the evaluation of an exceedingly large numbers of Feynman diagrams. *e.g.*

Process	$n = 7$	$n = 8$	$n = 9$	$n = 10$
$g\ g \to n\ g$	559,405	10,525,900	224,449,225	5,348,843,500
$q\bar{q} \to n\ g$	231,280	4,016,775	79,603,720	1,773,172,275

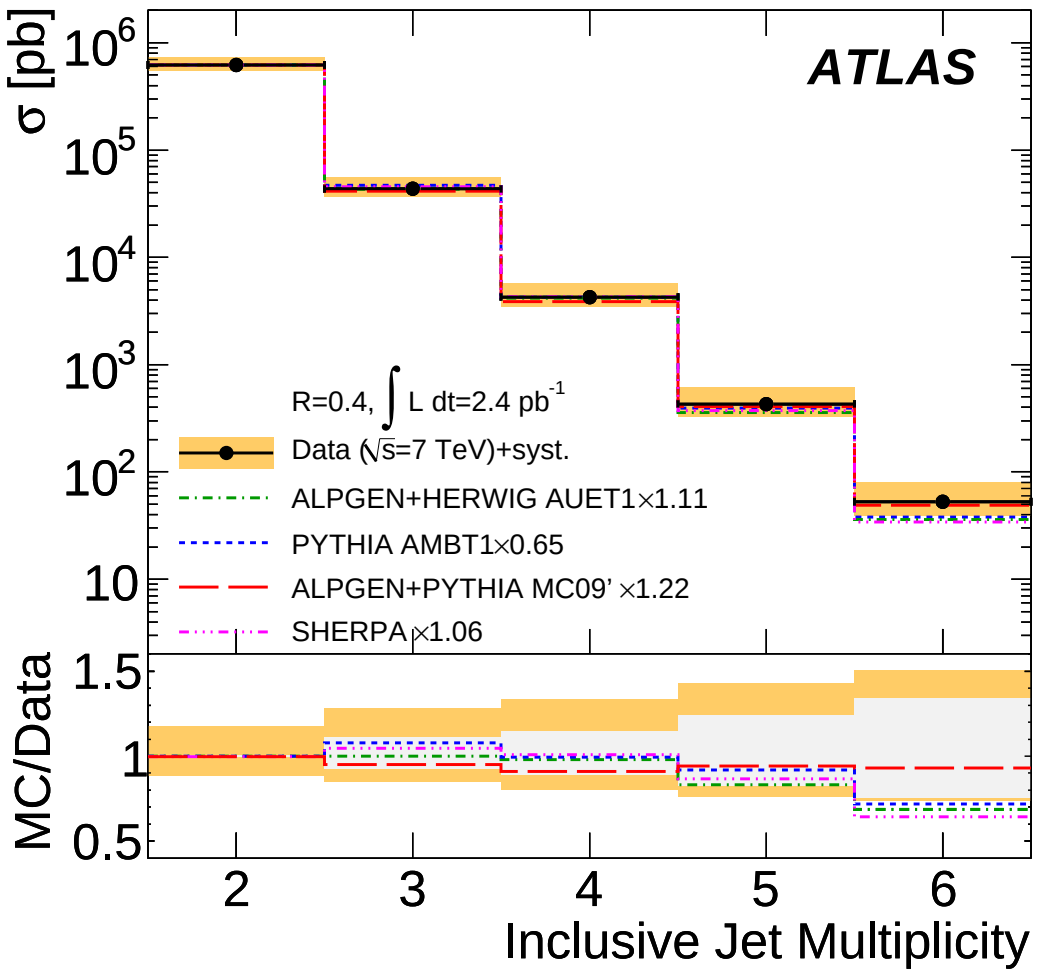
Number of Feynman diagrams corresponding to amplitudes with different numbers of quarks and gluons

F. Caravaglios *et al.* , NPB 539 (1999) 215

- A pure numerical approach to the calculations of transition amplitudes is welcome. This can be done with the ALPHA algorithm

F. Caravaglios and M. Moretti, PLB 358 (1995) 332

Jet cross-sections at the LHC



Eur.Phys.J. C71 (2011) 1763

The Dyson Schwinger approach

- The Dyson-Schwinger equation expresses *recursively* the n-point Green's functions in terms of the $1-, 2-, \ldots, (n-1)$-point functions.
  Suitable for a *numerical approach*

F. Dyson, Phys. Rev. 75 (1949) 1736
J. Schwinger, PNAS 37 (1951) 452–459
F. A. Berends and W. Giele, NPB 306 (1988) 759
A. Kanaki and C. G. Papadopoulos, hep-ph/0012004

- A 0-dimensional QFT is completely specified by all possible p-point (Euclidean) Green Functions

E. N. Argyres *et al.*, Eur.Phys.J. C19 (2001) 567-582

$$G_p \equiv < \phi^p > = N \int d\phi \, \phi^p \, e^{-S(\phi)}$$

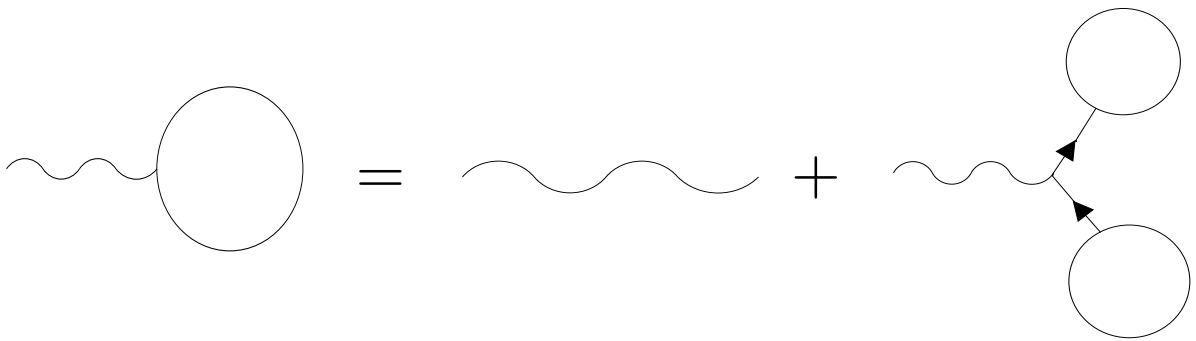
- They can be obtained from a *Generating Functional*

$$Z(J) = N \int d\phi \, e^{-S(\phi)+J\phi} = \sum_{p \geq 0} G_p \frac{J^p}{p!}$$

- The DS Equation follows from the standard identity

$$\begin{aligned} 0 &= N \int d\phi \frac{d}{d\phi} \left\{ e^{-S(\phi)+J\phi} \right\} \\ &= N \int d\phi \left\{ -S'(\phi) + J \right\} e^{-S(\phi)+J\phi} \\ &= \left\{ -S'(\partial_J) + J \right\} Z(J) \end{aligned}$$

- For instance in QED this equation can be graphically represented as



- This gives rise to a **recursive algorithm** easily implementable in computer codes

ALPGEN: M. L. Mangano *et al.*, JHEP 0307 (2003) 001

HELAC: A. Cafarella *et al.*, Comp.Phys.Comm. 180 (2009) 1941-1955

The NLO ME computation (The NLO Revolution)

A typical $2 \to m$ process at 1-loop

$$\sigma^{NLO} = \int_m d\sigma^B + \int_m \left(d\sigma^V + \int_1 d\sigma^A \right) + \int_{m+1} \left(d\sigma^R - d\sigma^A \right)$$

1. $d\sigma^B$ is the Born cross section
2. $d\sigma^V$ is the Virtual correction (loop diagrams: **Bottleneck**)
3. $d\sigma^R$ is the Real correction
4. $d\sigma^A$ and $\int_1 d\sigma^A$ are *unintegrated* and *integrated* counterterms (allowing to compute the Real emission of massless particles in 4 dimensions)

Approaches to counterterms evaluation

1. CS: S. Catani, M.H. Seymour, Nucl.Phys. B485 (1997) 291-419
2. FKS: S. Frixione, *et al.*, Nucl.Phys. B467 (1996) 399442
3. Nagy-Soper: JHEP 0709 (2007) 114
   (matching with PS including quantum intereferences)
4. POWHEG: S. Frixione, P. Nason, C. Oleari, JHEP 0711 (2007) 070
   (matching with PS)

The Virtual corrections $d\sigma^V$ and the decomposition of any 1-loop amplitude

$$
\begin{aligned}
A \quad = \quad & \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}\bar{D}_{i_3}} \\
& + \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}} \\
& + \sum_{i_0<i_1}^{m-1} b(i_0 i_1) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}} \\
& + \sum_{i_0}^{m-1} a(i_0) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}} + R
\end{aligned}
$$

The problem is getting the set $\mathcal{S} = \left\{ \begin{array}{lll} d(i_0 i_1 i_2 i_3), & c(i_0 i_1 i_2), & \\ b(i_0 i_1), & a(i_0), & R \end{array} \right.$

The OPP Method

G. Ossola, C. Papadopoulos, R. P., Nucl.Phys.B763:147-169,2007

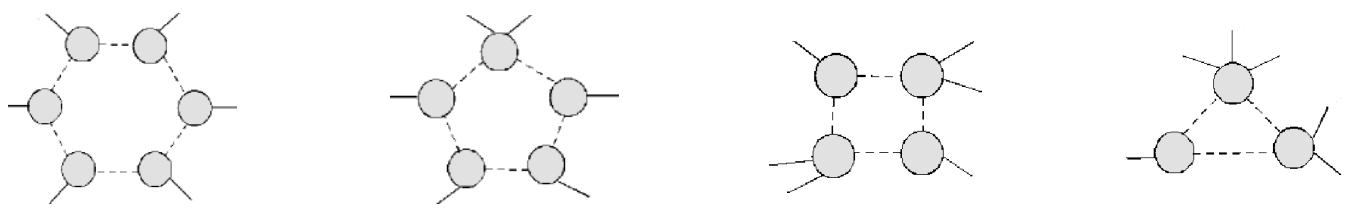
Working at the *integrand* level

$$A = \int d^n\bar{q}\ \left[\mathcal{A}(q) + \tilde{A}(q,\tilde{q},\epsilon)\right]$$

$$\left(\begin{array}{c} \bar{q} = q + \tilde{q} \\ n = 4 + \epsilon \end{array}\right)$$

- For example, in the case of $2 \to 6$

$$\mathcal{A}(q) = \sum \underbrace{\frac{N_i^{(6)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_5}}} + \underbrace{\frac{N_i^{(5)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_4}}} + \underbrace{\frac{N_i^{(4)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_3}}} + \underbrace{\frac{N_i^{(3)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}}} + \cdots$$



The function to be sampled *numerically* to extract the coefficients

$$
\begin{aligned}
N_i^{(6)}(q) &= \sum_{i_0<i_1<i_2<i_3}^{5} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \\
&+ \sum_{i_0<i_1<i_2}^{5} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] D_{i_3} D_{i_4} D_{i_5} \\
&+ \sum_{i_0<i_1}^{5} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] D_{i_2} D_{i_3} D_{i_4} D_{i_5} \\
&+ \sum_{i_0}^{5} \left[a(i_0) + \tilde{a}(q; i_0) \right] D_{i_1} D_{i_2} D_{i_3} D_{i_4} D_{i_5} \\
&+ \tilde{P}(q) D_{i_0} D_{i_1} D_{i_2} D_{i_3} D_{i_4} D_{i_5}
\end{aligned}
$$

- The general functional form of the *spurious* terms is known

Example ($p_0 = 0$)

$$\tilde{d}(q;0123) = \tilde{d}(0123)\,\epsilon(qp_1p_2p_3)$$

$$\int d^n\bar{q}\frac{\tilde{d}(q;0123)}{\bar{D}_0\bar{D}_1\bar{D}_2\bar{D}_3} = \tilde{d}(0123)\int d^n\bar{q}\frac{\epsilon(qp_1p_2p_3)}{\bar{D}_0\bar{D}_1\bar{D}_2\bar{D}_3} = 0$$

- The coefficients $\{d_i, c_i, b_i, a_i\}$ and $\{\tilde{d}_i, \tilde{c}_i, \tilde{b}_i, \tilde{a}_i\}$ are extracted **by solving linear systems of equations**

The use of special values of q helps (Unitarity)

$$D_0(q^\pm) = D_1(q^\pm) = D_2(q^\pm) = D_3(q^\pm) = 0$$

$$N^{(m-1)}(q^\pm) \quad = \Big[d(0123) + \tilde{d}(q^\pm; 0123)\Big] \prod_{i\neq 0,1,2,3}^{m-1} D_i(q^\pm)$$

$$d(0123) = \frac{1}{2}\left[\frac{N^{(m-1)}(q^+)}{\prod_{i\neq 0,1,2,3}^{m-1} D_i(q^+)} + \frac{N^{(m-1)}(q^-)}{\prod_{i\neq 0,1,2,3}^{m-1} D_i(q^-)}\right]$$

About R ($= R_1 + R_2$)

The origin of R_1

$$\frac{1}{\bar{D}_i} = \frac{1}{D_i}\left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right)$$

$\Rightarrow$ predicted within OPP
if the denominator structure is known

The origin of R_2

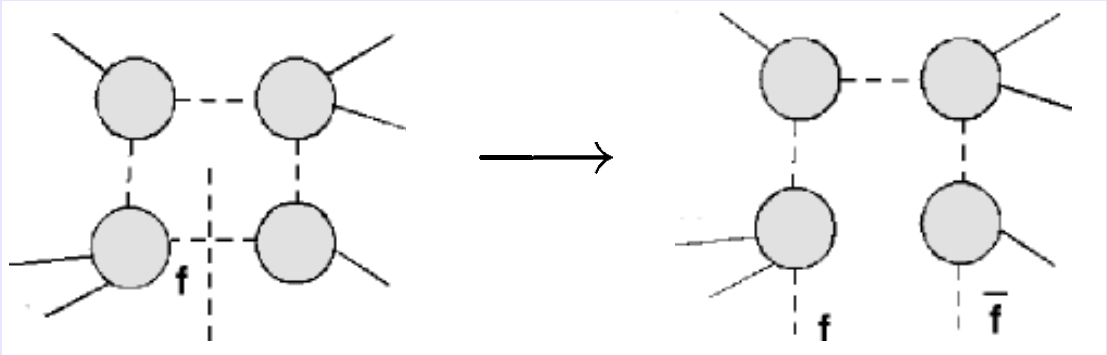
$$R_2 = \int d^n\bar{q}\frac{\tilde{N}(q,\tilde{q},\epsilon)}{\bar{D}_0\cdots\bar{D}_{m-1}}$$

$\Rightarrow$ effective tree-level Feynman Rules
up to 4 points *

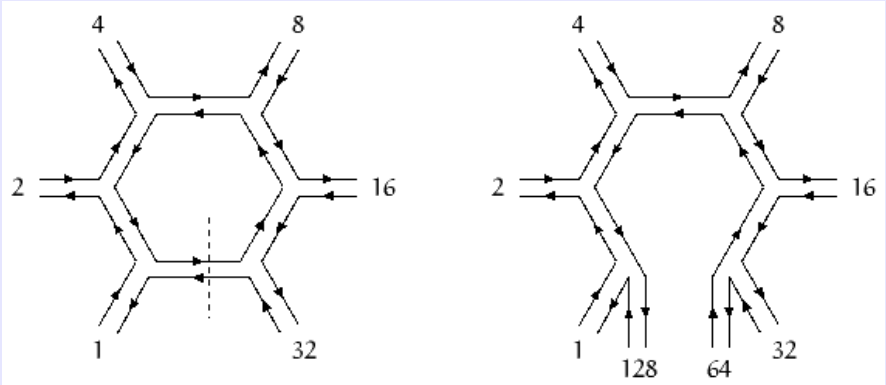
* QCD: Draggiotis, Garzelli, Papadopoulos, R. P., JHEP 0904:072,2009
EW: Garzelli, Malamos, R. P., JHEP 1001:040,2010
Effective GGH theory: Page, R. P., arXiv:1307.6142 [hep-ph]

Recursion Relations at 1-loop

- OPP + 1 hard-cut allow to use *the same tree-level Recursion Relations* for $m + 2$ tree-like structures



- The color can be treated *as at the tree level*



Tree level codes can be transformed into 1-loop ones

The Helac-NLO System

Bevilacqua *et al.*, Comput.Phys.Commun. 184 (2013) 986-997

1. CutTools
   $\{d_i, c_i, b_i, a_i\}$ and R_1
2. HELAC-1LOOP
   $N(q)$ and R_2
3. OneLOop
   scalar 1-loop integrals
4. HELAC-DIPOLES
   Real correction and CS dipoles

- Ossola, Papadopoulos, R. P., JHEP 0803 (2008) 042
- van Hameren, Papadopoulos, R. P., JHEP 0909 (2009) 106
- van Hameren, Comput.Phys.Commun. 182 (2011) 2427-2438
- Czakon, Papadopoulos, Worek, JHEP 0908 (2009) 085

V. Hirschi *et al.*, JHEP 1105 (2011) 044

The MadGraph Solution: MadLoop

- Need to upgrade MadGraph so to generate loop diagrams and numerical code for the integrand N(q)

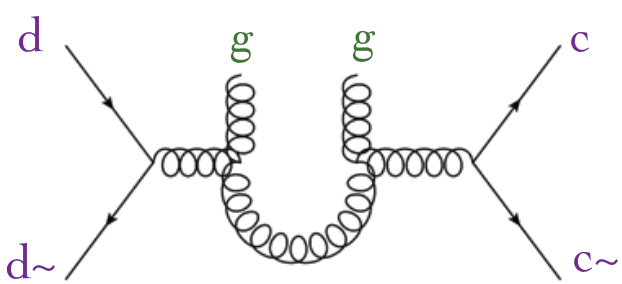
Remember:

➡ For a given phase-space point, we have to compute the numerator function several times (~50 or so for a box loop) per external phase-space point.

➡ Cut the Loop and use HELAS (with no denominator)

➡ Generic

➡ recycling



OpenLoop: [S. Pozzorini & al.(2011)]

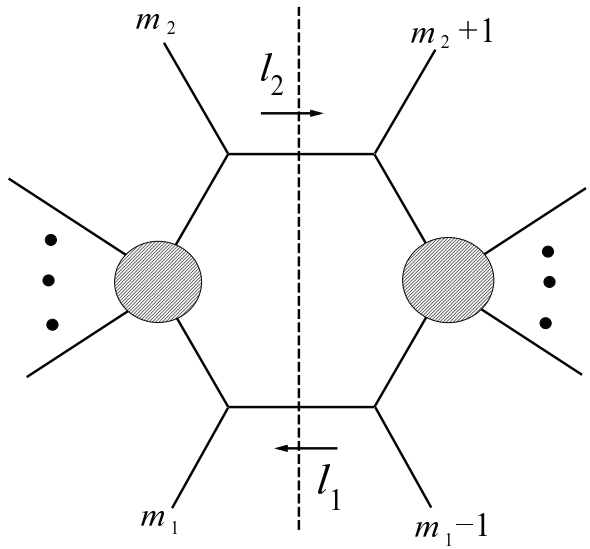
$$\mathcal{N}(l^\mu) = \sum_{r=0}^{r_{max}} C^{(r)}_{\mu_0\mu_1\cdots\mu_r} l^{\mu_0} l^{\mu_1} \cdots l^{\mu_r}$$

5-10 times faster

coefficient computed iteratively by ALOHA

Unitarity: Cutting (Gluing)

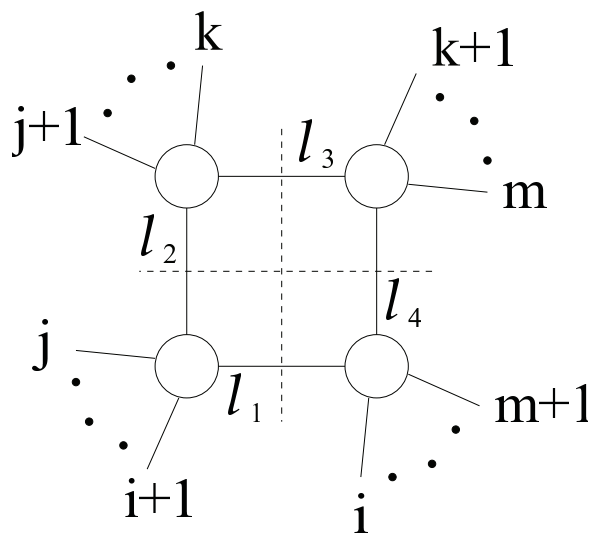
1. Double cuts ⇔ gluing 2 tree-level, *gauge invariant*, amplitudes (Bern, Dixon, Dunbar, Kosower 1994)



2. Different double cuts are applied to disentangle 1-loop scalar functions *by looking at the analytic structure of the result*
3. R is reconstructed by looking at collinear and infrared limits

Generalized Unitarity: more Cutting (more Gluing)

1. Quadruple cuts $\Leftrightarrow$ gluing 4 tree-level, *gauge invariant*, amplitudes (Britto, Cachazo, Feng, hep-th/0412103)



2. q integration frozen $\Rightarrow$ *coefficient d_i of the box extracted*
3. 3 bubbles are connected together, the box contributions subtracted and the *coefficients c_i of the triangles extracted*
4. ...

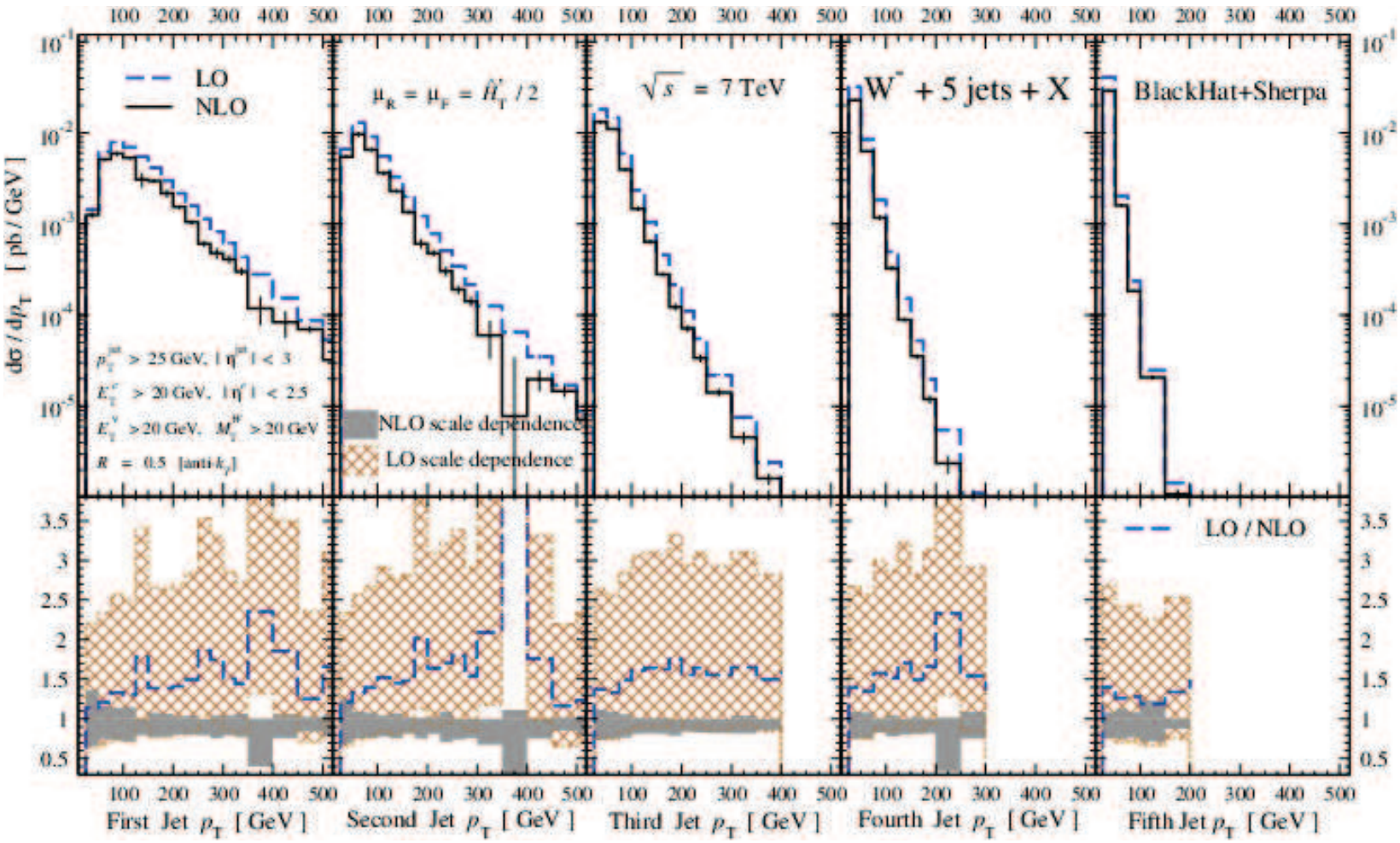
Generalized Unitarity: Relevant References

- Bern, Dixon, Dunbar, Kosower, hep-ph/9403226 and hep-ph/9409265
- Forde,0704.1835 [hep-ph]
- Ellis, Giele, Kunszt, 0708.2398 [hep-ph]
- Ellis, Giele, Kunszt, Melnikov, 0806.3467 [hep-ph]
- Berger et al. (**BlackHat**), 0803.4180 [hep-ph]

Between OPP and GU:

- G. Cullen, *et al.*, (**GoSam**), Eur.Phys.J. C72 (2012) 1889
- P. Mastrolia, *et al.*, (**SAMURAI**), JHEP 1008 (2010) 080

Largest multiplicity world record (BlackHat)

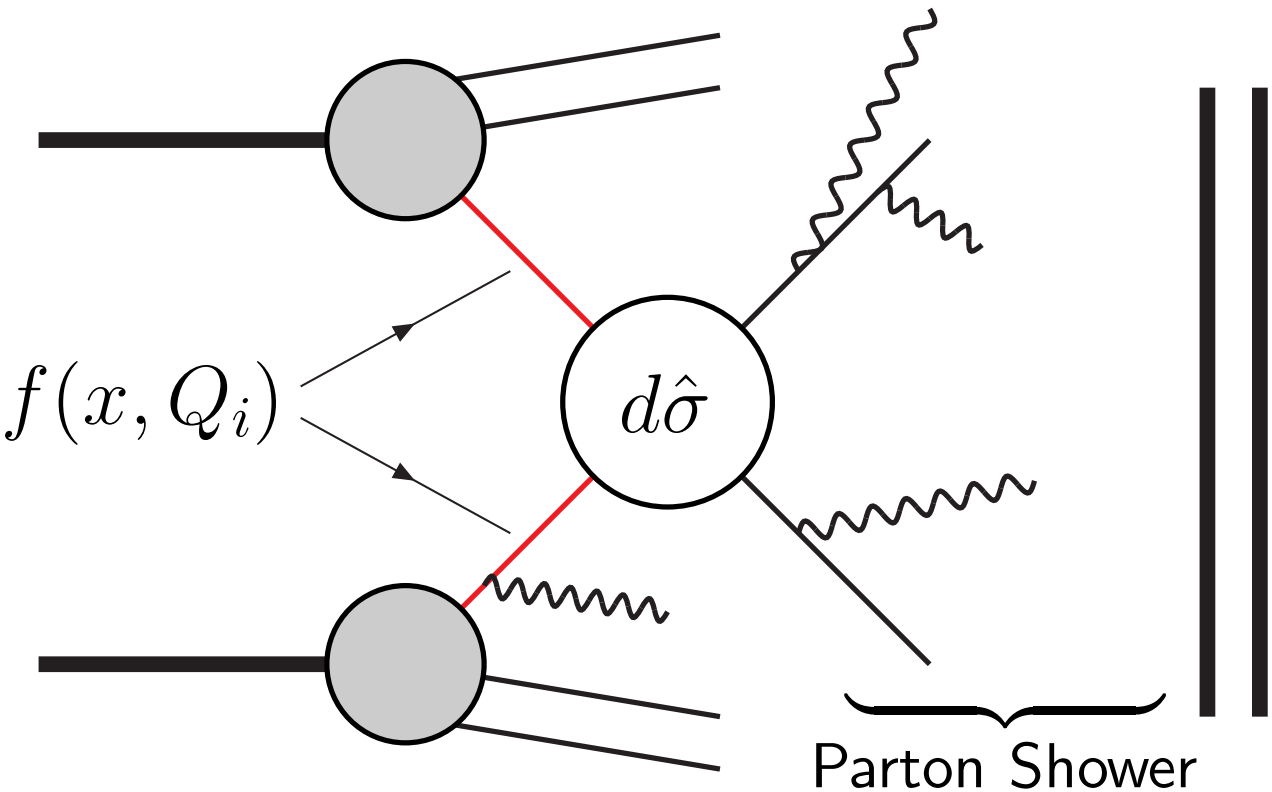


Less automatic approaches

1. MCFM: J.M. Campbell, R.K. Ellis, Phys. Rev. D62:114012 (2000)
2. MC@NLO: S. Frixione, B. R. Webber, JHEP 0206 (2002) (matching with PS)

Subprocesses selection

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1, Q_i) f_k(x_2, Q_i) \frac{d\hat{\sigma}_{j,k}(Q_i, Q_f)}{d\hat{X}} F(\hat{X} \to X; Q_i, Q_f)$$



$$\hat{X} \to F \to X$$

- The calculation of the cross section for multi-parton final states involves typically the sum over a large set of subprocesses and flavour configurations.
  E.g. for the $Wb\bar{b}$ final state:

jp	subprocess	jp	subprocess	jp	subprocess
1	$q\bar{q}' \to WQ\overline{Q}$	2	$qg \to q'WQ\overline{Q}$	3	$gq \to q'WQ\overline{Q}$
4	$gg \to q\bar{q}'WQ\overline{Q}$	5	$q\bar{q}' \to WQ\overline{Q}q''\bar{q}''$	6	$qq'' \to WQ\overline{Q}q'q''$
7	$q''q \to WQ\overline{Q}q'q''$	8	$q\bar{q} \to WQ\overline{Q}q'\bar{q}''$	9	$q\bar{q}' \to WQ\overline{Q}q\bar{q}$
10	$\bar{q}'q \to WQ\overline{Q}q\bar{q}$	11	$q\bar{q} \to WQ\overline{Q}q\bar{q}'$	12	$q\bar{q} \to WQ\overline{Q}q'\bar{q}$
13	$qq \to WQ\overline{Q}qq'$	14	$qq' \to WQ\overline{Q}qq$	15	$qq' \to WQ\overline{Q}q'q'$
16	$qg \to WQ\overline{Q}q'q''\bar{q}''$	17	$gq \to WQ\overline{Q}q'q''\bar{q}''$	18	$qg \to WQ\overline{Q}qq\bar{q}'$
19	$qg \to WQ\overline{Q}q'q\bar{q}$	20	$gq \to WQ\overline{Q}qq\bar{q}'$	21	$gq \to WQ\overline{Q}q'q\bar{q}$
22	$gg \to WQ\overline{Q}q\bar{q}'q''\bar{q}''$	23	$gg \to WQ\overline{Q}q\bar{q}q\bar{q}'$		

- Each of these subprocesses receives contributions **from several possible flavour configurations**
  (e.g. $u\bar{d} \to WQ\overline{Q}gg$, $u\bar{s} \to WQ\overline{Q}gg$, etc.)

- The subdivision in subprocesses can be designed to allow to sum the contribution of different flavour configurations **by simply adding trivial factors such as parton densities or CKM factors**, which factorize out of a single, flavour independent, matrix element

- For example the overall contribution from the 1st process in the list is given by

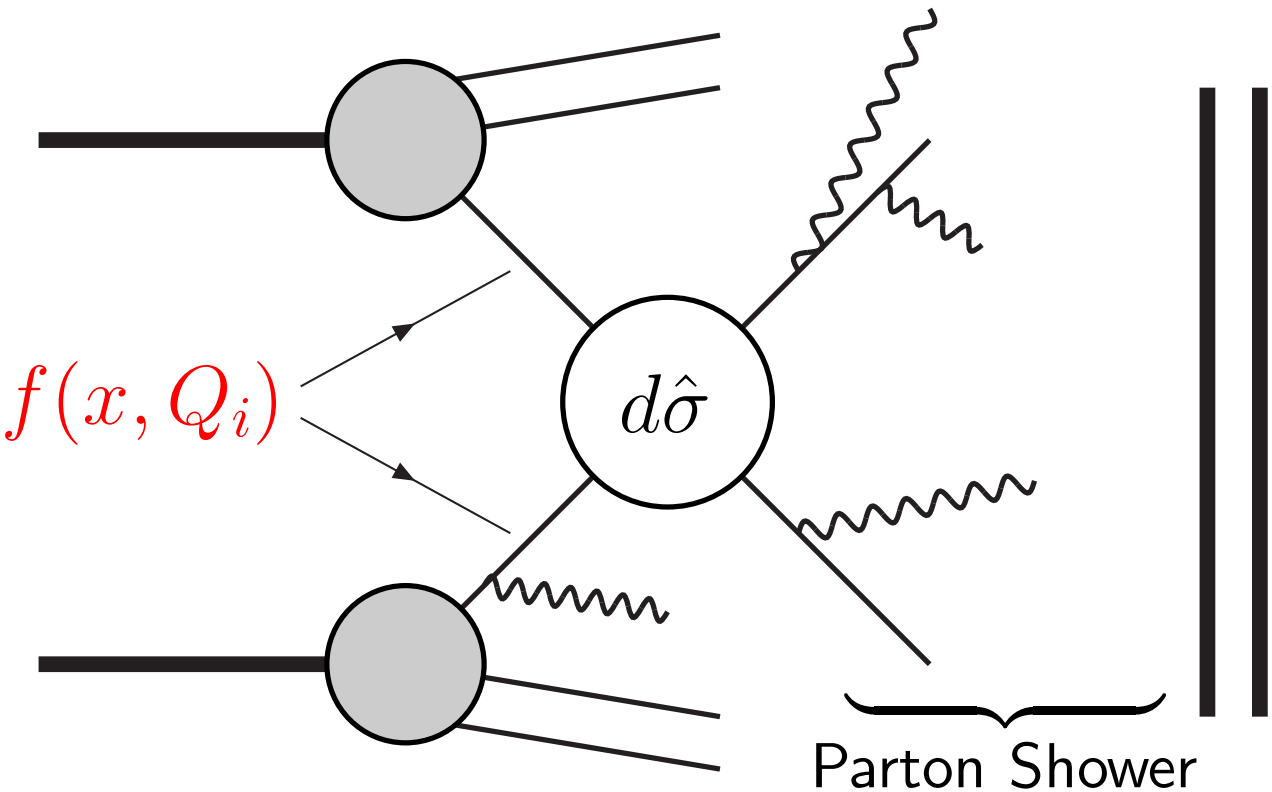
$$\left[u_1 \bar{d}_2 \cos^2\theta_c + u_1 \bar{s}_2 \sin^2\theta_c + c_1 \bar{s}_2 \cos^2\theta_c + c_1 \bar{d}_2 \sin^2\theta_c\right] \times |M(q\bar{q}' \to WQ\overline{Q}gg)|^2$$

  where $q_i = f(x_i),\ i = 1, 2$, are the parton densities for the quark flavour q

- Event by event, flavour configuration **selected with a probability proportional to the relative size of the individual contributions** to the luminosity, weighted by the Cabibbo angles

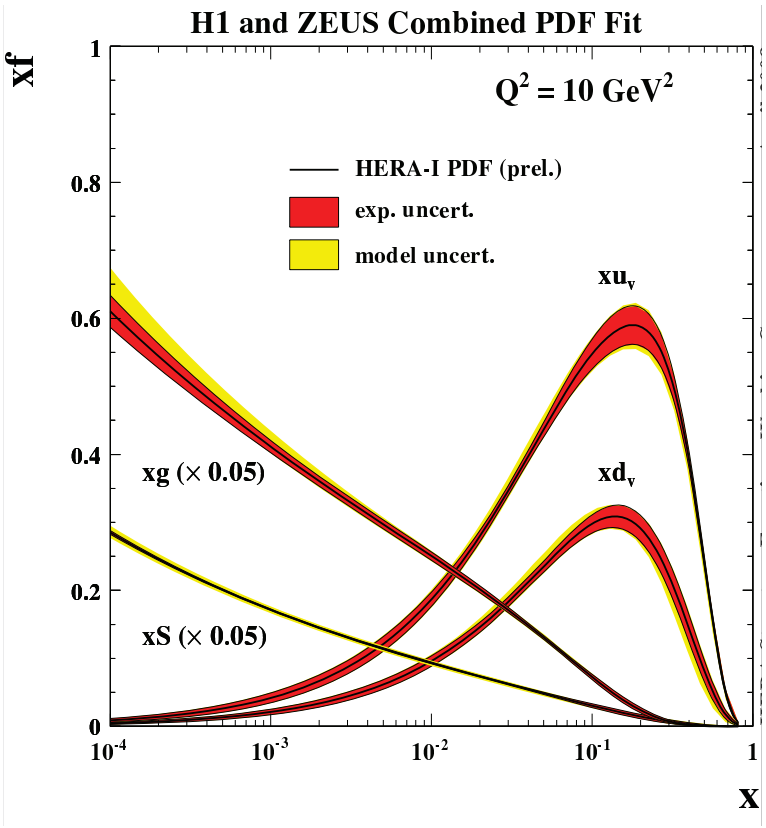
The convolution with the PDFs

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1, Q_i) f_k(x_2, Q_i) \frac{d\hat{\sigma}_{j,k}(Q_i, Q_f)}{d\hat{X}} F(\hat{X} \to X; Q_i, Q_f)$$



$$\hat{X} \to F \to X$$

- To complete the calculation, one needs to perform the integration over the *parton distribution functions* (PDFs)
- Those distribution functions are **traditionally extracted from deep inelastic scattering**



- **LHC data** are nowadays also used to measure them

- PDFs are available as **prepackaged computer programs**, which can be used to perform the integral numerically

The most used sets are:

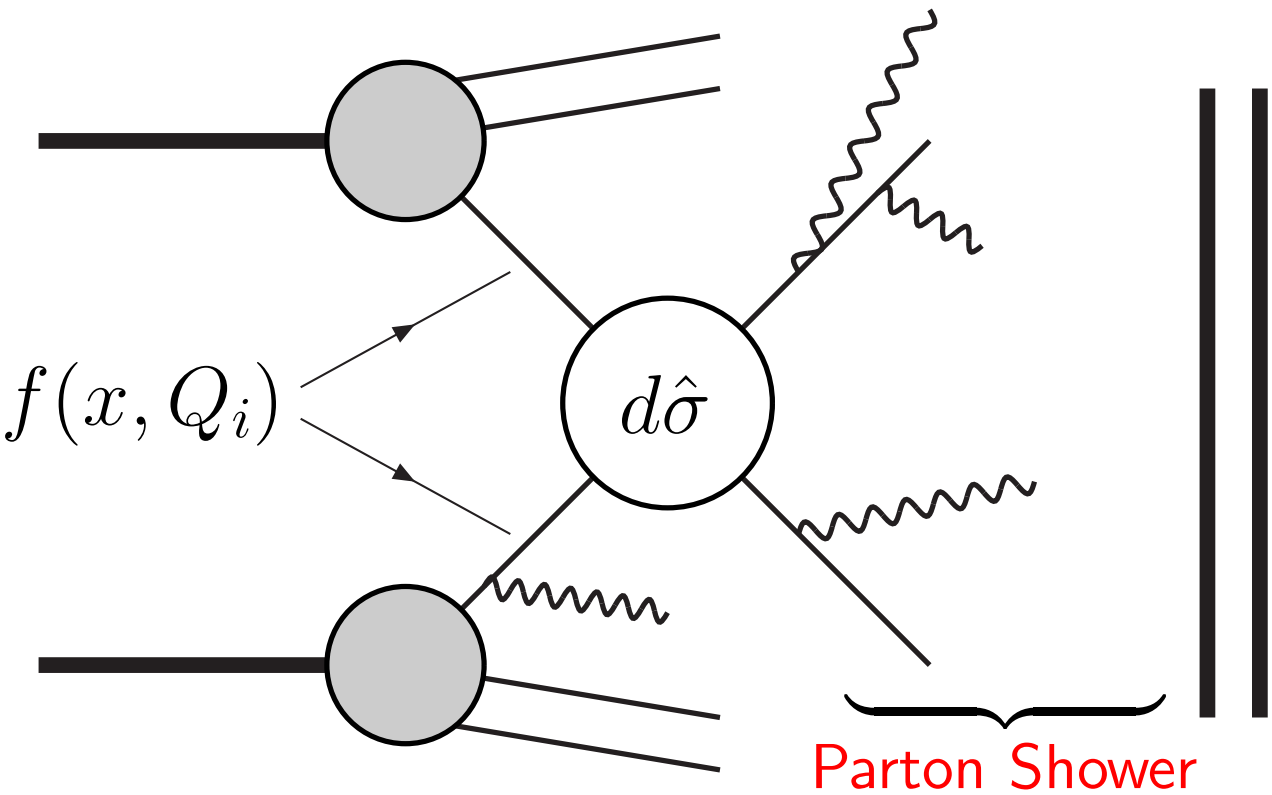
- CTEQ
  J. Pumplin *et al.*, JHEP 0207, 012 (2002)
- MSTW
  A. D. Martin *et al.*, Eur. Phys. J. C 14, 133 (2000)
- NNPDF (unbiased and at NLO)
  R. D. Ball *et al.*, Nucl.Phys. B838 (2010) 136-206

Other sets:

- ABKM09
- GJR08
- HERAPDF

Parton Shower

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1, Q_i) f_k(x_2, Q_i) \underbrace{\frac{d\hat{\sigma}_{j,k}(Q_i, Q_f)}{d\hat{X}}}_{\textbf{Showered}} F(\hat{X} \to X; Q_i, Q_f)$$



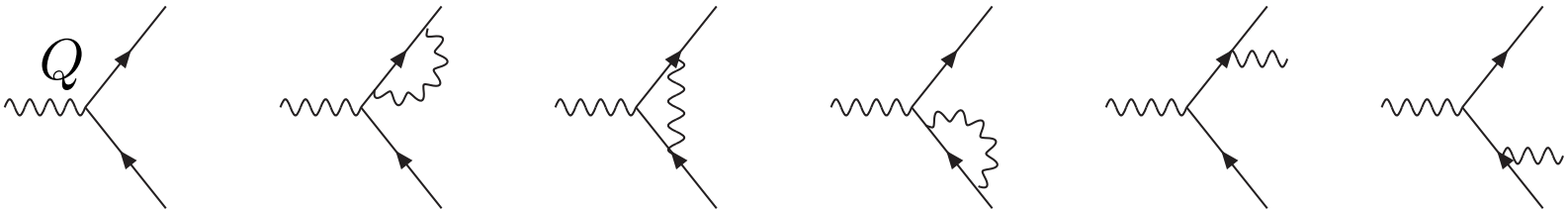
$$\hat{X} \to F \to X$$

1. Sudakov Form Factors
2. Color Flow
3. Matching

Sudakov Form Factors

- PS is important because a lot of final state jets are typically observed, in hadronic collisions, coming from QCD radiation that, subsequently, hadronizes. **Still pertubatively calculable** by introducing the **Sudakov Form Factors**

- In QED (at the LL):



$$= d\sigma_0 \left(1 - \frac{\alpha}{\pi} \log^2 \frac{Q}{Q_0}\right) \qquad (Q > Q_0 > 0)$$

$\log^2$ from **overlap of soft and collinear emissions**

- Thanks to factorization theorems one can exponentiate this result

$$d\sigma_{rad} = d\sigma_0 \; exp\left\{-\frac{\alpha}{\pi}\log^2\frac{Q}{Q_0}\right\}$$

- In QCD (at the LL): $\alpha \to C_F\alpha_s, \quad C_F = \frac{N_c^2-1}{2N_c} = \frac{4}{3}$ and

$$d\sigma_{rad} = d\sigma_0 \; exp\left\{-\frac{\alpha_s C_F}{\pi}\log^2\frac{Q}{Q_0}\right\}$$

- The term

$$exp\left\{-\frac{\alpha_s C_F}{\pi}\log^2\frac{Q}{Q_0}\right\}$$

represents the probability for a quark of **NOT** radiating any gluon when passing from a scale Q to a scale $Q_0 < Q$

- In reality $\alpha_s = \alpha_s(Q)$ and one defines the Sudakov Form Factor for a quark as

$$\Delta_q(Q_0, Q) \equiv exp\left\{-\int_{Q_0}^{Q} dq\, \Gamma_q(q, Q)\right\}$$

$$\Gamma_q(q, Q) = \frac{2C_F}{\pi}\frac{\alpha_s}{q}\left(\log\frac{Q}{q} - \frac{3}{4}\right) = P(q \to qg)$$

is the $q \to qg$ **Altarelli-Parisi splitting function**

When α_s is a constant taking the integral reproduces the previous expression (at the LL)

- The Sudakov Form factor for the gluon is

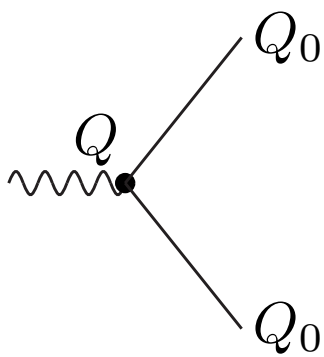
$$\Delta_g(Q_0, Q) \equiv exp\left\{-\int_{Q_0}^{Q} dq\left[\Gamma_g(q, Q) + \Gamma_f(q)\right]\right\}$$

$$\Gamma_g(q, Q) = P(g \to gg)\,, \quad \Gamma_f(q) = P(g \to q\bar{q})$$

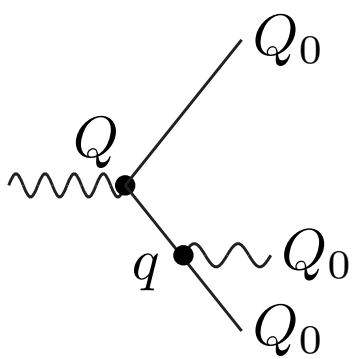
are the $g \to gg$ and $g \to q\bar{q}$ splitting functions

- $\Delta_{q,g}(Q_0, Q)$ allow to construct probabilistic MCs which generate the **perturbative Parton Shower cascade** (infinite emissions)

- Reliable in the soft/collinear regime **ONLY**. Outside one should calculate the **EXACT** Matrix elements (double counting problem $\Rightarrow$ Matching)



$$\Rightarrow \qquad R_2 = \Delta_q(Q_0, Q)^2$$

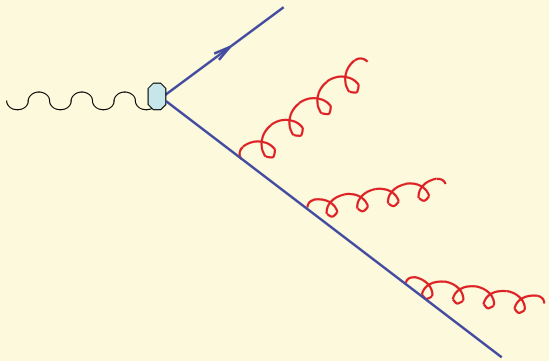


$$\Rightarrow \qquad R_3 = 2\,\Delta_q(Q_0, Q)^2 \int_{Q_0}^{Q} dq\, \Gamma_q(q, Q)\Delta_g(Q_0, q)$$

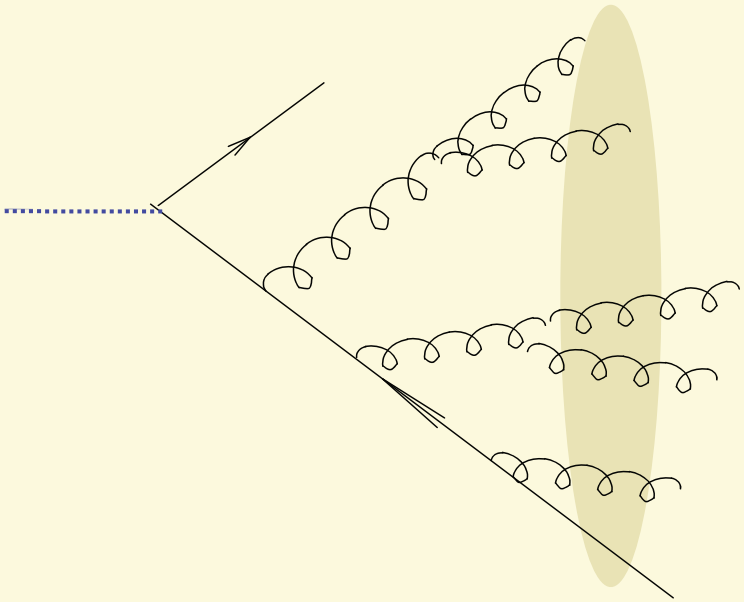
Probabilistic diagrams (NOT Feynman diagrams)

Color Flow

- The color flow is the set of color connections among the partons which defines **the set of dipoles for a given event**
- In QCD, at each color flow **corresponds a different structure of Sudakov Form Factors**
- The emission of soft gluon radiation in shower MC programs accounts for **quantum coherence**, which is implemented via the prescription of **angular ordering in the parton cascade**



The construction can be iterated to the next emission, with the result that emission angles keep getting smaller and smaller => **jet structure**



Total colour charge of the system is equal to the quark colour charge. Treating the system as the incoherent superposition of N gluons would lead to artificial growth of gluon multiplicity. Angular ordering enforces coherence, and leads to the proper evolution with energy of particle multiplicities.

- To reliably evolve a multi-parton state into a multi-jet configuration **necessary to associate a specific color-flow pattern to each generated parton-level event**

- A multigluon processes:

$$M(\{p_i\},\{\epsilon_i\},\{a_i\}) = \sum_{P(2,3,\ldots,n)} \mathrm{tr}(\lambda^{a_{i_1}}\lambda^{a_{i_2}}\ldots\lambda^{a_{i_n}})A(\{p_{i_1}\},\{\epsilon_{i_1}\};\ldots\{p_{i_n}\},\{\epsilon_{i_n}\})$$

F.A. Berends and W. Giele, NPB 294 (1987) 700
M. Mangano, S. Parke and Z. Xu, NPB 298 (1988) 653

$A(\{P_i\})$ are *dual* or *color-ordered* amplitudes: gauge-invariant, cyclically-symmetric functions of the gluons' momenta and helicities

- Each $A(\{P_i\})$ corresponds to a set of diagrams where color flows from one gluon to the next, according to the ordering specified by the permutation of indices

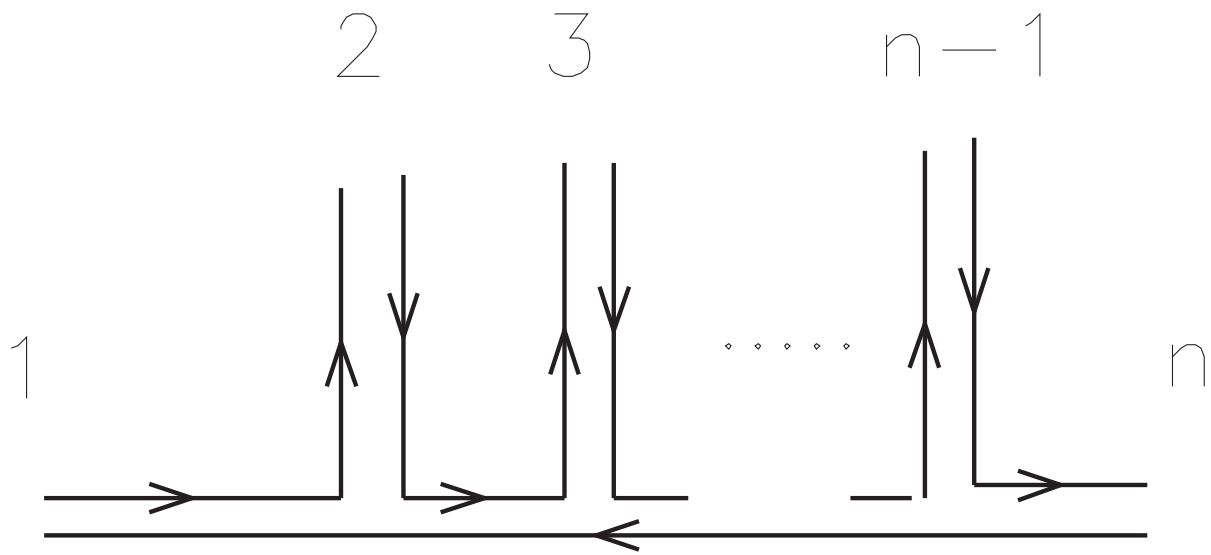


Figure 1: Colour structure of the n-gluon amplitude in the large-N limit.

- When summing over colors the amplitude squared, different orderings of dual amplitudes are orthogonal at the leading order in $1/N^2$

$$\sum_{\mathrm{col's}} |M(\{p_i\},\{\epsilon_i\},\{a_i\})|^2 \;=\; N^{n-2}(N^2-1)\sum_{P_i}\left[|A(\{P_i\})|^2 + \frac{1}{N^2}(\mathrm{interf.})\right]$$

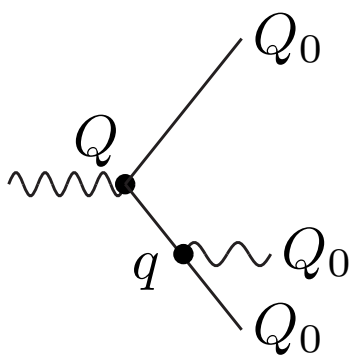
- At the leading order in $1/N^2$, therefore, the square of each dual amplitude is proportional to the relative probability of the corresponding color flow

- Each flow defines the set of color currents necessary and sufficient to implement the color ordering prescription for the coherent evolution of the gluon shower

Matching

1. CKKW (LO) S. Catani *et al.*, JHEP 0111 (2001) 063
2. MLM (LO) M. Mangano *et al.*, Eur.Phys.J. C53 (2008) 473-500
3. FxFx (NLO) Frederix and Frixione, JHEP 1212 (2012) 061

CKKW

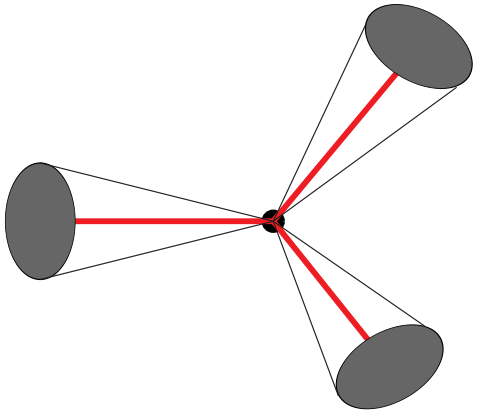


$$\Rightarrow \quad R_3 = 2\,\Delta_q(Q_0, Q)^2 \int_{Q_0}^{Q} dq\,\Gamma_q(q, Q)\Delta_g(Q_0, q)$$

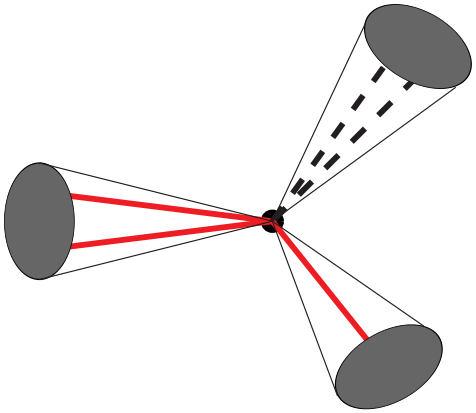
Outside the soft/collinear region $\Gamma_q(q, Q) \to |M_{q\bar{q}g}|^2$

MLM $(n_{jets} = 3)$ —— hard parton

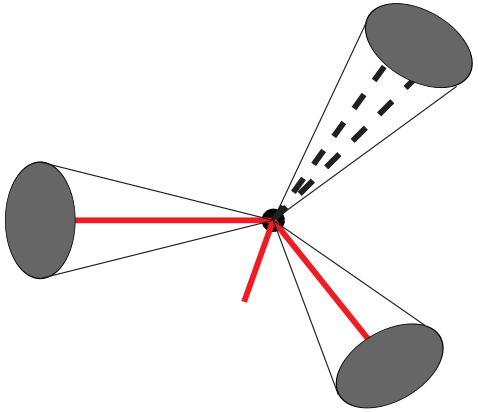
- - - - parton from shower



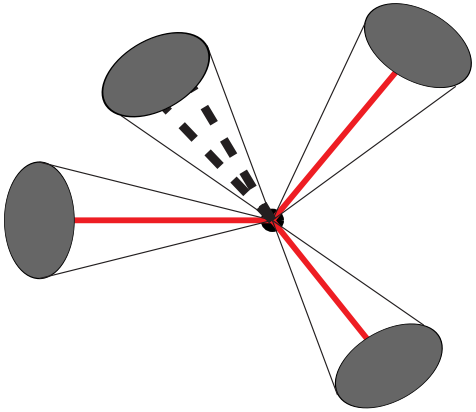
matched



NOT matched

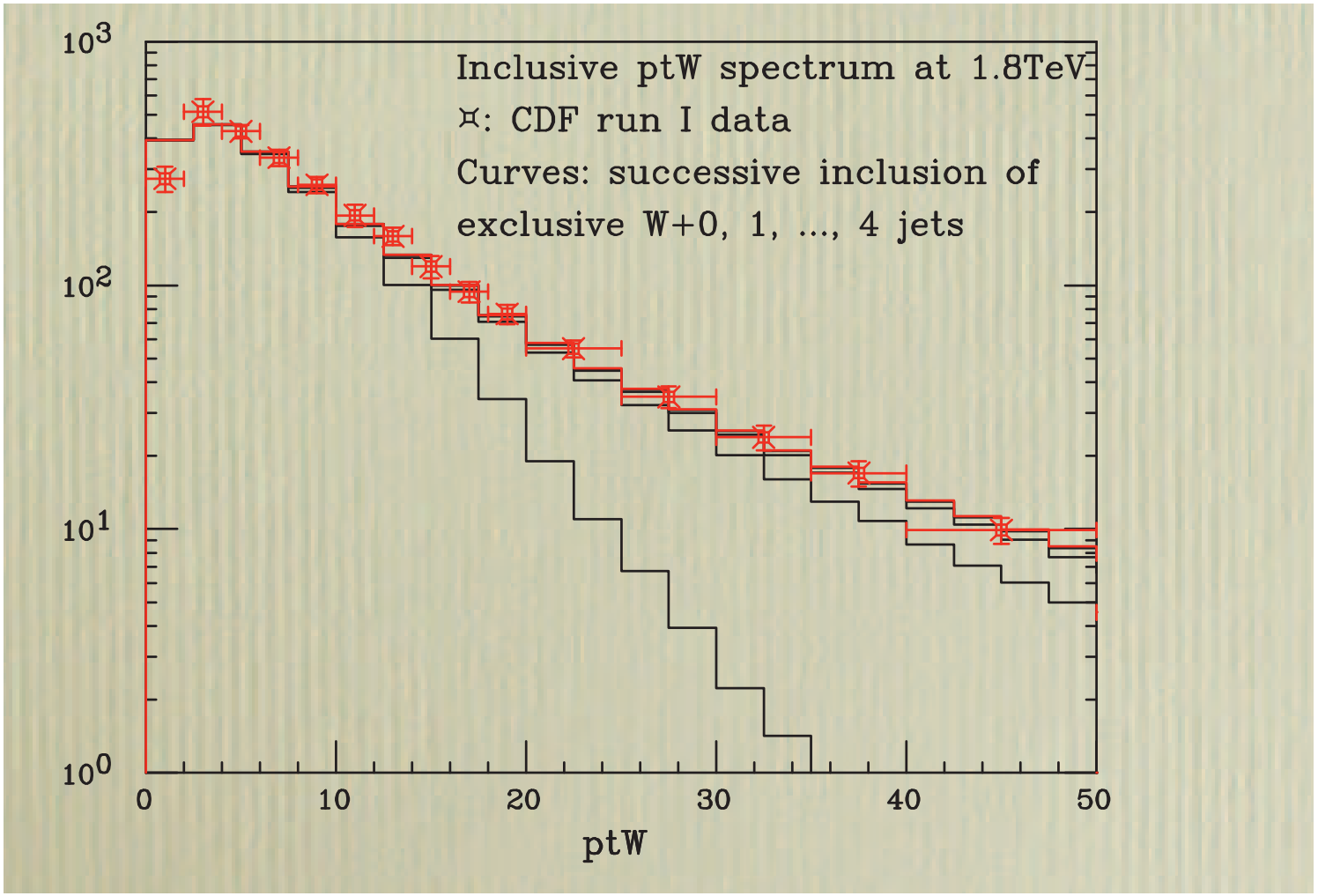


NOT matched



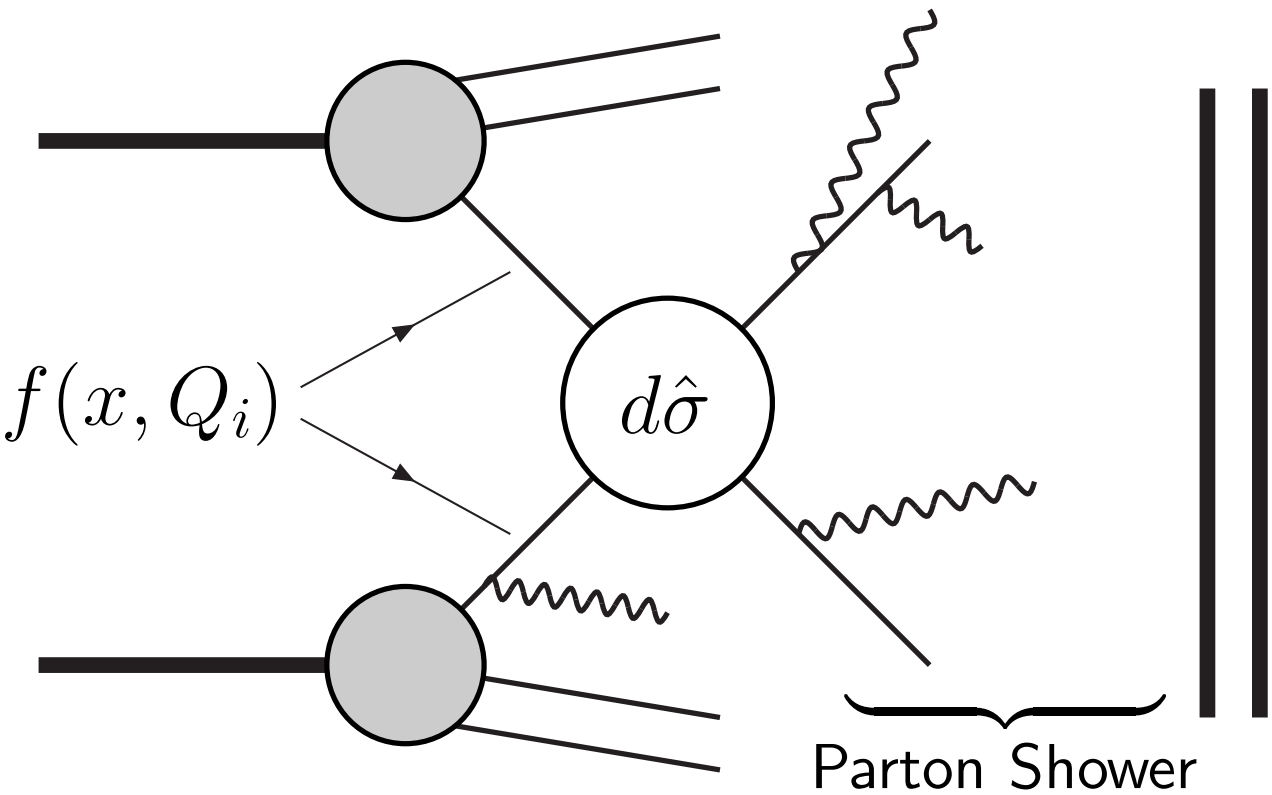
matched (if $n_{jets} = N_{max}$)

- Events obtained by applying this procedure to the parton level with increasing multiplicity can then be combined to obtain **fully inclusive samples** spanning a large multiplicity range (e.g. ALPGEN)



Hadronization

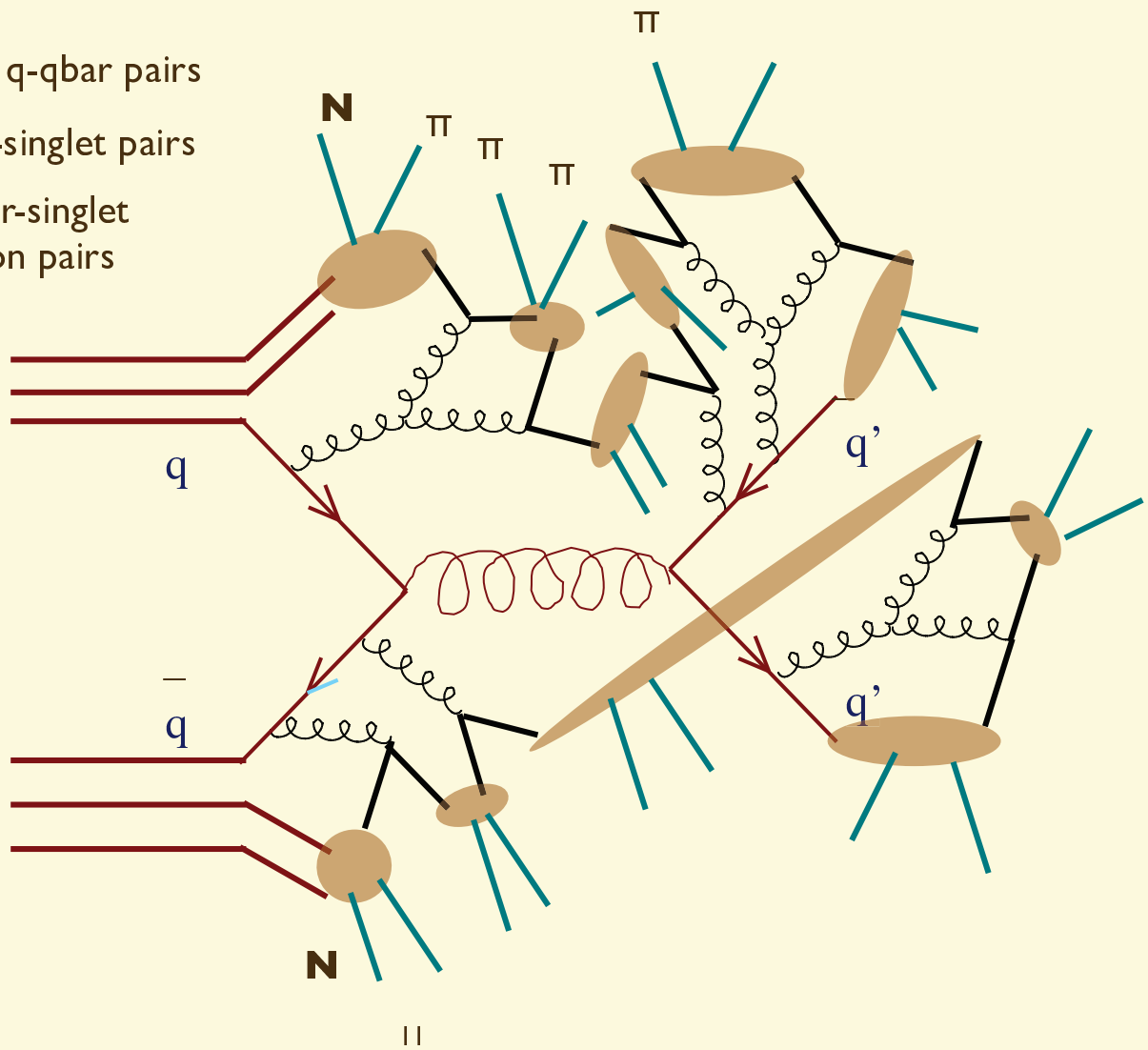
$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1, Q_i) f_k(x_2, Q_i) \underbrace{\frac{d\hat{\sigma}_{j,k}(Q_i, Q_f)}{d\hat{X}}}_{\textbf{Showered}} F(\hat{X} \to X; Q_i, Q_f)$$



$$\hat{X} \to F \to X$$

III: Hadronize partons

1. Split gluons into q-qbar pairs
2. Connect colour-singlet pairs
3. Decay the colour-singlet clusters into hadron pairs



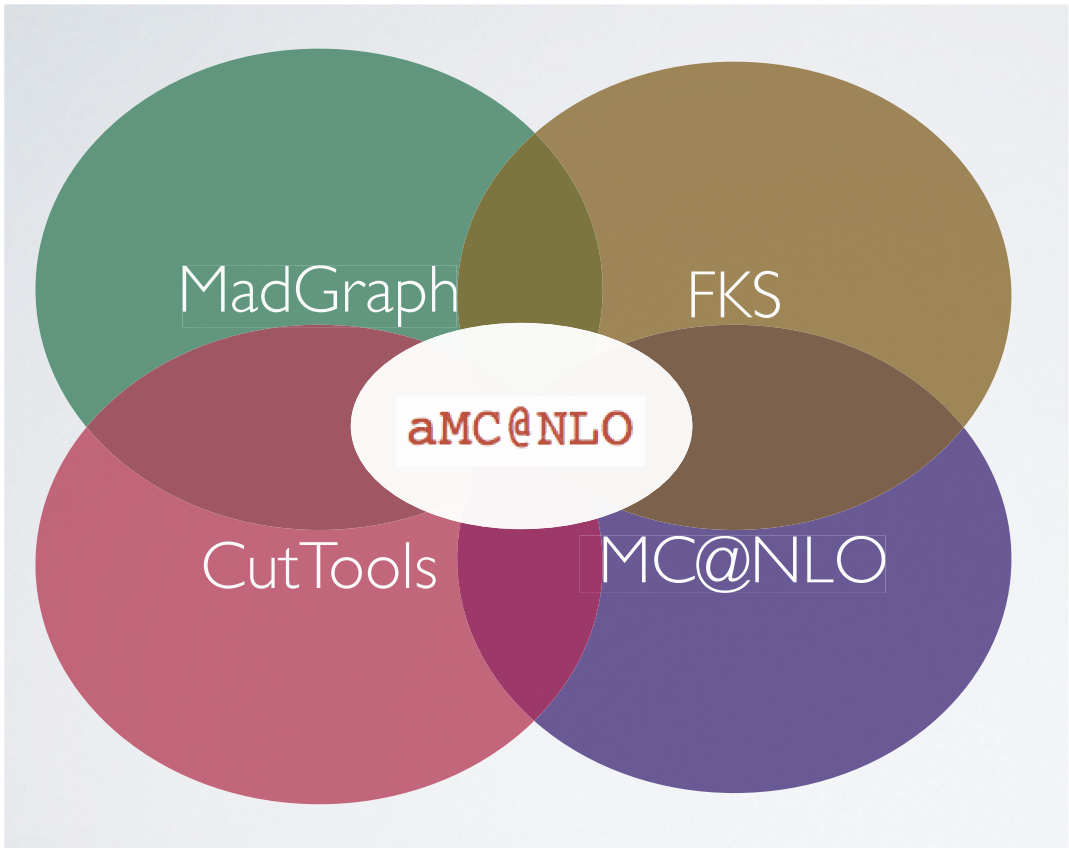
M. Mangano
"Introduction to hadronic collisions"

Main hadronization programs

1. HERWIG — Eur.Phys.J. C58 (2008) 639-707
2. PYTHIA — JHEP 0605 (2006) 026
3. SHERPA — JHEP 0902 (2009) 007

A complete automatic NLO approach

aMC@NLO (http://amcatnlo.cern.ch)



e. g. (2 weeks in a 150 node cluster)

	Process	μ	n_{lf}	Cross section (pb) LO	NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	25.62 ± 0.01	30.96 ± 0.06
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	8.195 ± 0.002	8.91 ± 0.01
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e\, j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e\, jj$	m_W	5	298.8 ± 0.4	300.3 ± 0.6
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^-\, j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^-\, jj$	m_Z	5	54.24 ± 0.02	56.69 ± 0.07
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^-\, j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+\, jj$	$2m_W$	4	0.07048 ± 0.00004	0.1377 ± 0.0005
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+\, j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZ\, j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.036 ± 0.002

NLO QCD corrections to $\mathrm{H} \to \mathrm{gg}$ in FDR

- An explicit example of NLO calculation in QCD is given
- A fully Four Dimensional approach is used (no DR)

- FDR - a recently introduced **Four Dimensional** approach to quantum field theories - is used to compute the NLO QCD corrections to $\mathbf{H} \to \mathbf{gg}$ in the large top mass limit

- The well known fully inclusive result

$$\Gamma(\mathbf{H} \to \mathbf{gg}) = \Gamma^{(0)}(\alpha_S(M_H^2)) \left[1 + \frac{95}{4}\frac{\alpha_S}{\pi}\right]$$

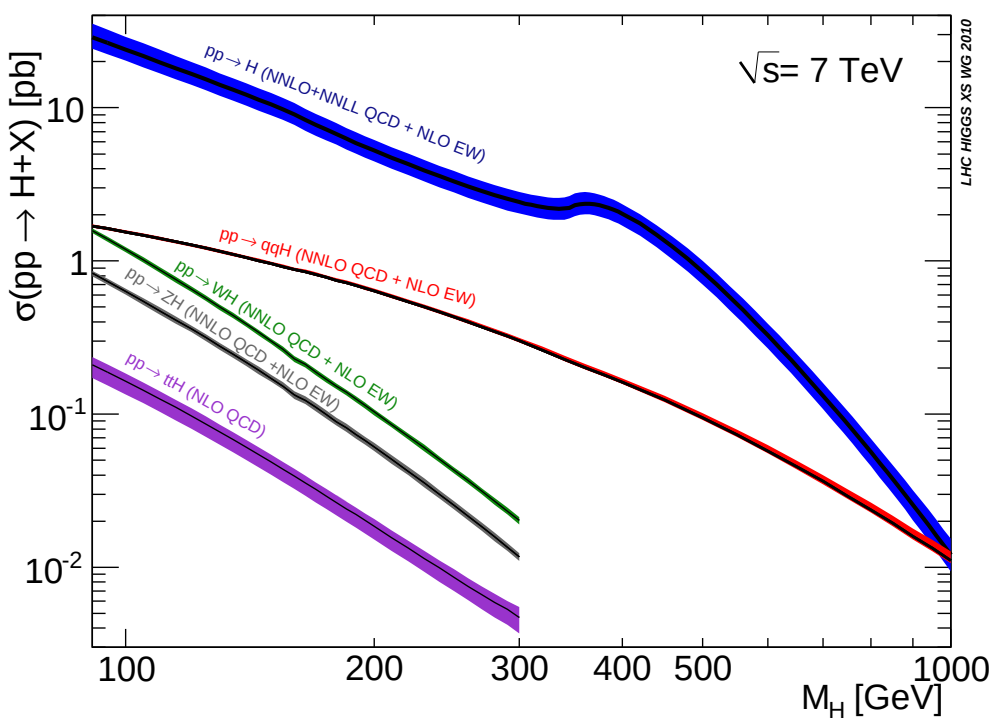
is re-derived, where

$$\Gamma^{(0)}(\alpha_S(M_H^2)) = \frac{G_F \alpha_S^2(M_H^2)}{36\sqrt{2}\pi^3} M_H^3$$

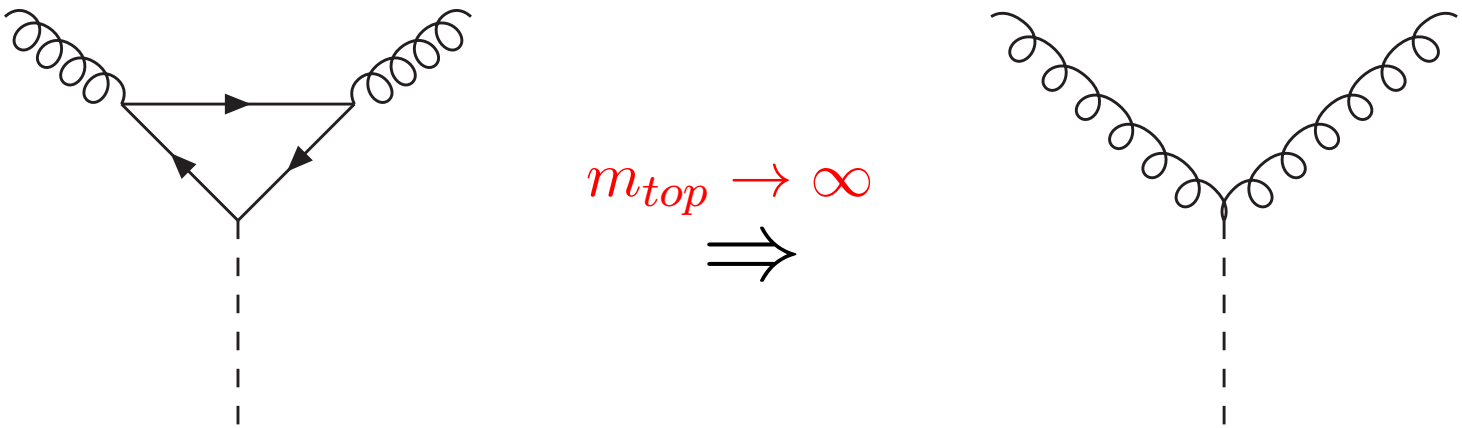
R. P., arXiv:1307.0705

Interesting because . . .

1. Many features of QCD are better understood within FDR
2. It provides an example of realistic 1-loop calculation in QCD since it involves **all** key ingredients, namely **UV**, **IR** and **CL** divergences, besides α_S **renormalization**
3. Furthermore, the crossed **QCD induced** $\mathbf{gg \to H}$ amplitude is the **main Higgs production mechanism at the LHC**



The Model



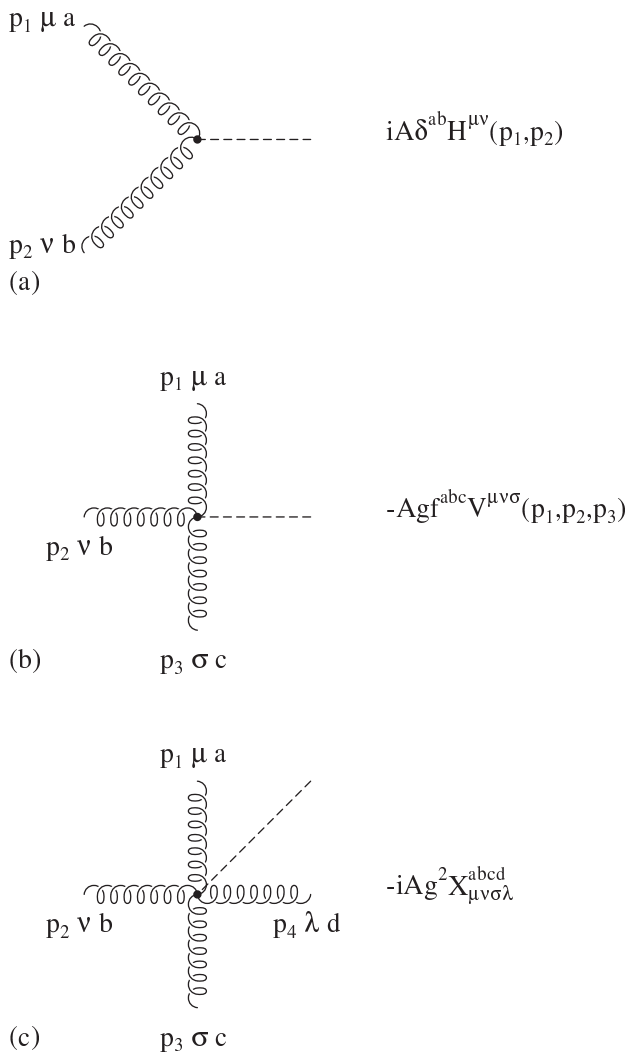
$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} A H G^a_{\mu\nu} G^{a,\mu\nu}$$

$$A = \frac{\alpha_S}{3\pi v}\left(1 + \frac{11}{4}\frac{\alpha_S}{\pi}\right)$$

where v is the vacuum expectation value, $v^2 = (G_F\sqrt{2})^{-1}$

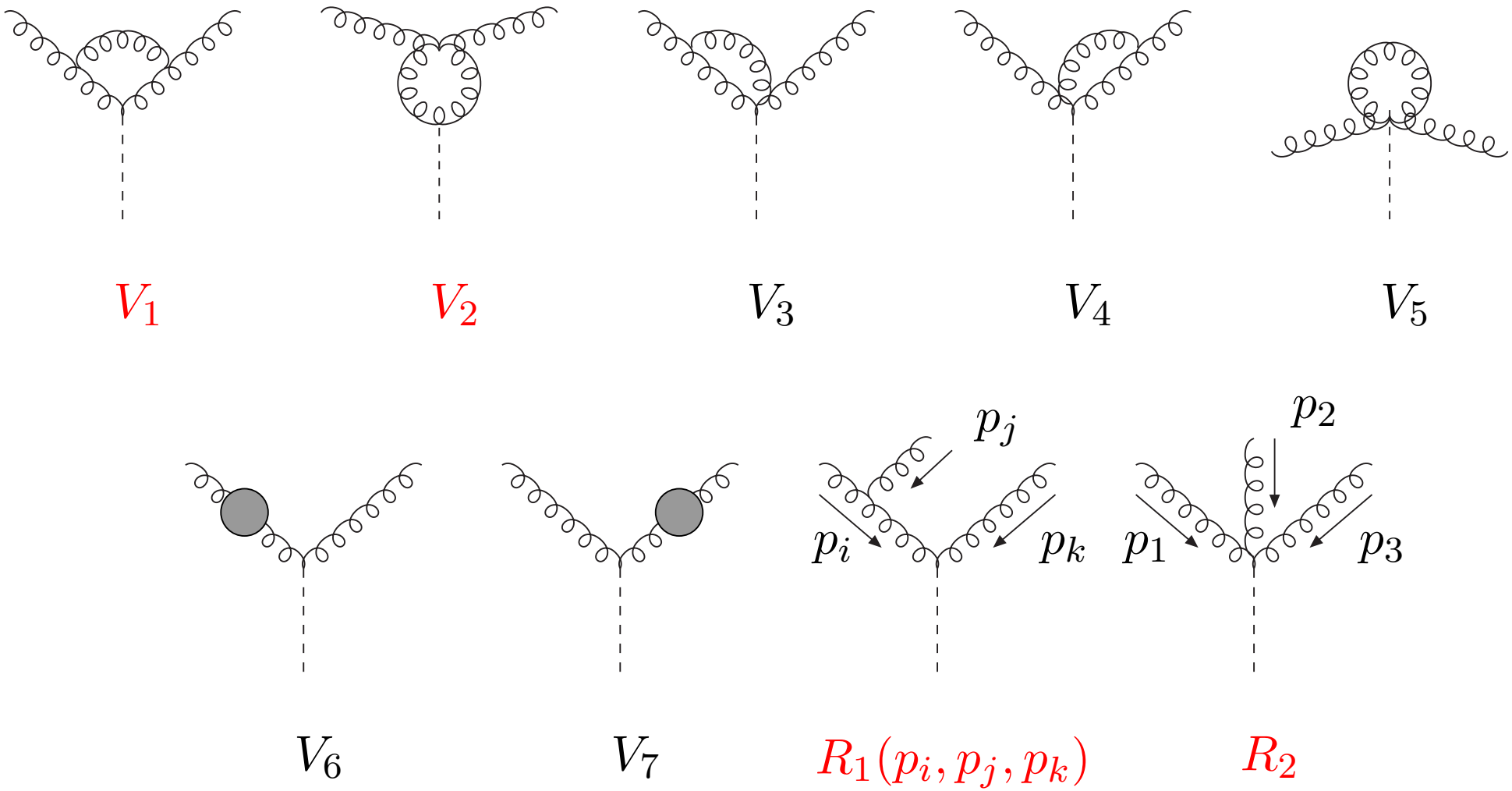
Generated Feynman Rules

F. Maltoni (2012)



V and X as in QCD and $\mathrm{H}^{\mu\nu}(\mathrm{p}_1, \mathrm{p}_2) = \mathrm{g}^{\mu\nu}\mathrm{p}_1 \cdot \mathrm{p}_2 - \mathrm{p}_1^{\nu}\mathrm{p}_2^{\mu}$

Contributing Diagrams



FDR vs Infinities

- Consider

$$\int d^4q \frac{q_\alpha q_\beta}{D_0 D_1}$$

$$D_0 = q^2 - M_0^2$$

$$D_1 = (q + p_1)^2 - M_1^2$$

$$D_i \;=\; q^2 - d_i\,, \quad d_i \;=\; M_i^2 - p_i^2 - 2(q \cdot p_i)\,, \quad p_0 = 0$$

- UV convergence *"improved"* by $\mathbf{D_i \to \bar{D}_i \;=\; D_i - \mu^2}$ and

$$\frac{1}{\bar{D}_i} = \frac{1}{\bar{q}^2}\left(1 + \frac{d_i}{\bar{D}_i}\right), \qquad \bar{q}^2 \;=\; q^2 - \mu^2$$

$$\frac{q_\alpha q_\beta}{\bar{D}_0 \bar{D}_1} = q_\alpha q_\beta \left(\left[\frac{1}{\bar{q}^4}\right] + \left[\frac{d_0 + d_1}{\bar{q}^6}\right] + \left[\frac{d_1^2}{\bar{q}^8}\right] + \frac{d_1^3}{\bar{q}^8 \bar{D}_1} + \frac{d_0 d_1}{\bar{q}^6 \bar{D}_1} + \frac{d_0^2}{\bar{q}^4 \bar{D}_0 \bar{D}_1} \right)$$

The FDR integral and the Virtual Part

$$B_{\alpha\beta}(p_1^2, M_0^2, M_1^2) =$$

$$\int [d^4q] \frac{q_\alpha q_\beta}{\bar{D}_0 \bar{D}_1} \equiv \lim_{\mu\to 0} \int d^4q \, q_\alpha q_\beta \left(\frac{d_1^3}{\bar{q}^8 \bar{D}_1} + \frac{d_0 d_1}{\bar{q}^6 \bar{D}_1} + \frac{d_0^2}{\bar{q}^4 \bar{D}_0 \bar{D}_1} \right)$$

- UV divergences subtracted before integration
- CL/IR singularities also regulated by μ^2, e.g.

$$\begin{aligned} B_{\alpha\beta}(0,0,0) &= \lim_{\mu\to 0} \int d^4q \, \frac{q_\alpha q_\beta d_1^3}{\bar{q}^8 \bar{D}_1} = \\ &\quad -8 p_1^\rho p_1^\sigma p_1^\tau \lim_{\mu\to 0} \int d^4q \frac{q_\alpha q_\beta q_\rho q_\sigma q_\tau}{\bar{q}^8 \bar{D}_1} = \mathbf{0}! \end{aligned}$$

Analogously $B_\alpha(0,0,0) = B(0,0,0) = 0$

Only V_1 and V_2 contribute to the Virtual Part

- $B_{\alpha\beta}(0,0,0)$, $B_\alpha(0,0,0)$ and $B(0,0,0)$ vanish
  due to a cancellation between UV *and* CL *regulators*

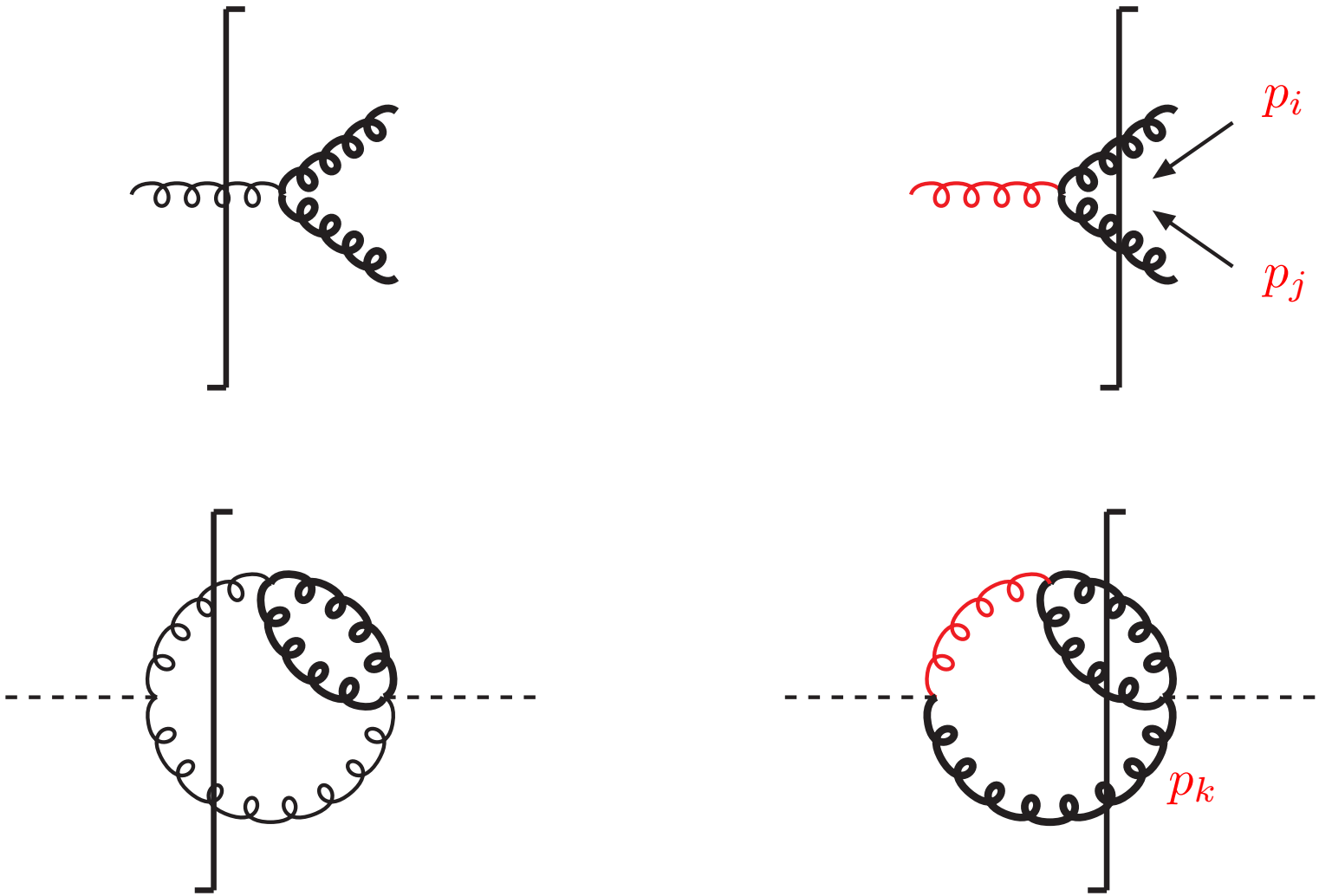
$$B(p^2,0,0) = -i\pi^2 \lim_{\mu\to 0} \int_0^1 dx \, \left[\ln(\mu^2 - p^2 x(1-x)) - \ln(\mu^2)\right]$$

- Furthermore $[s = -2(p_1 \cdot p_2),\ \mu_0 = \mu^2/s]$

$$\begin{aligned} C(s) &= \int [d^4q] \frac{1}{\bar{D}_0 \bar{D}_1 \bar{D}_2} = \lim_{\mu\to 0} \int d^4q \frac{1}{\bar{D}_0 \bar{D}_1 \bar{D}_2} \\ &= \frac{i\pi^2}{s} \left[\frac{\ln^2(\mu_0) - \pi^2}{2} + i\,\pi \ln(\mu_0)\right] \end{aligned}$$

$$\Gamma_V(\mathbf{H} \to \mathbf{gg}) = -3\frac{\alpha_S}{\pi}\,\Gamma^{(0)}(\alpha_S)\, M_H^2\, \mathcal{R}e\left[\frac{C(M_H^2)}{i\pi^2}\right]$$

The Real Part



$$\frac{1}{2(p_i \cdot p_j)} \to \frac{1}{(p_i + p_j)^2} = \frac{1}{s_{ij}} \quad \text{with } p_{i,j,k}^2 = \mu^2 \to 0 \ (\mu\text{−massive PS})$$

- The matrix element squared reads (diagrams R_1 and R_2)

$$|M|^2 = 192\,\pi\alpha_S A^2 \left[\frac{s_{23}^3}{s_{12}s_{13}} + \frac{s_{13}^3}{s_{12}s_{23}} + \frac{s_{12}^3}{s_{13}s_{23}} + \frac{2(s_{13}^2+s_{23}^2)+3s_{13}s_{23}}{s_{12}} + \frac{2(s_{12}^2+s_{23}^2)+3s_{12}s_{23}}{s_{13}} + \frac{2(s_{12}^2+s_{13}^2)+3s_{12}s_{13}}{s_{23}} + 6(s_{12}+s_{13}+s_{23}) \right]$$

- To be integrated over the μ-massive 3-body PS

$$\int d\Phi_3 = \frac{\pi^2}{4s} \int ds_{12} ds_{13} ds_{23}\, \delta(s - s_{12} - s_{13} - s_{23} + 3\mu^2)$$

- $\frac{1}{s_{ij}s_{jk}}$ generate $\ln^2(\mu^2)$ terms of IR/CL origin

  $\frac{1}{s_{ij}}$ collinear $\ln(\mu^2)$s

- By introducing the dimensionless variables $(x + y + z = 1)$

$$x = \frac{s_{12}}{s} - \mu_0 \,, \quad y = \frac{s_{13}}{s} - \mu_0 \,, \quad z = \frac{s_{23}}{s} - \mu_0$$

$$I(s) = \int_R dxdy \, \frac{1}{(x+\mu_0)(y+\mu_0)} \,, \quad J_p(s) = \int_R dxdy \, \frac{x^p}{(y+\mu_0)}$$

- As promised $(\mu_0 = \mu^2/s)$

$$\begin{aligned} I(s) &\sim \frac{\ln^2(\mu_0) - \pi^2}{2} \\ J_p(s) &\sim -\frac{1}{p+1}\ln(\mu_0) - \frac{1}{p+1}\left[\frac{1}{p+1} + 2\sum_{n=1}^{p+1}\frac{1}{n}\right] \end{aligned}$$

- Finally

$$\Gamma_R(\mathbf{H} \to \mathbf{ggg}) = 3\frac{\alpha_S}{\pi}\Gamma^{(0)}(\alpha_S) \times \left[\frac{1}{4} + I(M_H^2) - \frac{3}{2}J_0(M_H^2) - J_2(M_H^2)\right]$$

and

$$\begin{aligned}\Gamma(\mathbf{H} \to \mathbf{gg}) &= \Gamma_V(\mathbf{H} \to \mathbf{gg}) + \Gamma_R(\mathbf{H} \to \mathbf{ggg}) \\ &= \Gamma^{(0)}(\alpha_S)\left[1 + \frac{\alpha_S}{\pi}\left(\frac{95}{4} - \frac{11}{2}\ln\frac{M_H^2}{\mu^2}\right)\right]\end{aligned}$$

α_S renormalization

- The residual μ^2 is interpreted, in FDR, as a universal dependence on the renormalization scale
- $\ln(\mu^2)$ can be reabsorbed in the gluonic running of the strong coupling constant (Finite Renormalization)

$$\begin{aligned} \Gamma^{(0)}(\alpha_S) &\rightarrow \Gamma^{(0)}(\alpha_S(\mu^2)) \\ \alpha_S(M_H^2) &= \frac{\alpha_S(\mu^2)}{1+\frac{\alpha_S}{2\pi}\frac{11}{2}\ln\frac{M_H^2}{\mu^2}} \end{aligned}$$

$$\Gamma(\mathbf{H} \rightarrow \mathbf{gg}) = \Gamma^{(0)}(\alpha_S(M_H^2))\left[1+\frac{95}{4}\frac{\alpha_S}{\pi}\right]$$

$o\pi\epsilon\rho\ \epsilon\delta\epsilon\iota\ \delta\epsilon\iota\xi\alpha\iota$ (quod erat demostrandum)

Contribution of $\mathrm{H} \to \mathrm{gg}$ to $\Gamma(\mathrm{H} \to \mathrm{anything})$